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Design of training sequences and channel estimation for amplify-and-forward two-path relaying networks

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Abstract

This paper proposes a new channel estimation scheme for two-path relaying networks where two amplify-and-forward (AF) half-duplex relay nodes alternatively transmit the signals received from a source node. The channel estimation and interference cancellation would be mutually conditional in two-path relaying networks. To overcome such a difficulty, we design new training sequences and propose precoding matrices for use at the source and relays. Deriving the Cramer-Rao lower bound (CRLB), it is shown that the proposed estimator is efficient, i.e., the proposed channel estimation scheme achieves the CRLB. The performance of the proposed channel estimation is evaluated through the computer simulations where the validity of the theoretical analysis is also demonstrated.

Keywords: Amplify-and-forward; Channel estimation; Cramer-Rao lower bound; Two-path relay

Introduction

During the last decade, the cooperative communications have attracted lots of attention due to its ability to improve the system performance without the use of multiple antennas which increase the hardware complexity. In particular, among the various relaying protocols, an amplify-and-forward (AF) half-duplex (HD) relaying has been widely researched in the literature since it has less processing burden than the other schemes [1,2]. However, due to the use of orthogonal time slot (or frequency band), the HD relaying basically causes a loss of the spectral efficiency. Recently, in order to overcome such a problem of the HD relaying, a two-path relaying protocol was proposed in [3,4]. In two-path relaying networks, a pair of relays is introduced for alternatively forwarding the signal received from a source to a destination during odd and even time slots (see Figure 1). The main advantage of the two-path relaying is that its transmission rate becomes close to that of a full-duplex (FD) relaying. For instance, with M data blocks, a bandwidth efficiency of $M/(M+1)$ can be achieved by two-path relaying, which approaches 1 when M is sufficiently large. Unfortunately, the two-path relaying suffers from the inter-relay interference (IRI) due to the round-trip between relays.

In order to suppress such IRI, many works have been accomplished [4-8]. In [4], a partial interference cancellation (PIC) performed at the destination node was proposed. However, a complete suppression of IRI requires high computational complexity, and a direct link between the source and destination is neglected in the derivation of PIC algorithm. In many practical situations, especially for mobile cooperative communications, the direct link may not be ignored. Furthermore, in [5,6], cancellation of IRI at one of the relays was proposed. However, such a method requires the knowledge of the channel gain from the source to other relay, and thereby it increases the system overhead and complexity. To resolve such limitations, considering all possible transmission links, [7] developed a full interference cancellation (FIC) technique by which the IRI as well as inter-symbol interference (ISI) can be completely eliminated at the destination. The IRI cancellation in the FIC scheme utilizes the fact that the IRI term is essentially a transformation of the signal received in the previous time slot at the destination. As a result, the IRI can be perfectly suppressed at the destination. However, the performance of the FIC scheme is significantly degraded due to a severe error propagation during the

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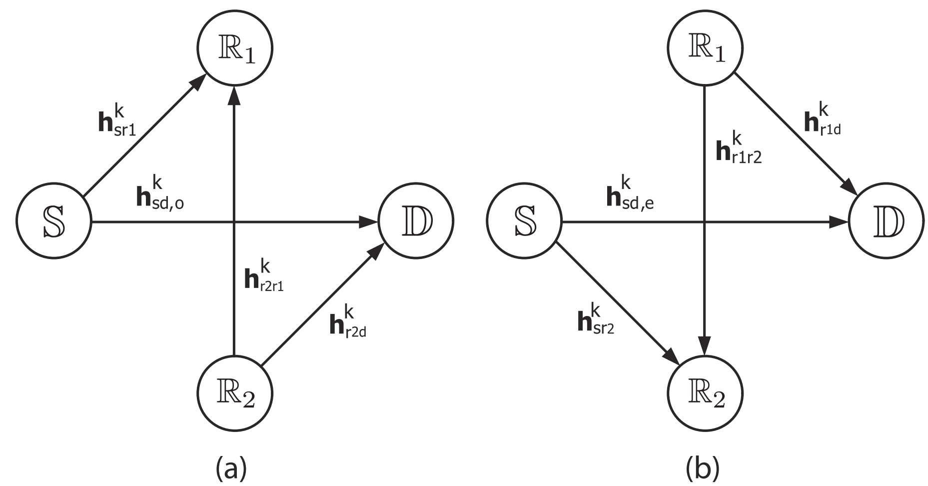

Figure 1 Operation of a two-path relaying network. (a) During odd time slot. **(b)** During even time slot.

ISI cancellation where the ISI is mitigated by performing a forward-and-backward successive interference cancellation (FB-SIC). To improve the performance of FIC, a robust SIC algorithm was proposed where the ISI cancellation is performed iteratively by using two consecutive block signals, which ultimately reduces the error propagation of the FB-SIC [8]. However, such works assumed that the perfect channel knowledge is available at the destination.

To get all the benefits of the two-path relaying, an accurate channel estimation is required. For the cooperative relaying networks, many works focusing on the channel estimation can be found in [9-12]. Lalos et al. [9] proposed a hybrid channel estimation scheme utilizing both training sequence and channel output correlation information for three-node cooperative relaying networks. Gao et al. [10] presented a Cramer-Rao lower bound (CRLB)-based training design and channel estimation for maximizing the signal-to-noise ratio (SNR) at the receiver in two-way relaying networks. Under the situation where multiple relays exist, the performance of the best linear unbiased estimation (BLUE) and linear minimum mean square error (LMMSE) estimation was respectively investigated in [11,12]. Although the various channel estimation schemes have been studied for use in cooperative relaying networks [9-12], a practical channel estimation for the two-path relaying networks has not been extensively studied in the literature. In particular, since the interference cancellation and channel estimation are mutually conditional, the conventional estimation methods in [9-12] cannot be applied to the two-path relaying networks. In [13], a channel estimation for AF-based two-path relaying was proposed. However, it has several practical limitations as follows:

- The channel estimation in [13] can be only applied to frequency flat fading channels. In addition, the authors assumed that the channels between nodes remain L consecutive data blocks (each data block consists of data symbols) to preserve the orthogonality between the pilot sequences transmitted by the source and relays. Specifically, in the simulations, it was assumed that $L = 80$. Thus, the reliable channel estimation cannot be achieved in mobile environment which is sometimes modeled by block fading channels [8,14].
- For the channel estimation purpose, authors in [13] slightly modified the relaying mechanism by making the relays forward no information to the destination in some time slots. Therefore, the loss of spectral efficiency is unavoidable.
- More importantly, authors in [13] assumed the channel reciprocity between relays, i.e., $\mathbf{h}^k_{r1r2} = \mathbf{h}^k_{r2r1}$ in Figure 1. The reciprocity principle is based on the property that electromagnetic waves traveling in both directions will undergo the same physical perturbations such as reflection, refraction, and diffraction [15]. However, such a property would not be maintained in most of the realistic cases, and the validity of the estimation algorithm must be guaranteed in every expected situations. In this regard, it is needed to develop a channel estimation that overcomes the limitations of [13].

As a result, such limitations motivate us to develop a practical channel estimation for two-path relaying networks over frequency selective fading channels without any modification of the relaying protocol. Moreover, the unrealistic assumptions such as the channel reciprocity and much too long coherence time are not considered. In two-path relaying networks, the channel estimation and interference cancellation would be mutually conditional. To overcome this difficulty, we design new training sequences and precoding matrices for use at the source and relays. It is shown that our proposed estimator is efficient, i.e., it achieves the CRLB. The accuracy of the theoretical results and the performance with the proposed channel estimation are demonstrated by the simulations.

The remainder of this paper is organized as follows. In 'Review of two-path relaying network' section, we briefly review the two-path relaying protocol. The design of training sequences and precoding matrices are described in 'Design and analysis of training-based channel estimation' section, where the CRLB for the proposed channel estimation is also derived. The numerical results are presented in 'Numerical results' section, and finally the paper is concluded in 'Conclusions' section.

Notations

Symbols $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\dagger}$ denote the conjugate, transpose, and Hermitian transpose operations, respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent the real and imaginary parts of its argument, respectively. $[\mathbf{A}]_{k,l}$ denotes the (k,l)th entry of a matrix $\mathbf{A}$ and $[\mathbf{a}]_k$ denotes the kth entry of a vector $\mathbf{a}$. $\det[\cdot]$ and $\mathrm{tr}\{\cdot\}$ stand for a determinant and trace of the matrix, respectively. $\mathrm{E}\{\cdot\}$ means a statistical expectation. $\mathbf{0}_{M\times K}$ denotes $M\times K$ all-zero matrix. $\mathbf{I}_M$ is an identity matrix of size $M\times M$. The modulo-N operation is denoted by $(\cdot) \mod N$.

Review of two-path relaying network

Let us consider a two-path relay network composed of one source ($\mathbb{S}$), one destination ($\mathbb{D}$), and two relays ($\mathbb{R}_1$, $\mathbb{R}_2$) as illustrated in Figure 1. Each node is equipped with a single antenna and relays operate with AF mode [7,8].

System parameters

We define data blocks transmitted by $\mathbb{S}$ during odd and even time slots in the kth block time as $\mathbf{x}^k_{\mathrm{s,o}} = \left[x^k_{\mathrm{s,o}}(0),\ldots,x^k_{\mathrm{s,o}}(N-1)\right]^{\mathrm{T}}$ and $\mathbf{x}^k_{\mathrm{s,e}} = \left[x^k_{\mathrm{s,e}}(0),\ldots,x^k_{\mathrm{s,e}}(N-1)\right]^{\mathrm{T}}$, respectively. The channel impulse response (CIR) in the kth block time between nodes $\mathbb{A}$ and $\mathbb{B}$ is denoted by $\mathbf{h}^k_{AB} = \left[h^k_{AB}(0),\ldots,h^k_{AB}(L_{AB}-1)\right]^{\mathrm{T}}$ where L_{AB} is the channel length. Let $\mathbf{H}^k_{AB}$ be an $N\times N$ circulant matrix, whose entries are given by the relation of $\left[\mathbf{H}^k_{AB}\right]_{m,l} = \mathbf{h}^k_{AB}((m-l) \mod N)$. It is assumed that $\mathbf{h}^k_{AB}$ is a normalized random vector having a zero-mean complex Gaussian distribution. Throughout this paper, we consider a quasi-static fading where the CIRs remain constant within a single block interval, i.e., a single time slot, but vary from block to block [14]. Therefore, to distinguish the CIR generated in the odd time slot between $\mathbb{S}$ and $\mathbb{D}$ from that generated in the even time slot, we define them $\mathbf{h}^k_{\mathrm{sd,o}}$ and $\mathbf{h}^k_{\mathrm{sd,e}}$, respectively. The transmit powers of $\mathbb{S}$ and $\mathbb{R}_i$ $(i=1,2)$ are defined as E_s and E_{r_i}, respectively. In addition, the additive white Gaussian noise (AWGN) vector generated at $\mathbb{R}_i$ in the kth block time is described by $\mathbf{n}^k_{r_i}$ with each entry having zero-mean and variance $\sigma^2_{nr_i}$. Similarly, the zero-mean AWGN vector at $\mathbb{D}$ during odd (even) time slot in the kth block time is defined as $\mathbf{n}^k_{\mathrm{d,o}}$ $(\mathbf{n}^k_{\mathrm{d,e}})$ with the covariance matrix $\sigma^2_{\mathrm{nd}}\mathbf{I}_N$.

Input-output relations

With the notations defined above, we describe input-output relations of two-path relaying systems. Specifically, in order to simplify the descriptions, without loss of generality, we equivalently use the cyclic-prefix (CP)-removed expressions in the rest of this paper. Practically, to eliminate the inter-block interference during the first time slot, a CP is appended to the block of data. This CP is discarded at the destination and the relay. The aforementioned operation transforms Toeplitz channel matrix into a circulant one.

During odd time slot in the kth block time

In this time slot, both $\mathbb{S}$ and $\mathbb{R}_2$ transmit their own signals to $\mathbb{R}_1$ and $\mathbb{D}$. The received signal at $\mathbb{R}_1$ is then given by

$$\mathbf{y}^k_{r_1} = \lambda_{\mathrm{s}}\mathbf{H}^k_{sr_1}\mathbf{x}^k_{\mathrm{s,o}} + \mathbf{H}^k_{r_2r_1}\mathbf{x}^k_{r_2} + \mathbf{n}^k_{r_1} \tag{1}$$

where the scaling factor is $\lambda_{\mathrm{s}} = \sqrt{E_{\mathrm{s}}}$ and the transmitted signal from $\mathbb{R}_2$, denoted by $\mathbf{x}^k_{r_2}$, is given by

$$\begin{aligned}\mathbf{x}^k_{r_2} &= \lambda_{r_2}\mathbf{y}^{k-1}_{r_2}\\ &= \lambda_{\mathrm{s}}\lambda_{r_2}\mathbf{H}^{k-1}_{sr_2}\mathbf{x}^{k-1}_{\mathrm{s,e}} + \lambda_{r_2}\mathbf{H}^{k-1}_{r_1r_2}\mathbf{x}^{k-1}_{r_1} + \lambda_{r_2}\mathbf{n}^{k-1}_{r_2}\end{aligned} \tag{2}$$

where $\mathbf{y}^{k-1}_{r_2}$ denotes the received signal at $\mathbb{R}_2$ in the previous time slot, and $\lambda_{r_2} = \sqrt{E_{r_2}/(E_{\mathrm{s}}+E_{r_1}+\sigma^2_{nr_2})}$. At this time, the received signal at $\mathbb{D}$ can be expressed as

$$\mathbf{y}^k_{\mathrm{d,o}} = \lambda_{\mathrm{s}}\mathbf{H}^k_{\mathrm{sd,o}}\mathbf{x}^k_{\mathrm{s,o}} + \mathbf{H}^k_{r_2d}\mathbf{x}^k_{r_2} + \mathbf{n}^k_{\mathrm{d,o}}. \tag{3}$$

By substituting (2) into (3), $\mathbf{y}^k_{\mathrm{d,o}}$ can be rewritten as [7,8]

$$\begin{aligned}\mathbf{y}^k_{\mathrm{d,o}} = &\underbrace{\lambda_{\mathrm{s}}\mathbf{H}^k_{\mathrm{sd}}\mathbf{x}^k_{\mathrm{s,o}}}_{\text{current block}} + \underbrace{\lambda_{\mathrm{s}}\lambda_{r_2}\mathbf{H}^k_{r_2d}\mathbf{H}^{k-1}_{sr_2}\mathbf{x}^{k-1}_{\mathrm{s,e}}}_{\text{past block}}\\ &+ \underbrace{\lambda_{r_2}\mathbf{H}^k_{r_2d}\mathbf{H}^{k-1}_{r_1r_2}\mathbf{x}^{k-1}_{r_1}}_{\text{IRI}} + \tilde{\mathbf{n}}^k_{\mathrm{d,o}}\end{aligned} \tag{4}$$

where the effective noise $\tilde{\mathbf{n}}^k_{\mathrm{d,o}}$ is defined as $\tilde{\mathbf{n}}^k_{\mathrm{d,o}} = \lambda_{r_2}\mathbf{H}^k_{r_2d}\mathbf{n}^{k-1}_{r_2} + \mathbf{n}^k_{\mathrm{d,o}}$. It is found from (4) that the received signal at $\mathbb{D}$ is disturbed by the past block received from $\mathbb{S}\rightarrow\mathbb{R}_2\rightarrow\mathbb{D}$ link and IRI received from $\mathbb{R}_1\rightarrow\mathbb{R}_2\rightarrow\mathbb{D}$ link.

During even time slot in the kth block time

In this time slot, $\mathbb{S}$ and $\mathbb{R}_1$ transmit the signals. Based on (1), the transmit signal from $\mathbb{R}_1$ can be written as

$$\begin{aligned}\mathbf{x}_{r_1}^k &= \lambda_{r_1}\mathbf{y}_{r_1}^k \\ &= \lambda_{\mathrm{s}}\lambda_{r_1}\mathbf{H}_{sr_1}^k\mathbf{x}_{\mathrm{s,o}}^k + \lambda_{r_1}\mathbf{H}_{r_2r_1}^k\mathbf{x}_{r_2}^k + \lambda_{r_1}\mathbf{n}_{r_1}^k\end{aligned} \tag{5}$$

where $\lambda_{r_1} = \sqrt{E_{r_1}/(E_{\mathrm{s}} + E_{r_2} + \sigma_{nr_1}^2)}$.

Referring to (3) and (5), the received signal at $\mathbb{D}$ can be easily formulated as

$$\mathbf{y}_{\mathrm{d,e}}^k = \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}^k\mathbf{x}_{\mathrm{s,e}}^k + \mathbf{H}_{r_1d}^k\mathbf{x}_{r_1}^k + \mathbf{n}_{\mathrm{d,e}}^k. \tag{6}$$

Substituting (5) into (6) produces

$$\begin{aligned}\mathbf{y}_{\mathrm{d,e}}^k =& \underbrace{\lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}^k\mathbf{x}_{\mathrm{s,e}}^k}_{\text{current block}} + \underbrace{\lambda_{\mathrm{s}}\lambda_{r_1}\mathbf{H}_{r_1d}^k\mathbf{H}_{sr_1}^k\mathbf{x}_{\mathrm{s,o}}^k}_{\text{past block}} \\ &+ \underbrace{\lambda_{r_1}\mathbf{H}_{r_1d}^k\mathbf{H}_{r_2r_1}^k\mathbf{x}_{r_2}^k}_{\text{IRI}} + \tilde{\mathbf{n}}_{\mathrm{d,e}}^k\end{aligned} \tag{7}$$

where the effective noise $\tilde{\mathbf{n}}_{\mathrm{d,e}}^k$ is defined as $\tilde{\mathbf{n}}_{\mathrm{d,e}}^k = \lambda_{r_1}\mathbf{H}_{r_1d}^k\mathbf{n}_{r_1}^k + \mathbf{n}_{\mathrm{d,e}}^k$. It should be noted from (4) and (7) that the interference cancellation and signal combining should be exploited in order to obtain the desired responses.

Inter-relay interference cancellation

The effects of IRI in (4) and (7) can be mitigated at $\mathbb{D}$ by utilizing the signals received in the previous time slots [7,8]. Using (6), the output of the IRI cancellation for $\mathbf{y}_{\mathrm{d,o}}^k$ is given by

$$\begin{aligned}\tilde{\mathbf{y}}_{\mathrm{d,o}}^k =& \mathbf{y}_{\mathrm{d,o}}^k - \left[\lambda_{r_2}\mathbf{H}_{r_2d}^k\mathbf{H}_{r_1r_2}^{k-1}\left(\mathbf{H}_{r_1d}^{k-1}\right)^{-1}\right]\mathbf{y}_{\mathrm{d,e}}^{k-1} \\ =& \underbrace{\lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,o}}^k\mathbf{x}_{\mathrm{s,o}}^k}_{\text{desired response}} \\ &\underbrace{+ \lambda_{\mathrm{s}}\lambda_{r_2}\mathbf{H}_{r_2d}^k\left[\mathbf{H}_{sr_2}^{k-1} - \mathbf{H}_{r_1r_2}^{k-1}\left(\mathbf{H}_{r_1d}^{k-1}\right)^{-1}\mathbf{H}_{\mathrm{sd,e}}^{k-1}\right]\mathbf{x}_{\mathrm{s,e}}^{k-1}}_{\text{interference}} \\ &+ \omega_{\mathrm{d,o}}^k\end{aligned} \tag{8}$$

where $\omega_{\mathrm{d,o}}^k$ is the effective noise observed at the output of the IRI cancellation which is given by

$$\omega_{\mathrm{d,o}}^k = \tilde{\mathbf{n}}_{\mathrm{d,o}}^k - \lambda_{r_2}\mathbf{H}_{r_2d}^k\mathbf{H}_{r_1r_2}^{k-1}\left(\mathbf{H}_{r_1d}^{k-1}\right)^{-1}\mathbf{n}_{\mathrm{d,e}}^{k-1}. \tag{9}$$

During the even time slot, a similar procedure of IRI cancellation described in (8) is performed. By using (3) to eliminate the IRI in (7), the output of the IRI cancellation for $\mathbf{y}_{\mathrm{d,e}}^k$ is obtained as

$$\begin{aligned}\tilde{\mathbf{y}}_{\mathrm{d,e}}^k =& \mathbf{y}_{\mathrm{d,e}}^k - \left[\lambda_{r_1}\mathbf{H}_{r_1d}^k\mathbf{H}_{r_2r_1}^k\left(\mathbf{H}_{r_2d}^k\right)^{-1}\right]\mathbf{y}_{\mathrm{d,o}}^k \\ =& \underbrace{\lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}^k\mathbf{x}_{\mathrm{s,e}}^k}_{\text{desired response}} \\ &\underbrace{+ \lambda_{\mathrm{s}}\lambda_{r_1}\mathbf{H}_{r_1d}^k\left[\mathbf{H}_{sr_1}^k - \mathbf{H}_{r_2r_1}^k\left(\mathbf{H}_{r_2d}^k\right)^{-1}\mathbf{H}_{\mathrm{sd,o}}^k\right]\mathbf{x}_{\mathrm{s,o}}^k}_{\text{interference}} \\ &+ \omega_{\mathrm{d,e}}^k\end{aligned} \tag{10}$$

where $\omega_{d,e}^k$ is defined as

$$\omega_{\mathrm{d,e}}^k = \tilde{\mathbf{n}}_{\mathrm{d,e}}^k - \lambda_{r_1}\mathbf{H}_{r_1d}^k\mathbf{H}_{r_2r_1}^k\left(\mathbf{H}_{r_2d}^{k-1}\right)^{-1}\mathbf{n}_{\mathrm{d,o}}^k. \tag{11}$$

We can see that the ISI components still remain in (8) and (10). Thus, after the IRI cancellation, the detection procedures such as the interference suppression and signal combining have to be performed [7,8].

Required channel knowledge

For a successful implementation of the detection procedures at $\mathbb{D}$, it is noticed from (8) and (10) that we need the knowledge on all of the possible transmission links as listed in Table 1. The first and second columns in Table 1 stand for the symbolic notations of the required CIRs and their propagation links, respectively. In particular, the bracket in the second column means whether the corresponding CIR can be estimated in the current or previous time slot. Finally, the last column stands for the time slot in which the denoted CIRs are used in the detection procedures. For example, the last row in Table 1 notifies us that the channel information of $\mathbb{R}_2 \to \mathbb{D}$ link denoted by $\mathbf{H}_{r2d}^k$ is used in the even time slot for the detection purpose, and it can be estimated in the previous time slot, i.e., odd time slot. From Table 1, we can see that $\lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}^{k-1}$ and $\lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,o}}^k$, needed in the odd and even time slots, are estimated in the

Table 1 Required channel knowledge at the destination

Symbols	Link (current/previous)	Time slot
$\lambda_s\mathbf{H}_{sd,o}^k$	$\mathbb{S} \to \mathbb{D}$ (current)	Odd
$\lambda_s\lambda_{r2}\mathbf{H}_{r2d}^k\mathbf{H}_{sr2}^{k-1}$	$\mathbb{S} \to \mathbb{R}_2 \to \mathbb{D}$ (current)	Odd
$\lambda_{r2}\mathbf{H}_{r2d}^k\mathbf{H}_{r1r2}^{k-1}$	$\mathbb{R}_1 \to \mathbb{R}_2 \to \mathbb{D}$ (current)	Odd
$\lambda_s\mathbf{H}_{sd,e}^{k-1}$	$\mathbb{S} \to \mathbb{D}$ (previous)	Odd
$\mathbf{H}_{r1d}^{k-1}$	$\mathbb{R}_1 \to \mathbb{D}$ (previous)	Odd
$\lambda_s\mathbf{H}_{sd,e}^k$	$\mathbb{S} \to \mathbb{D}$ (current)	Even
$\lambda_s\lambda_{r1}\mathbf{H}_{r1d}^k\mathbf{H}_{sr1}^k$	$\mathbb{S} \to \mathbb{R}_1 \to \mathbb{D}$ (current)	Even
$\lambda_{r1}\mathbf{H}_{r1d}^k\mathbf{H}_{r2r1}^{k-1}$	$\mathbb{R}_2 \to \mathbb{R}_1 \to \mathbb{D}$ (current)	Even
$\lambda_s\mathbf{H}_{sd,o}^k$	$\mathbb{S} \to \mathbb{D}$ (previous)	Even
$\mathbf{H}_{r2d}^k$	$\mathbb{R}_2 \to \mathbb{D}$ (previous)	Even

previous even and odd time slots, respectively. Therefore, four CIRs should be estimated in each time slot.

Design and analysis of training-based channel estimation

In this section, we propose a channel estimation in order to estimate the CIRs in Table 1. For that purpose, we design training sequences and precoding matrices for use at the source and relays. Then, we demonstrate that the proposed estimator is efficient by deriving the CRLB [16]. For the ease of notation and without loss of generality, we drop the block indices k and $k+1$ in the channel matrices.

Problem statement

In symbol recovery aspects, after the channel estimation, interference signals can be handled at $\mathbb{D}$ [7,8]. However, in channel estimation aspects, such procedures cannot be achieved since the channel estimation and interference cancellation would be mutually conditional. In other words, in order to properly estimate the CIRs at $\mathbb{D}$, the interference cancellation is a prerequisite. On the other hand, the interference cancellation before the channel estimation needs a reliable channel information.

As shown in the previous section, the channel knowledge of $\mathbb{S} \rightarrow \mathbb{R}_1 \rightarrow \mathbb{R}_2$ and $\mathbb{S} \rightarrow \mathbb{R}_2 \rightarrow \mathbb{R}_1$ links is useless at $\mathbb{D}$. Thus, it is not necessary for relays to retransmit the signal received from $\mathbb{S} \rightarrow \mathbb{R}_1 \rightarrow \mathbb{R}_2$ or $\mathbb{S} \rightarrow \mathbb{R}_2 \rightarrow \mathbb{R}_1$ link. Furthermore, the round-trip signal, propagated continuously between relays, must be eliminated at the relays prior to retransmission. Otherwise, the transmit power of the relays cannot be efficiently used for amplifying the desired signal since it is divided into interference signals as well as the desired one. As a result, the channel estimation suffers from a strong interference and noise, which makes a reliable channel estimation before the interference cancellation at $\mathbb{D}$ difficult to be achieved.

To resolve such technical challenges, in what follows, we present an efficient estimator design for use in two-path relay networks. Similar problems of the round-trip are observed in data transmission as well. However, in this case, it is difficult to treat such problems since it is impossible for the relays to have the knowledge of the transmit signal of $\mathbb{S}$. Furthermore, to suppress the interference components, the instantaneous responses of $\mathbb{S} \rightarrow \mathbb{R}_i \rightarrow \mathbb{R}_k$, $i, k \in \{1, 2\}$, and $\mathbb{R}_i \rightarrow \mathbb{R}_k$ links must be available at the relays, which significantly increases the system overhead and complexity. On the other hand, in channel estimation, we could overcome such difficulties by using a well-designed signaling protocol which is predefined.

Design of training sequence and precoder

We construct a repeated structure of training sequences (TSs), which consists of two groups. Each group has four subgroups, and each subgroup is constructed based on a base sequence defined as $\mathbf{u} = [u(0), \ldots, u(K-1)]^{\mathrm{T}}$. During the odd time slot, the lth subgroup is generated at $\mathbb{S}$ by multiplying $K \times K$ unitary precoding matrix $\mathbf{P}_{\mathrm{s,o}}^{(l)}$ to $\mathbf{u}$ as $\mathbf{T}_{\mathrm{sub,o}}^{(l)} = \left[\mathbf{u}^{\mathrm{T}}\mathbf{P}_{\mathrm{s,o}}^{(l)\mathrm{T}}, \mathbf{u}^{\mathrm{T}}\mathbf{P}_{\mathrm{s,o}}^{(l)\mathrm{T}}\right]^{\mathrm{T}}$. Note that the ith subgroups in the first and second groups are identical each other. Likewise, during the even time slot, the lth subgroup is defined as $\mathbf{T}_{\mathrm{sub,e}}^{(l)} = \left[\mathbf{u}^{\mathrm{T}}\mathbf{P}_{\mathrm{s,e}}^{(l)\mathrm{T}}, \mathbf{u}^{\mathrm{T}}\mathbf{P}_{\mathrm{s,e}}^{(l)\mathrm{T}}\right]^{\mathrm{T}}$. The first part in $\mathbf{T}_{\mathrm{sub,o}}^{(l)}$ ($\mathbf{T}_{\mathrm{sub,e}}^{(l)}$) prevents inter-block interference and second part will be used for the channel estimation, i.e., the first part acts as a CP. Therefore, we explain the proposed channel estimation by only considering the second part without loss of generality.

The basic structures of the training sequences at the relays are similar to that of $\mathbb{S}$. However, for the suppression of the unwanted resources, the relays substitute new precoded sequences for one of the received groups, which will be used for the estimation of $\mathbb{R}_i \rightarrow \mathbb{D}$ and $\mathbb{R}_i \rightarrow \mathbb{R}_k \rightarrow \mathbb{D}$ links. The lth subgroup of the newly inserted group at $\mathbb{R}_i$ is defined as $\mathbf{T}_{\mathrm{sub},r_i}^{(l)} = \left[\mathbf{u}^{\mathrm{T}}\mathbf{P}_{r_i}^{(l)\mathrm{T}}, \mathbf{u}^{\mathrm{T}}\mathbf{P}_{r_i}^{(l)\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{P}_{r_i}^{(l)}$ denotes the precoding matrix used for the lth subgroup at $\mathbb{R}_i$.

During odd time slot

$\mathbb{R}_2$ simply scales the first group received in the previous time slot and forwards it to both $\mathbb{R}_1$ and $\mathbb{D}$. At this time, the second group is transmitted as well. Let $\mathbf{r}_{r2}^{(k,l)}$ be the second part of the lth subgroup in the kth group transmitted from $\mathbb{R}_2$, then the corresponding received part at $\mathbb{D}$ is given by

$$\mathbf{y}_{\mathrm{d,o}}^{(k,l)} = \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,o}}\mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{u} + \mathbf{H}_{r2d}\mathbf{r}_{r2}^{(k,l)} + \mathbf{n}_{\mathrm{d,o}}^{(k,l)} \tag{12}$$

where $\mathbf{r}_{r2}^{(2,l)} = \sqrt{E_{r_2}}\mathbf{P}_{r_2}^{(l)}\mathbf{u}$ and $\mathbf{r}_{r2}^{(1,l)}$ will be derived later. Meanwhile, $\mathbb{R}_1$ receives the second part of the lth subgroup in the kth group which is formulated as

$$\mathbf{y}_{r_1}^{(k,l)} = \lambda_{\mathrm{s}}\mathbf{H}_{sr1}\mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{u} + \mathbf{H}_{r2r1}\mathbf{r}_{r2}^{(k,l)} + \mathbf{n}_{r_1}^{(k,l)}. \tag{13}$$

$\mathbb{R}_1$ replaces $\mathbf{y}_{r_1}^{(1,l)}$ with $\mathbf{r}_{r1}^{(1,l)} = \sqrt{E_{r_1}}\mathbf{P}_{r_1}^{(l)}\mathbf{u}$, and scales the second group as $\mathbf{r}_{r1}^{(2,l)} = \lambda_{r_1}\mathbf{y}_{r_1}^{(2,l)}$ to be transmitted in the next time slot.

During even time slot

By the definition of $\mathbf{r}_{r1}^{(k,l)}$ $(k = 1, 2)$ and $\mathbf{r}_{r2}^{(2,l)}$, referring to (13), the received second part of the lth subgroup at $\mathbb{D}$ can be represented as

$$\begin{aligned}
\mathbf{y}_{\mathrm{d,e}}^{(1,l)} &= \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}\mathbf{P}_{\mathrm{s,e}}^{(l)}\mathbf{u} + \sqrt{E_{r_1}}\mathbf{H}_{r1d}\mathbf{P}_{r1}^{(l)}\mathbf{u} + \mathbf{n}_{\mathrm{d,e}}^{(1,l)}, \\
\mathbf{y}_{\mathrm{d,e}}^{(2,l)} &= \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,e}}\mathbf{P}_{\mathrm{s,e}}^{(l)}\mathbf{u} + \lambda_{\mathrm{s}}\lambda_{r_1}\mathbf{H}_{r1d}\mathbf{H}_{sr1}\mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{u} \\
&\quad + \sqrt{E_{r_2}}\lambda_{r_1}\mathbf{H}_{r1d}\mathbf{H}_{r2r1}\mathbf{P}_{r_2}^{(l)}\mathbf{u} + \tilde{\mathbf{n}}_{\mathrm{d,e}}^{(2,l)}
\end{aligned} \tag{14}$$

where the effective noise vector is defined as $\tilde{\mathbf{n}}_{\mathrm{d,e}}^{(2,l)} = \lambda_{r_1}\mathbf{H}_{r1d}\mathbf{n}_{r_1}^{(2,l)} + \mathbf{n}_{\mathrm{d,e}}^{(2,l)}$.

The received signal at $\mathbb{R}_2$ can be derived as

$$\mathbf{y}_{r_2}^{(k,l)} = \lambda_{\mathrm{s}}\mathbf{H}_{sr2}\mathbf{P}_{\mathrm{s,e}}^{(l)}\mathbf{u} + \mathbf{H}_{r1r2}\mathbf{r}_{r1}^{(k,l)} + \mathbf{n}_{r_2}^{(k,l)}. \tag{15}$$

$\mathbb{R}_2$ substitutes $\mathbf{y}_{r_2}^{(2,l)}$ with $\mathbf{r}_{r2}^{(2,l)} = \sqrt{E_{r_2}}\mathbf{P}_{r_2}^{(l)}\mathbf{u}$ and scales the first group as $\mathbf{r}_{r2}^{(1,l)} = \lambda_{r_2}\mathbf{y}_{r_2}^{(1,l)}$. Plugging $\mathbf{r}_{r2}^{(k,l)}$ into (12) and referring to (15), (12) can be rewritten as

$$\begin{aligned}\mathbf{y}_{\mathrm{d,o}}^{(1,l)} &= \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,o}}\mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{u} + \lambda_{\mathrm{s}}\lambda_{r_2}\mathbf{H}_{r2d}\mathbf{H}_{sr2}\mathbf{P}_{\mathrm{s,e}}^{(l)}\mathbf{u} \\ &\quad + \sqrt{E_{r_1}}\lambda_{r_2}\mathbf{H}_{r2d}\mathbf{H}_{r1r2}\mathbf{P}_{r_1}^{(l)}\mathbf{u} + \tilde{\mathbf{n}}_{\mathrm{d,o}}^{(1,l)}, \\ \mathbf{y}_{\mathrm{d,o}}^{(2,l)} &= \lambda_{\mathrm{s}}\mathbf{H}_{\mathrm{sd,o}}\mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{u} + \sqrt{E_{r_2}}\mathbf{H}_{r2d}\mathbf{P}_{r2}^{(l)}\mathbf{u} + \mathbf{n}_{\mathrm{d,o}}^{(2,l)}\end{aligned} \tag{16}$$

where $\tilde{\mathbf{n}}_{\mathrm{d,o}}^{(1,l)} = \lambda_{r_2}\mathbf{H}_{r2d}\mathbf{n}_{r_2}^{(1,l)} + \mathbf{n}_{\mathrm{d,o}}^{(1,l)}$.

Let us define a $K \times K$ circulant matrix $\mathbf{U}$ whose first column is equal to $\mathbf{u}$. We also define $\mathbf{y}_{\mathrm{d,o}}^{(l)}$ as the summation of $\mathbf{y}_{\mathrm{d,o}}^{(1,l)}$ and $\mathbf{y}_{\mathrm{d,o}}^{(2,l)}$, i.e., $\mathbf{y}_{\mathrm{d,o}}^{(l)} = \mathbf{y}_{\mathrm{d,o}}^{(1,l)} + \mathbf{y}_{\mathrm{d,o}}^{(2,l)}$. Then, $\mathbf{y}_{\mathrm{d,o}}^{(l)}$ is given by

$$\begin{aligned}\mathbf{y}_{\mathrm{d,o}}^{(l)} &= \mathbf{P}_{\mathrm{s,o}}^{(l)}\mathbf{U}\left(2\lambda_2\bar{\mathbf{h}}_{\mathrm{sd,o}}\right) + \mathbf{P}_{\mathrm{s,e}}^{(l)}\mathbf{U}\left(\lambda_{\mathrm{s}}\lambda_{r_2}\bar{\mathbf{h}}_{sr2d}\right) \\ &\quad + \mathbf{P}_{r_1}^{(l)}\mathbf{U}\left(\sqrt{E_{r_1}}\lambda_{r_2}\bar{\mathbf{h}}_{r1r2d}\right) + \mathbf{P}_{r_2}^{(l)}\mathbf{U}\left(\sqrt{E_{r_2}}\bar{\mathbf{h}}_{r2d}\right) + \mathbf{w}_{\mathrm{d,o}}^{(l)}\end{aligned} \tag{17}$$

where $\bar{\mathbf{h}}_X = \left[\mathbf{h}_X^{\mathrm{T}}, \mathbf{0}_{(K-L_X)\times 1}^{\mathrm{T}}\right]^{\mathrm{T}}$ and we use the identity of $\mathbf{H}_X\mathbf{P}_Y^l\mathbf{u} = \mathbf{P}_Y^l\mathbf{U}\bar{\mathbf{h}}_X$, where X denotes the subscript of the channel terms and Y is the subscript of the precoding matrices. The commutative law can be met if and only if the precoding matrices have circulant structures. In (17), $\mathbf{w}_{\mathrm{d,o}}^{(l)} = \tilde{\mathbf{n}}_{\mathrm{d,o}}^{(1,l)} + \mathbf{n}_{\mathrm{d,o}}^{(2,l)}$ denotes the effective noise vector whose covariance matrix conditioned on a specific realization of $\mathbf{h}_{r2d}$ is given by $\mathrm{Cov}\left(\mathbf{w}_{\mathrm{d,o}}^{(l)}|\mathbf{h}_{r2d}\right) = \left(2\sigma_{nd}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2\right)\mathbf{I}_K$ where $P_{r2d} = \sum_{i=0}^{L_{r2d}} |h_{r2d}(i)|^2$ [9-11]. Consequently, the effective noise under a specific realization of $\mathbf{h}_{r2d}$ is still Gaussian distributed [10,11]. Stacking $\mathbf{y}_{\mathrm{d,o}}^{(l)}$ (for $l = 1, 2, 3, 4$), we have

$$\underbrace{\begin{bmatrix}\mathbf{y}_{\mathrm{d,o}}^{(1)} \\ \mathbf{y}_{\mathrm{d,o}}^{(2)} \\ \mathbf{y}_{\mathrm{d,o}}^{(3)} \\ \mathbf{y}_{\mathrm{d,o}}^{(4)}\end{bmatrix}}_{\mathbf{y}_1} = \underbrace{\begin{bmatrix}\mathbf{P}_{\mathrm{s,o}}^{(1)}\mathbf{U} & \mathbf{P}_{\mathrm{s,e}}^{(1)}\mathbf{U} & \mathbf{P}_{r_1}^{(1)}\mathbf{U} & \mathbf{P}_{r_2}^{(1)}\mathbf{U} \\ \mathbf{P}_{\mathrm{s,o}}^{(2)}\mathbf{U} & \mathbf{P}_{\mathrm{s,e}}^{(2)}\mathbf{U} & \mathbf{P}_{r_1}^{(2)}\mathbf{U} & \mathbf{P}_{r_2}^{(2)}\mathbf{U} \\ \mathbf{P}_{\mathrm{s,o}}^{(3)}\mathbf{U} & \mathbf{P}_{\mathrm{s,e}}^{(3)}\mathbf{U} & \mathbf{P}_{r_1}^{(3)}\mathbf{U} & \mathbf{P}_{r_2}^{(3)}\mathbf{U} \\ \mathbf{P}_{\mathrm{s,o}}^{(4)}\mathbf{U} & \mathbf{P}_{\mathrm{s,e}}^{(4)}\mathbf{U} & \mathbf{P}_{r_1}^{(4)}\mathbf{U} & \mathbf{P}_{r_2}^{(4)}\mathbf{U}\end{bmatrix}}_{\mathbf{M}_1} \times \underbrace{\begin{bmatrix}2\lambda_2\bar{\mathbf{h}}_{\mathrm{sd,o}} \\ \lambda_{\mathrm{s}}\lambda_{r_2}\bar{\mathbf{h}}_{sr2d} \\ \sqrt{E_{r_1}}\lambda_{r_2}\bar{\mathbf{h}}_{r1r2d} \\ \sqrt{E_{r_2}}\bar{\mathbf{h}}_{r2d}\end{bmatrix}}_{\mathbf{h}_1} + \underbrace{\begin{bmatrix}\mathbf{w}_{\mathrm{d,o}}^{(1)} \\ \mathbf{w}_{\mathrm{d,o}}^{(2)} \\ \mathbf{w}_{\mathrm{d,o}}^{(3)} \\ \mathbf{w}_{\mathrm{d,o}}^{(4)}\end{bmatrix}}_{\mathbf{w}_1}. \tag{18}$$

Least square (LS) estimate of $\mathbf{h}_1$ is given by [16]

$$\hat{\mathbf{h}}_1 = \mathbf{h}_1 + \left(\mathbf{M}_1^{\dagger}\mathbf{M}_1\right)^{-1}\mathbf{M}_1^{\dagger}\mathbf{w}_1. \tag{19}$$

The theoretical mean square error (MSE) of $\hat{\mathbf{h}}_1$ can be readily derived as

$$\begin{aligned}\mathrm{MSE}_1 &= \left(\frac{2\sigma_{nd}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2}{4K}\right)\mathrm{tr}\left\{\left(\mathbf{M}_1^{\dagger}\mathbf{M}_1\right)^{-1}\right\} \\ &\geq \frac{2\sigma_{nd}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2}{4K}\end{aligned} \tag{20}$$

where the equality holds if and only if $(\mathbf{M}_1^{\dagger}\mathbf{M}_1)^{-1}$ is a diagonal matrix and all elements on the diagonal are equal to $1/(4K)$ [10,11]. In order to obtain the minimum MSE of the channel estimate, we must have

$$\mathbf{U}^{\dagger}\mathbf{P}_X^{(i)\dagger}\mathbf{P}_Y^{(j)}\mathbf{U} = \begin{cases} K\mathbf{I}_K\ , & i = j \text{ and } X = Y \\ \mathbf{0}_{K\times K}, & \text{otherwise.}\end{cases} \tag{21}$$

To achieve (21), we design a base sequence in accordance with the Chu sequence which is a polyphase sequence that has a perfect periodic autocorrelation property leading to a constant magnitude in both time and frequency domains. The nth element of length-K Chu sequence is defined as [17]

$$u(n) = \begin{cases}\exp\left(j\pi t n^2/K\right), & \text{for even } K \\ \exp\left(j\pi t n(n+1)/K\right), & \text{for odd } K\end{cases} \tag{22}$$

where t is an integer which is relative prime to K. With the help of the Chu sequences, we have $\mathbf{U}^{\dagger}\mathbf{U} = K\mathbf{I}_K$. Next, let us decompose $\mathbf{M}_1$ as $\mathbf{M}_1 = \mathbf{P}\tilde{\mathbf{U}}$ where $\tilde{\mathbf{U}}$ is a block diagonal matrix of size $4K \times 4K$ composed of $\mathbf{U}$. Then, to achieve (21), we design the precoding matrices $\mathbf{P}$ by a block-wise orthogonal matrix defined as

$$\mathbf{P} = \begin{bmatrix}\mathbf{I}_K & \mathbf{I}_K & \mathbf{I}_K & \mathbf{I}_K \\ \mathbf{I}_K & -\mathbf{I}_K & \mathbf{I}_K & -\mathbf{I}_K \\ \mathbf{I}_K & \mathbf{I}_K & -\mathbf{I}_K & -\mathbf{I}_K \\ \mathbf{I}_K & -\mathbf{I}_K & -\mathbf{I}_K & \mathbf{I}_K\end{bmatrix}. \tag{23}$$

As a result, each node uses $\pm\ \mathbf{I}_K$ as the precoding matrix which satisfies the cumulative law used in (17). Consequently, the equality in (20) is satisfied. Introducing the CIR vector $\mathbf{h}_2 = \left[2\lambda_2\bar{\mathbf{h}}_{\mathrm{sd,e}}^{\mathrm{T}}, \lambda_{\mathrm{s}}\lambda_{r_1}\bar{\mathbf{h}}_{sr1d}^{\mathrm{T}}, \sqrt{E_{r_2}}\lambda_{r_1}\bar{\mathbf{h}}_{r2r1d}^{\mathrm{T}}, \sqrt{E_{r_1}}\bar{\mathbf{h}}_{r1d}^{\mathrm{T}}\right]^{\mathrm{T}}$, the estimation of $\mathbf{h}_2$ is performed based on (14). The estimation procedures including the design of training sequence and precoder are exactly the same as those presented from (17) to (23); add two groups, stack subgroups, and perform the LS estimation. We can also achieve the minimum MSE of $(\mathbf{h}_2 - \hat{\mathbf{h}}_2)$ since the column-shifted version of $\mathbf{P}$ is still unitary matrix. Therefore, we can omit the description of such processing. Theoretical MSE of $\hat{\mathbf{h}}_2$ is given by

$$\mathrm{MSE}_2 = \frac{2\sigma_{nd}^2 + \lambda_{r_1}^2 P_{r1d}\sigma_{nr_1}^2}{4K}. \tag{24}$$

From (20) and (24), the total MSE of the proposed channel estimation can be formulated as follows:

$$\begin{aligned}\mathrm{MSE} &= \frac{1}{2}\left(\mathrm{MSE}_1 + \mathrm{MSE}_2\right)\\ &= \frac{4\sigma_{nd}^2 + \lambda_{r_1}^2 P_{r1d}\sigma_{nr_1}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2}{8K}.\end{aligned} \tag{25}$$

From Table 1, we can see that the estimates in (19) are the scaled version of the required CIRs used in the detection process. Luckily, the scaling factors do not depend on the statistical quantities such as noise variances at the relays. Therefore, the proposed algorithm would be feasible in practice.

Next, we investigate the theoretical error floor performance by considering two link conditions. Let us first consider the case of $\sigma_{nd}^2 \to 0$. Then, the error floor is given by

$$\mathrm{MSE}_{c1} = \frac{\lambda_{r_1}^2 P_{r1d}\sigma_{nr_1}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2}{8K}. \tag{26}$$

Similarly, considering the noise-free reception condition at the relays, the error floor for $\sigma_{nr_i}^2 \to 0$ $(i = 1, 2)$ is given by

$$\mathrm{MSE}_{c2} = \frac{\sigma_{nd}^2}{2K}. \tag{27}$$

Cramer-Rao lower bound

From (19), it is obvious that the proposed estimator is unbiased. Thus, it is worth deriving the CRLB which is a lower bound on the variance of estimators and comparing it to the MSE performance of the proposed estimator. Here, we derive the CRLB for the proposed channel estimation. By definition, the LS estimation assumes the deterministic unknown channels, i.e., the LS estimation is the classical estimation [10,11,16]. Let us define $\mathbf{C} = \psi \mathbf{I}_{4K}$ where $\psi = (2\sigma_{nd}^2 + \lambda_{r_2}^2 P_{r2d}\sigma_{nr_2}^2)$. Then, the likelihood function of $\mathbf{y}_1$ is given by [10,11,16]

$$P(\mathbf{y}_1|\mathbf{h}_1) = \frac{1}{\pi^{4K}\det[\mathbf{C}]}\exp\left\{-\frac{\|\mathbf{y}_1 - \mathbf{M}_1\mathbf{h}_1\|^2}{\psi}\right\} \tag{28}$$

where the scalar term does not depend on $\mathbf{h}_1$. Omitting $(\pi^{4K}\det[\mathbf{C}])^{-1}$, the log-likelihood function can be written as

$$\begin{aligned}\log P(\mathbf{y}_1|\mathbf{h}_1) = & -\frac{1}{\psi}\Big(\mathbf{y}_1^\dagger\mathbf{y}_1 - \mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{y}_1 - \mathbf{y}_1^\dagger\mathbf{M}_1\mathbf{h}_1\\ & +\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{M}_1\mathbf{h}_1\Big)\end{aligned} \tag{29}$$

from which, except for the first term which is not a function of $\mathbf{h}_1$, we expand $\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{y}_1$, $\mathbf{y}_1^\dagger\mathbf{M}_1\mathbf{h}_1$, and $\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{M}_1\mathbf{h}_1$ as follows [11]:

$$\begin{aligned}\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{y}_1 &= \sum_{p=0}^{4K-1}\sum_{q=0}^{4K-1}[\mathbf{h}_1^*]_p\,[\mathbf{M}_1]_{q,p}^*\,[\mathbf{y}_1]_q,\\ \mathbf{y}_1^\dagger\mathbf{M}_1\mathbf{h}_1 &= \sum_{p=0}^{4K-1}\sum_{q=0}^{4K-1}[\mathbf{h}_1]_p\,[\mathbf{M}_1]_{q,p}\,[\mathbf{y}_1]_q^*,\\ \mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{M}_1\mathbf{h}_1 &= \sum_{l=0}^{4K-1}\sum_{p=0}^{4K-1}\sum_{q=0}^{4K-1}[\mathbf{h}_1]_l^*\,[\mathbf{M}_1]_{q,l}^*\,[\mathbf{M}_1]_{q,p}\,[\mathbf{h}_1]_p\\ &= 4K\sum_{l=0}^{4K-1}[\mathbf{h}_1]_l^*\,[\mathbf{h}_1]_l\end{aligned} \tag{30}$$

where the second line of the last equation in (30) results from the orthogonal structure of $\mathbf{M}_1$. The CRLB can be obtained from the inverse matrix of Fisher information matrix (FIM) $\mathbf{J}(\mathbf{h}_1)$ which is identified as

$$[\mathbf{J}(\mathbf{h}_1)]_{p,q} = -\mathrm{E}\left\{\left(\frac{\partial^2 \log P(\mathbf{y}_1|\mathbf{h}_1)}{\partial[\mathbf{h}_1]_p\,\partial[\mathbf{h}_1]_q^*}\right)\right\} \tag{31}$$

From (30), it is obvious that the second order derivatives of $\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{y}_1$ and $\mathbf{y}_1^\dagger\mathbf{M}_1\mathbf{h}_1$ with respect to $[\mathbf{h}_1]_p$ and $[\mathbf{h}_1]_q^*$ become zero. On the other hand, by defining the complex derivatives $\frac{\partial}{\partial\theta} = \frac{1}{2}\left(\frac{\partial}{\partial\Re\{\theta\}} - j\frac{\partial}{\partial\Im\{\theta\}}\right)$ and $\frac{\partial}{\partial\theta^*} = \frac{1}{2}\left(\frac{\partial}{\partial\Re\{\theta\}} + j\frac{\partial}{\partial\Im\{\theta\}}\right)$, we get

$$\begin{aligned}\frac{\partial^2\left(\mathbf{h}_1^\dagger\mathbf{M}_1^\dagger\mathbf{M}_1\mathbf{h}_1\right)}{\partial[\mathbf{h}_1]_p\,\partial[\mathbf{h}_1]_q^*} &= 4K\frac{\partial^2}{\partial[\mathbf{h}_1]_p\,\partial[\mathbf{h}_1]_q^*}\left(\sum_{l=0}^{4K-1}[\mathbf{h}_1]_l^*\,[\mathbf{h}_1]_l\right)\\ &= 4K\frac{\partial[\mathbf{h}_1]_p^*}{\partial[\mathbf{h}_1]_q^*},\quad 0 \le p, q \le 4K-1\\ &= \begin{cases} 4K, & p = q\\ 0, & \text{otherwise}\end{cases}\end{aligned} \tag{32}$$

From (29) to (32), the FIM is given by

$$[\mathbf{J}(\mathbf{h}_1)]_{p,q} = \begin{cases}\frac{4K}{\psi}, & p = q\\ 0, & \text{otherwise}\end{cases} \tag{33}$$

where we can see that MSE_1 achieves the CRLB represented by the inverse matrix of (33). Thus, the proposed estimator is efficient. Note that the CRLB for MSE_2 can be readily formulated by replacing ψ with $2\sigma_{nd}^2 + \lambda_{r_1}^2 P_{r1d}\sigma_{nr_1}^2$ in the result of (33).

Numerical results

Throughout the simulations, we consider the frequency-selective channels with memory lengths $L_{SD} = L_{SR_1} =$

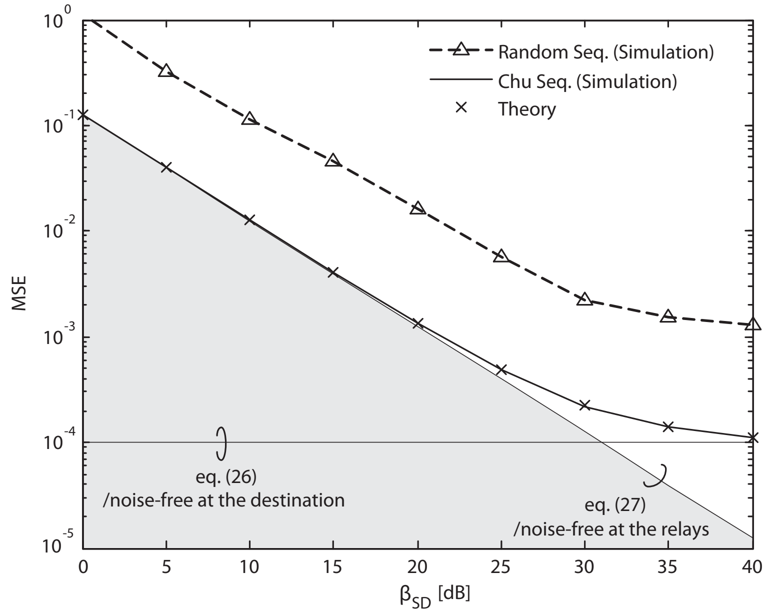

Figure 2 MSE performance of the proposed channel estimation using various training sequences. MSE performance of the proposed channel estimation using various training sequences with the proposed precoding matrices, when $\beta_{SR} = 25$ dB.

$L_{SR_2} = L_{R_1R_2} = L_{R_2R_1} = L_{R_1D} = L_{R_2D} = 2$. In particular, we assume that the gain of post-echo is 10 dB lower than that of a main path. It is also assumed that a frame consists of 50 data blocks to be constructed by QPSK modulation, and the length of each data block is 64, i.e., $N = 64$. The CP length is 4, which is larger than $\max\{L_{SD}, L_{SR_1}+L_{R_1D}-1, L_{SR_2}+L_{R_2D}-1, L_{R_1R_2}+L_{R_2D}-1, L_{R_2R_1}+L_{R_1D}-1\} = 3$. The root index of the Chu sequence is set to $t = 1$, and its length is equal to that of CP. For the illustration purpose, we introduce new parameters $\beta_{SD} = E_s/\sigma_{nd}^2$, $\beta_{SR_i} = E_s/\sigma_{nr_i}^2$ $(i = 1, 2)$ and $\beta_{R_iD} = E_{r_i}/\sigma_{nd}^2$. Assuming that $E_s = E_{r_1} = E_{r_2}$ and $\sigma_{nr_1}^2 = \sigma_{nr_2}^2$, it is further defined that $\beta_{SD} = \beta_{R_iD}$ and $\beta_{SR_1} = \beta_{SR_2} \triangleq \beta_{SR}$. In all experiments, we assume that $\beta_{SR} = 25$ dB.

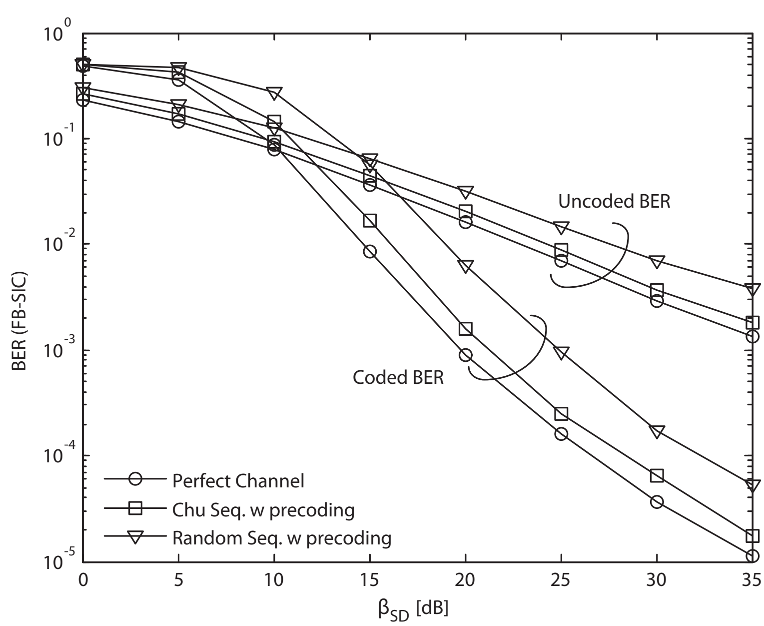

Figure 3 Coded and uncoded BER performance of the FB-SIC and MRC detection [7] when $\beta_{SR} = 25$ dB.

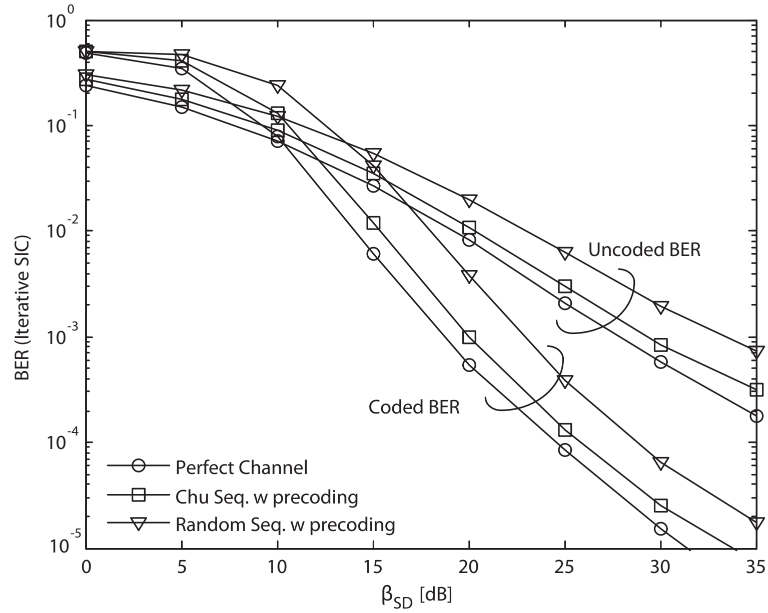

Figure 4 Coded and uncoded BER performance of the iterative SIC and MRC detection [8] when $\beta_{SR} = 25$ dB.

Figure 2 shows the MSE performance of the channel estimation using two different TSs, i.e., Chu sequence and random sequence. The precoding matrices given by (23) are applied to both cases in order to accomplish the channel estimation. It is shown that the suboptimal random sequence provides a good estimation performance. Nevertheless, the Chu sequence outperforms the random sequence. The accuracy of the theoretical results of (25), equal to the CRLB, is demonstrated through the simulations. As predicted, (27) becomes a performance bound of MSE as a function of β_{SD}; i.e., the MSE corresponding to the shaded region in Figure 2 is never achievable even if the reliability of relay links is much more improved. Meanwhile, the improvement of the relay link condition alleviates the MSE floor, which can be explained by (26).

Coded and uncoded bit error rate (BER) performances are investigated in Figures 3 and 4. To recover the symbol information, we apply FB-SIC or iterative SIC followed by maximal ratio combining and MMSE equalizer [7,8]. In order to evaluate the coded BER performance, we use the convolutional encoder with the generator polynomials (133, 171) and constraint length of 7 [18]. The Viterbi algorithm is applied for data decoding. We also include the BER performance with the perfect channel knowledge as a benchmark. From Figures 3 and 4, it is shown that the coded and uncoded BER performance of the proposed channel estimation scheme using the optimal sequence is very close to that of the perfect channel knowledge. In contrast, a tremendous performance degradation is observed with the random sequence. Finally, it is verified that with the practical channel estimation and error correction coding techniques applied, the iterative SIC scheme outperforms FB-SIC as presented in [8] where the uncoded system was considered. It should be noted that the channel estimation cannot be achieved without using (23), and such cases are not considered in the simulations.

Conclusions

In this paper, we proposed a training-based channel estimation for two-path cooperative relaying networks. The problem of the channel estimation in two-path relaying network is that the channel estimation and interference cancellation are mutually conditional. To resolve such a technical limitation, we designed a new sequence structure and proposed precoding matrices for use at the source and relays. The CRLB, a theoretical minimum bound of MSE, for the proposed channel estimation was also derived. Our simulation results demonstrate the accuracy of the theoretical analysis and the performance of the proposed channel estimation.

Competing interests
The authors declare that they have no competing interests.

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References

1. A Sendonaris, E Erkip, B Aazhang, User cooperation diversity. Part I. System description. IEEE Trans. Commun. **51**(11), 1927–1938 (2003)
2. RU Nabar, H Boelcskei, FW Kneubhueler, Fading relay channels: performance limits and space-time signal design. IEEE J. Sel. Areas Commun. **22**(6), 1099–1109 (2004)
3. B Rankov, A Wittneben, Spectral efficient signaling for half-duplex relay channels, in *Proceedings of Asilomar Conference on Signals, Systems and Computers* (Pacific Grove CA, USA, 28 October to 1 November 2005), pp. 1066–1071
4. B Rankov, A Wittneben, Spectral efficient protocols for half-duplex fading relay channels. IEEE J. Select. Areas Commun. **25**(2), 379–389 (2007)
5. H Wicaksana, SH Ting, CK Ho, WH Chin, YL Guan, AF two-path half-duplex relaying with inter-relay self interference cancellation: diversity analysis and its improvement. IEEE Trans. Wireless. Commun. **8**(9), 4720–4729 (2009)
6. H Wicaksana, SH Ting, M Motani, YL Guan, On the diversity-multiplexing tradeoff of amplify-and-forward half-duplex relaying. IEEE Trans. Commun. **58**(12), 3621–3630 (2010)
7. C Luo, Y Gong, F Zheng, Full interference cancellation for two-path relay cooperative networks. IEEE Trans. Veh. Technol. **60**(1), 343–347 (2011)
8. JS Baek, JS Seo, Efficient iterative SIC and detection for two-path cooperative block transmission relaying. IEEE Commun. Lett. **16**(2), 199–201 (2012)
9. AS Lalos, AA Rontogiannis, K Berberidis, Frequency domain channel estimation for cooperative communication networks. IEEE Trans. Signal Process. **58**(6), 3400–3405 (2010)
10. F Gao, R Zhang, Y-C Liang, Optimal channel estimation and training design for two-way relay networks. IEEE Trans. Commun. **57**(10), 3024–3033 (2009)
11. M PremKumar, VN SenthilKumaran, SJ Thiruvengadam, BLUE-based channel estimation technique for amplify-and-forward wireless relay networks. ETRI J. **34**(4), 511–517 (2012)
12. K Yan, S Ding, Y Qiu, Y Wang, H Liu, A low-complexity LMMSE channel estimation method for OFDM-based cooperative diversity systems with multiple amplify-and-forward relays. EURASIP J. Wireless Commun. Netw. **2008**, 1–9 (2008)
13. B Li, D Lin, S Li, A channel estimation scheme for AF-based two-path relay cooperative networks, in *International Conference on Computational Problem-Solving* (Chengdu, China, 21- -23 October 2011, pp. 77–81
14. R McEliece, W Stark, Channels with block interference. IEEE Trans. Inf. Theory **30**(1), 44–53 (1984)
15. M Guillaud, DTM Slock, R Knopp, A practical method for wireless channel reciprocity exploitation through relative calibration, in *Proceedings of the 8th International Symposium on Signal Processing and Its Applications* (Sydney, Australia, 28–31 August 2005), pp. 403–406
16. SM Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, (Prentice Hall, New Jersey, 1993)
17. DC Chu, Polyphase codes with good periodic correlation properties. IEEE Trans. Inform. Theory **IT-18**, 531–532 (1972)
18. M Huemer, A Onic, C Hofbauer, Classical and Bayesian linear data estimators for unique word OFDM. IEEE Trans. Signal Process. **59**(12), 6073–6085 (2011)

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