



## Memristive chaotic circuits and systems

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Memristor has been widely regarded as the fourth fundamental circuit element after resistor, inductor and capacitor, which was theoretically postulated by Chua in 1971 [1] and physically realized by the HP Labs team in 2008 [2]. Since its discovery, it has sparked unprecedented broad interest due to its significant application values in many engineering fields, such as intelligent computing, non-volatile memory, image processing, and chaotic circuits. The unique nonlinear features of the memristor actively promote the generation of chaos and other interesting dynamical behaviors, thus often used in chaotic circuits and systems for improving their chaotic characteristics and extending their application value [3–5]. The emergence of memristor has brought new enthusiasm into the study of chaotic circuits and systems, but it also comes with new challenges. So we organize this special issue to collect some new ideas, methods, and results on memristor chaotic circuits and systems and to shed light on the future research directions of design, analysis, and application of memristor chaotic circuits and systems.

This special issue accepted 13 representative papers. The corresponding research topics cover model construction, dynamical analysis, circuit implementation, control, and image encryption application of memristive chaotic circuits and systems. Some papers considered the design of multistable chaotic systems by introducing the memristor to the existing chaotic oscillator [6–10]. Authors in paper [6] proposed a three-dimensional memristive oscillator based on a simple damped Duffing oscillator and numerically showed its interesting dynamics of attractor growing with infinite coexisting attractors. Authors in paper [7] constructed a high-order memristive oscillation circuit from Van der Pol–Duffing oscillator and found its symmetric coexisting chaotic (or periodic) attractors by using the bifurcation diagrams and attraction basins. Authors in paper [8] established a new memristive jerk system with symmetric and asymmetric coexisting attractors. The dynamical analysis, multistability control and circuit implementation comprehensively demonstrate the proposed system. Authors in paper [9] proposed a second-order memristive emulator consisting of two diodes, two resistors and two capacitors, and applied it to jerk circuit for obtaining new memristive chaotic circuit. The simulation results illustrated the coexistence of bifurcations and attractors from different initial conditions. Authors in paper [10] introduced the voltage-controlled multistable memristor to the classic jerk system and achieved the purpose of constructing a new memristive chaotic system with line equilibria and extreme multistability. The PSpice circuit and microcontroller realizations of the system were studied. A new image encryption algorithm based on DNA coding and the corresponding system was designed, whose security performance was verified with the help of simulation testing and comparison. Some of the papers focused on generating a memristive chaotic system with different equilibria and dynamics from simple autonomous polynomial systems [11–13]. Liu and Tu constructed no-equilibria memristive chaotic system with hidden attractors, coexisting attractors and offset boosting properties for different initial values and parameters by adding ideal flux-controlled memristor to five-terms system [11]. Jia and Lai introduced a non-ideal memristor to Sprott A system to generate a new memristive system with three equilibria, coexisting chaotic attractors and amplitude control features [12]. Singh et al. studied the hyperchaos, coexisting attractors, hardware circuit implementation and active-adaptive projective synchronization control of memristor-based hyperchaotic system with line equilibria and established some theoretical and numerical results [13].

It is widely acknowledged that the memristor is a perfect device for mimicking biological nerve synapses and will play an essential role in the comprehensive and accurate realization of brain-like intelligence. The discovery of memristor opens the way for studying memristive neural networks. Introducing the memristor as the synapse in the neural network or using the memristor to model the neuron and neural network affected by electromagnetic radiation, many interesting neurodynamic behaviors can be found, which have attracted extensive discussion and in-depth research in academia. This special issue selects three papers related to the modeling, analysis and application of memristive neurons and neural networks [14–16]. Ma et al. analyzed chaotic bursting, rotation control attractors and offset-boosted attractors of Hindmarsh–Rose neurons with locally active memristors [14]. Chen and Min constructed bi-neuron Hopfield neural network with electromagnetic induction inspired by a flux-controlled memristor and numerically analyzed its complex dynamics [15]. Yu et al. applied the memristive synapse to neural networks for yielding multi-

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scroll attractors and coexisting attractors [16]. Based on the network, a medical digital image watermarking algorithm was designed to demonstrate its availability in engineering applications.

The study of discrete memristor which can be used to yield discrete chaotic and hyperchaotic maps has been of recent interest. Liang et al. in this issue reported a novel discrete memristive chaotic map with coexisting attractors and studied its dynamic behaviors, permutation entropy complexity, digital circuit implementation and NIST tests which well assessed its suitability for cryptographic applications [17]. Ramadoss et al. constructed some discrete memristive chaotic maps without fixed points and found the coexisting attractors in the maps [18]. The intervention of the discrete memristor can greatly improve the dynamic properties of the chaotic maps and its ease of implementation enables the engineering application to become a reality, causing the emergence of more research results.

Memristive chaotic circuits and systems have many interesting nonlinear dynamics and important engineering application values that attract increasing concerns. This special issue presents some new results of memristive chaotic circuits and systems, which provides some inspiration for future research directions. The design of the theoretical and physical model will remain a vital research foundation for memristive circuits and systems. The memristive circuits and neural network with special function (such as emotional expression, associative memory, and intelligent computing) will become key research topics. How to relate the functional performance of memristive systems to their internal dynamic behaviors will lead to more discussions in the scientific community. An important topic is the application of memristive chaotic systems, which will always be in the spotlight and greatly expanded in the future.

The successful organization of this focus point issue needs to sincerely thank all the authors, editors and reviewers for their hard work and professional advice. Also, we highly recommend this special issue to readers and hope this special issue brings inspiration to your research work. Finally, we hope it can make valuable contributions to advancing research methods and technologies in memristive circuits and chaos theory.

**Data availability** This paper has no associated data or the data will not be deposited. There is no data because all obtained results are in the paper. All authors confirm that there is no part of the paper that requires data.

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