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## A novel stochastic photo-thermoelasticity model according to a diffusion interaction processes of excited semiconductor medium

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Abstract A novel technique under the impact of stochastic heating due to the thermal effect of photothermal theory is investigated. Realistically, stochastic processes are taken on the boundary of the semiconductor medium. The interactions between optical, thermal, and mechanical waves in a half-space of the medium are studied according to the photo-thermoelasticity theory. The governing equations are described in one-dimensional elastic-electronic deformation. Laplace transforms with short-time approximation are used to analyze the main physical fields. To study the problem more realistically, some conditions are taken as random with white noise on the free surface of the elastic medium. The deterministic physical quantities are obtained with a stochastic calculus when a numerical inversion of the Laplace transform is applied. The silicon material is utilized to make the stochastic numerical simulation. The comparisons are carried out between the distributions of deterministic and stochastic (statistically, the mean and variance) the main physical quantities along different sample paths graphically and discussed.

## **1** Introduction

The stochastic models in recent years have many applications in a wide range of modern prominent science. The stochastic processes are used to describe the new mathematical and statistical models of biology, microbiology, biochemistry, epidemiology (the science of random spread of diseases like COVID-19), modern physics, mechanics, and microelectronics. The view of modern mathematical models that describe many problems in nature has differed depending on the surrounding random changes. However, when studying some physical mathematical models, adding randomness to the phenomenon will make the phenomenon more realistic. In this case, the stochastic technique (random process) is used to describe some deterministic processes in nature. On the other hand, such deterministic models represent ideal situations, which are unrealistic, so such models are often improved by including random effects. In fact, stochastic techniques are mathematical-statistical tools that help us deal with the random nature of the new models. Many reasons require that we replace the deterministic models with random (stochastic) ones [1, 2]. The first reason is that the systems under study are not completely isolated, so it is added noise. The second reason, not all the variables found in the phenomenon under study can be taken into account when making a mathematical model for the phenomenon, so these variables lead to additional noise. The third reason, the devices used in the measurements are not 100% accurate. According to these previous reasons, stochastic simulation processes should be taken into account when studying many modern physical problems.

The theory of thermoelasticity investigates the wave's behavior of the elastic bodies according to the impact of the temperature and mechanical fields. Many reasons that led to the interest in the theory of thermoelasticity, an example the fields of aeronautics, are greatly exposed to many thermal stresses on their metal structures (which are mainly elastic materials) as a result of their increased speed; this calls for a study on these structures. In the second half of the twentieth century, Biot [3] introduced the coupled thermoelasticity theory (CD theory) which can be obtained from the heat Fourier law by resolving the contradiction in the classical uncoupled theory (CUCT). The CUCT has a physical paradox, in which a parabolic behavior of the thermal wave with infinite speed

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is assumed. To solve this contradiction, a single relaxation time was inserted in the heat equation [4]. On the other hand, Green and Lindsay (G–L) [5] developed the heat equation when inserting into it a double relaxation time and introduced the generalized thermoelasticity theory. Many authors [5–8] used the generalized thermoelasticity theory widely to describe many physical problems. Many external fields with the thermal shock problem were used to investigate a 2D generalized thermoelasticity theory based on the two-temperature theory [9, 10]. Hobiny and Abbas [11] used the theoretical analysis to study the thermal damages in skin tissue when the induced intense moving heat source is used. On the other hand, Abbas et al. [12–16] studied the eigenvalue approach for spherical infinite elastic medium when the fractional order theory is applied to investigate the wave propagation using many techniques for many thermoelastic media.

On the other hand, the great importance of semiconductor materials has been taken into account, especially in modern industries, especially electrical circuits that are used in electronic industries. As a result of the internal excitation processes, some changes in the displacements of the particles result in what is known as thermoelastic (TE) deformation. Also, as a result of optical (photothermal transport processes) excitation, a transfer of surface electrons (carrier density appears with plasma wave) occurs in a process called electronic deformation (ED) (photo-excitation). Gordon et al. [17] was one of the first to study the properties of semiconductors by studying the acoustic waves due to electromagnetic radiation. A sensitive analysis of photoacoustic spectroscopy was used to investigate the effect of laser light sources on a semiconductor medium that falls on it [18]. After that, many researchers used the photothermal theory and photoacoustic spectroscopy to investigate the impact of laser beams falling on a sample of solid and fluid [19–22]. The photothermal diffusion process is used with the gradient in temperature of semiconductor samples to study the variable thermal conductivity [23]. The photothermal theory is used to explain the interaction between the plasma and thermal-elastic waves for semiconductors [24]. Lotfy and Tantawi [25] investigated the interaction which occurs between the thermoelasticity theory and photothermal theory in a non-homogeneous nano-composite semiconductor, called a photo-thermoelasticity theory. Recently, various scientists [26–29] studied the behavior of plasma-thermal-elastic waves in semiconductors when the magneto-thermoelasticity theory in general form is considered when intricate with other new theories as to the two-temperature theory and the hyperbolic two temperature.

A differential equations model which has one or more stochastic terms is called a stochastic differential equation (SDE), and this model has a resulting solution which is itself a random process. A stochastic differential equation can be used to formulate a physical system subject to thermal fluctuations. Recently, the techniques of stochastic simulation are used for describing the thermoelastic problem during the heat conduction transport process. Sherief et al. [30] developed a stochastic thermal shock problem according to the generalized thermoelasticity theory. Allam et al. [31, 32] studied a stochastic half-space problem under the effect of an internal heat source in the theory of generalized diffusion—thermoelasticity theory in an infinitely long annular cylinder. Gupta and Mukhopadhyay [33] investigated the stochastic thermoelastic interaction with random temperature distribution at the free surface of an elastic solid medium under a dual phase-lag model. On the other hand, Kant and Mukhopadhyay [34] used the stochastic temperature distribution at the boundary to describe an elastic half-space problem.

A novel theoretical technique is studied when the thermal shock problem in the ideal case is deterministic together with the realistic case while some noise exists in the context of the photo-excitation processes. The governing equation is described in one-dimensional (1D) deformation under the impact of random heating on the waves behavior of the semiconductor medium. The analytical solutions of temperature (thermal wave), displacement, stress (mechanical wave), and carrier density (plasma wave) are obtained using the 1D Laplace transform. The inverse of Laplace transforms with some numerical approximation is used to obtain the complete solutions. Statistically, the mean and the variance of the main physical fields are derived. Some comparisons are represented graphically and discussed.

#### 2 Mathematical formulation of the problem

The main variables according to the semiconductor medium are the temperature distribution (thermal wave)  $T(\vec{r}, t)$ , the carrier density distribution (plasma wave)  $N(\vec{r}, t)$ , and the displacement distribution  $u(\vec{r}, t)$  (elastic wave). The governing equations which describe the coupled system in semiconductor are [35–41]

$$\frac{\partial N(\vec{r},t)}{\partial t} = D_E \nabla^2 N(\vec{r},t) - \frac{N(\vec{r},t)}{\tau} + \kappa T(\vec{r},t), \tag{1}$$

$$\rho C_e \frac{\partial T(\vec{r}, t)}{\partial t} = k \nabla^2 T(\vec{r}, t) - \frac{E_g}{\tau} N(\vec{r}, t) + \gamma T_0 \nabla \cdot \frac{\partial u(\vec{r}, t)}{\partial t},$$
(2)

$$\rho \frac{\partial^2 T(\vec{r}, t)}{\partial t^2} = \mu \nabla^2 \vec{u}(\vec{r}, t) + (\mu + \lambda) \nabla (\nabla . \vec{u}(\vec{r}, t)) - \gamma \nabla T(\vec{r}, t) - \delta_n \nabla N(\vec{r}, t).$$
(3)

On the other hand, the consecutive equation which is described by the stress components is [41]:

$$\sigma_{ij} = \mu u_{i,jj} + (\mu + \lambda) u_{i,jj} - \gamma T_{,i} - \delta_n N_{,i} \big\}.$$
(4)

where  $\kappa = \frac{\partial N_0}{\partial T} \frac{T}{\tau}$  is a parameter describes the nonzero activation coupling case at the equilibrium carrier concentration  $N_0$ . The thermal-optical-elastic physical constants are  $D_E$ ,  $\tau$ ,  $E_g$ ,  $\mu$ ,  $\lambda$ ,  $\rho$ , k and  $T_0$  which represent the carrier diffusion coefficient, the

life-time relaxation time, the gap energy, the Lame's constants for elastic medium, the density, the thermal conductivity and the reference temperature, respectively. The volume thermal expansion is  $\gamma = (3\lambda + 2\mu)\alpha_T$ ,  $\alpha_T$  is the linear thermal expansion parameter,  $C_e$  is the specific heat coefficient,  $\delta_n = (2\mu + 3\lambda)d_n$  is the difference between valence band and conductive potential, and  $d_n$  is the coefficient of electronic deformation.

For more simplicity, the problem is taken in 1D deformation. The displacement vector can be represented as  $\vec{u} = (u(x, t), 0, 0)$ . On the other hand, the following non-dimensional variables in 1D are introduced as:

$$(x', u') = \frac{(x, u)}{C_T t^*}, \ t' = \frac{t}{t^*}, \ (T', N') = \frac{(\gamma T, \delta_n N)}{2\mu + \lambda}, \ \sigma' = \frac{\sigma}{\mu}.$$
(5)

The dashed is dropped, and all variables in 1D are taken (in the direction of x-axis). Therefore, the above system of Eqs. (1-4)with helping Eq. (14) yields:

$$\left(\frac{\partial^2}{\partial x^2} - A_2 - A_1 \frac{\partial}{\partial t}\right) N + \varepsilon_3 T = 0,$$
(6)

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial t}\right)T - \alpha_2 N + \alpha_3 \frac{\partial^2 u}{\partial x \partial t} = 0,$$
(7)

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right)u - \frac{\partial N}{\partial x} - \frac{\partial T}{\partial x} = 0,$$
(8)

$$\sigma = \sigma_{xx} = \frac{2\mu + \lambda}{\mu} \left( \frac{\partial u}{\partial x} - (T + N) \right).$$
(9)

where.

$$\alpha_2 = \frac{\alpha_T E_{gt}^*}{d_n \rho \tau C_e}, \alpha_3 = \frac{\gamma^2 T_{0t}^*}{\rho k}, A_1 = \frac{k}{D_E \rho c_e}, A_2 = \frac{kt^*}{D_E \rho c_e \tau}, \varepsilon_3 = \frac{k \kappa d_n t^*}{\alpha_t D_E \rho c_e}, C_T = \sqrt{\frac{2\mu + \lambda}{\rho}}, t^* = \frac{k}{\rho C_e C_T^2}.$$

## 3 The solution of the problem

Using the Laplace transform to convert the partial differential equations (PDE) into ordinary differential equations (ODE), Laplace transform for any function  $\zeta(x, t)$  takes the following form:

$$L(\zeta(x, t)) = \overline{\zeta}(x, s) = \int_{0}^{\infty} \zeta(x, t)e^{-st} dt.$$
 (10)

The homogenous initial conditions can be considered in this problem as:

$$u(x, t)|_{t=0} = \left. \frac{\partial u}{\partial t} \right|_{t=0} = 0, \quad T(x, t)|_{t=0} = \left. \frac{\partial T}{\partial t} \right|_{t=0} = 0, \\ \sigma(x, t)|_{t=0} = \left. \frac{\partial \sigma}{\partial t} \right|_{t=0} = 0, \quad N(x, t)|_{t=0} = \left. \frac{\partial N}{\partial t} \right|_{t=0} = 0. \right\}.$$
(11)

Appling the Laplace transform Eqs. (10) for the system of main 1D Eqs. (6-9) yields:

$$\left(D^2 - \alpha_1\right)\overline{N} + \varepsilon_3\overline{T} = 0,\tag{12}$$

$$\left(D^2 - s\right)\overline{T} - \alpha_2 \overline{N} + \alpha_3 D s \overline{u} = 0, \tag{13}$$

$$\left(D^2 - s^2\right)\overline{u} - D\overline{T} - D\overline{N} = 0,\tag{14}$$

$$\overline{\sigma} = \overline{\sigma}_{xx} = \alpha_4 \left( D\overline{u} - (\overline{T} + \overline{N}) \right). \tag{15}$$

where  $\alpha_1 = A_2 + sA_1, \alpha_4 = \frac{2\mu + \lambda}{\mu}$ . Eliminating  $\overline{T}, \overline{\mu}$  and  $\overline{N}$  in the system of ODE Eqs. (12), (14) yields:

$$D^{6} - \prod_{1} D^{4} + \prod_{2} D^{2} - \prod_{3} = 0.$$
 (16)

where the coefficients of Eq. (16) can be obtained as:

$$\prod_{1} = (s + s^{2} + \alpha_{1} - s\alpha_{3}),$$

$$\prod_{2} = (s^{3} + s\alpha_{1} + s^{2}\alpha_{1} + \varepsilon_{3}\alpha_{2} - s\alpha_{1}\alpha_{3} - s\alpha_{3}\varepsilon_{3}),$$

$$\prod_{3} = -(-s^{3}\alpha_{1} - s^{2}\varepsilon_{3}\alpha_{2}).$$
(17)

The solution of the thermal distribution according to the linearity case can be written as:

$$\overline{T}(x,s) = \sum_{i=1}^{3} M_i(s) e^{-k_i x}.$$
(18)

The other solutions of the main fields can be represented as:

$$\overline{u}(x,s) = \sum_{i=1}^{3} M'_{i}(s) e^{-k_{i}x} = \sum_{i=1}^{3} h_{1i} M_{i}(s) e^{-k_{i}x},$$
(19)

$$\overline{N}(x,s) = \sum_{i=1}^{3} M_i''(s) e^{-k_i x} = \sum_{i=1}^{3} h_{2i} M_i(s) e^{-k_i x},$$
(20)

$$\overline{\sigma}(x,s) = \sum_{i=1}^{3} M''' e^{-k_i x} = \sum_{i=1}^{3} h_{3i} M_i(s) e^{-k_i x}.$$
(21)

where  $M_i$ ,  $M'_i$ ,  $M''_i$  and  $M'''_i$  (i = 1, 2, 3) represent the unknown parameters which depend on the Laplace parameter s only, and

$$h_{1i} = \frac{-k_i^2 + s + h_{2i}\alpha_2}{k_i s \alpha_3}, h_{2i} = -\frac{\varepsilon_3}{k_i^2 - \alpha_1}, h_{3i} = -(1 + h_{2i} - h_{1i}k_i)\alpha_4.$$

$$(22)$$

#### **4** Boundary conditions

To obtain the unknown parameters  $M_i$ , some conditions can be applied on the surface of the stochastic semiconductor medium. (I) The system is exposed to thermal shock (Danilovskaya problem;  $T(t) = T_0(t) = T^*h(t)$ ) at the outer surface (x = 0) as:

$$\overline{T}(0,s) = \overline{T}_0(s) = T^*R(s) \Rightarrow \sum_{n=1}^3 M_i(s) = \overline{T}_0(s) = \frac{T^*}{s}.$$
(23)

(II) The traction is free at the outer surface (x = 0) which leads to:

$$\overline{\sigma}(0,s) = 0 \Rightarrow \sum_{i=1}^{3} M_i h_{3i} = 0.$$
(24)

(III) The recombination processes during the plasma distribution at the free surface (x = 0) lead to

$$\overline{N}(0, s) = \frac{\overline{\lambda}}{D_e} \overline{R}(s) \Rightarrow \sum_{i=1}^3 h_{2i} M_i(x, s) = \frac{\overline{\lambda}}{s D_e}.$$
(25)

where R(s) is a function representing the Heaviside unit step,  $T^* = T_0$  is a constant reference temperature, and  $\lambda$  is a chosen constant. From the above system of boundary conditions, the parameters  $M_i$  must be determined. In this case, the temperature distribution can be rewritten as:

$$\overline{T}(x,s) = M_1 e^{-k_1 x} + M_2 e^{-k_2 x} + M_3 e^{-k_3 x}.$$
(26)

#### 5 Invers of the Laplace transforms

The complete deterministic solution of the basic fields is obtained in the domain of space–time coordinates when the Riemannsum approximation technique is used numerically by inversion Laplace transform [42]. The main field in case of deterministic distributions according to the short time approximation is taken into consideration for the large value of *s*. The inverse of any function  $\overline{\zeta}(x, s)$  can be obtained as:

$$\zeta(x, t') = L^{-1}\{\overline{\zeta}(x, s)\} = \frac{1}{2\pi i} \int_{n-i\infty}^{n+i\infty} e^{st'} \overline{\zeta}(x, s) ds.$$
(27)

where s = n + iM ( $n, M \in R$ ), the solution of Eq. (26) takes the form:

$$\zeta(x, t') = \frac{\exp(nt')}{2\pi} \int_{\infty}^{\infty} \exp(i\beta t)\overline{\zeta}(x, n+i\beta)d\beta.$$
(28)

Using the numerical Fourier series expansion for Eq. (27) in the closed interval [0, 2t'] yields:

$$\zeta(x, t') = \frac{e^{nt'}}{t'} \left[ \frac{1}{2} \overline{\zeta}(x, n) + Re \sum_{k=1}^{N} \overline{\zeta}\left(x, n + \frac{ik\pi}{t'}\right) (-1)^n \right].$$
<sup>(29)</sup>

where *i* represents the imaginary unit, *N* can be chosen free as a large integer, and *Re* refers to the real part and the notation  $nt' \approx 4.7$  [42].

## 6 Stochastic main physical fields

#### 6.1 Stochastic temperature (thermal wave)

Now, a stochastic distribution of the temperature (thermal) at the boundary can be defined on the following form [32, 33]:

$$T_0(t) = T(t) + \varphi_0(t).$$
(30)

where  $T(t) = \frac{T^*}{h(t)}$ ,  $T^*$  is a constant temperature, h(t) is the Heaviside unit function, and a stochastic process is  $\varphi_0(t)$  which is based on the time-parameter *t* and satisfies:

$$E[\varphi_0(t)] = 0. (31)$$

The stochastic process  $\varphi_0(t)$  can be chosen from the white noise type (the most common type), and the stochastic process for the function x(t) satisfies the following relation:

$$E[L\{x(t)\}] = L[E\{x(t)\}].$$
(32)

On the other hand, the physical fields involve a boundary condition during a stochastic process, and in this case, the physical fields have a stochastic process mainly due to the random function  $\varphi_0(t)$ . Therefore, the following equation from Eqs. (26) and (30) is obtained:

$$E[\overline{T}(x,s)] = L[E\{T(x,t)\}] = \overline{T}(x,s).$$
(33)

Accordingly, the mean of all the sample paths of the temperature field, E[T(x, t)], is similar to the solution for the deterministic case.

But in the case of placing the temperature distribution on the following form:

$$T(x, s) = \theta(x, s) + \phi(x, s)T_0(s), \text{ or} 
\overline{T}(x, s) = (A_1 + A_2\overline{T_0}(s)) \exp(-k_1x) + (B_1 + B_2\overline{T_0}(s)) \exp(-k_2x) + \begin{cases} . \\ . \end{cases}$$
(34)  

$$(C_1 + C_2\overline{T_0}(s)) \exp(-k_3x)$$

According to Eq. (34), it yields:

$$\overline{\theta}(x,s) = A_1 \exp(-k_1 x) + B_1 \exp(-k_2 x) + C_1 \exp(-k_3 x),$$
(35)

$$\overline{\phi}(x,s) = A_2 \exp(-k_1 x) + B_2 \exp(-k_2 x) + C_2 \exp(-k_3 x).$$
(36)

On the other hand, when using Eqs. (30) and (34), it can be rewritten as [32, 33]:

$$\overline{T}(x,s) = \overline{\theta}(x,s) + \overline{\phi}(x,s) [\overline{T}(s) + \overline{\phi}_0(s)].$$
(37)

Applying the invers of Laplace transform technique of Eq. (37), in this case Eq. (37) can be written as:

$$\overline{T}(x,s) = \left\{\overline{\theta}(x,s) + \overline{\phi}(x,s)\overline{T}(s)\right\} + \overline{\phi}(x,s)\overline{\varphi}_0(s),$$
(38)

$$\overline{T}(x,s) = T^{1}(x,s) + \overline{\phi}(x,s)\overline{\varphi}_{0}(s),$$
(39)

where  $\overline{T^1}(x, s) = \overline{\theta}(x, s) + \overline{\phi}(x, s)\overline{T}(s)$ . According to the invers of Laplace transform method, Eq. (39) takes the form:

$$T(x,t) = T^{1}(x,t) + \int_{0}^{t} \phi(x,t-u)\varphi(u)du,$$
(40)

where  $T^1(x, s)$  expresses the deterministic temperature distribution and  $\phi(x, t)$  is the Laplace inverse of Eq. (36). However, Eq. (40) can be reduced as:

$$T(x,t) = T^{1}(x,t) + \int_{0}^{t} \phi(x,t-u)dW(u),$$
(41)

where W(u) is the Wiener process.

To complete the stochastic properties, the variance of the problem should be obtained; in this case, squaring Eq. (40) yields:

$$[T(x,t)]^{2} = [T^{1}(x,t)]^{2} + \int_{0}^{t} \int_{0}^{t} \varphi(u_{1})\varphi(u_{2})du_{1}du_{2}\phi(x,t-u_{1})\phi(x,t-u_{2}) + 2\int_{0}^{t} T^{1}(x,t)\varphi(u)\phi(x,t-u)du$$

$$(42)$$

Applying the expectation process to both sides of Eq. (42) yields:

$$E[T(x,t)]^{2} = E[T^{1}(x,t)]^{2} + \int_{0}^{t} \int_{0}^{t} E[\varphi(u_{1})\varphi(u_{2})]du_{1}du_{2}\phi(x,t-u_{1})\phi(x,t-u_{2}) \\ + 2\int_{0}^{t} E[\varphi(u)]T^{1}(x,t)\phi(x,t-u)du$$

$$(43)$$

Taking into consideration the following relations:

$$E[\varphi(u)] = 0, \quad E[\varphi(u_1)\varphi(u_2)] = \delta(u_1 - u_2).$$
(44)

The variance can be written in the following form:

$$Var[T(x,t)] = \int_{0}^{t} \int_{0}^{t} \phi(x,t-u_1)\phi(x,t-u_2)\delta(u_2-u_1)du_1du_2.$$
(45)

Using the following relation:

$$\int_{a}^{b} f(x)f(x-x_0)dx = f(x_0), \quad a < x_0 < b.$$
(46)

In this case, the variance can be rewritten as:

$$Var[T(x,t)] = \int_{0}^{t} \phi(x,t-u_1)\phi(x,t-u_2)du_1.$$
(47)

During the path  $u_1 = u_2$ , it yields:

$$Var[T(x,t)] = \int_{0}^{t} [\phi(x,t-u_1)]^2 du_1.$$
(48)

Substituting  $t - u_1 = \vartheta$ , the variance of temperature field can be written as:

$$Var[T(x,t)] = -\int_{t}^{0} [\phi(x,\vartheta)]^{2} d\vartheta = \int_{0}^{t} [\phi(x,\vartheta)]^{2} d\vartheta.$$
(49)

## 6.2 Deterministic stress distribution

Using the same technique which is described in Sect. 6.1, under the deterministic boundary condition Eqs. (24) and (21), the solution of the deterministic stress field can be described as [32, 33]:

$$\overline{\sigma}_{xx}(x,s) = h_{31}M_1 \exp(-k_1 x) + h_{32}M_2 \exp(-k_2 x) + h_{33}m_3 \exp(-k_3 x).$$
(50)

6.3 Stochastic stress distribution

Using the same form of Eq. (30) and proceeding in the same manner as in Sect. 6.1, the stochastic stress distribution can be obtained as:

$$E[\overline{\sigma}_{xx}(x,s)] = L[E\{\sigma_{xx}(x,s)\}] = \overline{\sigma}_{xx}(x,s).$$
(51)

Therefore, the mean of all the sample paths of the stress field distribution  $E[\sigma_{xx}(x, s)]$  can be taken in a similar manner of Eq. (30) for the deterministic case which can be put as:

$$\overline{\sigma}_{xx}(x,s) = \overline{\Omega}(x,s) + \overline{\omega}(x,s)\overline{T_0}, \text{ or}$$

$$\overline{\sigma}_{xx}(x,s) = \left(D_1 + D_2\overline{T_0}(s)\right) \exp(-k_1x) + \left(E_1 + E_2\overline{T_0}(s)\right) \exp(-k_2x) + \left(F_1 + F_2\overline{T_0}(s)\right) \exp(-k_3x)$$
(52)

On the other hand, from Eq. (52),  $\overline{\Omega}(x, s)$  can be rewritten as:

(

$$\Omega(x,s) = D_1 \exp(-k_1 x) + E_1 \exp(-k_2 x) + F_1 \exp(-k_3 x),$$
(53)

and  $\overline{\omega}(x, s)$  can be written as:

$$\overline{\omega}(x,s) = D_2 \exp(-k_1 x) + E_2 \exp(-k_2 x) + F_2 \exp(-k_3 x).$$
(54)

Therefore, using Eq. (30) yields:

$$\overline{\sigma}_{xx}(x,s) = \overline{\Omega}(x,s) + \overline{\omega}(x,s) \left[\overline{T} + \overline{\varphi}_0(t)\right].$$
(55)

Using the Laplace transform inverse of Eq. (55) under the influence of the convolution property of Laplace inverse:

$$\sigma_{xx}(x,t) = \sigma^{1}(x,t) + \int_{0}^{t} \omega(x,t-u)\varphi(u)\mathrm{d}u,$$
(56)

where  $\sigma^1(x, t)$  is the stress distribution in the deterministic case and  $\omega(x, t)$  is the Laplace inverse of  $\overline{\omega}(x, s)$ . By the same way used in Sect. 6.1, the variance for the stress distribution can be written as:

$$Var[\sigma_{xx}(x,t)] = \int_{0}^{t} \omega^{2}(x,\vartheta) d\vartheta.$$
(57)

#### 6.4 Deterministic displacement distribution

In view of the deterministic boundary conditions (19), where the displacement distribution is a part of stress distribution, however by the same technique described in Sect. 4, in this case the solution of the displacement field can be written in the following form:

$$\overline{u}(x,s) = h_{11}M_1 \exp(-k_1 x) + h_{12}M_2 \exp(-k_2 x) + h_{13}M_3 \exp(-k_3 x).$$
(58)

6.5 Stochastic displacement distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in Sect. 6.1, the mean displacement distribution can be written as [32, 33]:

$$E[\overline{u}(x,s)] = L[E\{u(x,s)\}] = \overline{u}(x,s).$$
(59)

where the mean E[u(x, s)] of all the displacement field sample paths has the similar solution of the deterministic case. Considering,

$$\overline{u}(x,s) = \overline{\Gamma}(x,s) + \overline{U}(x,s)\overline{T}_0, \text{ or}$$

$$\overline{u}(x,s) = \left(G_1 + G_2\overline{T}_0(s)\right)\exp(-k_1x) + \left(H_1 + H_2\overline{T}_0(s)\right)\exp(-k_2x) + \left(K_1 + K_2\overline{T}_0(s)\right)\exp(-k_3x)$$
(60)

On the other hand, the values of  $\overline{\Gamma}(x, s)$  and  $\overline{U}(x, s)$  according to Eq. (60) can be written as:

$$\overline{\Gamma}(x,s) = G_1 \exp(-k_1 x) + H_1 \exp(-k_2 x) + G_1 \exp(-k_3 x),$$
(61)

$$\overline{U}(x,s) = G_2 \exp(-k_1 x) + H_2 \exp(-k_2 x) + G_2 \exp(-k_3 x).$$
(62)

Using Eqs. (30), (60) can be rewritten as:

$$\overline{u}(x,s) = \overline{\Gamma}(x,s) + \overline{U}(x,s) [\overline{T}(s) + \overline{\varphi}_0(s)].$$
(63)

Under the influence of the inverse of Laplace transform, Eq. (63) can be reduced as:

$$u(x,t) = u^{1}(x,t) + \int_{0}^{t} U(x,t-u)\varphi(u)du,$$
(64)

where  $u^{1}(x, t)$  is the deterministic displacement field and U(x, t) is the Laplace inverse of the function  $\overline{U}(x, s)$ .

By the same way, the variance for the displacement distribution can be obtained as in Sect. 6; in this case, the variance for the displacement distribution can be written as:

$$Var[\sigma_{xx}(x,t)] = \int_{0}^{t} U^{2}(x,\vartheta)d\vartheta.$$
(65)

6.6 Deterministic carrier density distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in the above deterministic sections, the deterministic carrier density field solution is obtained as:

$$N(x,s) = M_1 h_{31} \exp(-k_1 x) + M_2 h_{32} \exp(-k_2 x) + M_3 h_{33} \exp(-k_3 x).$$
(66)

6.7 Stochastic carrier density distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in the above stochastic sections, the mean displacement distribution can be written as [32, 33]:

$$E[\overline{N}(x,s)] = L[E\{N(x,s)\}] = \overline{N}(x,s), \tag{67}$$

where the mean E[N(x, s)] takes all carrier density field sample paths.

Considering the stochastic carrier density distribution, it can be written as:

$$N(x, s) = \overline{m}(x, s) + \overline{n}(x, s)T_0, \quad or$$
  

$$\overline{N}(x, s) = (a_1 + a_2\overline{T_0}(s)) \exp(-k_1x) + (b_1 + b_2\overline{T_0}(s)) \exp(-k_2x) + (c_1 + c_2\overline{T_0}(s)) \exp(-k_3x)$$
(68)

where the values of  $\overline{m}(x, s)$  and  $\overline{n}(x, s)$  can take the form:

$$\overline{m}(x,s) = a_1 \exp(-k_1 x) + b_1 \exp(-k_2 x) + c_1 \exp(-k_3 x),$$
(69)

$$\overline{n}(x,s) = a_2 \exp(-k_1 x) + b_2 \exp(-k_2 x) + c_2 \exp(-k_3 x).$$
(70)

Under the effect of stochastic term which is defined by Eq. (30), stochastic carrier density distribution can be rewritten as:

$$\overline{N}(x,s) = \overline{m}(x,s) + \overline{n}(x,s) \left[\overline{T} + \overline{\varphi}_0(s)\right].$$
(71)

Using the inverse of Laplace transform, it yields:

$$N(x,t) = N^{1}(x,t) + \int_{0}^{t} n(x,t-u)\varphi(u)du,$$
(72)

where  $N^1(x, t)$  is the deterministic carrier density field and n(x, t) is the Laplace inverse of the function  $\overline{n}(x, s)$ .

The variance for the carrier density field can be obtained as:

$$Var[N(x,t)] = \int_{0}^{t} n^{2}(x,\vartheta)d\vartheta$$
(73)

 Table 1 The Si input physical parameters in SI units

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Unit	Symbol	Si
N/m <sup>2</sup>	λ	$6.4 \times 10^{10}$
	$\mu$	$6.5  imes 10^{10}$
kg/m <sup>3</sup>	ρ	2330
К	$T_0$	800
sec (s)	τ	5 <sup>-5</sup>
m <sup>2</sup> /s	$D_e$	$2.5 \times 10^{-3}$
m <sup>3</sup>	$d_n$	$-9^{-31}$
eV	$E_g$	1.11
K <sup>-1</sup>	$\alpha_t$	$4.14^{-6}$
$Wm^{-1} K^{-1}$	k	150
J/(kg K)	$C_e$	695
m/s	$\tilde{s}$	2

Table 2         The values of the
variance of the main fields for
different times at $x = 0.1953$ and
x = 0.6328

	x = 0.1953			x = 0.6328		
	t = 0.05	t = 0.075	t = 0.10	t = 0.05	t = 0.075	t = 0.10
T(x,t)	0.0321088	0.0397147	0.0510153	0.0767555	0.177733	0.3002
$\sigma_{xx}(x,t)$	0.0451	0.0885	0.1725	0.00013	0.00086	0.0045
N(x, t)	$2.29\times10^{-13}$	$3.11 \times 10^{-13}$	$4.16 \times 10^{-13}$	$2.6 \times 10^{-13}$	$4.3 \times 10^{-13}$	$6.7 \times 10^{-13}$
u(x,t)	0.000069	0.000187	0.000467	$9.0394 \times 10^{-8}$	$1.10575 \times 10^{-6}$	0.00001

#### 7 Numerical results

This section analyzes the obtained results by considering the input parameters (physical constants) of silicon material which represents the semiconductor example. In this case, the numerical simulation of the propagation of the waves accompanying the basic physical quantities can be done and represented graphically under the impact of the noise intensity. Accordingly, the physical constants of silicon are shown in Table 1 which can be used in this problem [39–41].

Figure 1a to d shows the variation of the main deterministic physical variables (temperature, carrier density, normal stress and displacement) against the horizontal distance according to the different values of three dimensionless times, namely t = 0.02, t = 0.04 and t = 0.06. However, the behavior of waves propagation for different values of time (t = 0.02, t = 0.04 and t = 0.06) takes the same behavior but with a difference in magnitude. In Fig. 1a, the deterministic distribution of temperature (thermal wave) starts from a maximum point that fulfills the thermal condition and continues to decrease continuously taking the exponential behavior without any jumps until reaching the minimum before it coincides with the zero line. In Fig. 1b, the deterministic stress begins at zero satisfying the mechanical condition which decreases sharply near the surface until reaches the peak minimum point. With the increase in the thermal effect on the surface, the stress distribution begins to increase in a continuous and smooth manner until it reaches a state of stability by going deeper into the material and this is shown by matching with the zero line. In Fig. 1c, the carrier density in the deterministic case (plasma wave) starts from a maximum point that meets the state of plasma recombination on the surface and then begins to decrease continuously in the smooth state without any jumps until it disappears with increasing distance to reach the steady state. Finally, in Fig. 1d, the deterministic (elastic wave) displacement distribution on the surface starts from an extreme point with very sharp dips to a minimum value that increases by a small amount before coinciding with the zero line.

Figure 2a–d displays the dispersion of the distributions of the fundamental physical quantities around their mean which is described by variance. The numerical simulation was carried out at different values of the dimensionless time (t = 0.2, t = 0.4 and t = 0.6) and the behavior of all distributions (the variance of the main fields), due to a white noise in stochastic case. The impact of the values of time on the variance of main fields is presented in Table 2 at two values of the distance x. It can be seen that the variance travels like a wave and ends with a diffuse portion for both the stress and the carrier density distributions. Moreover, the variance on the free surface of semiconductor is zero except for the temperature and displacement distributions, where all physical properties are invariant on the boundary except for temperature distributions, where the maximum occurs at the free surface. In general, it occurs at the free surface. In general, the physical field distributions vanish after a finite distance (x = 1.3) for the stochastic as well as deterministic.

Fig. 1 The deterministic temperature, normal stress, carrier density and displacement against the distance for different values of times in the context of generalized thermoelasticity theory



mean when t = 0.04



Figure 3a–d illustrates the comparison between stochastic and deterministic temperature, normal thermal stress, carrier density and displacement distributions according to three different sample paths at t = 0.04. Figure 3a displays a combined plot of the temperature distribution in a deterministic case and a stochastic case for three sample paths. From this figure, it becomes clear a large variation between the stochastic temperature (thermal) distribution and the deterministic temperature distribution along three





different sample paths which is clearly visible near the boundary. On the other hand, the stochastic distribution of temperature coincides with the deterministic distribution with increasing distance to enter the depth of the semiconductor material. Figure 3b shows the deterministic normal stress distribution (mean) with stochastic normal stress distribution according to three different random sample paths. From this figure, a significant variation between the stochastic normal stress distribution and deterministic normal stress distribution is obtained at the beginning near the surface boundary. This difference continues with the increase in the distance until entering the depth of the material as a result of the thermal stresses on the semiconductor material and then matching occurs until the state of stability. Figure 3c shows the comparison between the deterministic carrier density (plasma) distribution with the stochastic plasma distribution. From this figure, it is clear that there is a discrepancy between the distributions, but in general the mean of all three sample paths of the stochastic plasma distribution coincides with the deterministic plasma distribution with increasing distance. Finally, Fig. 3d displays the deterministic displacement (elastic) distribution which is in its comparison with a displacement (elastic) distribution. Similarly in Fig. 1, there is a large variation near the boundary surface, and then, there is a correspondence with the increase in the distance by entering into the depth of the material.

## 8 Conclusion

The present work is based on the interaction between the photothermal theory and thermoelasticity theory when a semiconductor material is used. The main concept of this problem depends on two types of analyses: deterministic and stochastic types. The used stochastic type is defined when the Gaussian white noise stochastic process is added to the deterministic conditions. The numerical deterministic solutions of temperature, normal stress, carrier density, and displacement are obtained according to the Laplace transform method. The mathematical mean and variance of the main fields are derived due to the stochastic nature of the solution. Through the mathematical deduction of the variance, it turns out that its value is directly proportional to the squared noise intensity. In addition, it was concluded that the noise intensity converges to zero, and the solution paths, in this case, converge to the expected sample path. It was also noted that in the case of the deterministic distribution of the physical fields under study as well as in the case of the stochastic distribution, it was found that the waves propagating to all the fields vanish with the increasing distance, which is consistent with the physical assumptions. The magnitudes of deterministic distributions of the physical variables increase with increasing time. Unlike normal stress distribution, the temperature, carrier density, and displacement distributions show no significant differences (negligible differences) between the stochastic and the deterministic distributions. All physical distributions are obtained to be continuous according to the physical domain. This physical–mathematical model can be used to improve product quality and production efficiency of semiconductor materials. The analysis and obtained results in this work will be very much significant in studying the uses of semiconductor such as diodes, triodes, and modern electronic devices.

Data availability The information applied in this research is ready from the authors at request.

#### Declarations

Conflict of interest The authors have declared that no competing interests exist.

A

#### Appendix

The coefficients of Eqs. (26) are:

$${}_{1} = -\frac{\lambda(h_{32} - h_{33})}{sD_{e}(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(74)

$$A_2 = -\frac{(-h_{23}h_{32} + h_{22}h_{33})}{(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(75)

$$B_1 = \frac{-\chi(h_{31} - h_{33})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(76)

$$B_2 = \frac{(h_{23}h_{31} - h_{21}h_{33})}{(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(77)

$$C_1 = \frac{\bar{\lambda}(h_{31} - h_{32})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(78)

$$C_2 = \frac{(-h_{22}h_{31} + h_{21}h_{32})}{(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}.$$
(79)

The coefficients of Eqs. (50) are:

$$D_1 = -\frac{\lambda h_{31}(h_{32} - h_{33})}{sD_e(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(80)

$$E_1 = \frac{\lambda h_{32}(-h_{31} + h_{33})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(81)

. .

$$F_1 = \frac{\lambda h_{33}(h_{31} - h_{32})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(82)

$$D_2 = -\frac{h_{31}(-h_{23}h_{32} + h_{22}h_{33})t_0}{(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(83)

$$E_2 = \frac{h_{32}(h_{23}h_{31} - h_{21}h_{33})t_0}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(84)

$$F_2 = \frac{h_{33}(-h_{22}h_{31} + h_{21}h_{32})t_0}{(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}.$$
(85)

The coefficients of Eqs. (58) are:

$$G_1 = -\frac{\lambda h_{11}(h_{32} - h_{33})}{sD_e(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(86)

$$H_1 = \frac{\lambda h_{12}(-h_{31} + h_{33})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(87)

$$K_1 = \frac{\lambda h_{13}(h_{31} - h_{32})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(88)

$$G_2 = -\frac{h_{11}(-h_{23}h_{32} + h_{22}h_{33})}{(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$
(86)

$$H_2 = \frac{h_{12}(h_{23}h_{31} - h_{21}h_{33})}{(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(87)

$$K_2 = \frac{h_{13}(-h_{22}h_{31} + h_{21}h_{32})}{(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}.$$
(88)

The coefficients of Eqs. (66) are:

$$= -\frac{\lambda h_{21}(h_{32} - h_{33})}{(89)}$$

$$a_{1} = -\frac{m_{21}(h_{32} - h_{33})}{sD_{e}(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})},$$

$$(89)$$

$$\lambda h_{22}(-h_{31} + h_{33})$$

$$b_1 = \frac{1}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$

$$(90)$$

$$\overline{\lambda}h_{23}(h_{31} - h_{32})$$

$$c_1 = \frac{h_{23}(h_{31} - h_{32})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})},$$
(91)

$$a_2 = -\frac{h_{21}(h_{32}h_{23} - h_{33}h_{22})}{((h_{22} - h_{23})h_{31} - (h_{21} - h_{23})h_{32} + (h_{21} - h_{22})h_{33})},$$
(92)

$$b_2 = \frac{h_{22}(h_{23}h_{31} - h_{21}h_{33})}{((-h_{22} + h_{23})h_{31} + (h_{21} - h_{23})h_{32} - (h_{21} - h_{22})h_{33})},$$
(93)

$$= \frac{h_{23}(h_{22}h_{31} - h_{21}h_{32})}{(421 - 422)(431 - 422)(433)}$$
(94)

$$c_2 = \frac{h_{23}(h_{22}+h_{13})h_{21}(h_{21}-h_{23})h_{32}}{((-h_{22}+h_{23})h_{31}+(h_{21}-h_{23})h_{32}-(h_{21}+h_{22})h_{33})}.$$
(94)

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