



Logarithmic-corrected Einstein–Gauss–Bonnet inflation compatible with GW170817

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Abstract In this paper, we investigate the inflationary phenomenology of an Einstein–Gauss–Bonnet theory with the extension of a logarithmic modified $f(R)$ gravity, compatible with the GW170817 event. The main idea of our work is to study different results for an almost linear Ricci scalar through logarithmic corrections and examine whether such model is viable. First of all, the theoretical framework under slow-roll evolution of the scalar field is presented and also the formalism of the constant-roll evolution making predictions for the non-Gaussianities of the models is developed, since the constant-roll evolution is known to enhance non-Gaussianities. As shown, the non-Gaussianities are of the order $\mathcal{O} \sim (10^{-1})$. Furthermore, the slow-roll indices and the observational indices of inflation are calculated for several models of interest. As demonstrated, the phenomenological viability of the models at hand is achieved for a wide range of the free parameters and the logarithmic term has a minor contribution to numerical calculations, as expected.

1 Introduction

The recent years, one of the most remarkable achievements in theoretical and observational Cosmology is, without a doubt, the detection of the gravitational waves about one hundred years after Einstein's predictions. In 2017, the LIGO–Virgo detectors observed a gravitational wave coming from the merging of two neutron stars [1]. The interesting fact of the observation was that the gravitational wave arrived almost equal to the gamma-ray burst. This means that the speed of the gravitational wave c_T is approximately equal to the speed of light, namely $c_T^2 = 1$ in natural units. This constraint imposes stringent conditions on modified theories of gravity that may describe successfully the nature on such scales. Many generalized theories of gravity provide viable models compatible with this astrophysical event, see Ref. [2].

One attractive class of theories, in order to approach the inflationary era of the universe or other astrophysical phenomena, is the Einstein–Gauss–Bonnet theories [3–45]. These theories involve the Gauss–Bonnet term, more specifically, quadratic terms of the Ricci scalar and of the Riemann and Ricci tensors, in the context of Einstein's gravity. Our motivation to utilize this class of gravitational theories in order to describe the inflationary era of the universe originates from the string property. In detail, the whole theory is string-corrected

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canonical scalar field theory minimally coupled to gravity. In our previous work [36], we proved that this theory can be rectified in view of the GW170817 event, by simply setting the gravitational speed wave equal to unity and expressing all the physical quantities in terms of the scalar field.

In this paper, we shall extend our previous works [36,46], presenting a modified theory of gravity $f(R)$ with a logarithmic term, based on Ref. [47], and also modified gravity with logarithmic terms is presented in Ref. [48]. First of all, we present in detail the theoretical framework of the evolution of the scalar field under slow-roll and constant-roll approximations in the background of Friedmann–Robertson–Walker spacetime. As mentioned before, we express all the involved physical quantities in terms of functions of the scalar field and their higher derivatives with respect to the scalar field. By imposing the slow-roll conditions into the gravitational equations of motion, the slow-roll indices and the observational indices have quite simple and elegant final expressions. Afterwards, we consider the constant roll evolution of the scalar field, by imposing the condition $\ddot{\phi} = \beta H \dot{\phi}$ and we repeat the same process. Moreover, the amount of non-Gaussianities is predicted and as expected is not significantly altered in comparison with the R gravity. Finally, we shall examine the compatibility of the theoretical framework with the latest Planck Data, see Ref. [49] in the context of the two possible ways of the scalar field evolution. In the last section, viable models for the inflationary era are constructed by considering coupling scalar field functions $\xi(\phi)$ with compatible numerical values with the observations.

Before we begin our analysis, it is necessary to explain the reason why the theory needs massless gravitons in order to be consistent with the GW170817 event. In nature, the interactions between particles and fields are achieved through the propagators of the fundamental forces. The graviton is the propagator of the gravitational waves produced either in the early universe or from astrophysical events. From the perspective of the Elementary Particle Physics, the graviton must be massless during the inflationary and post-inflationary era. Despite the appealing string property of the Einstein–Gauss–Bonnet theory, there is a serious drawback. The theory predicts that the primordial tensor perturbations are incompatible with the GW170817 event. As we have proved in our previous works [32], it is possible to get a massless graviton under certain constraints. Only if the coupling scalar function $\xi(\phi)$ satisfies the differential equation $\ddot{\xi} - H\dot{\xi} = 0$, we can demand $c_T^2 = 1$. Hence, we can obtain a viable model for the inflationary era in the context of Einstein–Gauss–Bonnet gravity.

2 Theoretical framework of logarithmic-corrected $f(R)$ gravity

We begin our analysis by considering the following gravitational theory of a scalar field ϕ , since all the information about the universe in the era of inflation is encoded in it. Let us assume that the action is defined as,

$$S = \int d^4x \sqrt{-g} \left(\frac{f(R)}{2\kappa^2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \xi(\phi) \mathcal{G} \right), \quad (1)$$

where g is the determinant of the metric tensor, $\kappa = \frac{1}{M_P}$ is the gravitational constant while M_P denotes the reduced Planck mass, $V(\phi)$ is the scalar potential and $\xi(\phi)$ signifies the Gauss–Bonnet coupling scalar function. We assume a modified theory of gravity $f(R)$ where $f(R) = R^{1+\delta}$ with $\delta \ll 1$. Considering a power-law model, it becomes abundantly clear that for small deviations from unity, one could easily obtain logarithmic corrections to gravity via Taylor expansion; such terms are known for describing quantum corrections. Finally, the Gauss–Bonnet term \mathcal{G} is given by the expression $\mathcal{G} = R^2 - 4R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$,

with $R_{\alpha\beta}$ and $R_{\alpha\beta\gamma\delta}$ being the Ricci and Riemann tensor, respectively. Furthermore, the line-element is assumed to have the Friedmann–Robertson–Walker form,

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \tag{2}$$

where $a(t)$ is the scale factor of the universe and the metric tensor has the form of $g_{\mu\nu} = \text{diag}(-1, a(t)^2, a(t)^2, a(t)^2)$. The effective Lagrangian of inflation is not specified by the data at present time. Thus, although the inflationary era is a classical era of our universe, which is described by a four-dimensional spacetime, it still is possible that the quantum era may have a direct imprint on the effective Lagrangian of inflation. Therefore, the two most simple corrections of the inflationary effective Lagrangian may be provided by higher curvature terms, like $f(R)$ gravity corrections, and Einstein–Gauss–Bonnet corrections. As long as the metric is flat, the Ricci scalar and the Gauss–Bonnet term are topological invariant and can be written as $R = 12H^2 + 6\dot{H}$, $\mathcal{G} = 24H^2(\dot{H} + H^2)$, respectively. H is Hubble’s parameter, and in addition, the “dot” denotes differentiation with respect to the cosmic time. We expand the modified gravitational function $f(R)$ as follows:

$$f(R) \simeq R + \delta R \ln(\alpha R), \tag{3}$$

where α is a constant with mass dimensions $[m]^{-2}$ for consistency. It is expected that the logarithmic term $\ln(\alpha R)$ has minor contribution to the equations of motion because represents quantum corrections. Differentiating Eq. (3) with respect to the Ricci scalar gives,

$$F \simeq 1 + \delta + \delta \ln(\alpha R). \tag{4}$$

Implementing the variation principle with respect to the metric tensor and the scalar field in Eq. (1) generates the field equations of gravity and the continuity equation of the scalar field. By splitting the field equations in time and space components, the gravitational equations of motion are then derived which read,

$$\frac{3FH^2}{\kappa^2} = \frac{1}{2}\dot{\phi}^2 + V + 24\dot{\xi}H^3 + \frac{FR - f}{2\kappa^2} - \frac{3H\dot{F}}{\kappa^2}, \tag{5}$$

$$\frac{-2F\dot{H}}{\kappa^2} = \dot{\phi}^2 - 16\dot{\xi}H\dot{H} - 8H^2(\ddot{\xi} - H\dot{\xi}) + \frac{\ddot{F} - H\dot{F}}{\kappa^2}, \tag{6}$$

$$\ddot{\phi} + 3H\dot{\phi} + V' + \xi' \mathcal{G} = 0. \tag{7}$$

As proved in a recent work of ours [36], certain additional constraints on the gravitational wave speed need to be imposed so as to achieve compatibility with recent striking observations from GW170817. Gravitational waves are perturbations in the metric which travel through spacetime with the speed of light. The gravitational wave speed in natural units for Einstein–Gauss–Bonnet theories has the form,

$$c_T^2 = 1 - \frac{Q_f}{2Q_t}, \tag{8}$$

where $Q_f = 16(\ddot{\xi} - H\dot{\xi})$ and $Q_t = \frac{F}{\kappa^2} - 8\dot{\xi}H$ are auxiliary functions depending on the scalar field and the Ricci scalar. Compatibility can be achieved by equating the velocity of gravitational waves with unity, or making it infinitesimally close to unity. In other words, we demand $Q_f = 0$. The constraint leads to an ordinary differential equation $\ddot{\xi} = H\dot{\xi}$. Now we will solve this equation in terms of the derivatives of scalar field. Assuming that $\dot{\xi} = \xi' \dot{\phi}$ and $\frac{d}{dt} = \dot{\phi} \frac{d}{d\phi}$ the constraint equation has the form,

$$\xi'' \dot{\phi}^2 + \xi' \ddot{\phi} = H \xi' \dot{\phi}. \tag{9}$$

Considering the approximation

$$\xi' \ddot{\phi} \ll \xi'' \dot{\phi}^2, \tag{10}$$

Equation (9) can be solved easily with respect to the derivative of the scalar field,

$$\dot{\phi} \simeq \frac{H \xi'}{\xi''}. \tag{11}$$

In order to study the inflationary era of the universe, it is necessary to solve analytically the system of equations of motion. It is obvious that this system is very difficult to study analytically. Thus, we assume the slow-roll approximations during inflation. Mathematically speaking, the following conditions are assumed to hold true,

$$\dot{H} \ll H^2, \quad \frac{1}{2} \dot{\phi}^2 \ll V, \quad \ddot{\phi} \ll 3H\dot{\phi}, \tag{12}$$

thus, the equations of motion can be simplified greatly. Hence, after imposing the constraint of the gravitational wave and considering the slow-roll approximations the equations of motion have the following elegant forms,

$$\frac{3FH^2}{\kappa^2} = V + 24\dot{\xi}H^3 + \frac{FR-f}{2\kappa^2} - \frac{3H\dot{F}}{\kappa^2}, \tag{13}$$

$$\frac{-2F\dot{H}}{\kappa^2} = -16\dot{\xi}H\dot{H} + \frac{\ddot{F}-H\dot{F}}{\kappa^2}, \tag{14}$$

$$3H\dot{\phi} + V' + \xi' \mathcal{G} = 0. \tag{15}$$

However, even with the slow-roll approximations holding true, the system of differential equations still remains intricate and cannot be solved. Further approximations are needed in order to derive the inflationary phenomenology, so we neglect string corrections themselves. This is a reasonable assumption since even though the Gauss–Bonnet scalar coupling function is seemingly neglected, it participates indirectly from the gravitational wave condition. Also, in many cases, sting corrections are proven to be subleading. Moreover, under slow-roll assumptions the Ricci scalar is written as, $R \simeq 12H^2$. Recalling that $f(R) \simeq R + \delta R \ln(\alpha R)$, one obtains elegant simplifications and functional expressions for the equations of motion. The first and the second derivatives of the function F are, respectively, $\dot{F} = 2\delta(\frac{\dot{H}}{H})$, $\ddot{F} = 2\delta(\frac{\ddot{H}}{H} - \frac{\dot{H}^2}{H^2})$. The last two terms in the first equation of motion are quite smaller in order of magnitude than the scalar potential of the field hence, as we will prove in the fourth section numerically, these terms can be neglected. The same approximation can be applied in the second equation of motion for the last term of the right hand side. Thus, the final simplified equations of motion are,

$$H^2 \simeq \frac{\kappa^2 V}{3(1 - \delta)}, \tag{16}$$

$$\dot{H} \simeq -\frac{1}{2}(\kappa\dot{\phi})^2, \tag{17}$$

$$3H\dot{\phi} + V' \simeq 0, \tag{18}$$

where the parameter δ can be discarded for $|\delta| \ll 1$. In the next section, we shall prove that the discarded terms are quite smaller in order of magnitude. The dynamics of inflation can

be described by six parameters named the slow-roll indices, defined as follows [3,36],

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE}, \quad \epsilon_5 = \frac{\dot{F} + \kappa^2 Q_a}{2H\kappa^2 Q_t}, \quad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t}, \tag{19}$$

where the auxiliary functions are defined as,

$$Q_a = -8\dot{\xi}H^2, \quad Q_b = -16\dot{\xi}H, \quad Q_e = -32\dot{\xi}\dot{H}, \quad E = \frac{F}{\kappa^2\dot{\phi}^2} \left(\dot{\phi}^2 + 3\frac{(\dot{F} + \kappa^2 Q_a)^2}{2\kappa^4 Q_t} \right), \tag{20}$$

in detail, the auxiliary functions and the slow-roll indices can be written as follows:

$$Q_a = \frac{8\kappa^2 V \left(\frac{\kappa^2 V}{3-3\delta}\right)^{\frac{1}{2}} \xi'^2}{3(\delta-1)\xi''}, \tag{21}$$

$$Q_b = -\frac{16\kappa^2 V (\xi')^2}{3(1-\delta)\xi''}, \tag{22}$$

$$Q_e = -\frac{16\kappa^4 V \left(\frac{\kappa^2 V}{3-3\delta}\right)^{\frac{1}{2}} (\xi')^4}{3(\delta-1)(\xi'')^3} \tag{23}$$

$$Q_t = \frac{\delta + 1 + \delta \ln\left(-\frac{4a\kappa^2 V}{\delta-1}\right)}{\kappa^2} + \frac{8\kappa^2 V (\xi')^2}{3(\delta-1)\xi''} \tag{24}$$

$$\epsilon_1 \simeq \frac{\kappa^2}{2} \left(\frac{\xi'}{\xi''}\right)^2, \tag{25}$$

$$\epsilon_2 \simeq 1 + \frac{\xi'(V'\xi'' - 2V\xi''')}{2V(\xi'')^2}, \tag{26}$$

$$\epsilon_3 = -\frac{\kappa^2 \delta (\xi')^2}{2(1 + \delta + \delta \ln\left(-\frac{4a\kappa^2 V}{\delta-1}\right))(\xi'')^2}, \tag{27}$$

$$\epsilon_5 = \frac{(\kappa\xi')^2(-3(\delta-1)\delta + 8V\xi''\kappa^2)}{2\xi''(8\kappa^4 V (\xi')^2 + 3(\delta-1)(1 + \delta + \delta \ln\left(-\frac{4a\kappa^2 V}{\delta-1}\right))\xi'')}, \tag{28}$$

$$\epsilon_6 = \frac{\xi'(\kappa^2 V \xi' \xi''(8k^2 \xi' V' - 3(\delta-1)\delta) - 8\kappa^4 V^2 \xi'(\xi^{(3)} \xi' - 2(\xi'')^2) + 3(\delta-1)\delta(\xi'')^2 V')}{2V(\xi'')^2 \left(3(\delta-1)\xi'' \left(\delta \ln\left(-\frac{4a\kappa^2 V}{\delta-1}\right) + \delta + 1\right) + 8\kappa^4 V (\xi')^2\right)}, \tag{29}$$

where the index ϵ_4 is omitted due to the perplexed expression. The scalar potential with an unspecified scalar coupling function $\xi(\phi)$ can be written as,

$$V(\phi) = V_0 e^{\int \frac{\kappa^2 \xi'(\phi)}{(\delta-1)\xi''(\phi)} d\phi}. \tag{30}$$

with V_0 being the amplitude of the scalar potential with mass dimensions $[m]^4$. In order to examine the validity of a model, the results which the model produces must be confronted to the recent Planck observational data [49]. In the following model, we shall derive the values for the quantities, namely the spectral index of primordial curvature perturbations n_S , the tensor-to-scalar-ratio r and finally, the tensor spectral index n_S [3,36]. These quantities are connected with the slow-roll indices introduced previously, as shown below,

$$n_s = 1 - 2\frac{2\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4}{1 - \epsilon_1}, \quad n_T = -2\frac{\epsilon_1 + \epsilon_6}{1 - \epsilon_1},$$

$$r = 16 \left| \left(\frac{\kappa^2 Q_e}{4HF^2} - \epsilon_1 - \epsilon_3 \right) \frac{Fc_A^3}{\kappa^2 Q_t} \right|, \quad (31)$$

where c_A the sound wave velocity defined as,

$$c_A^2 = 1 + \frac{\kappa^4 (\dot{F} + \kappa^2 Q_a) Q_e}{2\kappa^4 Q_t \dot{\phi}^2 + 3(\dot{F} + Q_a)^2}. \quad (32)$$

Based on the latest Planck observational data [49] the spectral index of primordial curvature perturbations is $n_S = 0.9649 \pm 0.0042$ and the tensor-to-scalar-ratio r must be $r < 0.064$. Our goal now is to evaluate the observational indices during the first horizon crossing. However, instead of using wavenumbers, we shall use the values of the scalar potential during the initial stage of inflation. Taking it as an input, we can obtain the actual values of the observational quantities. We can do so by firstly evaluating the final value of the scalar field. This value can be derived by equating slow-roll index ϵ_1 in Eq. (25) to unity. Consequently, the initial value can be evaluated from the e -foldings number, defined as $N = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$, where the difference $t_f - t_i$ signifies the duration of the inflationary era. Recalling the definition of $\dot{\phi}$ in Eq. (11), one finds that the proper relation from which the initial value of the scalar field can be derived is,

$$N = \int_{\phi_i}^{\phi_f} \frac{\xi''}{\xi'} d\phi. \quad (33)$$

From this equation, as well as Eq. (25), it is obvious that choosing an appropriate coupling function is the key in order to simplify the results.

3 Constant-roll evolution of the scalar field in logarithmic-corrected $f(R)$ gravity and primordial non-Gaussianities

In the following section, we shall analyse the theoretical framework of the evolution of the scalar field under constant-roll condition $\ddot{\phi} = \beta H \dot{\phi}$, where β is defined as the constant-roll parameter to be specified later. After the specific assumption and neglecting string corrections, the equations of motion are modified as follows:

$$\dot{\phi} = H(1 - \beta) \frac{\xi'}{\xi''}, \quad (34)$$

as a result, the equations of motion are,

$$H^2 \simeq \frac{\kappa^2 V}{3(1 - \delta)}, \quad (35)$$

$$\dot{H} \simeq -\frac{1}{2} \kappa^2 H^2 (1 - \beta)^2 \left(\frac{\xi'}{\xi''} \right)^2, \quad (36)$$

$$(3 + \beta) H \dot{\phi} + V' \simeq 0. \quad (37)$$

In the context of the constant-roll evolution of the scalar field, the slow-roll indices for an arbitrary coupling scalar function $\xi(\phi)$ are given by the following expressions:

$$\epsilon_1 = \frac{(\beta - 1)^2 (\kappa \xi')^2}{2(\xi'')^2}, \quad (38)$$

$$\epsilon_2 = \beta. \quad (39)$$

$$\epsilon_3 = -\frac{(\beta - 1)^2 \delta (\kappa \xi')^2}{2(\xi'')^2 \left(\delta \ln \left(-\frac{4a\kappa^2 V}{\delta - 1} \right) + \delta + 1 \right)}, \tag{40}$$

$$\epsilon_5 = -\frac{(\beta - 1)\kappa^2 (\xi')^2 \left(3(\beta - 1)(\delta - 1)\delta + 8\kappa^2 V(\xi'') \right)}{2\xi'' \left(3(\delta - 1)\xi'' \left(\delta \ln \left(-\frac{4a\kappa^2 V}{\delta - 1} \right) + \delta + 1 \right) - 8(\beta - 1)\kappa^4 V(\xi')^2 \right)}, \tag{41}$$

$$\begin{aligned} \epsilon_6 &= \frac{(\beta - 1)\xi' \left((\beta - 1)\kappa^2 V \xi' \xi'' \left(3(\delta - 1)\delta - 8\kappa^2 \xi' V' \right) + 8(\beta - 1)\kappa^4 V^2 \xi' \left(\xi^{(3)} \xi' - 2(\xi'')^2 \right) + 3(\delta - 1)\delta (\xi'')^2 V' \right)}{2V(\xi'')^2 \left(8(\beta - 1)\kappa^4 V(\xi')^2 - 3(\delta - 1)\xi'' \left(\delta \ln \left(-\frac{4a\kappa^2 V}{\delta - 1} \right) + \delta + 1 \right) \right)}, \end{aligned} \tag{42}$$

where again as before, the index ϵ_4 is omitted due to the lengthy expression. The auxiliary functions is obvious that can be found easily. The e -folding number can be written as,

$$N = \frac{1}{1 - \beta} \int_{\phi_i}^{\phi_f} \frac{\xi''}{\xi'} d\phi. \tag{43}$$

When the scalar field evolves with a constant-rate of roll enhances the non-Gaussianities features. Until now, the perturbations in the cosmic microwave background (CMB) are described perfectly as Gaussian distributions, since no practical evidence is found pointing out a non-Gaussian pattern in the CMB. It is possible though, not evident for the moment, that in the following years the observations may reveal a non-Gaussian pattern in the CMB primordial power spectrum. In this section, we shall discuss how to evaluate the non-Gaussianities quantitatively in the context of the GW170817-compatible Einstein–Gauss–Bonnet gravity, using the formalism and notation of [50]. Even though a $f(R)$ logarithmic gravity is assumed, we consider the same equations as in the [50]. We first define the following quantities,

$$\delta_\xi = \kappa^2 H \dot{\xi}, \quad \delta_X = \frac{\kappa^2 \dot{\phi}^2}{H^2}, \quad \epsilon_s = \epsilon_1 - 4\delta_\xi, \quad n_s = \frac{\dot{\epsilon}_s}{H\epsilon_s}, \quad s = \frac{\dot{c}_A}{Hc_A}. \tag{44}$$

Here, we shall implement a different formula for the sound wave speed, however equivalent to the previous, which is based on these newly defined quantities for convenience and reads,

$$c_A^2 \simeq 1 - \frac{64\delta_\xi^2 (6\delta_\xi + \delta_X)}{\delta_X}. \tag{45}$$

Recalling Eqs. (34), (36) and (37), one finds that the aforementioned auxiliary terms have the following forms for an unspecified scalar coupling function $\xi(\phi)$,

$$\delta_\xi \simeq \frac{(\beta - 1)\kappa^4 V(\xi')^2}{3(\delta - 1)\xi''} \tag{46}$$

$$\delta_X \simeq \frac{(\beta - 1)^2 (\kappa \xi')^2}{(\xi'')^2}, \tag{47}$$

$$\epsilon_s \simeq \frac{(\beta - 1)(\xi')^2 \left(3(\beta - 1)(\delta - 1)\kappa^2 - 8\kappa^4 V\xi'' \right)}{6(\delta - 1)(\xi'')^2}, \tag{48}$$

$$n_s \simeq \frac{2(\beta - 1) \left(\xi^{(3)} \xi' \left(4\kappa^2 V \xi'' - 3(\beta - 1)(\delta - 1) \right) - (\xi'')^2 \left(-3(\beta - 1)(\delta - 1) + 4\kappa^2 \xi' V' + 8\kappa^2 V \xi'' \right) \right)}{(\xi'')^2 \left(8\kappa^2 V \xi'' - 3(\beta - 1)(\delta - 1) \right)}, \tag{49}$$

$$s = (1 - \beta) \frac{\xi' c'_A}{\xi'' c_A}. \tag{50}$$

In certain examples, we shall demonstrate that by choosing appropriately the coupling function, the quantity ϵ_s is simplified greatly, and it shall also coincide with ϵ_1 , along with δ_X .

No matter the form of the sound wave velocity, the derivative c_A is very complex; hence, for obvious reasons its analytic expression is omitted. These forms are very useful due to the fact that the power spectra \mathcal{P}_S of the primordial curvature perturbations and the equilateral momentum approximation term $f_{\text{NL}}^{\text{eq}}$ can be derived from such terms. These quantities are defined as,

$$\mathcal{P}_S = \frac{\kappa^4 V}{24\pi^2 \epsilon_s c_A}, \tag{51}$$

$$f_{\text{NL}}^{\text{eq}} \simeq \frac{55}{36} \epsilon_s + \frac{5}{12} n + \frac{10}{3} \delta_\xi. \tag{52}$$

In the following, we shall appropriately specify the value of the term $f_{\text{NL}}^{\text{eq}}$ during the first horizon crossing, to see what the constant-roll condition brings along. The evaluation shall be performed by using the values of the free parameters in such a way so that the viability of the observational indices of inflation is achieved according to the 2018 Planck data [49].

4 Testing the theoretical framework with the latest observational data

In this section, we shall present explicitly examples of GW170817 compatible Einstein–Gauss–Bonnet models, with the $f(R)$ extension, that can yield a phenomenologically viable inflationary era. First of all, the Gauss–Bonnet coupling scalar function $\xi(\phi)$ must be defined properly, aiming for a simple ratio ξ'/ξ'' so as to facilitate our study.

4.1 Model compatible with the latest Planck Data under the slow-roll assumption

Consider the coupling scalar function $\xi(\phi)$ being equal to,

$$\xi(\phi) = \lambda_1 \int^{\kappa\phi} e^{\gamma_1(x)^n} dx, \tag{53}$$

where λ_1 , γ_1 and n are dimensionless constants to be specified later while x is an auxiliary integration variable. This particular coupling scalar function is chosen due to the simplicity of the ratio ξ'/ξ'' , specifically

$$\frac{\xi'}{\xi''} = \frac{(\kappa\phi)^{1-n}}{\kappa n \gamma_1}, \tag{54}$$

even though the coupling itself seems superfluous. Nevertheless, it has been proven that it is a viable candidate for a Gauss–Bonnet scalar coupling function for the minimally coupled case, so it is interesting to examine the impact of logarithmic corrections. One can specify the scalar potential from Eq. (18), which has the following form,

$$V(\phi) = V_1 e^{\frac{(\kappa\phi)^{2-n}}{(2-n)n\gamma_1(\delta-1)}}, \tag{55}$$

where V_1 is the integration constant. Let us now proceed with the evaluation of the slow-roll indices,

$$\epsilon_1 \simeq \frac{(\kappa\phi)^{2-2n}}{2n^2\gamma_1^2}, \tag{56}$$

$$\epsilon_2 \simeq \frac{(\kappa\phi)^{-n} (\phi V' - 2(n-1)V)}{2\gamma_1 n V}, \tag{57}$$

$$\epsilon_3 \simeq -\frac{\delta(\kappa\phi)^{2-2n}}{2\gamma_1^2 n^2 \left(\delta \ln \left(-\frac{4q\kappa^2 V}{\delta-1} \right) + \delta + 1 \right)}, \tag{58}$$

$$\epsilon_5 \simeq \frac{(\kappa\phi)^{1-n} \left(8\gamma_1 \lambda_1 n \kappa^4 V (\kappa\phi)^n e^{\gamma_1 (\kappa\phi)^n} - 3(\delta-1)\delta\kappa\phi \right)}{2\gamma_1 n \left(3\gamma_1 (\delta-1)n(\kappa\phi)^n \left(\delta \ln \left(-\frac{4q\kappa^2 V}{\delta-1} \right) + \delta + 1 \right) + 8\lambda_1 \kappa \phi \kappa^4 V e^{\gamma_1 (\kappa\phi)^n} \right)}, \tag{59}$$

$$\epsilon_6 \simeq \frac{(\kappa\phi)^{1-n} \left(\kappa \phi \kappa^4 V \left(8\lambda_1 \kappa^3 V' e^{\gamma_1 (\kappa\phi)^n} - 3(\delta-1)\delta \right) + 8\gamma_1 (\kappa^4 V)^2 e^{\gamma_1 (\kappa\phi)^n} (n(\gamma_1 (\kappa\phi)^n - 1) + 1) + 3\gamma_1 (\delta-1)\delta n(\kappa\phi)^n \kappa^3 V' \right)}{2\gamma_1 n \kappa^4 V \left(3\gamma_1 (\delta-1)n(\kappa\phi)^n \left(\delta \ln \left(-\frac{4q\kappa^2 V}{\delta-1} \right) + \delta + 1 \right) + 8\lambda_1 \kappa \phi \kappa^4 V e^{\gamma_1 (\kappa\phi)^n} \right)}. \tag{60}$$

It is obvious that the first three slow-roll indices have quite simple expressions in contrast to the next three due to the derivatives of the scalar potential and the derivatives of the coupling scalar function. Again, ϵ_4 is omitted due to the length expression. Now, we can determine the final value of the field when the inflationary era ends, by setting the first slow-roll index Eq. (56) into unity,

$$\phi_f = \frac{(2n^2 \gamma_1^2)^{\frac{1}{2-2n}}}{\kappa}. \tag{61}$$

The initial value of the scalar field can be calculated from the e-folds number, see Eq. (33)

$$\phi_i = \frac{((\kappa\phi_f)^2 - \frac{N}{\gamma_1})^{\frac{1}{n}}}{\kappa}, \tag{62}$$

where we assumed that the number of e -folds is $N \simeq 60$. Considering the following values for the free parameters in natural units, namely, $\kappa^2 = 1$, $(N, n, \gamma_1, V_1, \lambda_1, \delta, \alpha) = (60, -2, -5, 1, 1, 0.001, 1)$ then, the spectral index of primordial curvature perturbations n_S , the tensor-to-scalar-ratio r and finally, the tensor spectral index n_T are, respectively, $n_S = 0.966381$, $r = 0.000014$ and $n_T = -1.7 \times 10^{-6}$, which are acceptable according to the latest Planck Data. Moreover, we found that the initial value of the field is $\phi_i = 0.236704$ and the final numerical value of the field is $\phi_f = 2.41827$ which means that with the passage of time, based on the continuity, the field increases until the inflationary era ends. In addition, the numerical values of the slow-roll indices are $\epsilon_1 = 8.79 \times 10^{-7}$, $\epsilon_2 = 0.016$, $\epsilon_3 = \epsilon_4 = \epsilon_5 = -8.7 \times 10^{-10}$, and $\epsilon_6 = -1.75 \times 10^{-9}$, where it becomes apparent that all of them are subleading compared to the second slow-roll index (Fig. 1).

Lastly, we examine the validity of our approximations. Based on the slow-roll approximations, we note that $\dot{H} \sim \mathcal{O}(10^{-7})$ compared to $H^2 \sim \mathcal{O}(10^{-1})$, similarly $\frac{1}{2}\dot{\phi}^2 \sim \mathcal{O}(10^{-7})$ in contrast to $V \sim 1$. Indeed, the approximations in (12) are valid. Moreover, the string terms in the equations of motion (13) and (14) are $24\dot{\xi}H^3 \sim \mathcal{O}(10^{-42})$ and $16\dot{\xi}H\dot{H} \sim \mathcal{O}(10^{-48})$ which justifies the reason why the terms were neglected. Furthermore, from the first modified Friedmann equation numerically speaking, $\frac{FR-f}{2\kappa^2} \sim \mathcal{O}(10^{-3})$ and $\frac{3H\dot{F}}{\kappa^2} \sim \mathcal{O}(10^{-9})$ are quite smaller than the scalar potential justifying our assumption. The term $\frac{\ddot{F}-H\dot{F}}{\kappa^2} \sim \mathcal{O}(10^{-10})$ in the second Friedmann equation is also minor in contrast to the term $\frac{2\dot{H}\dot{F}}{\kappa^2} \sim \mathcal{O}(10^{-7})$.

4.2 Model with power-law coupling under the constant-roll approximation

In this subsection, it is considered a much simpler Gauss–Bonnet coupling scalar function $\xi(\phi)$ that follows a power-law form,

$$\xi(\phi) = \lambda_2 (\kappa\phi)^m. \tag{63}$$

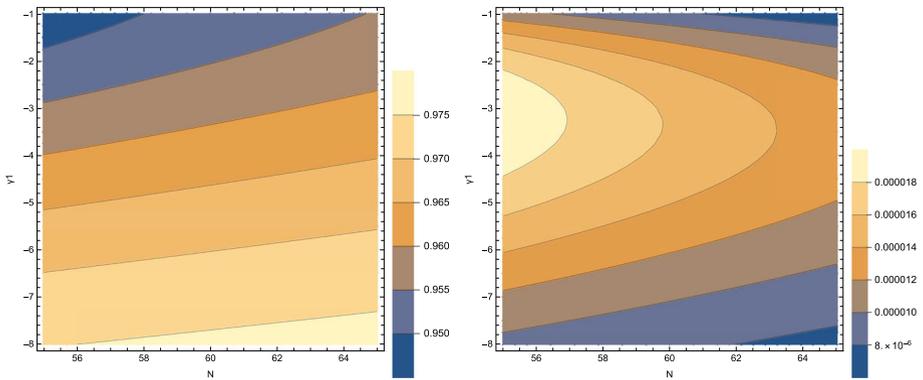


Fig. 1 Contour plots of the spectral index of primordial curvature perturbations (left) and the tensor-to-scalar ratio (right) depending on parameters N and γ_1 ranging from $[55, 65]$ and $[-8, -1]$, respectively. It can be inferred that both parameters influence their values, but the spectral index changes with a lesser rate

where λ_2 and m are dimensionless constants to be specified later. This is a very appealing function since the ratio $\frac{\xi'}{\xi''}$ which appears in our calculations is greatly simplified, since,

$$\frac{\xi'}{\xi''} = \frac{\phi}{m - 1}. \tag{64}$$

This model was also studied in Ref. [46]. Since the scalar coupling function is specified, one can find the scalar potential from Eq. (7),

$$V(\phi) = V_2 e^{-\frac{(\beta^2+2\beta-3)(\kappa\phi)^2}{6(m-1)(\delta-1)}}, \tag{65}$$

where V_2 is an integration constant. For this model, the constant-roll indices are given by the following equations,

$$\epsilon_1 = \frac{(\beta - 1)^2(\kappa\phi)^2}{2(m - 1)^2}, \tag{66}$$

$$\epsilon_2 = \beta, \tag{67}$$

$$\epsilon_3 = -\frac{(\beta - 1)^2\delta(\kappa\phi)^2}{2(m - 1)^2 \left(\delta \ln \left(-\frac{4\alpha\kappa^2 V(\phi)}{\delta - 1} \right) + \delta + 1 \right)}, \tag{68}$$

$$\epsilon_5 = \frac{(\beta - 1) \left(3(\beta - 1)(\delta - 1)\delta(\kappa\phi)^2 + 8\lambda_2(m - 1)m\kappa^4 V(\kappa\phi)^m \right)}{2(m - 1) \left(8(\beta - 1)\lambda_2 m\kappa^4 V(\kappa\phi)^m - 3(\delta - 1)(m - 1) \left(\delta \ln \left(-\frac{4\alpha\kappa^2 V}{\delta - 1} \right) + \delta + 1 \right) \right)}, \tag{69}$$

$$\epsilon_6 = -\frac{(\beta - 1) \left(8(\beta - 1)\lambda_2 m\kappa^4 V(\kappa\phi)^m \left(m\kappa^4 V + \kappa\phi\kappa^3 V' \right) - 3(\delta - 1)\delta\phi \left((\beta - 1)\kappa\phi\kappa^4 V + (m - 1)\kappa^3 V' \right) \right)}{2(m - 1)\kappa^4 V \left(8(\beta - 1)\lambda_2 m\kappa^4 V(\kappa\phi)^m - 3(\delta - 1)(m - 1) \left(\delta \ln \left(-\frac{4\alpha\kappa^2 V}{\delta - 1} \right) + \delta + 1 \right) \right)}, \tag{70}$$

where ϵ_4 is not written analytically due to the perplexed form. Despite the fact that the indices ϵ_4 to ϵ_6 are quite perplexed, the indices ϵ_1 to ϵ_3 have very simple forms. By setting the index

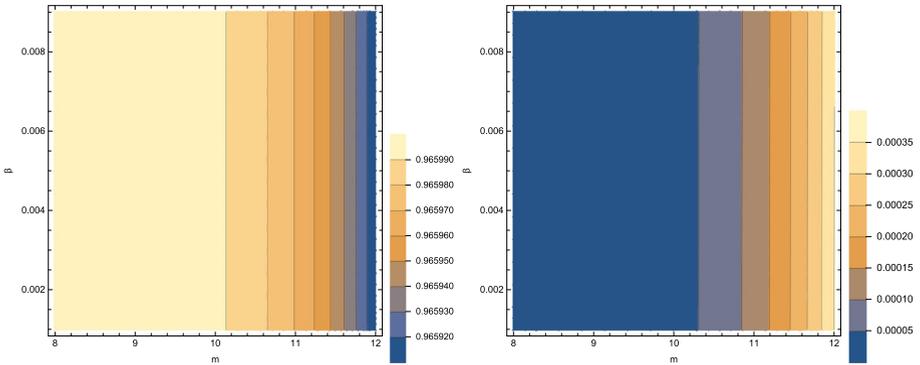


Fig. 2 Contour plots of the spectral index of primordial curvature perturbations (left) and the tensor-to-scalar ratio (right) depending on parameters β and m ranging from $[0.001, 0.009]$ and $[8, 12]$, respectively. Concerning the spectral index, it is clear that the dominant parameter which defines its value is the constant-roll parameter

ϵ_1 to unity, the final value of the field when inflationary era ends is equal to,

$$\phi_f = \pm \frac{\sqrt{2}}{\kappa \sqrt{\frac{(\beta-1)^2}{(m-1)^2}}}, \tag{71}$$

while, the initial value of the field is,

$$\phi_i = \pm \phi_f e^{-\frac{N(1-\beta)}{m-1}}. \tag{72}$$

Our following analysis is based in the case of the positive value of the scalar field. Assigning the following values to the free parameters, always in reduced Planck units, $(\kappa, \delta, \lambda_2, m, N, \beta, V_2, \alpha) = (1, 0.003, 1, 8, 60, 0.017, 1, 1)$ then, the observational indices take the following values $n_S = 0.966$, $r = 7.6 \times 10^{-7}$ and $n_T = -9.5 \times 10^{-8}$, which are acceptable according to the latest Planck Data. Furthermore, $c_A = 1$; hence, the model is free of ghosts. The numerical values of the slow-roll indices are $\epsilon_1 \simeq 4.8 \times 10^{-8}$, $\epsilon_2 = 0.017$, $\epsilon_3 = -2.3 \times 10^{-10}$ and the rest indices are all equal to ϵ_3 , a feature which is expected at least for ϵ_5 and ϵ_6 when F is more dominant than string corrections. Moreover, we mention that the initial and final values of the scalar field are $\phi_i \simeq 0.002$ and $\phi_f = 10.0707$ which indicates an increase in the scalar field. In Fig. 2, we plot the spectral index of primordial curvature perturbations n_S (left) and the tensor-to-scalar ratio r (right) depending on parameters β and m ranging $[0.001, 0.009]$ and $[8, 12]$, respectively.

Moreover, we make predictions for the amount of non-Gaussianities in the primordial power spectrum of the curvature perturbations. The auxiliary terms of the specific model are defined as,

$$\delta_\xi \simeq \frac{(\beta - 1)\lambda_2 m \kappa^4 V(\kappa\phi)^m}{3(\delta - 1)(m - 1)} \tag{73}$$

$$\epsilon_s \simeq \frac{(\beta - 1) (3(\beta - 1)(\delta - 1)(\kappa\phi)^2 - 8\lambda_2(m - 1)m\kappa^4 V(\kappa\phi)^m)}{6(\delta - 1)(m - 1)^2}, \tag{74}$$

$$n_s \simeq \frac{8(\beta - 1)\lambda_2(m - 1)m(\kappa\phi)^m (m\kappa^4 V + \kappa\phi\kappa^3 V') - 6(\beta - 1)^2(\delta - 1)(\kappa\phi)^2}{(m - 1) (3(\beta - 1)(\delta - 1)(\kappa\phi)^2 - 8\lambda_2(m - 1)m\kappa^4 V(\kappa\phi)^m)}. \tag{75}$$

From Eq. (52), the expected value of $f_{\text{NL}}^{\text{eq}}$ for the exact same set of parameters we used to obtain the viability of the model with the Planck data, is $f_{\text{NL}}^{\text{eq}} = 0.117024$ which is also an accepted value and may explain why non-Gaussianities have yet to be observed. Finally, the parameters used to derive such values are $\delta\xi \simeq 2.1 \times 10^{-22}$, $\epsilon_s = 4.80305 \times 10^{-8}$ and $\eta = 0.280857$ which means that one of them is in fact dominant. These results imply that $\epsilon_s = \epsilon_1$.

At this point, it is important to mention that the observed quantities n_S and r experience different changes after the alteration of the free parameters. Specifically, the primordial spectral index is affected only by the constant-roll parameter β , while the tensor-to-scalar ratio is affected by the exponent m of the coupling scalar function along with the constant-roll parameter with the first being more dominant factor. This can easily be observed in Fig. 2 where one sees that the spectral index of scalar perturbations is depicted by a simple plot resembling vertical lines. In addition, while the term $f_{\text{NL}}^{\text{eq}}$ is independent of parameter λ_2 , it can be enhanced by decreasing the exponent m , but such a change leads to a subsequent decrease in the tensor-to-scalar ratio. For instance, numerically speaking, choosing $m = 2$ leads to $f_{\text{NL}}^{\text{eq}} = 0.819167$, $n_S = 0.966$ and the effective value of the tensor-to-scalar ratio is 0, since $r \sim \mathcal{O}(10^{-51})$.

Finally, we examine the validity of the approximations which were made during this model at the first horizon crossing. Firstly, the slow-roll approximations in Eq. (12) hold true since $\dot{H} \sim \mathcal{O}(10^{-8})$ compared to $H^2 \sim \mathcal{O}(10^{-1})$ and $\frac{1}{2}\dot{\phi}^2 \sim \mathcal{O}(10^{-8})$ compared to $V \sim \mathcal{O}(10^1)$ are negligible. Also, the terms which were omitted in Eqs. (5) and (5) are of the order (in reduced Planck units) $24\dot{\xi}H^3 \sim \mathcal{O}(10^{-21})$ while $16\dot{\xi}H\dot{H} \sim \mathcal{O}(10^{-29})$, which explains why these terms, compared to the scalar potential and the kinetic term, can be neglected and $V' \sim \mathcal{O}(10^{-4})$, whereas $\xi'G \sim \mathcal{O}(10^{-18})$ which explains the neglect of string term. Lastly, the following two terms in Eqs. (5) $\frac{FR-f}{2\kappa^2} \sim \mathcal{O}(10^{-3})$ and $\frac{3H\dot{F}}{k^2} \sim \mathcal{O}(10^{-10})$ are quite smaller than the scalar potential and the term $\frac{\ddot{F}-H\dot{F}}{k^2} \sim \mathcal{O}(10^{-11})$ in the second Friedmann equation is also minor in contrast to the term $\frac{2\dot{H}F}{\kappa^2} \sim \mathcal{O}(10^{-8})$, justifying our approximations.

5 Conclusions

In this work, an alteration of our previous work [36] is presented, by using a modified $f(R)$ logarithmic gravity instead of Einstein's gravity, consistent with the GW170817 event. We focused our analysis on the inflationary era of the universe, by considering that the scalar field evolves with either under slow-roll assumptions or with a constant rate of roll. In both of these cases, the slow-roll indices and the observational quantities of inflation were evaluated along with the predicted amount of non-Gaussianities, in the case of constant-roll evolution of the scalar field. After the theoretical framework, we confronted two models with the observational data coming from the Planck 2018 collaboration, considering logarithmic corrections. As shown, the resulting inflationary phenomenology can be compatible with the latest Planck data, for a wide range of the free parameters of the theory, after neglecting string corrections, which is shown to be a reasonable approach. In our analysis, we demonstrated that all the assumptions made were satisfied for all the models examined and for the values of the free parameters that yield inflationary viability with respect to the latest Planck data. In the constant-roll case, we also investigated the amount of non-Gaussianities that are predicted from the model, by calculating the nonlinear term $f_{\text{NL}}^{\text{eq}}$ in the equilateral momentum approximation. Interestingly enough, we demonstrated that the amount of non-Gaussianities is quite small. Finally, we performed an analytic approximation in the differential equation

that connects the scalar field potential and the scalar coupling function, and we examined the phenomenology of inflation in this case too. As evinced, the models can also be compatible with the Planck 2018 too, even with a logarithmic modified gravity $f(R)$ with an almost linear Ricci scalar through logarithmic corrections.

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