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Dry unit weight of compacted soils prediction using GMDH-type neural network

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Abstract. Dry unit weight (γ_d) of soils is usually determined by *in situ* tests, such as rubber balloon, sand cone, nuclear density measurements, etc. The elastic wave method using compressional wave has been broadly used to determine various geotechnical parameters. In the present paper, the polynomial neural network (NN) is used to estimate the γ_d of compacted soils indirectly depending on P-wave velocity (V_p), moisture content (ω) and plasticity index (PI) as well as fine-grained particles (FC). Eight natural soil samples (88 data) were applied for developing a polynomial representation of model. To determine the performance of the proposed model, a comparison was carried out between the predicted and experimentally measured values. The results show that the developed GMDH-type NN has a great ability ($R^2 = 0.942$) to predict the γ_d of the compacted soils and is more efficient (53% to 73% improvement) than the previous reported methods. Finally, the derived model sensitivity analysis has been performed to evaluate the effect of each input variable on the proposed model output and shows that the P-wave velocity is the most influential parameter on the predicted γ_d.

List of symbols

E_r	Scaled relative error	R^2	Absolute fraction of variance
FC	Fine-grained particles	SCF	Scaled cumulative frequency
LL	Liquid limit (%)	V_p	P-wave velocity
M	Total numbers of data sets	X	Input variable
MAD	Mean absolute deviation	y	Actual output
$MAPE$	Mean absolute percent error	ω	Moisture content (%)
$MARE$	Mean absolute relative error	ω_{opt}	Optimum moisture content
m_i	Input parameter	γ_d	Dry unit weight
m_j	output parameter	$\gamma_{d\,\mathrm{max}}$	Maximum dry unit weight
PI	Plastic index (%)	$\gamma_{d\mathrm{mi}}$	Actual measured dry unit weight
PL	Plastic limit	$\gamma_{d\mathrm{pi}}$	Predicted dry unit weight
$RMSE$	Root mean square error	$\overline{\gamma}_{dm}$	Mean of the actual dry unit weight

1 Introduction

The mechanical improvement of soil through the compaction using mechanical energy is a cost effective stabilization technique. During the course of a proper compaction process, as the strength of the soil increases, settlement potential, hydraulic conductivity and void ratio decrease [1].

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The compaction parameters of soil are determined using the standard Proctor test [2]. In laboratory, the maximum dry unit weight ($\gamma_{d\,\max\text{-lab}}$) and the corresponding optimum moisture content (ω_{opt}) are determined by plotting a graph of dry unit weight (γ_d) against moisture contents (ω) according to ASTM D698 [3]. Soil is compacted in the field according to the compaction parameters ($\gamma_{d\,\max\text{-lab}}$ and ω_{opt}). In most specifications for earthwork, the contractor is instructed to achieve a compacted field dry unit weight ($\gamma_{d\text{-field}}$) range of 95–98 percent of the $\gamma_{d\,\max\text{-lab}}$. This is characteristic of relative compaction (R), which can be expressed as [4]

$$R(\%) = \frac{\gamma_{d\text{-field}}}{\gamma_{d\,\max\text{-lab}}} \times 100. \tag{1}$$

The measuring $\gamma_{d\text{-field}}$ of earth fill for construction of engineering earthwork for roads, embankments, earth dams, retaining walls, soil liners, etc., is a major challenge in geotechnical engineering. There are a number of test methods to determine the $\gamma_{d\text{-field}}$ of the compacted earth fill, such as nuclear density measurement, sand cone, rubber balloon, seismic velocity, and plate loading tests, but the most preferred methods are the nuclear and sand cone tests. Although the nuclear density method is more practical and faster, more dependable results are obtained by destructive and time-consuming methods, such as the sand cone and balloon methods. During the course of the sand cone test, vibrations in the vicinity of the test area and moisture content of material largely affect the results. The measurements obtained from nuclear densitometers are significantly affected by the grain size distribution and extreme care must be displayed during the application in the site because of radioactivity [5].

The moisture content (ω) has a significant impact on the γ_d of compacted soil. When water is added to the soil through compaction, it performs as a softening agent on the soil particles. The soil particles slip over each other and move into a densely packed situation. The γ_d, after the compaction, first enhances as the ω increases. However, beyond the ω_{opt} (corresponding $\gamma_{d\,\max}$), any increase in the ω tends to reduce the γ_d [5].

Besides ω, other important factor that affect the compaction is the soil type [5]. The soil compaction process has important differences for cohesionless against cohesive soils. The shapes and the positions of the compaction curves change as the texture of the soils varies from coarse to fine [6]. The main difference is that the cohesive soils are naturally highly dependent on water and cohesionless soils are not [7]. Problems in the compaction of clays are strongly related to their state of consistency index (liquid limit (LL), plastic limit (PL) and natural ω) [8]. Several researchers have illustrated methods to estimate the compaction parameters of soils using index properties [9–15]. Jesmani *et al.* [16] investigated the effect of clay content on the compaction parameters.

The compressional wave velocity (V_p) is one of the parameters used to predict the mechanical, dynamic and physical characteristics of soils and rocks [17–25].

Measuring the V_p using ultrasonic testing is a simple and fast approach to determine specifications of compacted soils. Ultrasonic waves propagation in a material depends on the properties and condition of the substance. Wave travel times through the fastest possible paths in soil masses are measured using these tests [26]. ASTM D2845 [27] standard provides guidelines for the preparation of the specimen's surfaces, where the transducers are placed. Several researchers applied V_p by using ultrasonic testing to describe physical properties of soils, *e.g.*, density, plasticity, clay content, Atterberg limits and porosity [28–35].

Due to the disadvantages of general methods to determine γ_d in the field, indirect assessment of the γ_d of compacted soils from other geotechnical tests carried out more simply can be used. Kolay and Baser [36] used the general linear model (GLM) to model the γ_d of the compacted soils using V_p and ω in laboratory conditions and proposed the following equation:

$$\gamma_d = 23 - 0.005V_p - 0.27\omega + 0.0001V_p \cdot \omega. \tag{2}$$

The equation is developed based on statistical analysis with notable modeling drawbacks. Such models are not efficiently able to consider the complex interactions between the soil parameters and γ_d. Therefore, more complicated methods are necessary to consider the complex behavior of γ_d.

Computational intelligence (CI) methods, such as support vector machines (SVM), artificial neural network (ANN), adaptive neuro-fuzzy system (ANFIS), fuzzy inference system (FIS), etc., can be considered as efficient techniques. CI methods have been used in many geotechnical engineering problems for modeling complex correlations between input and output parameters [37–48]. Najjar *et al.* [49], Sinha and Wang [50], Günaydın [51] and Sinivasulu *et al.* [52] proposed models based on ANN to predict the compaction parameters of soils. Kolay and Baser [36] used the multilayer perceptron (MLP) neural network (NN) to model the γ_d of the compacted soils using V_p and ω in laboratory conditions. The basic disadvantage that limits the practicability of SVM, ANN, ANFIS, FIS models is that they are black-box models and have not the ability to generate practicable equations [53].

In recent years, self-organizing kinds of NN are used in a wide range of applications. One of such NN models is the group method of data handling (GMDH). The purpose of this method, which was first developed by Ivakhnenko [54], is to identify the functional structure of a model hidden in the experimental data. The GMDH polynomial NN model has the capability to choose the most important input parameters that affect the model and results in a regression relation relating the input parameters to the output ones [55]. By GMDH polynomial NN complex models can be gradually produced based, on an assessment of their behavior, on a set of multi-input single-output data pairs (X_i, y_i)

Table 1. Soils physical properties [36]. NP: Non-plastic. CH: Fat clay. CL: Lean clay. SM: Silty sand. SC: Clayey sand.

Symbol	Soil type	FC (%)	G_s	LL (%)	PL (%)	PI (%)
$F1$	CH	75.80	2.70	56.90	30.75	26.15
$F2$	CH	71.93	2.44	61.60	29.05	32.55
$F3$	CL	67.61	2.63	42.35	25.36	16.99
$F4$	CL	60.67	2.54	49.50	32.51	16.99
$C1$	SM	19.82	2.59	NP	NP	NP
$C2$	SM	24.70	2.59	30.80	24.30	6.5
$C3$	SC	43.72	2.63	42.35	25.36	16.99
$C4$	SC	40.06	2.51	30.00	16.65	13.35

Table 2. Detailed information of parameters. σ: Standard deviation. CV: Coefficient of variation.

Parameter	Minimum	Mean	Maximum	σ	CV
$\omega\,(\%)$	3.61	16.05	36.85	7.43	0.46
$V_p\,(\frac{\text{m}}{\text{s}})$	105.31	575.00	960.45	235.38	0.41
$FC\,(\%)$	19.82	50.65	75.80	20.18	0.40
$PI\,(\%)$	0	16.19	32.55	9.57	0.59
$\gamma_d\,(\frac{\text{KN}}{\text{m}^3})$	13.22	17.10	21.58	2.04	0.12

$(i = 1, 2, \ldots, M)$. With this polynomial algorithm, a model can be described as a set of neurons in which various pairs connected in every layer through a quadratic polynomial, and hence new neurons in the subsequent layer, are produced. With this algorithm, a model can be described as a set of neurons in which different pairs connected in each layer may be provided. Such representation can be used to map inputs to outputs [56]. The major advantage of GMDH is that the analytical equations can be easily analyzed and interpreted by users. Furthermore, in GMDH, highly short and noisy data can be efficiently handled [55].

Nowadays, genetic algorithms optimizations have been applied in the GMDH NN. In recent years, the genetic algorithms optimized GMDH has been utilized in geotechnical engineering problems and has shown some improvement [57–63].

The purpose of present study is the development of a GMDH polynomial NN to predict the γ_d of compacted soils. As previously mentioned, ω and soil type have a strong influence on γ_d. Also, V_p has been widely used to predict soil parameters. In the present article, it is assumed that the γ_d of the soils is influenced by moisture content (ω), P-wave velocity (V_p) and plasticity index (PI) as well as fine-grained particles (FC). ω and V_p are the characteristics of the soil state, while the PI and FC are the characteristics of the soil nature or type.

2 Database compilations

This study analyzes eight natural soil samples (four types) using an already published data set taken from different locations in the province of Yozgat, Turkey [36]. Soils used in the study were classified according to the unified soil classification system (USCS) by consistency limits (LL, PL) [64] and sieve analysis results [65]. The samples were sieved using a #4 mesh size before the standard Proctor test. Physical properties of soil samples are given in table 1. Standard Proctor tests were performed with eleven different moisture contents (ω) for each soil sample. The tests were numbered according to increasing ω. Eleven distinct points for each soil sample were obtained on the compaction curve. Afterward, ω, V_p and γ_d were measured for all the test points during standard Proctor tests ($8 \times 11 = 88$ tests). An ultrasonic test device was used to measure V_p of the compacted soil and in accordance with the direct method [27]. The V_p was calculated dividing the length of the soil by transit time. In the database, the γ_d of compacted soils were supposed to be affected by plasticity index (PI) and fine-grained particles (FC) in addition to P-wave velocity (V_p) and moisture content (ω) as summarized in table 2.

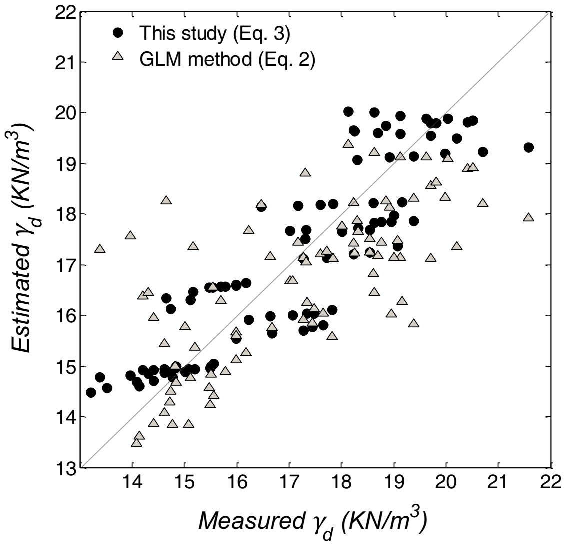

Fig. 1. Comparison between the measured and predicted γ_d using empirical formulas.

Table 3. Suggested empirical equations and statistical results.

Equation	Reference	R^2	$RMSE$	$MARE$
$\gamma_d = 23 - 0.005V_p - 0.27\omega + 0.0001V_p \cdot \omega$	Kolay and Baser [39]	0.44	1.52	7.09
$\gamma_d = 20.89 - 0.002\omega + 0.0009V_p - 0.07FC - 0.05PI$	This study	0.78	0.95	4.70

3 Regression analysis

Kolay and Baser [39] used V_p and ω to predict γ_d by the GLM method (eq. (2)). V_p and ω describe the soil condition or compaction in the field. The γ_d can be affected by other soil parameters such as PI [13–15] and FC [16], for these two parameters are representative of the type or nature of soils. Thus, In this article multiple regression analysis was performed to predict γ_d as a function of ω, V_p, FC and PI. The derived equation is as follows:

$$\gamma_d = 20.89 - 0.002\ \omega + 0.0009V_p - 0.07\ FC - 0.05\ PI. \tag{3}$$

The comparison between the predicted (using eqs. (2) and (3)) and measured (from Proctor test) γ_d is shown in fig. 1. In table 3 the predictabilities of the proposed equation and GLM for all data set are given.

To determine the performance of the equations, absolute fraction of variance (R^2), root mean squared error ($RMSE$) and mean absolute relative error ($MARE$) were used as follows:

$$R^2 = 1 - \left[\frac{\sum_1^M (\gamma_{d\mathrm{mi}} - \gamma_{d\mathrm{pi}})^2}{\sum_1^M (\gamma_{d\mathrm{mi}} - \overline{\gamma}_{dm})^2}\right] \tag{4}$$

$$RMSE = \sqrt{\frac{1}{M}\sum_1^M (\gamma_{d\mathrm{mi}} - \gamma_{d\mathrm{pi}})^2} \tag{5}$$

$$MARE = \frac{1}{M}\cdot\sum_1^M \left|\frac{\gamma_{d\mathrm{mi}} - \gamma_{d\mathrm{pi}}}{\gamma_{d\mathrm{mi}}}\right| \times 100, \tag{6}$$

where M is the total number of data, the $\gamma_{d\mathrm{mi}}$ and $\gamma_{d\mathrm{pi}}$ are the measured and predicted γ_d at the i-th test number.

As seen from fig. 1 and table 3, although the suggested equation is simpler, it has better performance than the GLM method. This is for appearing PI and FC those characterize the soil type in eq. (3).

Table 4. Detailed information of parameters used in the model.

Parameter		Train (72 data sets)			Test (16 data sets)		
		minimum	mean	maximum	minimum	mean	maximum
Input	$\omega\,(\%)$	3.61	16.35	36.85	4.31	17.22	32.90
	$V_p\left(\frac{\text{m}}{\text{s}}\right)$	105.31	583.79	960.45	126.90	535.47	942.62
	$FC\,(\%)$	19.82	50.54	75.80	19.82	50.54	75.80
	$PI\,(\%)$	0	2.64	32.55	0	2.64	32.55
Output	$\gamma_d\left(\frac{\text{KN}}{\text{m}^3}\right)$	13.22	17.16	21.58	13.39	16.85	20.70

4 Evaluation of dry unit weight using GMDH polynomial NN

The advantage of the ANNs is that they are very useful in learning complex relationships between multi-dimensional data. Despite the good performance of ANNs, they are not capable of generating prediction equations. This is a fundamental disadvantage that limits their practicability. In order to overcome this disadvantage, group method of data handling (GMDH) type neural network (NN) has been used in this article. The GMDH-type NN is aimed at identifying the functional structure of a model hidden in the empirical data. For inaccurate, noisy, or small data sets, the GMDH is the best optimal simplified model, with a higher accuracy and a simpler structure than typical full physical models [66].

The main goal of the present study is to derive a polynomial equation for estimating γ_d. For this purpose, the data are divided randomly into two separate data sets including the training and testing data set. In this research, among 88 data sets, 16 randomly data sets (2 for each sample type) have been used to test and 72 data sets have been used to train. Testing and training data are statistically similar (table 4).

Several parameters including number of generations, population size, number of hidden layers, mutation probability and crossover probability involved in GMDH predictive. The selection of these parameters will influence the model generalization ability of polynomial NN. In this study, a population of 100 individuals with a mutation probability of 0.01 and crossover probability of 0.90 is used in 300 generations for the population size of which no further improvement is achieved. Although the triple hidden layer behavior has shown a little better, however, in order to avoid over-fitting and obtaining simpler equations, it was decided to choose double hidden layer. The flowchart of the proposed polynomial method is shown in fig. 2.

For evaluating γ_d by GMDH models, various combinations of input parameters (for all data sets) were trained to determine a proper combination of them (table 5). The fundamental methodology used involves removing parameters of the input layer and then do the same analysis again [67].

As seen in table 5, the model with all parameters (ω, V_p, FC and PI) has the best performance in comparison with other combinations of GMDH models. One concludes that all considered input parameters do influence the value of γ_d. In the following, all four parameters will be used as input parameters to the GMDH modeling.

The evolved GMDH structure is shown in fig. 3. The polynomial equation corresponding to such model to predict γ_d is as follows:

$$\gamma_d = 7.5 - 1.75Y_2 + 1.803Y_3 + 0.537Y_2^2 + 0.367Y_3^2 - 0.875Y_2 \cdot Y_3 \tag{7a}$$

$$Y_3 = 0.075 + 0.0094V_p + 0.63Y_1 - 0.0000012V_p^2 + 0.0176Y_1^2 - 0.0004V_p \cdot Y_1 \tag{7b}$$

$$Y_2 = -64.657 + 1.951\omega + 6.871Y_1 - 0.0148\omega^2 - 0.134Y_1^2 - 0.086\omega \cdot Y_1 \tag{7c}$$

$$Y_1 = 27.29 - 0.634FC + 0.972PI + 0.0114FC^2 + 0.0342PI^2 - 0.04FC \cdot PI, \tag{7d}$$

where FC is the fine-grained particles, PI the plasticity index, ω the moisture content, V_p the P-wave velocity and γ_d the dry unit weight.

Figure 4 shows the scattergram for the estimated γ_d from the model and the measured γ_d from the Proctor test. According to this figure, very good correlation for both the training and testing data is shown by the GMDH model. The developed model predictability is statistically given in table 6. It is obvious that the proposed model can effectively predict the testing output data that has not been trained.

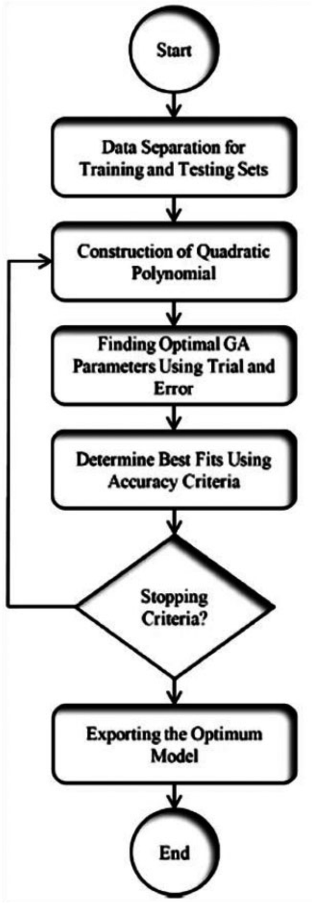

Fig. 2. The flowchart of the proposed method.

Table 5. GMDH models of various combinations of input variables.

Combination of input parameters	R^2	$RMSE$	$MARE$
$V_p + FC + PI$	0.90	0.63	2.84
$V_p + \omega + PI$	0.83	0.84	3.94
$V_p + \omega + FC$	0.86	0.75	3.50
$\omega + FC + PI$	0.89	0.66	3.25
$V_p + \omega + FC + PI$	0.942	0.49	1.88

In table 7, the GMDH polynomial model predictability (for all data set) is statistically compared with the GLM equations and MLP model. As Kolay and Baser [36] did not provide equation for the MLP method, its R^2, $RMSE$ and $MARE$ cannot be determined for all data set. Therefore, in table 7, R^2, $RMSE$ and $MARE$ for the MLP method are given in table 4 in Kolay and Baser [36]. It can be seen that the MLP method has a better performance than the proposed equation in this study (eq. (3)) and the GLM method (eq. (1)). However, the best fit is obtained by the GMDH method (53% to 73% improvement).

Graphs of the scaled relative error (E_r) *versus* scaled cumulative frequency (SCF) can be used to compare the accuracy of the equations. E_r is determined by the following equation:

$$E_r(\%) = \frac{(\gamma_{d\mathrm{pi}} - \gamma_{d\mathrm{mi}})}{\gamma_{d\mathrm{mi}}} \times 100. \tag{8}$$

As seen in fig. 5, wide ranges of prediction are given by GLM relationships in comparison to the proposed polynomial model. The GMDH model is more accurate with respect to the other two methods.

Before compaction of soils in the site, PI, FC and ω are measured in laboratory. Thus, the *in situ* γ_d can be accurately determined by measuring V_p of the compacted site and using eq. (7).

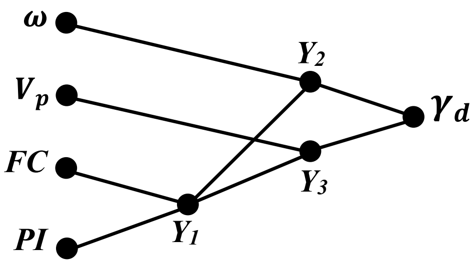

Fig. 3. Generalized GMDH structure to predict γ_d.

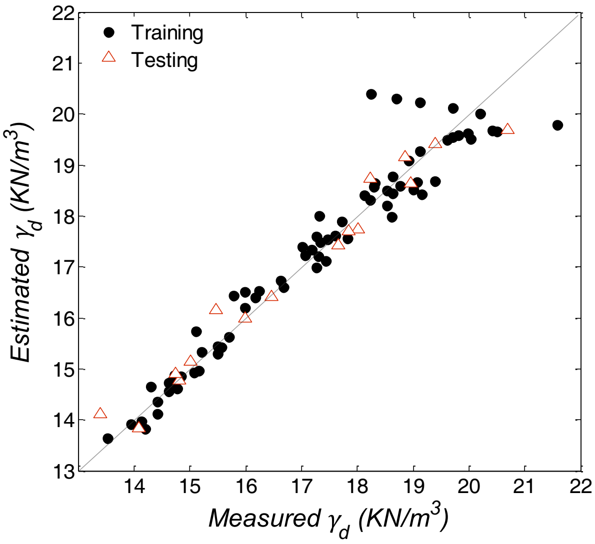

Fig. 4. Comparison between the measured and predicted γ_d using GMDH.

Table 6. Performance indices of GMDH model to predict γ_d.

Model (stage)	R^2	$RMSE$	$MARE$
GMDH (training)	0.938	0.50	1.87
GMDH (testing)	0.954	0.44	1.98

Table 7. Performance indices of different methods to predict γ_d.

Equation	R^2	$RMSE$	$MARE$
$\gamma_d = 23 - 0.005V_p - 0.27\omega + 0.0001V_p \cdot \omega$ [36]	0.44	1.52	7.09
$\gamma_d = 20.89 - 0.002\omega + 0.0009V_p - 0.07FC - 0.05PI$ (This study)	0.78	0.95	4.70
γ_d, MLP model [36]	≈ 0.80	≈ 0.87	≈ 4.03
γ_d, GMDH model (This study)	0.942	0.49	1.89

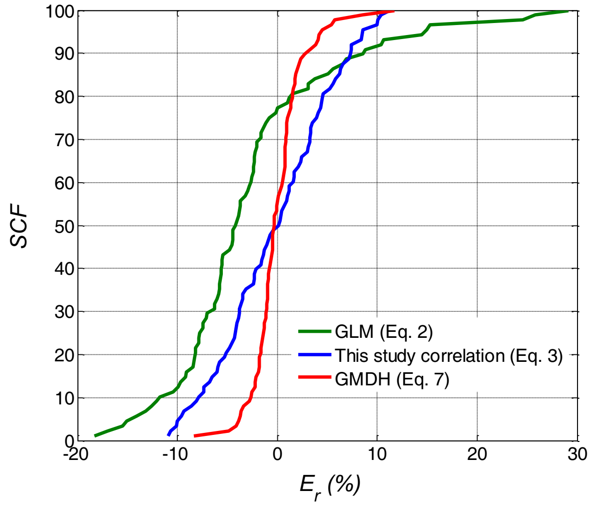

Fig. 5. Scaled relative errors of the estimated γ_d.

Table 8. Effect of each input variable on the model γ_d.

Input parameter	ω	V_p	FC	PI
R_{ij}	0.875	0.914	0.884	0.805

5 Sensitivity analysis

As the evolved polynomial equation is complex, the sensitivity analysis has been performed to determine the influence of each input parameter on the model output [62]. For this purpose, the cosine amplitude manner illustrated in the following equation is used [68]:

$$R_{ij} = \frac{\sum_{k=1}^{M} (m_{ik} \times m_{jk})}{\sqrt{\sum_{k=1}^{M} m_{ik}^2 \ \sum_{k=1}^{M} m_{jk}^2}}, \tag{9}$$

where M is the total number of data, m_i the input parameter and m_j the output parameter. The R_{ij} range is 0–1 and represents the strength of the relation between each input variable and the model output. Table 8 shows the achieved strength of relations for the proposed model. As presented in this table, the P-wave velocity is the most important parameter on the model γ_d.

Figure 6 shows the impact of changes in the input variables on the proposed model to predict γ_d. Various changes at constant rate (-15% to 15%) have been selected. For each input variable changes, $MARE$ changes in the model output has been determined. As seen in fig. 6, the developed polynomial model is significantly influenced by varying the FC value and the $MARE$ increases seriously by changing it. For example, 15% error in determining FC in the laboratory may lead to an error of about 12% ($MARE \approx 12$) in predicting γ_d by the proposed model. In other words, 1.3% error in FC is equal to 15% error in V_p, which affects the predicted γ_d. Then it requires more accuracy to measure FC in laboratory.

6 Conclusions

There are a number of test methods to determine the dry unit weight (γ_d) of the compacted soils, such as nuclear density measurement, sand cone, rubber balloon, seismic velocity, and plate loading tests. Group method of data handling (GMDH) polynomial is a self-organizing kind of neural network (NN) and can be applied for modeling complex problems. For inaccurate, noisy, or small data sets, the GMDH is the best optimal simplified model, with a higher accuracy and a simpler structure than typical full physical models. Due to the disadvantages of common methods to determine γ_d in the site, the present study developed a GMDH polynomial NN to predict the γ_d of compacted soils based on moisture content (ω), P-wave velocities (V_p) and fine-grained particles (FC) as well as plasticity index (PI). A published database containing 88 data sets from Yozgat, Turkey [36], were used to model the GMDH. New equations to predict γ_d are proposed using the regression technique and the GMDH model. Furthermore, in order to determine the effect of each input variable on model γ_d, the sensitivity analysis of the polynomial model was carried out.

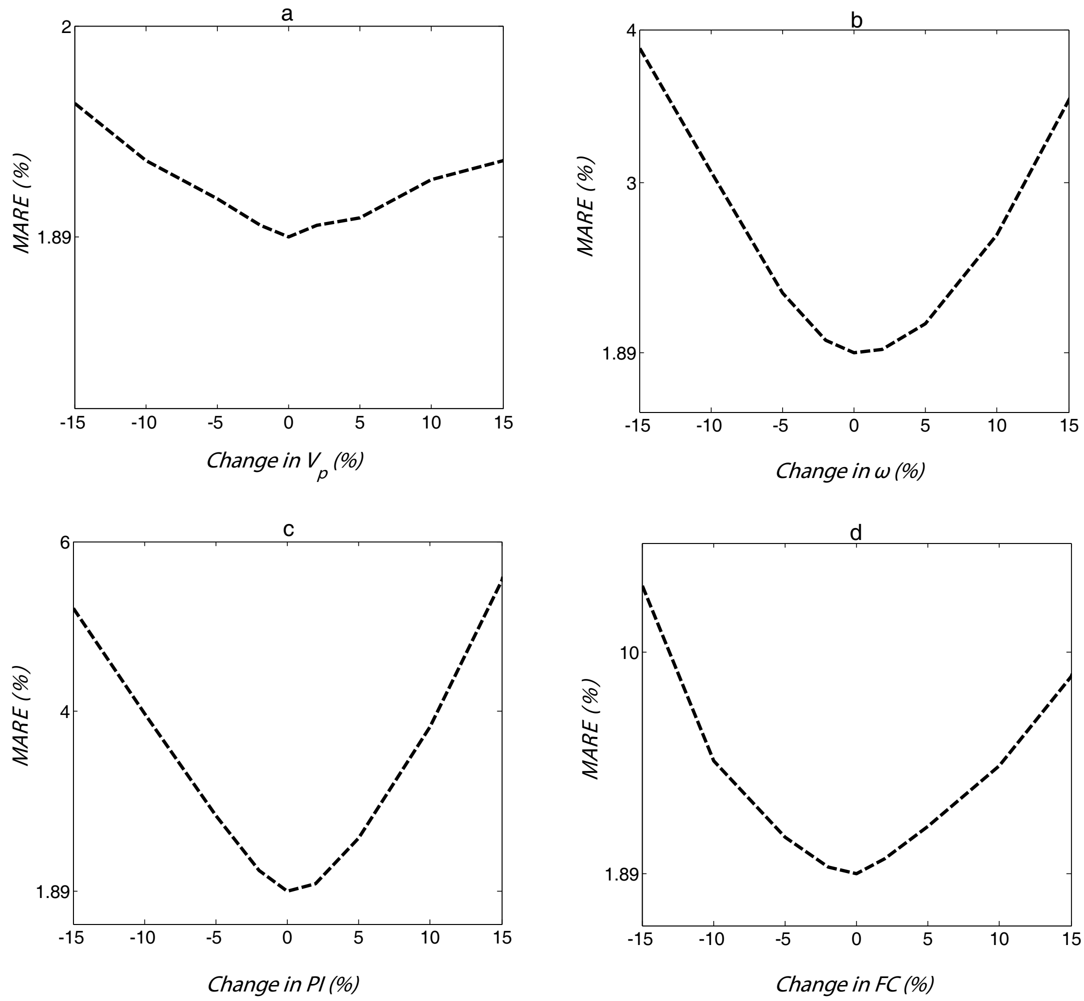

Fig. 6. The output $MARE$ variations due to changes in the input parameters.

The results indicate that:

- The suggested equation (eq. (3)) using the multiple regression analysis with four input parameters (ω, V_p, FC and PI) has a better performance than the GLM model with two input parameters (ω and V_p). The improvement is for appearing FC and PI those characterize the soil type.
- The MLP model proposed by Kolay and Baser [36] has a better performance than the proposed equations in this study and the GLM model.
- The evolved GMDH model in the form of simple polynomial equations has been effectively applied to predict γ_d.
- The proposed GMDH model is more efficient (53% to 73% improvement) than the previous reported methods and by using its corresponding equation, γ_d of the soils of this area (CH, CL, SC and SM) can be accurately predicted.
- The sensitivity analysis shows that the V_p of the compacted soil is the most important parameter on the model γ_d.
- The proposed polynomial model to predict γ_d is significantly affected by varying the FC and the mean absolute relative error ($MARE$) increases greatly by changing it. So, to determine this parameter in laboratory requires high accuracy.
- As PI, FC and ω of the soils are measured in laboratory before compacting in the site, the *in situ* γ_d can be accurately determined by measuring V_p in compacted site and using the developed GMDH equation (eq. (7)).

It should be mentioned here that the proposed GMDH equation to predict γ_d obtained from limited type (CH, CL, SM and SC) and number of soil samples within the range $13.39 \leq \gamma_d \leq 21.58$. Then, more researches are necessary to check the validity of the derived equation for other types of soils.

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