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Erratum

Erratum to: Stability in quadratic torsion theories

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A typing error has led to an incorrect expression in Eq. (18). The correct formula should read

$$\tilde{R}_{(\mu\nu)\rho\sigma} = \tilde{\nabla}_{[\sigma} Q_{\rho]\mu\nu} + T^{\lambda}_{\cdot\rho\sigma} Q_{\lambda\mu\nu}. \tag{18}$$

On the other hand, a missing -1 factor has been detected in some of the results presented in Appendix D. This factor affects only the pseudo-vectorial sector, where a change of sign must be applied to each term containing a contraction of the axial four-vector S^μ with itself. In the main body of the article, this alters part of the equations and results presented for that sector in Sects. 3.3 and 4.

In Sect. 3.3, the Eqs. (53), (54), (56), (59), (60), (61), (62), (63), (65), (66), (67), (68), (69a), (69b) and (69c) have the wrong sing in each term containing two contracted axial four-vectors S^μ. The correct expressions should read

$$\begin{aligned}\mathcal{L}_g = {} & \frac{16}{9}(p+s+t)\partial_\mu T_\nu \partial^\mu T^\nu + \frac{16}{9}(p-2r)\partial_\mu T_\nu \partial^\nu T^\mu \\ & + \frac{16}{9}(p-r+5s-t)\partial_\mu T^\mu \partial_\nu T^\nu + \frac{1}{9}t\partial_\mu S_\nu \partial^\nu S^\mu \\ & - \frac{1}{9}(2r+t)\partial_\mu S_\nu \partial^\mu S^\nu - \frac{1}{18}(3q-4r)\partial_\mu S^\mu \partial_\nu S^\nu \\ & + \frac{8}{9}(r+t)\varepsilon^{\mu\nu\rho\sigma}\partial_\rho T_\mu \partial_\nu S_\sigma - \mathcal{V}(T,S),\end{aligned} \tag{53}$$

$$\begin{aligned}\mathcal{L}_g = {} & \frac{8}{9}(p+s+t)F_{\mu\nu}(T)F^{\mu\nu}(T) \\ & - \frac{1}{18}(2r+t)F_{\mu\nu}(S)F^{\mu\nu}(S) - \frac{1}{6}q\partial_\mu S^\mu \partial_\nu S^\nu \\ & + \frac{16}{3}(p-r+2s)\partial_\mu T^\mu \partial_\nu T^\nu - \mathcal{V}(T,S),\end{aligned} \tag{54}$$

$$\pi_S^\mu \equiv \frac{\partial \mathcal{L}_g}{\partial(\partial_0 S_\mu)} = -\frac{2}{9}(2r+t)F^{0\mu}(S) - \frac{1}{3}\eta^{0\mu} q \partial_\alpha S^\alpha, \tag{56}$$

$$\pi_S^0 = -\frac{1}{3} q\, \partial_\alpha S^\alpha, \tag{59}$$

$$\pi_S^i = -\frac{2}{9}(2r+t)(\dot{S}^i - \partial^i S^0), \tag{60}$$

$$\begin{aligned}\mathcal{H}_g = {} & -\frac{9}{64}\frac{(\pi_T^i)^2}{(p+s+t)} - \frac{8}{9}(p+s+t)F_{ij}(T)F^{ij}(T) \\ & + \frac{9}{4}\frac{(\pi_S^i)^2}{2r+t} + \frac{1}{18}(2r+t)F_{ij}(S)F^{ij}(S) \\ & + \pi_T^i \partial_i T_o + \pi_S^i \partial_i S_o + \mathcal{V}(T,S),\end{aligned} \tag{61}$$

$$\mathcal{V}(T,S) = -\frac{2}{3}(c+3\lambda)T_\mu T^\mu + \frac{1}{24}(b+3\lambda)S_\mu S^\mu + \mathcal{O}(3), \tag{62}$$

$$\begin{aligned}\mathcal{V}^{(4)}(T,S) = {} & -\frac{64}{27}(p-r+2s)T_\alpha T^\alpha T_\beta T^\beta \\ & - \frac{1}{108}(p-r+2s)S_\alpha S^\alpha S_\beta S^\beta \\ & + \frac{8}{81}(2p+3q-4r+2s)T_\alpha S^\alpha T_\beta S^\beta \\ & + \frac{8}{81}(p+r+4s)T_\alpha T^\alpha S_\beta S^\beta,\end{aligned} \tag{63}$$

$$\lambda_1 = -\frac{257}{216}\left(p-r+2s+\sqrt{A}\right), \tag{65}$$

$$\lambda_2 = -\frac{257}{216}\left(p-r+2s-\sqrt{A}\right), \tag{66}$$

$$\lambda_3 = \frac{8}{81}(2p+3q-4r+2s), \tag{67}$$

$$A = \frac{1}{771^2}\Big(586249p^2 - 1168402pr + 586249r^2$$

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$$+ 2349092ps - 2332708rs + 2357284s^2\Big), \tag{68}$$

$$\lambda_1 = -\frac{8}{81}(p+3s), \tag{69a}$$

$$\lambda_2 = \frac{8}{81}(p+3s), \tag{69b}$$

$$\lambda_3 = -\frac{16}{81}(p+3s), \tag{69c}$$

respectively. Accordingly, the stability conditions for the axial mode, presented in the third column of Table 1, should read

	S^μ
Ghost-free	$q = 0$ $2r + t > 0$
Tachyon-free (Weak torsion)	$b + 3\lambda < 0$
Tachyon-free (General torsion)	$p + 3s = 0$ $b + 3\lambda < 0$
Reduction to GR when $T^{\alpha}_{\cdot\mu\nu} = 0$	$p - r = 0$ $s = 0$

Furthermore, Table 2 should be corrected as

Summary	T^μ	S^μ
$p = r = s = 0$	$t < 0$ $c + 3\lambda > 0$	$q = 0$ $t > 0$ $b + 3\lambda < 0$

The corrected results in Sect. 3.3 are now in agreement with Refs. [6, 7].

In Sect. 4, the conditions over the parameters b, c and λ mentioned below Eq. (70) should read:"$b + 3\lambda < 0$ and $c + 3\lambda > 0$". Additionally, the parameter t should be put to zero in Eq. (70). Then, the new conclusion are that the particular case where the Lagrangian in (24) reduces to GR in the absence of torsion cannot safely propagate the vector and pseudo-vector torsion modes simultaneously.

In Appendix D, the Eqs. (D.29), (D.30), (D.31), (D.32), (D.33), (D.34), (D.35), (D.36) and (D.37) should read

$$\widehat{R} = R - 4\nabla_\alpha T^\alpha - \frac{8}{3}T_\beta T^\beta + \frac{1}{6}S_\beta S^\beta, \tag{D.29}$$

$$\begin{aligned}\widehat{R}^2\Big|_{g=\eta} &= 16\partial_\alpha T^\alpha \partial_\beta T^\beta + \frac{64}{3}\partial_\alpha T^\alpha T_\beta T^\beta \\ &\quad - \frac{8}{6}\partial_\alpha T^\alpha S_\beta S^\beta - \frac{8}{9}T_\alpha T^\alpha S_\beta S^\beta \\ &\quad + \frac{1}{36}S_\alpha S^\alpha S_\beta S^\beta + \frac{64}{9}T_\alpha T^\alpha T_\beta T^\beta,\end{aligned} \tag{D.30}$$

$$\begin{aligned}\widehat{R}_{\nu\sigma}\widehat{R}^{\nu\sigma}\Big|_{g=\eta} &= \frac{16}{9}\partial_\mu T_\nu \partial^\mu T^\nu + \frac{32}{9}\partial_\alpha T^\alpha \partial_\beta T^\beta \\ &\quad - \frac{1}{18}(\partial_\alpha S_\beta \partial^\alpha S^\beta - \partial_\alpha S_\beta \partial^\beta S^\alpha) \\ &\quad - \frac{4}{9}\epsilon^{\mu\nu\rho\sigma}\partial_\mu S_\sigma \partial_\rho T_\nu + \frac{160}{27}\partial_\alpha T^\alpha T_\beta T^\beta \\ &\quad - \frac{64}{27}\partial_\mu T_\nu T^\mu T^\nu - \frac{10}{27}\partial_\alpha T^\alpha S_\beta S^\beta \\ &\quad + \frac{4}{27}\partial_\mu T_\nu S^\mu S^\nu + \frac{64}{27}T_\alpha T^\alpha T_\beta T^\beta \\ &\quad + \frac{1}{108}S_\alpha S^\alpha S_\beta S^\beta - \frac{16}{81}T_\alpha T^\alpha S_\beta S^\beta \\ &\quad - \frac{8}{81}T_\alpha S^\alpha T_\beta S^\beta,\end{aligned} \tag{D.31}$$

$$\begin{aligned}\widehat{R}_{\nu\sigma}\widehat{R}^{\sigma\nu}\Big|_{g=\eta} &= \frac{48}{9}\partial_\alpha T^\alpha \partial_\beta T^\beta - \frac{4}{9}\epsilon^{\mu\nu\rho\sigma}\partial_\mu S_\sigma \partial_\nu T_\rho \\ &\quad + \frac{1}{18}(\partial_\alpha S_\beta \partial^\alpha S^\beta - \partial_\alpha S_\beta \partial^\beta S^\alpha) \\ &\quad + \frac{160}{27}\partial_\alpha T^\alpha T_\beta T^\beta - \frac{64}{27}\partial_\alpha T_\beta T^\beta T^\alpha \\ &\quad - \frac{10}{27}\partial_\alpha T^\alpha S_\beta S^\beta + \frac{4}{27}\partial_\mu T_\nu S^\mu S^\nu \\ &\quad + \frac{64}{27}T_\alpha T^\alpha T_\beta T^\beta + \frac{1}{108}S_\alpha S^\alpha S_\beta S^\beta \\ &\quad - \frac{16}{81}T_\alpha T^\alpha S_\beta S^\beta - \frac{8}{81}T_\alpha S^\alpha T_\beta S^\beta,\end{aligned} \tag{D.32}$$

$$\begin{aligned}\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\mu\nu\rho\sigma}\Big|_{g=\eta} &= \frac{32}{9}\partial_\rho T_\nu \partial^\rho T^\nu + \frac{16}{9}\partial_\alpha T^\alpha \partial_\beta T^\beta \\ &\quad - \frac{2}{9}\partial_\alpha S_\beta \partial^\alpha S^\beta - \frac{1}{9}\partial_\alpha S^\alpha \partial_\beta S^\beta \\ &\quad + \frac{8}{9}\epsilon^{\mu\nu\rho\sigma}\partial_\nu S_\sigma \partial_\rho T_\mu - \frac{128}{27}\partial_\rho T_\nu T^\rho T^\nu \\ &\quad + \frac{128}{27}\partial_\alpha T^\alpha T_\beta T^\beta - \frac{8}{27}\partial_\alpha T^\alpha S_\beta S^\beta \\ &\quad - \frac{16}{27}\partial_\alpha S^\alpha T_\beta S^\beta + \frac{8}{27}\partial_\alpha T_\beta S^\alpha S^\beta \\ &\quad + \frac{8}{27}\partial_\alpha S_\beta T^\alpha S^\beta + \frac{8}{27}\partial_\alpha S_\beta S^\alpha T^\beta \\ &\quad + \frac{64}{27}T_\alpha T^\alpha T_\beta T^\beta + \frac{1}{108}S_\alpha S^\alpha S_\beta S^\beta \\ &\quad - \frac{8}{27}T_\alpha T^\alpha S_\beta S^\beta - \frac{16}{27}T_\alpha S^\alpha T_\beta S^\beta,\end{aligned} \tag{D.33}$$

$$\begin{aligned}\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\rho\sigma\mu\nu}\Big|_{g=\eta} &= \frac{32}{9}\partial_\rho T_\nu \partial^\nu T^\rho + \frac{16}{9}\partial_\alpha T^\alpha \partial_\beta T^\beta \\ &\quad + \frac{2}{9}(\partial_\alpha S_\beta \partial^\alpha S^\beta - \partial_\alpha S^\alpha \partial_\beta S^\beta) \\ &\quad + \frac{8}{9}\epsilon^{\mu\nu\rho\sigma}\partial_\nu S_\sigma \partial_\mu T_\rho - \frac{128}{27}\partial_\rho T_\nu T^\rho T^\nu \\ &\quad + \frac{128}{27}\partial_\alpha T^\alpha T_\beta T^\beta - \frac{8}{27}\partial_\alpha T^\alpha S_\beta S^\beta \\ &\quad + \frac{8}{27}\partial_\alpha T_\beta S^\alpha S^\beta - \frac{8}{27}\partial_\alpha S_\beta T^\alpha S^\beta \\ &\quad - \frac{8}{27}\partial_\alpha S_\beta S^\alpha T^\beta - \frac{8}{27}\partial_\alpha S^\alpha T_\beta S^\beta\end{aligned}$$

$$+\frac{64}{27}T_\alpha T^\alpha T_\beta T^\beta+\frac{1}{108}S_\alpha S^\alpha S_\beta S^\beta$$
$$+\frac{8}{81}T_\alpha T^\alpha S_\beta S^\beta-\frac{32}{81}T_\alpha S^\alpha T_\beta S^\beta, \tag{D.34}$$

$$\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\mu\rho\nu\sigma}\big|_{g=\eta}=\frac{8}{9}\partial_\rho T_\nu\partial^\nu T^\rho+\frac{8}{9}\partial_\rho T_\nu\partial^\rho T^\nu$$
$$+\frac{8}{9}\partial_\alpha T^\alpha\partial_\beta T^\beta+\frac{1}{6}\partial_\alpha S^\alpha\partial_\beta S^\beta$$
$$+\frac{32}{27}\partial_\alpha T^\alpha T_\beta T^\beta-\frac{32}{27}\partial_\alpha T_\beta T^\beta T^\alpha$$
$$-\frac{4}{27}\partial_\alpha T^\alpha S_\beta S^\beta+\frac{4}{27}\partial_\alpha T_\beta S^\alpha S^\beta$$
$$+\frac{12}{27}\partial_\alpha S^\alpha T_\beta S^\beta+\frac{32}{27}T_\alpha T^\alpha T_\beta T^\beta$$
$$+\frac{1}{216}S_\alpha S^\alpha S_\beta S^\beta+\frac{16}{81}T_\alpha S^\alpha T_\beta S^\beta$$
$$-\frac{4}{81}T_\alpha T^\alpha S_\beta S^\beta, \tag{D.35}$$

$$T_{\mu\nu\rho}T^{\mu\nu\rho}=\frac{2}{3}T_\mu T^\mu-\frac{1}{6}S_\nu S^\nu, \tag{D.36}$$

$$T_{\mu\nu\rho}T^{\nu\rho\mu}=-\frac{1}{3}T_\mu T^\mu-\frac{1}{6}S_\nu S^\nu, \tag{D.37}$$

respectively. Therefore, the potential in Eq. (D.39) should be corrected as

$$\mathcal{V}(T,S)=-\frac{2}{3}(c+3\lambda)T_\alpha T^\alpha+\frac{1}{24}(b+3\lambda)S_\alpha S^\alpha$$
$$+\frac{4}{27}(2p-2q+5s)\partial_\alpha T^\alpha S_\beta S^\beta$$
$$-\frac{8}{27}(p-r+s)\partial_\alpha T_\beta S^\alpha S^\beta$$
$$+\frac{4}{27}(3q-2r)\partial_\alpha S^\alpha T_\beta S^\beta$$
$$-\frac{8}{27}r\partial_\alpha S_\beta T^\alpha S^\beta-\frac{8}{27}r\partial_\alpha S_\beta S^\alpha T^\beta$$
$$-\frac{64}{27}(p-r+2s)T_\alpha T^\alpha T_\beta T^\beta$$
$$-\frac{1}{108}(p-r+2s)S_\alpha S^\alpha S_\beta S^\beta$$
$$+\frac{8}{81}(p+r+4s)T_\alpha T^\alpha S_\beta S^\beta$$
$$+\frac{8}{81}(2p+3q-4r+2s)T_\alpha S^\alpha T_\beta S^\beta. \tag{D.39}$$

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