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Gravitational form factors of the delta resonance in chiral EFT

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Abstract The leading one-loop corrections to the gravitational form factors of the delta resonance are calculated in the framework of chiral effective field theory. Various contributions to the energy–momentum tensor and the renormalization of the low-energy constants are worked out. Using the small scale expansion, expressions for static quantities are obtained and the real and imaginary parts of the gravitational form factors are calculated numerically.

1 Introduction

The linear response of the effective action to the change of the space-time metric specifies mechanical properties of particles with various spins. Static characteristics, like the mass, spin and the D-term correspond to the hadron gravitational form factors (GFFs) at zero momentum transfer [1,2]. Determining the third mechanical characteristics (the D-term) of a particle is a more difficult problem than the mass and spin, which are well-studied and well-measured quantities. The D-term of the nucleon has been related to the distribution of the internal forces in Ref. [3] (for a review see e.g. Ref. [4]). Poincaré covariance of the hadron states guarantees that the leading two GFFs at zero momentum-transfer correspond to mass and spin [5]. Adding total derivatives to the energy–momentum tensor (EMT) leaves the Poincaré group generators unaffected, i.e. it does not impact the particle's mass or spin. Therefore, these static characteristics are well constrained. However, Poincaré symmetry does not protect the D-term. By adding total derivatives to the EMT, the D-term can be significantly changed. For example, it has been shown in Ref. [6] that even infinitesimally small interactions (minimally coupled to gravity) can drastically impact the D-term: while the free Klein–Gordon theory predicts the D-term of a spin-0 particle to be $D = -1$, it becomes $D = -1/3$ after inserting an "improvement term" in the EMT. The improvement term is obtained by requiring that the action respects the conformal symmetry of the classical theory in the massless limit, which is equivalent to adding a term of non-minimal coupling $-\frac{1}{12} R\Phi^2$ to the action (where R is the Riemann scalar, and Φ is the free Klein–Gordon field). On a quantum level, the conformal symmetry is broken, but the improvement term is required to remove UV divergences up to three loops in dimensional regularization in the Klein–Gordon theory [7].

In recent years, GFFs have attracted increasing attention for characterizing properties of hadrons with different spins due to their connection to generalized parton distributions (GPDs). Parameterizations of the EMT matrix elements in terms of GFFs for hadrons have been considered for spin-0 [2], spin-1 [8–10], and for arbitrary-spin particles [11]. The mechanical properties, energy and spin densities as well as spatial distributions of pressure and shear forces have been introduced for spin-0 and spin-1/2 in Ref. [3], and later also for systems with higher spins [10,12,13].

The nucleon gravitational form factors can be measured experimentally in exclusive processes like deeply virtual

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Compton scattering (DVCS) [14,15] and hard exclusive meson production [16]. The connection to GFFs can be seen in the QCD description of these processes, where the *symmetric* energy–momentum tensor (EMT) appears naturally in the operator product expansion, see derivation, e.g., in Ref. [14]. The first results of measurements of the D-term in hard QCD processes became available in Refs. [17,18] for the nucleon, and in Ref. [19] for the pion. Profound studies of subtleties in the extraction of the D-term from hard exclusive processes can be found in Ref. [20]. The GFFs have also been studied in lattice QCD, see e.g. Refs. [21–25] and references therein.

In contrast to the electromagnetic properties of the delta resonances, which have been extensively investigated both experimentally and theoretically [26,27], it is very difficult to measure experimentally the GFFs of the Δ or to extract them from the corresponding GPDs because of the short-lived nature of the Δ resonance. Several calculations have been done to estimate these GFFs. For example, the SU(2) Skyrme model [13], the quark–diquark model [27] and a lattice QCD approach (for the gluonic part) [28] have been employed to calculate the GFFs of the spin-3/2 delta resonances and the corresponding pressure and shear forces. Except for the constraints that the mass and spin should be obtained from the zero momentum-transfer limits of the leading two GFFs, the other GFFs calculated in Refs. [13] and [27] show notable differences and even different signs for the D-term ($D = -3.53$ in Ref. [13] and $D = 0.986$ in Ref. [27]). Notice that the negative sign of the D-term is expected to follow from the stability condition. More systematic studies are thus required to investigate the properties of the GFFs of delta resonance, e.g., to what extent the expected stability condition holds, taking into account that the deltas are unstable.

For systematic studies of low-energy hadronic processes with delta resonances induced by gravity one may rely on the effective chiral Lagrangian for nucleons, pions and delta resonances in curved spacetime. Effective Lagrangian of pions in curved spacetime has been derived in Ref. [29], and the GFFs of the pion can be found in Ref. [30]. The effective chiral Lagrangian of order two for nucleons and pions in curved spacetime, along with the calculation of the leading one-loop contributions to the nucleon GFFs, can be found in Ref. [31]. In this work we apply chiral effective field theory (EFT) to calculate the one-particle matrix elements of EMT for delta resonances within the EOMS scheme to be discussed below.

Our paper is organized as follows: In Sect. 2 we specify the relevant terms of the effective Lagrangian for pions, nucleons and delta resonances in curved spacetime and the corresponding expression for the EMT. We calculate the gravitational form factors of the delta resonance in Sect. 3. The results of our work are summarized in Sect. 4.

2 Effective action in curved space time and the energy–momentum tensor

The action corresponding to the leading-order contributions to the effective Lagrangian of pions, nucleons and delta resonances interacting with an external gravitational field can be straightforwardly obtained from the corresponding expressions in the flat spacetime. It has the following form:

$$
\begin{aligned}
S_\pi^{(2)} &= \int d^4x\sqrt{-g}\left\{\frac{F^2}{4}\, g^{\mu\nu}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger)\right. \\
&\quad \left. + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\}, \qquad (1)\\
S_{\pi\mathrm{N}}^{(1)} &= \int d^4x\sqrt{-g}\left\{\bar\Psi\, i\gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu \Psi - m\bar\Psi\Psi\right. \\
&\quad \left. + \frac{g_A}{2}\,\bar\Psi\gamma^\mu\gamma_5 u_\mu\Psi\right\}, \qquad (2)\\
S_{\pi\Delta}^{(1)} &= -\int d^4x\sqrt{-g}\Big[g^{\mu\nu}\,\bar\Psi^i_\mu\, i\gamma^\alpha\overset{\leftrightarrow}{\nabla}_\alpha\Psi^i_\nu - m_\Delta\, g^{\mu\nu}\bar\Psi^i_\mu\Psi^i_\nu \\
&\quad - g^{\lambda\sigma}\left(\bar\Psi^i_\mu i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\lambda\Psi^i_\sigma + \bar\Psi^i_\lambda i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\sigma\Psi^i_\mu\right) \\
&\quad + i\bar\Psi^i_\mu\gamma^\mu\gamma^\alpha\gamma^\nu\overset{\leftrightarrow}{\nabla}_\alpha\Psi^i_\nu + m_\Delta\bar\Psi^i_\mu\gamma^\mu\gamma^\nu\Psi^i_\nu \\
&\quad + \frac{g_1}{2}\, g^{\mu\nu}\bar\Psi^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu + \frac{g_2}{2}\,\bar\Psi^i_\mu \\
&\quad \left(u^\mu\gamma^\nu + u^\nu\gamma^\mu\right)\gamma_5\Psi^i_\nu + \frac{g_3}{2}\,\bar\Psi^i_\mu u_\alpha\gamma^\mu\gamma^\alpha\gamma_5\gamma^\nu\Psi^i_\nu\Big], \qquad (3)
\end{aligned}
$$

where the delta resonance is represented by the Rarita–Schwinger field. Note that the delta fields Ψ_i^μ contain isospin projectors $\xi^{\frac{3}{2}}_{ij} = \delta_{ij} - \frac{1}{3}\tau_i\tau_j$, i.e. they satisfy the condition $\Psi_i^\mu = \xi^{\frac{3}{2}}_{ij}\Psi_j^\mu$. Here, μ and i are the Lorenz and isospin-indices, $g^{\mu\nu}$ is the metric with the signature $(+,-,-,-)$ and $\gamma_\mu \equiv e^a_\mu\gamma_a$, with e^a_μ denoting the vielbein gravitational fields. It follows from the consistency conditions, imposed on the Lagrangian with Rarita–Schwinger fields, that the following relations for the low-energy constants g_1, g_2 and g_3 must be satisfied: $g_2 = g_3 = -g_1$ [32].[1] Further, m, g_A and F refer to nucleon mass, the axial vector coupling of the nucleon and the pion decay constant in the chiral limit, respectively (also called bare parameters later). The action corresponding to the leading-order chiral Lagrangian containing pions, nucleons and deltas interacting with an external gravitational field is

[1] In this work we take the off-shell parameter A equal to -1, which removes the dependence on the spacetime dimension in the action $S^{(1)}_{\pi\Delta}$ in Eq. (3) and simplifies the free propagator of the delta resonance. See also the discussion in Ref. [33].

$$S^{(1)}_{\pi N\Delta} = -\int d^4x\sqrt{-g}\; g_{\pi N\Delta}\;\bar\Psi_{\mu,i}\left(g^{\mu\nu}-\gamma^\mu\gamma^\nu\right)u_{\nu,i}\Psi + \text{H.c.}, \tag{4}$$

where $u_\mu = \tau_i u_{\mu,i}$. The pion-field dependent matrix u_μ will be specified below. From the second-order chiral Lagrangian containing pions and deltas interacting with an external gravitational field we need to consider the following term [34]:

$$S^{(2)}_{\pi\Delta,a} = \int d^4x\sqrt{-g}\, a_1\,\bar\Psi^i_\mu\Theta^{\mu\alpha}(z)\,\langle\chi_+\rangle\, g_{\alpha\beta}\Theta^{\beta\nu}(z')\Psi^i_\nu, \tag{5}$$

where z and z' are two independent parameters. Further terms of the second order Lagrangian contributing (at tree order) to our calculations of the GFFs of delta resonance contain only the Riemann curvature tensor, the Ricci tensor and the Ricci scalar. The most general second-order Lagrangian of such terms involves either the curvature scalar or the Riemann and Ricci tensors and reduces to the following minimal form:

$$\begin{aligned}
S^{(2)}_{\pi\Delta,b} = \int d^4x\sqrt{-g}\Big[& h_1 R\, g^{\alpha\beta}\bar\Psi^i_\alpha\Psi^i_\beta + h_2 R\,\bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\Psi^i_\beta \\
&+ i h_3 R\left(g^{\alpha\lambda}\bar\Psi^i_\alpha\gamma^\beta\overrightarrow{\nabla}_\lambda\Psi^i_\beta - g^{\beta\lambda}\bar\Psi^i_\alpha\gamma^\alpha\overleftarrow{\nabla}_\lambda\Psi^i_\beta\right) \\
&+ h_4 R^{\mu\nu}\,\bar\Psi^i_\mu\Psi^i_\nu + 2 i h_5 R^{\mu\nu}\, g^{\alpha\beta}\bar\Psi^i_\alpha\gamma_\mu\overleftrightarrow{\nabla}_\nu\Psi^i_\beta \\
&+ i h_6 R^{\mu\nu} g^{\alpha\beta}\left(\bar\Psi^i_\alpha\gamma_\mu\overrightarrow{\nabla}_\beta\Psi^i_\nu - \bar\Psi^i_\nu\gamma_\mu\overleftarrow{\nabla}_\beta\Psi^i_\alpha\right) \\
&+ i h_7 R^{\mu\nu}\left(\bar\Psi^i_\alpha\gamma^\alpha\overrightarrow{\nabla}_\mu\Psi^i_\nu - \bar\Psi^i_\nu\gamma^\alpha\overleftarrow{\nabla}_\mu\Psi^i_\alpha\right) \\
&+ h_8 R^{\mu\nu}\left(\bar\Psi^i_\alpha\gamma^\alpha\gamma_\mu\Psi^i_\nu + \bar\Psi^i_\nu\gamma_\mu\gamma^\alpha\Psi^i_\alpha\right) \\
&+ i h_9 R^{\mu\nu}\left(\bar\Psi^i_\kappa\gamma^\kappa\gamma^\alpha\gamma_\mu\overrightarrow{\nabla}_\nu\Psi^i_\alpha - \bar\Psi^i_\alpha\gamma_\mu\gamma^\alpha\gamma^\kappa\overleftarrow{\nabla}_\nu\Psi^i_\kappa\right) \\
&+ i h_{10} R^{\mu\nu\alpha\beta}\bar\Psi^i_\alpha\sigma_{\mu\nu}\Psi^i_\beta \\
&+ i\left[h_{11}\, R^{\mu\nu\alpha\beta} + h_{12}\, R^{\mu\alpha\nu\beta}\right]\Big(\bar\Psi^i_\alpha\gamma_\mu\overrightarrow{\nabla}_\nu\Psi^i_\beta \\
&- \bar\Psi^i_\beta\gamma_\mu\overleftarrow{\nabla}_\nu\Psi^i_\alpha\Big) + h_{13} R^{\mu\alpha\nu\beta}\bar\Psi^i_\alpha\gamma_\mu\gamma_\nu\Psi^i_\beta \\
&+ i\left[h_{14}\, R^{\mu\nu\alpha\beta} + h_{15}\, R^{\mu\alpha\nu\beta}\right]\Big(\bar\Psi^i_\kappa\gamma^\kappa\gamma_\mu\gamma_\nu\overrightarrow{\nabla}_\alpha\Psi^i_\beta \\
&- \bar\Psi^i_\beta\gamma_\nu\gamma_\mu\gamma^\kappa\overleftarrow{\nabla}_\alpha\Psi^i_\kappa\Big)\Big],
\end{aligned} \tag{6}$$

where the h_i are coupling constants.

The various building blocks of the effective Lagrangian are defined as follows:

$$\begin{aligned}
& D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu, \\
& \overleftrightarrow{\nabla}_\mu = \frac{1}{2}(\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu), \\
& \Theta^{\mu\nu}(z) = g^{\mu\nu} + z\gamma^\mu\gamma^\nu, \\
& \overrightarrow{\nabla}_\mu\Psi^i_\nu = \nabla^{ij}_\mu\Psi^j_\nu = \Big[\delta^{ij}\partial_\mu + \delta^{ij}\Gamma_\mu - i\delta^{ij}v^{(s)}_\mu \\
& \quad - i\epsilon^{ijk}\mathrm{Tr}\left(\tau^k\Gamma_\mu\right) + \frac{i}{2}\delta^{ij}\omega^{ab}_\mu\sigma_{ab}\Big]\Psi^j_\nu - \Gamma^\alpha_{\mu\nu}\Psi^i_\alpha, \\
& \bar\Psi^i_\nu\overleftarrow{\nabla}_\mu = \nabla^{ij}_\mu\Psi^j_\nu = \bar\Psi^j_\nu\Big[\delta^{ij}\partial_\mu - \delta^{ij}\Gamma_\mu + i\delta^{ij}v^{(s)}_\mu \\
& \quad + i\epsilon^{ijk}\mathrm{Tr}\left(\tau^k\Gamma_\mu\right) - \frac{i}{2}\delta^{ij}\omega^{ab}_\mu\sigma_{ab}\Big] - \bar\Psi^i_\alpha\Gamma^\alpha_{\mu\nu}, \\
& \overrightarrow{\nabla}_\mu\Psi = \partial_\mu\Psi + \frac{i}{2}\omega^{ab}_\mu\sigma_{ab}\Psi + \left(\Gamma_\mu - i v^{(s)}_\mu\right)\Psi, \\
& \bar\Psi\overleftarrow{\nabla}_\mu = \partial_\mu\bar\Psi - \frac{i}{2}\bar\Psi\,\sigma_{ab}\,\omega^{ab}_\mu - \bar\Psi\left(\Gamma_\mu - i v^{(s)}_\mu\right), \\
& u_\mu = i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger v_\mu u - u v_\mu u^\dagger)\right], \\
& \chi = 2B_0(s+ip), \\
& \Gamma_\mu = \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger - i(u^\dagger v_\mu u + u v_\mu u^\dagger)\right], \\
& \omega^{ab}_\mu = -\frac{1}{2}g^{\nu\lambda}e^a_\lambda\left(\partial_\mu e^b_\nu - e^b_\sigma\Gamma^\sigma_{\mu\nu}\right), \\
& \Gamma^\lambda_{\alpha\beta} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right), \\
& R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}, \\
& R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}, \\
& R = g^{\mu\nu}R^\lambda{}_{\mu\lambda\nu}, \\
& \sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu,\gamma_\nu\right], \\
& \chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u,
\end{aligned} \tag{7}$$

where the 2×2 unitary matrix U represents the pion field, s, p, $l_\mu = v_\mu - a_\mu$, $r_\mu = v_\mu + a_\mu$ and $v^{(s)}_\mu$ refer to the external sources, $\chi = 2B_0(s+ip)$, and the parameter B_0 is related to the vacuum condensate in the chiral limit. The vielbein fields satisfy the following relations:

$$\begin{aligned}
& e^a_\mu e^b_\nu\eta_{ab} = g_{\mu\nu}, \quad e^\mu_a e^\nu_b\eta^{ab} = g^{\mu\nu}, \\
& e^a_\mu e^b_\nu g^{\mu\nu} = \eta^{ab}, \quad e^\mu_a e^\nu_b g_{\mu\nu} = \eta_{ab}.
\end{aligned} \tag{8}$$

Using the definition of the EMT for matter fields interacting with the gravitational metric field,

$$T_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}}\frac{\delta S_\mathrm{m}}{\delta g^{\mu\nu}}, \tag{9}$$

we obtain in flat spacetime from the action of Eq. (1):

$$\begin{aligned}
T^{(2)}_{\pi,\mu\nu} = & \frac{F^2}{4}\mathrm{Tr}(D_\mu U(D_\nu U)^\dagger) \\
& -\frac{\eta_{\mu\nu}}{2}\left\{\frac{F^2}{4}\mathrm{Tr}(D^\alpha U(D_\alpha U)^\dagger) + \frac{F^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\} \\
& + (\mu\leftrightarrow\nu),
\end{aligned} \tag{10}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor. For the fermion fields interacting with the gravitational vielbein fields we use the definition [35]

$$T_{\mu\nu}(g,\psi) = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\,e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\,e^a_\mu\right], \tag{11}$$

where e denotes the determinant of e^a_μ. The action of Eq. (2) leads to the following expression for the EMT in flat spacetime:

$$
\begin{aligned}
T^{(1)}_{\pi N,\mu\nu} \ = & \ \frac{i}{2}\bar\Psi\,\gamma_\mu \overleftrightarrow{D}_\nu\Psi + \frac{g_A}{4}\bar\Psi\,\gamma_\mu\gamma_5 u_\nu\Psi \\
& -\frac{\eta_{\mu\nu}}{2}\left(\bar\Psi\, i\gamma^\alpha\overleftrightarrow{D}_\alpha\Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\alpha\gamma_5 u_\alpha\Psi\right) \\
& +(\mu\leftrightarrow\nu)\,. \qquad (12)
\end{aligned}
$$

The actions of Eqs. (3)–(5) lead to the following expressions for the EMT in flat spacetime:

$$
\begin{aligned}
T^{(1)}_{\pi\Delta,\mu\nu} = & -\bar\Psi^i_\mu\, i\gamma^\alpha\overleftrightarrow{D}_\alpha\Psi^i_\nu + \bar\Psi^i_\alpha\, i\gamma^\alpha\overleftrightarrow{D}_\mu\Psi^i_\nu \\
& +\bar\Psi^i_\mu\, i\gamma^\alpha\overleftrightarrow{D}_\nu\Psi^i_\alpha + m_\Delta\bar\Psi^i_\mu\Psi^i_\nu - \frac{i}{2}\,\bar\Psi^i_\alpha\,\gamma_\mu\overleftrightarrow{D}_\nu\Psi^{i\alpha} \\
& +\frac{i}{2}\Big(\bar\Psi^i_\mu\,\gamma_\nu\overleftrightarrow{D}_\alpha\Psi^{i\alpha} + \bar\Psi^{i\alpha}\,\gamma_\nu\overleftrightarrow{D}_\alpha\Psi^i_\mu - \bar\Psi^i_\mu\,\gamma_\nu\gamma^\alpha\gamma_\beta\overleftrightarrow{D}_\alpha\Psi^{i,\beta} \\
& -\bar\Psi^i_\alpha\gamma^\alpha\gamma_\nu\gamma^\beta\overleftrightarrow{D}_\mu\Psi^i_\beta - \bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\gamma_\nu\overleftrightarrow{D}_\beta\Psi^i_\mu\Big) \\
& +\frac{i}{4}\,\partial^\lambda\Big[\bar\Psi^{i,\alpha}\Big(\gamma_\mu\eta_{\lambda[\alpha}\eta_{\beta]\mu} + \eta_{\lambda\mu}\eta_{\nu[\alpha}\gamma_{\beta]} + \eta_{\mu\nu}\eta_{\lambda[\beta}\gamma_{\alpha]}\Big)\Psi^{i,\beta}\Big] \\
& -\frac{m_\Delta}{2}\,\left(\bar\Psi^i_\mu\,\gamma_\nu\gamma^\alpha\Psi^i_\alpha + \bar\Psi^i_\alpha\,\gamma^\alpha\gamma_\nu\Psi^i_\mu\right) \\
& -\frac{g_1}{4}\left[2\bar\Psi^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu + \bar\Psi^{i,\alpha}u_\mu\gamma_\nu\gamma_5\Psi^i_\alpha\right] \\
& -\frac{g_2}{4}\Big[2\bar\Psi^i_\mu u_\nu\gamma^\alpha\gamma_5\Psi^i_\alpha + 2\bar\Psi^i_\alpha u_\nu\gamma^\alpha\gamma_5\Psi^i_\mu \\
& +\,\bar\Psi^{i,\alpha}u_\alpha\gamma_\nu\gamma_5\Psi^i_\mu + \bar\Psi^i_\mu u_\alpha\gamma_\nu\gamma_5\Psi^{i\alpha}\Big] \\
& -\frac{g_3}{4}\Big[\bar\Psi^i_\mu u_\alpha\gamma_\nu\gamma^\alpha\gamma_5\gamma^\beta\Psi^i_\beta + \bar\Psi^i_\beta u_\alpha\gamma^\beta\gamma^\alpha\gamma_5\gamma_\nu\Psi^i_\mu \\
& +\,\bar\Psi^i_\alpha u_\mu\gamma^\alpha\gamma_\nu\gamma_5\gamma^\beta\Psi^i_\beta\Big] + \frac{\eta_{\mu\nu}}{2}\Big[\bar\Psi^i_\alpha\, i\gamma^\beta\overleftrightarrow{D}_\beta\Psi^{i\alpha} \\
& -m_\Delta\,\bar\Psi^i_\alpha\Psi^{i\alpha} - \bar\Psi^i_\alpha i\gamma^\alpha\overleftrightarrow{D}_\beta\Psi^{i\beta} - \bar\Psi^{i\alpha}i\gamma^\beta\overleftrightarrow{D}_\alpha\Psi^i_\beta \\
& +i\bar\Psi^i_\rho\gamma^\rho\gamma^\alpha\gamma^\lambda\overleftrightarrow{D}_\alpha\Psi^i_\lambda + m_\Delta\bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\Psi^i_\beta \\
& +\frac{g_1}{2}\,\bar\Psi^i_\beta u_\alpha\gamma^\alpha\gamma_5\Psi^{i\beta} + \frac{g_2}{2}\,\bar\Psi^{i\alpha}\left(u_\alpha\gamma_\beta + u_\beta\gamma_\alpha\right)\gamma_5\Psi^{i\beta} \\
& +\frac{g_3}{2}\,\bar\Psi^i_\alpha u_\beta\gamma^\alpha\gamma^\beta\gamma_5\gamma^\lambda\Psi^i_\lambda\Big] + (\mu\leftrightarrow\nu)\,, \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
T^{(1)}_{\pi N\Delta,\mu\nu} = & \ \frac{1}{2}g_{\pi N\Delta}\ \eta_{\mu\nu}\Big[\bar\Psi^i_\alpha u^\alpha_i\Psi + \bar\Psi u^\alpha_i\Psi^i_\alpha - \bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta u^i_\beta\Psi \\
& -\bar\Psi\gamma^\beta\gamma^\alpha u^i_\beta\Psi^i_\alpha\Big] - g_{\pi N\Delta}\ \left(\bar\Psi^i_\mu u^i_\nu\Psi + \bar\Psi u^i_\nu\Psi^i_\mu\right) \\
& +\frac{1}{2}g_{\pi N\Delta}\ \Big[\bar\Psi^i_\mu\gamma_\nu\gamma^\alpha u^i_\alpha\Psi + \bar\Psi^i_\alpha\gamma^\alpha\gamma_\mu u^i_\nu\Psi \\
& +\bar\Psi\gamma^\alpha\gamma_\nu u^i_\alpha\Psi^i_\mu + \bar\Psi\gamma_\mu\gamma^\alpha u^i_\nu\Psi^i_\alpha\Big] + (\mu\leftrightarrow\nu)\,, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
T^{(2)}_{\pi\Delta,a,\mu\nu} = & \ a_1\ \bar\Psi^i_\mu\,\langle\chi_+\rangle\,\Psi^i_\nu + \frac{\tilde z}{2}a_1\ \Big(\bar\Psi^i_\mu\gamma_\nu\gamma^\alpha\,\langle\chi_+\rangle\,\Psi^i_\alpha \\
& +\bar\Psi^i_\alpha\gamma^\alpha\gamma_\mu\,\langle\chi_+\rangle\,\Psi^i_\nu\Big) - \frac{a_1}{2}\eta_{\mu\nu}\Big[\bar\Psi^i_\alpha\,\langle\chi_+\rangle\,\Psi^{i\alpha} \\
& +\tilde z\ \bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\,\langle\chi_+\rangle\,\Psi^i_\beta\Big] + (\mu\leftrightarrow\nu)\,, \qquad (15)
\end{aligned}
$$

where $A^{[\alpha}B^{\beta]} = A^\alpha B^\beta - A^\beta B^\alpha$, $A^{(\alpha}B^{\beta)} = A^{\%alpha}B^\beta + A^\beta B^\alpha$, $\tilde z = z + z' + nzz'$ and n is spacetime dimension.

The action of Eq. (6) leads to the following expression for the EMT in flat spacetime (we have dropped terms involving $h_2, h_3, h_7, h_8, h_9, h_{14}$ and h_{15}, because they do not give any contributions in our analysis of the delta GFFs):

$$
\begin{aligned}
T^{(2)}_{\pi\Delta,b,\mu\nu} = & \ h_1\left(\eta_{\mu\nu}\partial_\lambda\partial^\lambda - \partial_\mu\partial_\nu\right)\bar\Psi^i_\alpha\Psi^{i\alpha} \\
& +\frac{h_4}{2}\Big[\partial^\lambda\partial_\lambda\left(\bar\Psi^i_\nu\Psi^i_\mu\right) + \eta_{\mu\nu}\partial^\alpha\partial^\beta\left(\bar\Psi^i_\beta\Psi^i_\alpha\right) \\
& -\partial^\lambda\partial_\mu\left(\bar\Psi^i_{(\lambda}\Psi^i_{\nu)}\right)\Big] + ih_5\Big[\partial^\lambda\partial_\lambda\left(\bar\Psi^i_\alpha\gamma_\mu\overleftrightarrow{D}_\nu\Psi^{i\alpha}\right) \\
& +\eta_{\mu\nu}\partial^\kappa\partial^\beta\left(\bar\Psi^i_\alpha\gamma_\beta\overleftrightarrow{D}_\kappa\Psi^{i\alpha}\right) - \partial^\lambda\partial_\mu\left(\bar\Psi^i_\alpha\gamma_{(\lambda}\overleftrightarrow{D}_{\nu)}\Psi^{i\alpha}\right)\Big] \\
& +\frac{ih_6}{2}\Big[\partial^\lambda\partial_\lambda\left(\bar\Psi^{i\alpha}\gamma_\mu\overrightarrow{D}_\alpha\Psi^i_\nu - \bar\Psi_\nu\gamma_\mu\overleftarrow{D}_\alpha\Psi^{i\alpha}\right) \\
& +\eta_{\mu\nu}\partial^\kappa\partial^\beta\left(\bar\Psi^{i\alpha}\gamma_\beta\overrightarrow{D}_\alpha\Psi^i_\kappa - \bar\Psi^i_\kappa\gamma_\beta\overleftarrow{D}_\alpha\Psi^{i\alpha}\right) \\
& -\partial^\lambda\partial_\mu\left(\bar\Psi^{i\alpha}\gamma_{(\lambda}\overrightarrow{D}_\alpha\Psi^i_{\nu)} - \bar\Psi^i_{(\nu}\gamma_{\lambda)}\overleftarrow{D}_\alpha\Psi^{i\alpha}\right)\Big] \\
& +ih_{10}\,\partial^\kappa\partial^\beta\left(\bar\Psi^i_\kappa\sigma_{\beta\nu}\Psi^i_\mu - \bar\Psi^i_\mu\sigma_{\beta\nu}\Psi^i_\kappa\right) \\
& +\frac{ih_{11}}{2}\partial^\kappa\partial^\beta\Big[\bar\Psi^i_\kappa\gamma_\beta\overrightarrow{D}_\mu\Psi^i_\nu - \bar\Psi^i_\kappa\gamma_\nu\overrightarrow{D}_\beta\Psi^i_\mu \\
& +\bar\Psi^i_\mu\gamma_\nu\overrightarrow{D}_\beta\Psi^i_\kappa - \bar\Psi^i_\nu\gamma_\beta\overrightarrow{D}_\mu\Psi^i_\kappa - \bar\Psi^i_\nu\gamma_\beta\overleftarrow{D}_\mu\Psi^i_\kappa \\
& +\bar\Psi^i_\mu\gamma_\nu\overleftarrow{D}_\beta\Psi^i_\kappa \\
& -\bar\Psi^i_\kappa\gamma_\nu\overleftarrow{D}_\beta\Psi^i_\mu + \bar\Psi^i_\kappa\gamma_\beta\overleftarrow{D}_\mu\Psi^i_\nu\Big] + \frac{ih_{12}}{2}\partial^\kappa\partial^\beta \\
& \times\Big[\bar\Psi^i_\mu\gamma_\beta\overrightarrow{D}_\kappa\Psi^i_\nu - \bar\Psi^i_\beta\gamma_\nu\overrightarrow{D}_\kappa\Psi^i_\mu + \bar\Psi^i_\beta\gamma_\nu\overrightarrow{D}_\mu\Psi^i_\kappa \\
& -\bar\Psi^i_\mu\gamma_\beta\overrightarrow{D}_\nu\Psi^i_\kappa - \bar\Psi^i_\nu\gamma_\beta\overleftarrow{D}_\kappa\Psi^i_\mu + \bar\Psi^i_\mu\gamma_\nu\overleftarrow{D}_\kappa\Psi^i_\beta - \bar\Psi^i_\kappa\gamma_\nu\overleftarrow{D}_\mu\Psi^i_\beta \\
& +\bar\Psi^i_\kappa\gamma_\beta\overleftarrow{D}_\nu\Psi^i_\mu\Big] + \frac{h_{13}}{2}\partial^\kappa\partial^\beta\Big[\eta_{\mu\nu}\bar\Psi^i_\beta\Psi^i_\kappa - \bar\Psi^i_\beta\gamma_\nu\gamma_\kappa\Psi^i_\mu \\
& -\bar\Psi^i_\mu\gamma_\beta\gamma_\nu\Psi^i_\kappa + \bar\Psi^i_\mu\gamma_\beta\gamma_\kappa\Psi^i_\nu\Big] + (\mu\leftrightarrow\nu)\,. \qquad (16)
\end{aligned}
$$

The covariant derivatives D acting on spin-1/2 and spin-3/2 fields in $T^{(\pi N)}_{\mu\nu}$, $T^{(\pi\Delta)}_{\mu\nu}$ and $T^{(\Delta)}_{\mu\nu}$ coincide with ∇ in Eq. (7) with $g_{\mu\nu} = \eta_{\mu\nu}$, i.e. $\Gamma^\beta_{\mu\nu} = \omega^{ab}_\mu = 0$.

The above expressions of the EMT can be used for the calculations of various matrix elements between states containing one nucleon and/or delta resonance and an arbitrary number of pions at low energies. Below we consider the corrections to the GFFs of the delta resonance at leading one-loop order.

3 One-loop corrections to the gravitational form factors of the delta resonances

In this section we calculate leading one-loop contributions to the matrix elements of the EMT for the one-particle states of the delta resonance We extract these matrix elements from the residues of Green's functions at complex poles of the initial and final four-momenta squared, corresponding to the unstable delta-states [36].

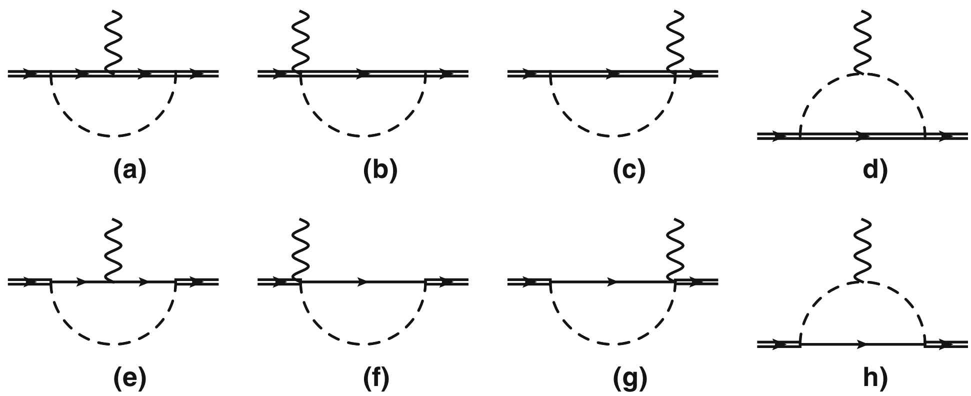

Fig. 1 One-loop diagrams contributing to the one-particle matrix elements of the EMT for the delta resonances. Dashed, solid and double lines correspond to pions, nucleons and delta resonances, respectively. The wiggly line indicates the EMT insertion

The one-loop diagrams contributing to our calculations are shown in Fig. 1. We apply the so called ϵ-counting scheme (also called the small scale expansion),[2] i.e. the pion lines count as of chiral order minus two, the nucleon and delta lines have order minus one, interaction vertices originating from the effective Lagrangian of order N count also as of chiral order N and the vertices generated by the EMT have the orders corresponding to the number of quark mass factors and derivatives acting on the pion fields. Derivatives acting on the nucleon and delta fields count as of chiral order zero. The momentum transfer between the initial and final states of the delta resonance also counts as of chiral order one, therefore in those terms of EMT which contain full derivatives, these derivatives count as of chiral order one. Integration over loop momenta is counted as of chiral order four. The delta-nucleon mass difference also counts as of order one within the ϵ-counting scheme.

Since we are interested in the delta matrix elements of order three in the chiral expansion, we need vertices with two delta lines, generated by the EMT, up to third order. From the effective Lagrangian specified above, we have obtained these vertices from the expressions of the EMT for the zeroth, first and second chiral orders, while there are no tree-order contributions at third order. Simple power counting arguments show that for all one-loop diagrams except (f) we only need vertices up to order one. Naively it seems that for diagrams of topology (a) and (f) we need also pion-baryon-baryon vertices of chiral order two, because the gravitational-source-baryon-baryon vertices originating from the EMT starts with chiral order zero. However, the leading contribution of the diagrams with the mentioned zeroth order vertices is exactly canceled by the wave function renormalization constant of the delta resonance multiplying the tree-order diagrams. Therefore, the formally zeroth order vertices in effect start contributing as vertices of order one. As a result, the diagrams with the pion-baryon-baryon vertex of order two only start contributing at chiral order four. For this reason we do not consider such diagrams in this work. It is understood that the above described power counting for loop diagrams is realized as the result of our manifestly Lorentz-invariant calculations only after performing an appropriate renormalization. To get rid off the divergent parts and power counting violating pieces from the expressions of one-loop diagrams we apply the EOMS scheme of Refs. [38,39].

3.1 Gravitational form factors of the delta resonance

The matrix element of the total EMT for the delta resonances can be parameterized in terms of seven form factors [11,13][3]:

$$
\begin{aligned}
&\langle p_f, s_f | T_{\mu\nu} | p_i, s_i \rangle \\
&= -\bar{u}_{\alpha'}(p_f, s_f) \Bigg[\frac{P^\mu P^\nu}{m} \left(\eta^{\alpha'\alpha} F_{1,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{1,1}(t) \right) \\
&\quad + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{4m} \left(\eta^{\alpha'\alpha} F_{2,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{2,1}(t) \right) \\
&\quad + \frac{i}{2m_\Delta} P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho \left(\eta^{\alpha'\alpha} F_{4,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{4,1}(t) \right) \\
&\quad - \frac{1}{m_\Delta} \left(\eta^{\alpha\{\mu}\Delta^{\nu\}}\Delta^{\alpha'} + \eta^{\alpha'\{\mu}\Delta^{\nu\}}\Delta^\alpha - 2\eta^{\mu\nu}\Delta^\alpha\Delta^{\alpha'} \right. \\
&\quad \left. - \Delta^2 \eta^{\alpha\{\mu}\eta^{\nu\}\alpha'} \right) F_{5,0}(t) \Bigg] u_\alpha(p_i, s_i), \qquad (17)
\end{aligned}
$$

where m_Δ is the physical mass of the delta resonances (we work in isospin symmetric limit), (p_i, s_i) and (p_f, s_f) are the momenta and polarizations of the incoming and outgoing particles, respectively, and $P = (p_i + p_f)/2$, $\Delta = p_f - p_i$, $t = \Delta^2$.

The tree-order diagrams contributing to the matrix element of the EMT up to third chiral order yield the following contributions to the formfactors:

[2] For an alternative power counting in an EFT with delta resonances, see Ref. [37].

[3] Straightforward calculation of the matrix elements in Eq. (17) within chiral EFT leads to nine invariant structures. It can, however, be shown that only seven structures are independent [11]. The two redundant structures can be eliminated using on-shell identities as given in Refs. [11,27].

$$
\begin{aligned}
F_{1,0,\text{tree}}(t) &= 1-\frac{t}{m_\Delta^2}+\frac{t\,(2h_5 m_\Delta+2h_{10}-h_{13})}{m_\Delta}\\
&\quad -\frac{(-2h_6+2h_{11}+h_{12})\,t^2}{2m_\Delta^2},\\
F_{1,1,\text{tree}}(t) &= -4-4m_\Delta\,(h_{12}m_\Delta-2h_{10}+h_{13})\\
&\quad +(4h_6-2\,(2h_{11}+h_{12}))\,t,\\
F_{2,0,\text{tree}}(t) &= -2-4\,(2h_1-2h_{10}+h_{13})\,m_\Delta\\
&\quad +(2h_6-2h_{11}-h_{12})\,t,\\
F_{2,1,\text{tree}}(t) &= 0,\\
F_{4,0,\text{tree}}(t) &= \frac{3}{2}-\frac{t}{2m_\Delta^2}+t\left(\frac{h_{10}}{m_\Delta}-\frac{h_{13}}{2m_\Delta}+h_5-h_6\right.\\
&\quad \left.+h_{11}+\frac{h_{12}}{2}\right)-\frac{(-2h_6+2h_{11}+h_{12})\,t^2}{4m_\Delta^2},\\
F_{4,1,\text{tree}}(t) &= -2-2m_\Delta\,(h_{12}m_\Delta-2h_{10}+h_{13})\\
&\quad +(2h_6-2h_{11}-h_{12})\,t,\\
F_{5,0,\text{tree}}(t) &= -\frac{1}{2}+\frac{1}{2}\,(h_4+4h_{10}-h_{13})\,m_\Delta\\
&\quad +\frac{1}{4}\,(2h_6-2h_{11}-h_{12})\,t,
\end{aligned}
\tag{18}
$$

where the h_i-terms are generated by the EMT of Eq. (16).

In the calculations of loop diagrams shown in Fig. 1, we apply dimensional regularization (see, e.g., Ref. [40]) and use the program FeynCalc [41,42]. The one-loop expressions of the form factors are too lengthy to be shown explicitly. They are available from the authors upon request.

To get rid of the power-counting violating contributions we split the bare low-energy parameters as the renormalized ones and counterterms. We specify the finite parts of counterterms by applying the EOMS scheme with the remaining renormalization scale chosen as $\mu = m_N$, where m_N is the mass of the nucleon. The one-loop finite parts of counter terms δh_i are given by:

$$
\begin{aligned}
\delta h_1 &= \frac{\delta h_{12} m_N}{2}-\frac{\left(1575\,g_{\pi N\Delta}^2+172\,g_1^2\right)m_N}{207360\pi^2F^2},\\
\delta h_4 &= -2\,\delta h_{10}-\delta h_{12}m_N-\frac{m_N(45\,g_{\pi N\Delta}^2+2336\,g_1^2)}{51840\pi^2F^2},\\
\delta h_5 &= -\frac{\delta h_{12}}{2}-\frac{11(135\,g_{\pi N\Delta}^2+124\,g_1^2)}{207360\pi^2F^2},\\
\delta h_{13} &= 2\,\delta h_{10}-\delta h_{12}m_N+\frac{\left(9\,g_{\pi N\Delta}^2+490\,g_1^2\right)m_N}{10368\pi^2F^2}.
\end{aligned}
\tag{19}
$$

After renormalization, we obtain the following expressions for the GFFs at $t = 0$, expanded in powers of the pion mass and the delta-nucleon mass difference in the chiral limit δ:

$$
\begin{aligned}
F_{1,0,\text{loop}}(0) &= 0,\\
F_{1,1,\text{loop}}(0) &= -\frac{5g_1^2 m_N(3\pi M-49\delta)}{648\pi^2F^2}\\
&\quad +\frac{g_{\pi N\Delta}^2 m_N}{144\pi^2F^2\left(M^2-\delta^2\right)}\left(-53\delta^3+24\delta\left(M^2-\delta^2\right)\ln\frac{M}{m_N}+24i\pi\delta^2\sqrt{\delta^2-M^2}-12i\pi M^2\sqrt{\delta^2-M^2}\right.\\
&\quad \left.+12\left(M^2-2\delta^2\right)\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}+53\delta M^2\right)+\mathcal{O}(\epsilon^2),\\
F_{2,0,\text{loop}}(0) &= -\frac{g_1^2 m_N(25\pi M-1068\delta)}{2160\pi^2F^2}\\
&\quad +\frac{g_{\pi N\Delta}^2 m_N\left(29\delta+48\delta\ln\frac{M}{m_N}-48i\pi\sqrt{\delta^2-M^2}+48\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{288\pi^2F^2}+\mathcal{O}(\epsilon^2),\\
F_{2,1,\text{loop}}(0) &= -\frac{g_1^2 m_N^3}{54\pi F^2 M}+\frac{g_{\pi N\Delta}^2 M m_N^3\sqrt{\frac{\delta^2}{M^2}-1}\left(\ln\left(\sqrt{\frac{\delta^2}{M^2}-1}+\frac{\delta}{M}\right)-i\pi\right)}{15\pi^2F^2\left(M^2-\delta^2\right)}+\mathcal{O}(\epsilon^0),\\
F_{4,0,\text{loop}}(0) &= 0,\\
F_{4,1,\text{loop}}(0) &= \frac{5g_{\pi N\Delta}^2 m_N^2}{576\pi^2F^2}+\frac{235g_1^2m_N^2}{2592\pi^2F^2}+\mathcal{O}(\epsilon),\\
F_{5,0,\text{loop}}(0) &= -\frac{g_1^2 m_N(150\pi M-3323\delta)}{25920\pi^2F^2}\\
&\quad +\frac{g_{\pi N\Delta}^2 m_N\left(5\delta+2\delta\ln\frac{M}{m_N}-2i\pi\sqrt{\delta^2-M^2}+2\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{96\pi^2F^2}+\mathcal{O}(\epsilon^2).
\end{aligned}
\tag{20}
$$

Next, we define the slopes of the GFFs by writing the form factors as:

$$F_{i,j}(t) = F_{i,j}(0) + s_{F_{i,j}} t + \mathcal{O}(t^2). \tag{21}$$

Calculating loop contributions to these quantities and expanding in powers of the pion mass and δ we obtain

$$\begin{aligned}
s_{F_{1,0}} &= \frac{g_1^2(8\delta - 255\pi M)}{10368\pi^2 F^2 m_N} \\
&+ \frac{g_{\pi N\Delta}^2}{576\pi^2 F^2 m_N \left(M^2-\delta^2\right)}\Bigg(25\delta(\delta^2 - M^2) + 24\delta\left(\delta^2 - M^2\right)\ln\frac{M}{m_N} - 12i\pi(2\delta^2 - M^2)\sqrt{\delta^2 - M^2} \\
&-12\left(M^2 - 2\delta^2\right)\sqrt{\delta^2 - M^2}\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M}\Bigg) + \mathcal{O}(\epsilon^2), \\
s_{F_{1,1}} &= \frac{g_1^2 m_N}{432\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\delta^3 + M^2\left(-\delta + i\pi\sqrt{\delta^2 - M^2}\right) - M^2\sqrt{\delta^2 - M^2}\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M}\right)}{120\pi^2 F^2\left(M^2 - \delta^2\right)^2} + \mathcal{O}(\epsilon^0), \\
s_{F_{2,0}} &= -\frac{g_1^2 m_N}{108\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M} - i\pi\right)}{60\pi^2 F^2\sqrt{\delta^2 - M^2}} + \mathcal{O}(\epsilon^0), \\
s_{F_{2,1}} &= \frac{g_{\pi N\Delta}^2 m_N^3\left(-\delta^3 + M^2\left(\delta - i\pi\sqrt{\delta^2 - M^2}\right) + M^2\sqrt{\delta^2 - M^2}\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M}\right)}{140\pi^2 F^2 M^2\left(M^2 - \delta^2\right)^2} - \frac{g_1^2 m_N^3}{504\pi F^2 M^3} + \mathcal{O}(\epsilon^{-2}), \\
s_{F_{4,0}} &= \frac{g_{\pi N\Delta}^2\left(163\delta^2 - 96\left(M^2 - \delta^2\right)\ln\frac{M}{m_N} - 96i\pi\delta\sqrt{\delta^2 - M^2} + 96\delta\sqrt{\delta^2 - M^2}\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M} - 163M^2\right)}{4608\pi^2 F^2\left(M^2 - \delta^2\right)} \\
&+ \frac{g_1^2\left(877 - 150\ln\frac{M}{m_N}\right)}{25920\pi^2 F^2} + \mathcal{O}(\epsilon), \\
s_{F_{4,1}} &= 0 + \mathcal{O}(\epsilon^{-1}), \\
s_{F_{5,0}} &= \frac{g_1^2 m_N}{3456\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\ln\frac{\delta + \sqrt{\delta^2 - M^2}}{M} - i\pi\right)}{960\pi^2 F^2\sqrt{\delta^2 - M^2}} + \mathcal{O}(\epsilon^0).
\end{aligned} \tag{22}$$

Note that the tree-order contributions to slopes are included in Eq. (18). Notice further that these expressions as well as the expressions in Eq. (20) contain (unphysical) singularities in the $M \to \delta$ limit, which is due to effect first uncovered, to the best of our knowledge, in Ref. [43] for the electromagnetic interaction. In particular, in this limit, the one-photon exchange approximation fails completely and one needs to resum the series of multiphoton exchange diagrams. The same is also true for the gravitational interaction manifested in the above mentioned singularities.

The full one-loop contributions to GFFs consist of real and imaginary parts also for spacelike transfer momenta. This is because of the deltas being unstable particles. Due to the lack of empirical data, from which we could fix the free parameters contributing at tree order in Fig. 2, we plot the contributions of renormalized (subtracted) one-loop diagrams as functions of $Q^2 = -t$.

4 Summary

In the framework of chiral EFT for pions, nucleons and delta resonances interacting with an external gravitational field, we calculated the leading one-loop contributions to the one-particle matrix elements of the EMT for delta resonances and extracted the corresponding contributions to the gravitational form factors. To get rid of the UV divergences and power counting violating pieces from the loop diagrams we applied the EOMS renormalization scheme of Refs. [38,39]. Since the delta resonances are unstable particles, the loop contributions to the gravitational form factors are complex valued quantities also for space-like momentum transfers. This is manifested in contributions of one-loop diagrams to the real and imaginary parts of the GFFs, see Fig. 2. Unfortunately, no empirical data are available, from which we could deduce the low-energy constants contributing at tree order. We also give analytic expressions of GFFs at zero transfers and slope parameters in the form of expansions in small parameters. Notice that the value as well as the sign of the D-term of delta resonances cannot be predicted/calculated within chiral EFT.

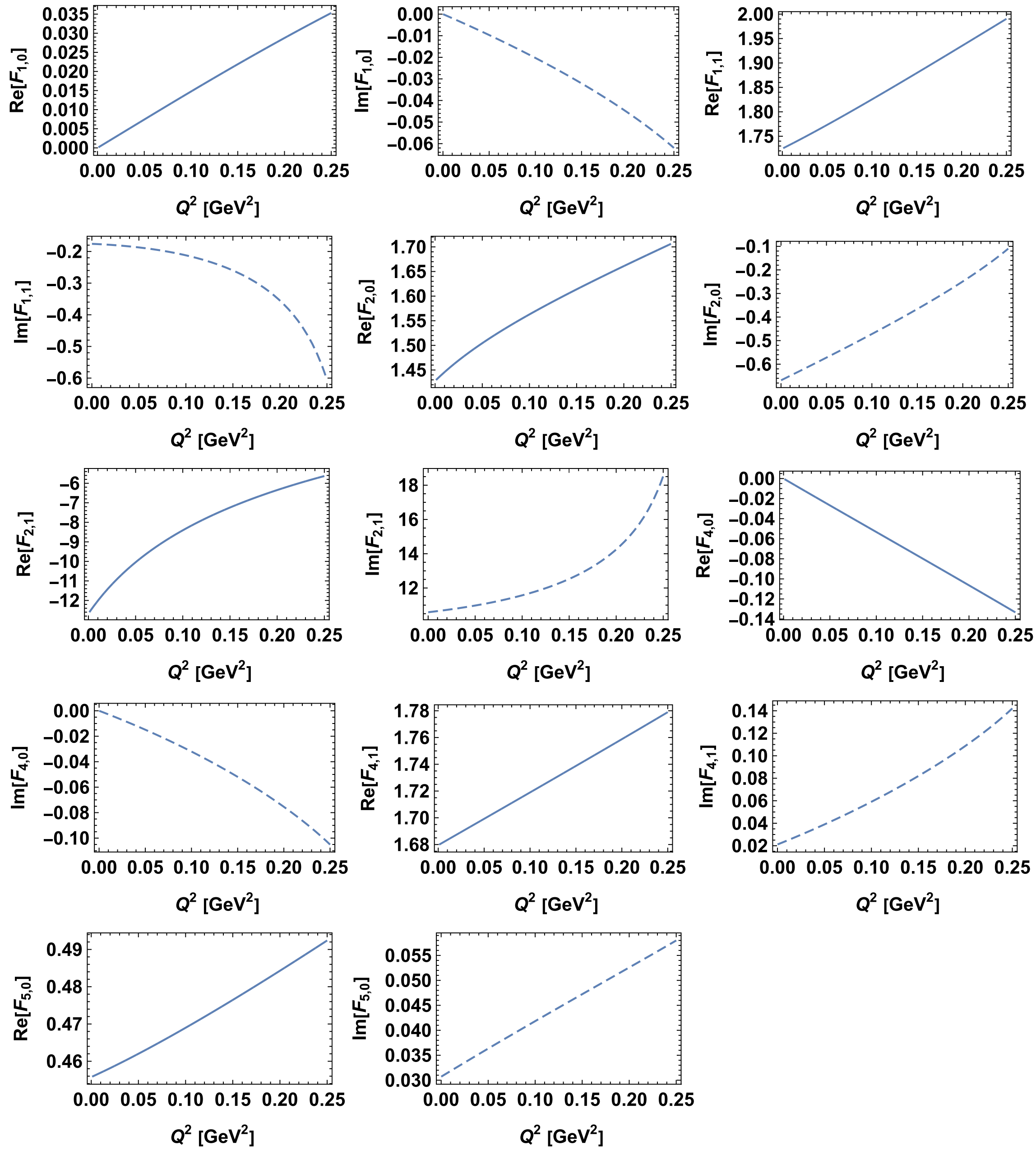

Fig. 2 Contributions of renormalized (subtracted) one-loop diagrams to the GFFs of the delta resonances. Solid and dashed lines correspond to real and imaginary parts, respectively

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