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Relativistic spin dynamics conditioned by dark matter axions

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Abstract The relativistic spin dynamics defined by the pseudoscalar field of dark matter axions is rigorously determined. The relativistic Hamiltonian in the Foldy–Wouthuysen representation is derived. It agrees with the previously obtained nonrelativistic Hamiltonians and the relativistic classical estimation of the axion wind effect. In the relativistic Hamiltonian, the biggest term describes the extraordinary (three orders of magnitude) enhancement of the axion wind effect in storage ring experiments as compared with experiments with immobile particles. This term defines the spin rotation about the longitudinal axis. The effects caused by the axion-induced oscillating EDM and the axion wind consist in the spin rotations about the different horizontal axes and phases of stimulating oscillations differ by $\pi/2$. Experiments in a search for dark matter axions has been discussed. The two experimental designs for axion search experiments in storage rings have been elaborated.

The axion is a hypothetical neutral pseudoscalar particle with a very low mass $m_a < 10^{-2}\mathrm{eV}/c^2$. Its existence has been postulated by Peccei and Quinn [1,2] to resolve the strong CP problem in quantum chromodynamics. If the axion exists, it can be a possible component of cold dark matter. The mass of dark matter axions is restricted by astrophysical observations [3,4] and cosmological arguments [5–7]. An experimental search for axions being quanta of the pseudoscalar field can result in a discovery of fifth force. The successful search needs the creation of the corresponding relativistic theory of spin dynamics.

We use the standard denotations of Dirac matrices (see, e.g., Ref. [8]) and the system of units $\hbar = 1$, $c = 1$. We include $\hbar$ and c explicitly when this inclusion clarifies the problem.

CP-noninvariant interactions caused by dark matter axions are time-dependent. Like photons, moving axions form a wave which pseudoscalar field reads

$$a(\boldsymbol{r}, t) = a_0 \cos\left(E_a t - \boldsymbol{p}_a \cdot \boldsymbol{r} + \phi_a\right). \tag{1}$$

Here $E_a = \sqrt{m_a^2 + \boldsymbol{p}_a^2}$, $\boldsymbol{p}_a$, and m_a are the energy, momentum, and mass of axions [9]. The Earth motion through our galactic defines its velocity relative to dark matter, $V \sim 10^{-3}c$. Therefore, $|\boldsymbol{p}_a| \approx m_a V$ [11] and axions and axion-like particles have momenta of the order of $|\nabla a| \sim 10^{-3}\dot{a}c$.

Axions manifest themselves in interactions with photons, gluons, and fermions (first of all, nucleons). The corresponding contributions to the Lagrangian density are defined by [10–15]

$$\begin{aligned} &\mathcal{L}_\gamma = -\tfrac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\widetilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \boldsymbol{E}\cdot\boldsymbol{B}, \\ &\mathcal{L}_g = \tfrac{g_{QCD}^2 C_g}{32\pi^2 f_a} a G_{\mu\nu}\widetilde{G}^{\mu\nu}, \qquad \mathcal{L}_N = g_{aNN}\gamma^\mu\gamma^5\partial_\mu a, \end{aligned} \tag{2}$$

where $g_{QCD}^2/(4\pi) \sim 1$ is the coupling constant for the color field, C_g, $g_{a\gamma\gamma}$ and g_{aNN} are model-dependent constants, f_a is the constant of interaction of axions with matter (axion decay constant), and the tilde denotes a dual tensor.

The approach based on Eq. (2) and following Eq. (4) is now generally accepted [9–26]. However, we should note the existence of alternative approaches to the description of axions. In particular, we mention Refs. [27–31] defining axion-induced particle motion and spin evolution in electromagnetic and gravitational fields. The paper [28] also considers the spin precession of relativistic charged particles in storage rings. While one follows the generally accepted approach, other approaches should also be taken into consideration under performing an experiment.

The constant f_a, the axion mass and their contribution to dark matter depend on time of a phase transition in the expanding Universe. This circumstance defines possibilities of a search for dark matter axions. If one supposes that all dark matter with the local density ρ_{DM} is formed by axions, the axion field amplitude (1) reads [11]

$$a_0 = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a}. \tag{3}$$

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The axion–photon interaction leads to the inverse Primakoff effect [32,33]. This effect is observable and axions are searched in experiments with haloscopes. We can mention the experiments ADMX [34,35], ADMX-HF [36–38], HAYSTAC [39], CAPP [40–42], and RADES [43] and the reviews [12,13,15,16,31,44,45]. We can also note experiments with helioscopes (see, e.g., Ref. [46,47]) and the light-shining-through-a-wall experiments [48].

Another possibility to search for axions is connected with their influence on the spin. Nucleon spins are effected, besides the direct interaction defined by the Lagrangian density $\mathcal{L}_N$, by the axion–gluon interaction. Its *CP*-noninvariance results in an appearance of oscillating nucleon electric dipole moments (EDMs) which are proportional to the axion field. The corresponding Lagrangian density can be written as follows [11,12,14,17]:

$$\mathcal{L}_{aEDM} = -\tfrac{i}{2} g_d a \sigma^{\mu\nu}\gamma^5 F_{\mu\nu}, \tag{4}$$

where the EDM is equal to $d_a = g_d a = g_d a_0 \cos[m_a(t - \boldsymbol{V}\cdot\boldsymbol{r}) + \phi_a]$.

We underline that Eq. (1) is used in this paper only for a specification of frequency of the axion field oscillation.

The two spin-dependent Lagrangian densities, $\mathcal{L}_{aEDM}$ and $\mathcal{L}_N$, substantially differ. The former Lagrangian density defines an interaction of oscillating EDM induced by the axion field with external electric and magnetic fields. The latter one depends only on the axion field and is independent of other fields. In the static limit, the corresponding Hamiltonians are given by (see Refs. [9,49]):

$$\mathcal{H}_{aEDM} = -d_a \boldsymbol{\sigma}\cdot\boldsymbol{E}, \qquad \mathcal{H}_N = g_{aNN}(\boldsymbol{\sigma}\cdot\nabla a), \tag{5}$$

where $\boldsymbol{\sigma}$ is the Pauli matrix. The Hamiltonian $\mathcal{H}_{aEDM}$ defines the EDM effect. The Hamiltonian $\mathcal{H}_N$ is proportional to the momentum of axions (axion wind effect). If there is a constant magnetic field and the spin rotation frequency in this field is close to the axion field frequency $\omega_a = m_a c^2/\hbar$, the magnetic resonance leading to the spin depolarization takes place. The method of magnetic resonance has been used in the CASPEr experiment [12,18,45] and other spin experiments with immobile nucleons and nuclei [14,19]. Effects caused by both the oscillating EDM and the axion wind have been studied. In Ref. [50], a search for the interaction of antiprotons with dark matter axions has been fulfilled in the Penning trap. Similar searches can be carried out for other antiparticles, such as positrons and antimuons [50]. Other experiments searching for the axion wind have been analyzed in Ref. [9].

It has been obtained in Refs. [10,22] by the elimination method (developed by Pauli [51]) that the *nonrelativistic* Hamiltonian $\mathcal{H}_N$ contains also the term proportional to the *particle* velocity $\boldsymbol{v}$:

$$\mathcal{H}_N^{(1)} = g_{aNN}\dot{a}(\boldsymbol{\sigma}\cdot\boldsymbol{v}). \tag{6}$$

We underline that taking into account the term (6) in the experiment [50] can be necessary despite of nonrelativistic velocities of antiprotons in this experiment.

The new approach to the experimental search for dark matter axions based on storage ring experiments has been proposed relatively recently [23]. Since particles in such experiments are relativistic, this approach needs a construction of relativistic quantum-mechanical dynamics of spin. A purely classical consideration shows that the axion wind effect depends on an axion motion relative to a spinning particle [24]. In the rest frame of a relativistic particle, the Hamiltonian $\mathcal{H}_N$ is proportional to the factor $\nabla a \sim \gamma m_a \boldsymbol{v}_a^{(0)} a$, where $\boldsymbol{v}_a^{(0)}$ is the axion velocity in this frame. After a transition to the laboratory frame, the γ factor however cancels due to the time transformation [24]. Thus, the axion wind effect amplifies $v_a^{(0)}/V \approx v/V \sim c/V \sim 10^3$ times in storage ring experiments. This extraordinary enhancement is the main preference of the use of storage ring experiments for an axion search. However, any relativistic quantum-mechanical analysis has not been made in Ref. [24] and other papers. Some experimental methods of the axion search in storage rings have been elaborated in Refs. [17,23–26].

Needed derivations are fulfilled in the present study. The Lagrangian $L = \overline{\psi}\mathcal{L}\psi$ describing electromagnetic and pseudoscalar interactions of a Dirac particle is defined by

$$\begin{aligned}\mathcal{L} &= \gamma^\mu(i\hbar\partial_\mu - eA_\mu) - m + \tfrac{\mu'}{2}\sigma^{\mu\nu}F_{\mu\nu} - i\tfrac{d}{2}\sigma^{\mu\nu}\gamma^5 F_{\mu\nu}\\ &\quad + g_{aNN}\gamma^\mu\gamma^5\Lambda_\mu, \qquad \Lambda_\mu = (\Lambda_0, \boldsymbol{\Lambda}) = \partial_\mu a = (\dot{a}, \nabla a),\\ \gamma^5 &= \begin{pmatrix} 0 & -1\\ -1 & 0\end{pmatrix},\end{aligned} \tag{7}$$

where μ' and $d = d_0 + g_d a$ are the anomalous magnetic and electric dipole moments and d_0 is the constant EDM.

The corresponding Hamiltonian in the Dirac representation is given by

$$\begin{aligned}\mathcal{H} &= \beta m + \boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) + e\Phi + \mu'(i\boldsymbol{\gamma}\cdot\boldsymbol{E} - \boldsymbol{\Pi}\cdot\boldsymbol{B})\\ &\quad - d(\boldsymbol{\Pi}\cdot\boldsymbol{E} + i\boldsymbol{\gamma}\cdot\boldsymbol{B}) + g_{aNN}(-\gamma^5\Lambda_0 + \boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}).\end{aligned} \tag{8}$$

This equation agrees with the nonrelativistic equation (5).

In the general case, it is convenient to present the Dirac Hamiltonian as follows:

$$\begin{aligned}&\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \qquad \beta\mathcal{M} = \mathcal{M}\beta,\\ &\beta\mathcal{E} = \mathcal{E}\beta, \qquad \beta\mathcal{O} = -\mathcal{O}\beta.\end{aligned} \tag{9}$$

The even operators $\mathcal{M}$ and $\mathcal{E}$ and the odd operator $\mathcal{O}$ are diagonal and off-diagonal in two spinors, respectively. Equation (9) is applicable for a particle with any spin if the number of components of a corresponding wave function is equal to $2(2s+1)$, where s is the spin quantum number.

To obtain the Schrödinger form of relativistic quantum mechanics (QM), one needs to fulfill the relativistic Foldy-Wouthuysen (FW) transformation (see Refs. [52–56] and ref-

erences therein). This is a relativistic extension of the transformation found by Foldy and Wouthuysen [57]. The FW transformation is one of basic methods of contemporary QM. An importance of the FW transformation for physics has significantly increased nowadays due to a great progress of the art of analytic computer calculations. A great advantage of the FW representation is a simple form of operators corresponding to classical observables. In this representation, the Hamiltonian and all operators are even, i.e., block-diagonal (diagonal in two spinors). The passage to the classical limit usually reduces to a replacement of the operators in quantum-mechanical Hamiltonians and equations of motion with the corresponding classical quantities. The possibility of such a replacement, explicitly or implicitly used in practically all works devoted to the FW transformation, has been rigorously proved for the stationary case in Ref. [58,59]. The probabilistic interpretation of wave functions lost in the Dirac representation is restored in the FW one [60]. Thanks to these properties, the FW representation provides the best possibility of obtaining a meaningful classical limit of relativistic QM not only for the stationary case but also for the nonstationary one [54]. The FW transformation is applicable not only for Dirac fermions but also for particles with any other spins [61,62].

Since our analysis is relativistic, we take into account all axion-dependent terms in Eq. (8). The corresponding terms in Eq. (9) are given by

$$\mathcal{M} = m - \mu'\boldsymbol{\Sigma}\cdot\boldsymbol{B} - d\boldsymbol{\Sigma}\cdot\boldsymbol{E}, \qquad \mathcal{E} = e\Phi + g_{aNN}\boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda},$$
$$\mathcal{O} = \boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) + i\mu'\boldsymbol{\gamma}\cdot\boldsymbol{E} - id\boldsymbol{\gamma}\cdot\boldsymbol{B} - g_{aNN}\gamma^5\Lambda_0. \tag{10}$$

The relativistic FW-transformed Hamiltonian is defined by [53,55]:

$$\mathcal{H}_{FW} = \beta\epsilon + \mathcal{E} + \frac{1}{4}\left\{\frac{1}{2\epsilon^2+\{\epsilon,\mathcal{M}\}}, (\beta\,[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},\mathcal{F}]])\right\}, \quad \mathcal{F} = \mathcal{E} - i\hbar\frac{\partial}{\partial t}, \quad \epsilon = \sqrt{\mathcal{M}^2+\mathcal{O}^2}. \tag{11}$$

It should be emphasized that Eq. (11) is applicable for arbitrarily strong external fields [53,55]. However, we can restrict ourselves by the weak-field approximation.

The straightforward derivation results in the relativistic FW Hamiltonian

$$\mathcal{H}_{FW} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3,$$
$$\mathcal{H}_1 = \beta\epsilon' + e\Phi + \frac{1}{4}\left\{\left(\frac{\mu_0 m}{\epsilon'+m}+\mu'\right)\frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma}\cdot[\boldsymbol{\pi}\times\boldsymbol{E}] - \boldsymbol{\Sigma}\cdot[\boldsymbol{E}\times\boldsymbol{\pi}]\right)\right\} - \frac{1}{2}\left\{\left(\frac{\mu_0 m}{\epsilon'}+\mu'\right), \boldsymbol{\Pi}\cdot\boldsymbol{B}\right\}$$
$$+\frac{\mu'}{4}\left\{\frac{1}{\epsilon'(\epsilon'+m)}, \left[(\boldsymbol{B}\cdot\boldsymbol{\pi})(\boldsymbol{\Pi}\cdot\boldsymbol{\pi}) + (\boldsymbol{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{B})\right]\right\},$$
$$\mathcal{H}_2 = -d\boldsymbol{\Pi}\cdot\boldsymbol{E} + \frac{d}{4}\left\{\frac{1}{\epsilon'(\epsilon'+m)}, \left[(\boldsymbol{E}\cdot\boldsymbol{\pi})(\boldsymbol{\Pi}\cdot\boldsymbol{\pi}) + (\boldsymbol{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{E})\right]\right\} - \frac{d}{4}\left\{\frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma}\cdot[\boldsymbol{\pi}\times\boldsymbol{B}] - \boldsymbol{\Sigma}\cdot[\boldsymbol{B}\times\boldsymbol{\pi}]\right)\right\}, \qquad \boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}, \tag{12}$$

where $\mathcal{H}_1$ defines the CP-conserving part of the total Hamiltonian $\mathcal{H}_{FW}$, $\mu_0 = e\hbar/(2m)$ is the Dirac magnetic moment, and $\epsilon' = \sqrt{m^2+\boldsymbol{\pi}^2}$. We have omitted spin-independent terms defining contact interactions. The quantity $\mathcal{H}_2$ which has been first derived in Ref. [49] characterizes the contribution of the EDM. We should note that $\mathcal{H}_2$ contains additional terms as compared with Ref. [49] owing to the noncommutativity of a with the operators ∇ and $\partial/(\partial t)$. However, these terms are negligible due to a very small mass of the axion and are also omitted.

The new terms describing the interaction with the axion wind are given by

$$\mathcal{H}_3 = \frac{g_{aNN}}{2}\left\{\frac{\boldsymbol{\Pi}\cdot\boldsymbol{p}}{\epsilon'}, \Lambda_0\right\} + \frac{g_{aNN}}{2}\left[\left\{\frac{m}{\epsilon'}, \boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}\right\} + \frac{(\boldsymbol{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}(\boldsymbol{p}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Lambda}\cdot\boldsymbol{p})\frac{(\boldsymbol{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}\right]. \tag{13}$$

The first and biggest term in $\mathcal{H}_3$ obtained in the present study leads to an extraordinary increase of the axion wind effect in storage ring experiments by three orders of magnitude as compared with experiments with immobile particles. This term is essentially the relativistic generalization of earlier results obtained in Refs. [10,22].

Equations (12), (13) confirm Eqs. (5), (6) previously obtained in the nonrelativistic approximation. Equation (13) also approves the relativistic classical consideration of the axion wind effect presented in Ref. [24].

In the semiclassical approximation, the angular velocity of the spin rotation has the form

$$\boldsymbol{\Omega}_s = \boldsymbol{\Omega}_1 + \boldsymbol{\Omega}_2 + \boldsymbol{\Omega}_3,$$
$$\boldsymbol{\Omega}_1 = -\frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)\boldsymbol{B} - \frac{(g-2)\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) - \left(g-2+\frac{2}{\gamma+1}\right)(\boldsymbol{\beta}\times\boldsymbol{E})\right\},$$
$$\boldsymbol{\Omega}_2 = -\frac{e\eta}{2m}\left[\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right],$$
$$\boldsymbol{\Omega}_3 = 2g_{aNN}\left[\Lambda_0\boldsymbol{\beta} + \frac{\boldsymbol{\Lambda}}{\gamma} + \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{\Lambda})\boldsymbol{\beta}\right]$$
$$= -2g_{aNN}\sin(m_a t - \boldsymbol{p}_a\cdot\boldsymbol{r} + \phi_a)a_0\left[m_a\boldsymbol{\beta} - \frac{\boldsymbol{p}_a}{\gamma} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{p}_a)\boldsymbol{\beta}\right], \tag{14}$$

where $\boldsymbol{\Omega}_1$ is determined by the Thomas-Bargmann-Michel-Telegdi equation [63–65], $\boldsymbol{\beta} = \boldsymbol{v}/c$, and the factors $g = 4(\mu_0+\mu')m/e$ and $\eta = 4(d_0+g_d a)m/e$ are introduced.

The analysis of spin precession of a relativistic charged particle in storage rings has been carried out in Ref. [28].

However, the result obtained disagrees with Eq. (14). This is very natural because of the use of alternative approach in Ref. [28]. As a rule, experimentalists apply the generally accepted approach.

Equation (14) describes the spin motion relative to the Cartesian coordinate axes. In accelerators and storage rings, the spin dynamics is usually considered relative to the beam direction. In this case, the angular velocity of spin rotation is given by [66] (see also Refs. [67,68] and references therein)

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_s - \boldsymbol{\Omega}_c = -\frac{e}{m}\left[G\boldsymbol{B} - \left(G - \frac{1}{\gamma^2-1}\right)(\boldsymbol{\beta}\times\boldsymbol{E})\right] + \boldsymbol{\Omega}_2 + \boldsymbol{\Omega}_3, \tag{15}$$

where $G = (g-2)/2$ and $\boldsymbol{\Omega}_c$ defines the angular velocity of the cyclotron motion. The (quasi)magnetic resonance takes place when Ω (more precisely, the vertical component of $\boldsymbol{\Omega}$) is close to the axion field frequency ω_a. In this resonance, the interactions caused by the oscillating EDM and the axion wind effect act like a spin flipper. As follows from Eq. (15), the resonance occurs under a definite connection between the axion mass and fields in storage rings. A comparison of oscillating terms in $\boldsymbol{\Omega}_2$ and $\boldsymbol{\Omega}_3$ shows that the relative importance of the axion wind effect is greater and less for a greater and a less axion mass, respectively.

It is now planned to search for dark matter axions in the frameworks of two EDM experiments which will be fulfilled in a magnetic storage ring with magnetic focusing [69–71] and in an electric storage ring with electric [72] or magnetic [73,74] focusing. The use of a storage ring with main E and B fields has also been proposed [23]. The JEDI collaboration will search for dark matter axions in a magnetic storage ring (see Refs. [23,25,26]). Varying the quantity Ω allows one to scan a studied interval of axion masses. In Refs. [23,25,26], only the effect of oscillating EDM has been considered because the large first term in Eq. (13) was unknown. Other terms in this equation are comparatively small and oscillate with the frequencies $\Omega \pm \Omega_c$. In a magnetic storage ring, $(e/m)GB \approx \omega_a$ near the resonance. We expect that the contributions of the oscillating EDM and the axion wind effect into the spin rotation are comparable under this condition. The axion wind effect conditioned by the first term in Eq. (13) can be even dominant. In the considered case, the quantities $g_d m$ and g_{aNN} are comparable. In units used, $e = \sqrt{\alpha} \approx 1/\sqrt{137}$. The constants g_d and g_{aNN} are not dimensionless. Evidently, the effects caused by the oscillating EDM and the axion wind consist in the spin rotations about the different axes (radial and azimuthal, respectively) and phases of stimulating oscillations differ by $\pi/2$. The theory developed in Ref. [75] is perfectly applicable for a quantitative description of the *general* case of (quasi)magnetic resonance, when both axion-induced effects are taken into account. The more special case when the spin rotates about one axis has been considered with respect to the EDM experiment in Refs. [76,77].

It is proposed in Refs. [23,25,26] to use a simultaneous decrease of the vertical magnetic field and the beam momentum for scanning studied intervals of spin rotation frequencies and axion masses. The ring radius should be conserved. Another possibility [23] consists in keeping the vertical magnetic field and applying a radial electric field. Simultaneous variations of the latter field and the beam momentum with the conservation of the ring radius allow one to search for the axions in a studied interval of their masses. The use of a radiofrequency Wien filter at the frequency of the sidebands of the axion and spin rotation $(g-2)$ frequency, $f_{WF} = (\omega_a \pm \Omega)/(2\pi)$, has been recently proposed in Ref. [17]. This proposition relates only to the search for an oscillating EDM because added oscillating electromagnetic fields are useless for a search for the axion wind effect. The product of two sinusoidal functions, $a(t)\boldsymbol{B}(t)$ or $a(t)\boldsymbol{E}(t)$, gives an oscillation at a needed angular frequency Ω.

It should be noted that ultralight axions ($f_a = \omega_a/(2\pi) < 10^{-4}$ Hz or $f_a < 1/\tau_{coh}$, where $\tau_{coh} \sim 10^4$ s is the spin coherence time) can be discovered by methods used for a search of static EDMs. One should only take into account that the axion wind effect, unlike the EDM one, leads to the spin turn about the longitudinal direction [23,24].

An advantage of the experiment in an electric storage ring is an opportunity to search for light and ultralight axions [23]. The experiment is fulfilled with protons. The proton spin is frozen ($\Omega = 0$) at the momentum $p = mc/\sqrt{G}$ [see Eq. (15)]. For light axions, freezing the spin allows one to search for the resonance $\Omega = \omega_a$ at rather small frequencies $1/\tau_{coh} < f_a < 1$ Hz. Higher frequencies are also manageable. It has been previously proposed [23] to use the (quasi)magnetic resonance. In this case, the radial electric field and the beam momentum can be varied and the ring radius should be conserved. Certainly, the above-mentioned radiofrequency Wien filter at the frequency $f_{WF} = (\omega_a \pm \Omega)/(2\pi)$ can also be applied. In this case, the mass of searched axions is not limited. We underline a high sensitivity of the axion search in an electric storage ring.

We have different propositions for conducting experiments on the axion search in storage rings. We propose to use a Wien filter with constant (but changeable) $\boldsymbol{B}$ and $\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B}$ fields. A previously proposed change of a main (magnetic or electric) field and the beam momentum *during one run* covering a studied interval of frequencies is not fully appropriate. Essential changes of main field and beam parameters can uncontrollably vary beam dynamics during the run. As a result, they distort the resonance curve and decrease the experimental sensitivity. On the contrary, Wien filters do not affect beam dynamics and influence only the spin motion. Of course, they can cause the well-known systematic error [78]. Indeed, an appearance of a vertical electric field in a ring with

magnetic focusing influences the magnetic moment and can lead to a false EDM effect [78]. However, this possible effect is always proportional to a vertical electric field in the Wien filter and cannot imitate a resonance on the resonance curve characterizing the run. Moreover, the effect can be significantly decreased [74,78].

We can add that fields in a Wien filter can be inverted ($\boldsymbol{B} \to -\boldsymbol{B},\ \boldsymbol{E} \to -\boldsymbol{E}$).

Our another proposition consists in an application of a radiofrequency cavity stimulating coherent synchrotron oscillations of the beam at the frequency $f_{rf} = 2\pi|\omega_a \pm \Omega|$. After the modulation, the beam velocity reads

$$v(t) = v_0 + \Delta v_{rf} \cos(\omega_{rf} t + \phi_{rf}) + \Delta v_{fr} \cos(\omega_{fr} t + \phi_{fr}), \tag{16}$$

where Δv_{rf}, $\omega_{rf} = 2\pi f_{rf}$, and ϕ_{rf} are the amplitude, angular velocity, and phase of the modulated synchrotron velocity oscillations and Δv_{fr}, ω_{fr}, and ϕ_{fr} are corresponding parameters of the free synchrotron velocity oscillations. The achievable amplitude of the velocity oscillations is 3.5×10^6 m/s [79] and therefore $\Delta v_{rf}/v_0$ is greater than 0.01. This ratio is roughly two orders of magnitude greater than the corresponding ratio of the effective field amplitude in the radiofrequency Wien filter to the main field in the storage ring in the method [17]. The latter quantities should be averaged over the ring circumference. Therefore, the application of our proposition for an axion-induced EDM search in a magnetic storage ring should increase the sensitivity two orders of magnitude as compared with the method [17]. Furthermore, our proposition (unlike the method [17]) ensures excellent conditions for a search for the axion wind effect. In an electric storage ring, our proposition can also be successfully used for such a search but is useless for a search for the axion-induced EDM.

Our second proposition is very similar to the resonance method developed by Y. Orlov [80,81] (see also Ref. [79]) for the EDM experiment. He proposed to stimulate coherent synchrotron oscillations at the angular frequency Ω. However, his method has not been accepted by the (sr)EDM collaboration since a nonlinearity of beam dynamics in storage rings can result in unwanted spin resonance effects. Unlike this method, similar oscillations at the angular frequency $|\omega_a \pm \Omega|$ cannot lead to such effects. We should specify that the storage (observation) time should not be less than $2\pi/\omega_a$.

A choice between different experimental propositions depends on a sensitivity and systematic errors and needs further investigations.

In summary, we have rigorously described the relativistic spin dynamics defined by the pseudoscalar field of dark matter axions. The relativistic Hamiltonian in the FW representation has been derived for the first time. It agrees with the known *nonrelativistic* Hamiltonians [9,10,22] and the *relativistic* classical estimation of the axion wind effect [24]. The biggest term in this Hamiltonian extraordinarily (by three orders of magnitude) amplifies the axion wind effect in storage rings as compared with experiments with immobile particles. It describes the spin rotation about the longitudinal axis with the angular frequency ω_a defined by the axion mass. The effects caused by the axion-induced oscillating EDM and the axion wind consist in the spin rotations about the different axes (radial and azimuthal, respectively) and phases of stimulating oscillations differ by $\pi/2$. The experimental search for dark matter axions has been discussed. The two experimental designs for axion search experiments in storage rings have been elaborated.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Associated data cannot be added because parameters of axion-matter interactions are still unknown.]

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