



Complexity and the departure from spheroidicity

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Abstract In this work we investigate the effect of spheroidicity on complexity in self-gravitating, static systems. Utilizing the anisotropic generalisation of the Vaidya–Tikekar superdense stellar model, we employ the complexity factor to connect the spheroidal parameter to the pressure anisotropy and density inhomogeneity. Our findings indicate that deviation from spherical symmetry lead to a higher degree of complexity within the stellar body. We further show the equation of state of parameter is inherently linked to the complexity factor thus demonstrating that the nature of matter in self-gravitating bounded systems plays an important role in the effect of pressure anisotropy and density inhomogeneities.

1 Introduction

In recent times, gravitational collapse has been vastly covered in great detail, which is the consequence of the work done by Oppenheimer and Snyder [1]. In various studies, gravitational collapse has been investigated with dissipation along with important components, namely stability, luminosity and temperature profiles. Key findings were established by Vaidya [2] in the form of an exterior spacetime which consisted of radiation. In different gravity theories, numerous equations of state (EoS) were utilized in order to complete or more so close off systems which resulted in a variety of models for gravitational collapse. At the forefront of gravitational collapse, the main focus has always been on the Einstein field equations. Considering the field equations, new exact solutions were obtained for a five-dimensional spherically sym-

metric static distribution of a perfect fluid. The Frobenius method was employed to obtain solutions which were presented in the form of an infinite series. This study showed that the models they looked at, all produced a barotropic equation of state of which all energy conditions were satisfied. The reader is directed to the works by Krupanandan and Hansraj [3] for other models which consisted of anisotropy and an MIT bag model equation of state $p_r = \gamma\rho - \beta$, which was the first attempt in higher curvature gravity. In an alternative study, solutions to the field equations and the deceleration parameter was found by Maharaj and Naidoo [4]. This was done by strategically using a form for the Hubble parameter which resulted in establishing various number of solutions to the Einstein field equations with variable cosmological constant and variable gravitational constant. Much success has been achieved when dealing with the field equations, which has given more insight into models in higher dimensions [5]. The EoS plays a key role in various models when dealing with stability and instability. Govender et al. [6] employed a perturbative approach in their model in which they studied the effects that an EoS has on the dynamical stability or instability of a spherically symmetric star undergoing dissipative collapse. In the analysis carried out, a linear EoS of the form $p_r = \gamma\mu$ was utilized on the perturbed radial pressure and density. This led to determining the gravitational behaviour of the collapsing star. Bogadi et al. [7] investigated a static model in which they used a perturbative approach. This led to the model changing into a dynamical heat dissipating model, which resulted in more insight into dynamical (in)stability. We shift our focus onto complexity, more so, a new definition for complexity, has been noted by Herrera [8]. In the investigation carried out, the definition for complexity was established for static and spherically symmetric self-gravitating systems. The complexity factor is a consequence of the orthogonal splitting of the Riemann tensor with regards to general relativity. Herrera highlighted that the definition

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that was established was a result of the measure of the departure with regards to the Tolman mass, with respect to its value for a zero complexity system. An important finding from this work gives rise to interior solutions to the Einstein equations which satisfy the case of zero complexity. A vital aspect prior to the complexity factor is that of the statistical complexity determined by Lopez-Ruiz et al. [9]. In the study conducted by Lopez-Ruiz et al., the statistical measure of complexity was focused on in which it was applied to numerous physical situations. Within the framework of classical physics, the idea of complexity can be used to, among other things, describe systems such as the perfect crystal, and the ideal gas, which both exhibit the factors of “order” (in terms of the symmetric atomic arrangement), and “information” which for the ideal gas, refers to the fact that it can be in any of its allowed states while having the same probability distribution. The much earlier work of Lopez-Ruiz et al. [9] presents complexity as the concept of “disequilibrium” which gives a measure of “how far away” a system is from all of its equiprobable allowed states. Additionally, they demonstrated that a more formal definition of complexity based on its quantization can make it possible for “disequilibrium” and “information” to be reconciled. This development subsequently enabled Sanudo and Pacheco [10], and de Avellar et al. [11] to put forward a definition of complexity in the context of relativistic fluids in strong gravitational fields. In these studies the probability distribution is replaced by the energy density of the fluid system. However, this construct may be somewhat inadequate for the desired objective since it is understood that other matter variables such as the pressure, and the components of the energy–momentum tensor are also crucial for a physically reasonable fluid in general relativity. Hence, the study presented in Herrera [8] have provided a vital improvement for static fluid spheres, by clearly defining complexity as a measurable quantity that is strongly coupled to density inhomogeneities and the spacetime geometry. The concept of the complexity factor for static anisotropic self-gravitating source with the generalized $f(R)$ metric was utilized in the works by Abbas and Nazar [12]. This concept was established by performing the orthogonal splitting of the Riemann-Christoffel tensor which resulted in scalar functions which in turn, led to the vanishing of the complexity condition for the self-gravitating system. The basis for this investigation was the formulation of three ordinary differential equations, which was determined by static spherically symmetric anisotropic self-gravitating geometry together with five unknown variables ($v, \mu, \rho, P_r, P_\perp$). The condition that was used ($Y_{TF} = 0$), gave rise to the vanishing complexity factor condition in which with an addition of another condition, they were able to achieve the result produced by Herrera [8]. New models have been determined for compact stars. Maurya et al. [13, 14] investigated a model for a spherically symmetric anisotropic compact star. In this investigation, a

new anisotropic solution for Einstein’s field equations was determined for embedding class one which proved viable as they uncovered a solution that is relevant to realistic objects such as Her X-1 and RXJ 1856-37. In their analysis, they were able to find various relationships namely, the surface and central density as well as the effective mass. Yousaf et al. [15] pursued the concept of complexity for static cylindrically symmetric matter configurations in $f(R, T, RT)$ gravity. This was achieved by coupling an irrotational static cylindrical spacetime together with a locally anisotropic relativistic fluid. Once again, the orthogonal splitting of the Riemann curvature tensor was carried out which emphasizes the rich results that can be achieved from the orthogonal splitting. In another study, the role of pressure anisotropy on relativistic compact stars were looked at. This study was carried out by Maurya et al. [16], of which the model was based on a spherically symmetric relativistic object consisting of anisotropic particle pressure. They employed a spatial metric potential of Korkina and Orlyanskii [17] in order to solve the Einstein field equations and thereafter commenting on the compact strange stars such as the Her X-1 and SMC X-1. Numerous models have given insight into compact stars with comparisons being made with realistic known objects [18, 19], though in much recent times, work has been considered in Einstein–Maxwell–Gauss–Bonnet (EMGB) theory of gravitation [20]. Herrera [21] engaged in the complexity for dynamical spherically symmetric dissipative self-gravitating fluid distributions. In this study, it was shown that the dissipative and non-dissipative cases were presented separately. For the dissipative case, the fluid distribution satisfies the vanishing complexity factor condition. The results obtained from various studies [8, 22, 23] produced different scalar functions with regards to the orthogonal splitting of the Riemann tensors. Bhar and Govender [20] expanded compact stars by using the Krori–Barua (KB) ansatz in conjunction with a linear equation of state of the form $p_r = \beta\rho - \gamma$ in which they obtained new exact solutions of the EMGB field equations. In their successful attempt, it was established that the Gauss-Bonnet terms have no effect on the various quantities namely, the density, the pressure and the anisotropy. We now focus our attention on Vaidya–Tikekar superdense stars. Thirukkanesh et al. [24] investigated the anisotropic generalization of Vaidya–Tikekar superdense stars. In their analysis carried out, they formulated a method to make an anisotropic generalization of the Vaidya–Tikekar superdense star model. This was achieved by solving the anisotropic Einstein field equations, which was done by carefully selecting hypergeometric functions for the gravitational potential and anisotropy. In conclusion, for this model, they established the basis for how the anisotropy affects the physical behaviour of a compact star. Sharma et al. [25] made use of the Vaidya–Tikekar metric ansatz in their study, which allowed for them to produce a new class of interior solutions which describe

a static and spherically symmetric anisotropic matter distribution that reveals a linear EoS. The solution obtained is utilized to determine the effects of deviation from sphericity of 3-surface geometry on the mass-radius relationship.

This paper is structured as follows. In Sect. 2 we focus on complexity in static spherically symmetric models in which we configure the quantities. In Sect. 3 we present our static compact stellar model, and employ an equation of state. In Sect. 4 we consider the evolution of the complexity factor in which we present various graphs for the behaviour of Y_{TF} . Concluding comments are made in Sect. 5.

2 Complexity in static spherically symmetric models

$$ds^2 = -e^{v(r)}(r)dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

We assume that the stellar composition is anisotropic in nature and accordingly the energy–momentum tensor of the stellar fluid is taken as

$$T_b^a = \text{diag}(-\rho, p_r, p_t, p_t), \tag{2}$$

where ρ , p_r and p_t are the energy density, radial pressure and tangential pressure, respectively. The fluid four-velocity \mathbf{u} is comoving and is given by

$$u^a = e^{-v/2}\delta_0^a. \tag{3}$$

The Einstein field equations for the line element (1) are obtained as (in system of units having $8\pi G = 1$ and $c = 1$)

$$\rho = \frac{(1 - e^{-\lambda})}{r^2} + \frac{\lambda'e^{-\lambda}}{r}, \tag{4}$$

$$p_r = \frac{v'e^{-\lambda}}{r} - \frac{(1 - e^{-\lambda})}{r^2}, \tag{5}$$

$$p_t = \frac{e^{-\lambda}}{4} \left(2v'' + v'^2 - v'\lambda' + \frac{2v'}{r} - \frac{2\lambda'}{r} \right), \tag{6}$$

where primes represent differentiation with respect to the radial coordinate r .

Herrera was the first to establish the definition for the complexity factor in static and spherically symmetric self-gravitating systems. The complexity factor is a scalar function which is given by Y_{TF} in which the anisotropy Π can be measured as well as the energy density gradient $\rho' = \frac{d\rho}{dr}$. In much recent works, the complexity concept was expanded to the setting of dynamical spherically symmetric dissipative self-gravitating fluid distributions. Using the definition constructed by Herrera, we utilize Y_{TF} as the complexity factor for spherically symmetric static self-gravitating systems (4-6) given by

$$Y_{TF} = 8\pi\Pi - \frac{4\pi}{r^3} \int_0^r x^3 \rho'(x) dx, \tag{7}$$

where $\Pi = p_r - p_t$.

Herrera in his study, concluded that the complexity factor Y_{TF} represents the influence of local anisotropy of pressure and density inhomogeneity on the Tolman mass (m_T). The relationship between Y_{TF} and the Tolman m_T is given by

$$m_T = (m_T)_\Sigma \left(\frac{r}{R} \right)^3 + r^3 \int_r^R \frac{e^{(v+\lambda)/2}}{x} Y_{TF} dx, \tag{8}$$

where M_T is the total Tolman mass of the fluid sphere of radius R ($R = r_\Sigma$). In a study carried out by Herrera [8], numerous observations have been made with regards to the complexity factor. Firstly, the complexity factor vanishes for isotropic fluid and all other configurations for which both the terms in (7) identically vanish. Secondly, considering the basis, it is clear that there exists many configurations with vanishing complexity factors and lastly, it is important to understand that the contribution of pressure anisotropy to Y_{TF} is local in nature but differs with regards to the case for density energy inhomogeneity.

3 Static compact stellar model

In a recent paper [26], the Vaidya and Tikekar (VT) [27] superdense stellar model was generalised to include pressure anisotropy and a linear equation of state of the form

$$p_r = \alpha\rho - \beta, \tag{9}$$

where α is a constant and β encodes the surface density. The solution is given in a simple closed form

$$v = \int r e^{\lambda} \left[\frac{(\alpha + 1)(1 - e^{-\lambda})}{r^2} + \frac{\alpha\lambda'e^{-\lambda}}{r} - \beta \right] dr, \tag{10}$$

$$e^{\lambda(r)} = \frac{1 - K(r^2/L^2)}{1 - (r^2/L^2)}. \tag{11}$$

which has a spheroidal geometry characterized by the parameters L (which has the dimension of a length) and K (which denotes departure from spherical geometry). The metric will be spherically symmetric and well behaved for $r < L$ and $K < 1$. For $K = 1$, the spheroidal 3-space degenerates into flat 3-space. In the case $K = 0$ (i.e., $b = L$), it becomes spherical. We note that the metric with $K = 0$ generates the constant density solution commonly referred to as the Schwarzschild interior solution. Following this approach, making use of (11), (4) and (5) into the equation of state, we solve the system and obtain

$$e^v = A \left(1 - \frac{r^2}{L^2} \right)^n \left(1 - \frac{Kr^2}{L^2} \right)^\alpha e^{K(L^2-r^2)\beta/2}, \tag{12}$$

where A is a constant of integration and we have defined

$$n = \frac{1}{2} \left[-1 - 3\alpha + L^2\beta + K(1 + \alpha - L^2\beta) \right].$$

The thermodynamical quantities assume the following form [26]

$$\rho = \frac{(1 - K) \left(3 - \frac{Kr^2}{L^2} \right)}{L^2 \left(1 - \frac{Kr^2}{L^2} \right)^2}, \tag{13}$$

$$p_r = \alpha\rho - \beta, \tag{14}$$

$$p_t = \frac{(K - 1)}{4(L^2 - r^2)(L^2 - Kr^2)^3(L^2 - KR^2)^4} \sum_{i=1}^7 F_i, \tag{15}$$

where each F_i ($i = 1, 2, \dots, 7$) are functions of parameters K, R, L, α and r . It is important to note that β is not a free parameter for this situation and is given by $\beta = \alpha\rho_R$, for which we have R as the radius of the star and ρ_R is the surface density which is given by

$$\rho_R = \frac{(1 - K) \left(3 - \frac{KR^2}{L^2} \right)}{L^2 \left(1 - \frac{KR^2}{L^2} \right)^2}. \tag{16}$$

In the case in which we have $\rho = \rho_R$, the radial pressure vanishes i.e., $p_r(r = R) = 0$, which defines the boundary.

We determine the central density from Eq. (13) which is given by

$$\rho_c = \frac{3(1 - K)}{L^2}. \tag{17}$$

From Eq. (17), we notice that we have the restriction $K < 1$. In our model, we consider anisotropy, thus we can determine the anisotropy by utilizing the following

$$\Pi = p_t - p_r. \tag{18}$$

In conclusion, we establish that the anisotropy vanishes at the centre by setting $r = 0$, we have $\Pi = 0$.

We now consider a sphere with mass $m(r)$ and radius r , where the mass inside the sphere is given by

$$m(r) = \frac{1}{2} \int_0^r \omega^2 \rho(\omega) d\omega. \tag{19}$$

Following this, we integrate equation (19) and obtain

$$m(r) = \frac{(1 - K)r^3}{2(L^2 - Kr^2)}. \tag{20}$$

Once again, we note that by setting $r = 0$, we have $m(r) = 0$.

In order to fix the constants not appearing in the solution we match the interior solution to the vacuum Schwarzschild exterior

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2$$

$$+ r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{21}$$

across the boundary R . The matching conditions determine the constants as

$$L = \frac{R\sqrt{2KM - KR + R}}{\sqrt{2M}}, \tag{22}$$

where $M = m(R)$ is the total mass and

$$A = \frac{L^2 - R^2}{L^2 - KR^2} \times \left(1 - \frac{R^2}{L^2} \right)^{\frac{1}{2} \left[-\alpha(K-3) + \frac{\alpha(K-1)^2 L^2 (KR^2 - 3L^2)}{(L^2 - KR^2)^2} - K + 1 \right]} \times \left(1 - \frac{KR^2}{L^2} \right)^{-\alpha} \times \exp \left[- \frac{KG(K, \alpha, R, L, r)}{2(L^2 - KR^2)^2} \right], \tag{23}$$

where

$$G(K, \alpha, R, L, r) = 2(L^2 - KR^2)^2 \times (L^2 - r^2) \left[\frac{\alpha(K-1)(KR^2 - 3L^2)}{2(L^2 - KR^2)^2} \right] + \alpha(K - 1)r^2(KR^2 - 3L^2) - \alpha(K - 1)R^2(KR^2 - 3L^2). \tag{24}$$

With the above model parameters, Sharma et al. [26] demonstrated that this solution obeyed the requirements for regularity, stability and causality. This model was also utilised by Bogadi et al. [7] and Govender et al. [28] to investigate the interplay among the hydrostatic, gravitational and anisotropic forces once a star loses equilibrium. It was further shown that the Tolman–Oppenheimer–Volkoff forces are sensitive to the spheroidal parameter, K as the collapse proceeds towards the horizon formation.

4 Evolution of the complexity factor

We now investigate the evolution of the complexity factor as a function of the radial coordinate. In order to do this we make use of (7) and the model discussed in Sect. 3. In order to achieve this, we vary the spheroidal parameter K for different fluid configurations (different α) and plot Y_{TF} as a function of r . The values of α chosen correspond to dark energy ($\alpha = -1$), dark radiation fluid ($\alpha = -\frac{1}{3}$), dust ($\alpha = 0$), radiation fluid ($\alpha = \frac{1}{3}$) and stiff fluid ($\alpha = 1$). While these matter profiles correspond to cosmological fluids, they have been discussed within astrophysical contexts (see for example [29–33]). From Fig. 1, we observe that for $K = -2$, the complexity factor is an increasing function from the center towards the boundary with the stiff EoS

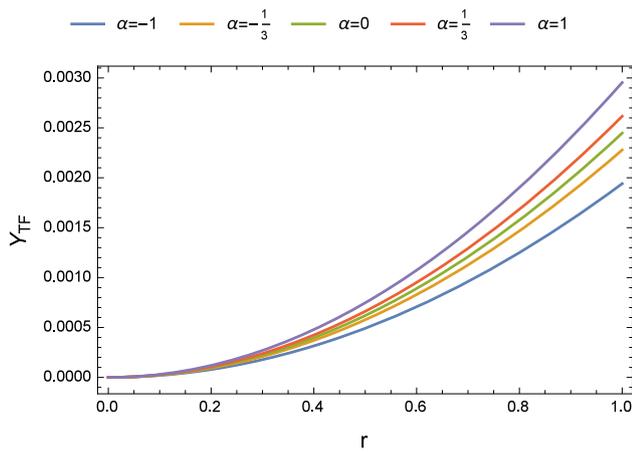


Fig. 1 Y_{TF} vs the radial coordinate r for $K = -2$ and varying α

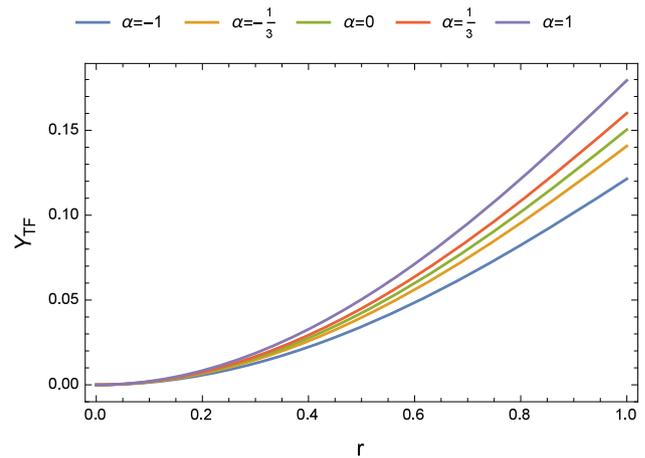


Fig. 2 Y_{TF} vs the radial coordinate r for $K = -20$ and varying α

dominating the others. For $K = -20$, (Fig. 2), we observe a similar trend in Y_{TF} , ie: monotonically increasing function of the radial coordinate as the boundary is approached. The magnitude of Y_{TF} has increased by a factor of 100 with an increase in 10% in the spheroidal parameter. In Fig. 3, we observe the evolution of the complexity factor as a function of the radial coordinate when $K = -1000$. It is interesting to observe an initial increase in Y_{TF} up to a radius, $r = r_0$, beyond of which the complexity factor decreases smoothly towards the boundary. It appears that the nature of matter (determined by the EoS parameter, α) does not influence Y_{TF} . By writing $Y_{TF} = \Pi + w(r)$, where $\Pi = p_r - p_t$ and $w(r) = -\frac{4\pi}{r^3} \int_0^r x^3 \rho'(x) dx$, we have plotted the components of Y_{TF} in Figs. 4 and 5. In Fig. 4, we consider the evolution of complexity factor for a dust sphere ($\alpha = 0$) for different values of K . It is clear from Fig. 4 that the contributions from the density inhomogeneities dominate at each interior point of the stellar distribution. We also observe that an increase in the magnitude of the spheroidal parameter K , results in a larger Y_{TF} . The contributions from the pressure anisotropy are quenched by the density inhomogeneities. For a stiff fluid ($\alpha = 1$), Fig. 5 shows a similar trend in the competing factors of Y_{TF} . The density inhomogeneities grow with increasing K and dominate any contributions from Π . For the stiff EoS ($\alpha = 1$), we note that the contributions from pressure anisotropy are more significant then their counterparts for the dust sphere. Note that for $\alpha = 0$, the radial pressure vanishes at each interior point of the sphere. The anisotropy is driven solely by the tangential pressure. This implies that a vanishing complexity factor is achievable when the tangential stresses are balanced by the density inhomogeneity of the stellar fluid.

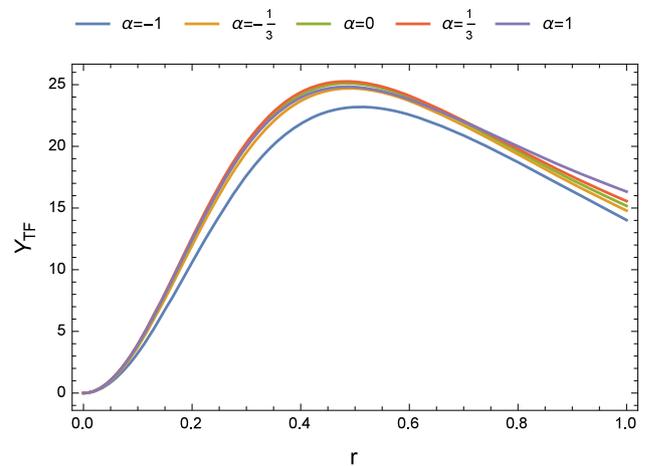


Fig. 3 Y_{TF} vs the radial coordinate r for $K = -1000$ and varying α

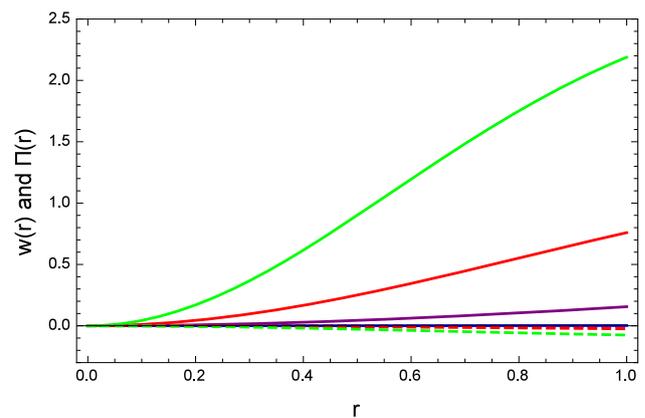


Fig. 4 $w(r)$ (solid lines) and $\Pi(r)$ (dashed lines) vs the radial coordinate r for fixed $\alpha = 0$ and $K = -2$ (Blue), $K = -20$ (Purple), $K = -50$ (Red), $K = -100$ (Green)

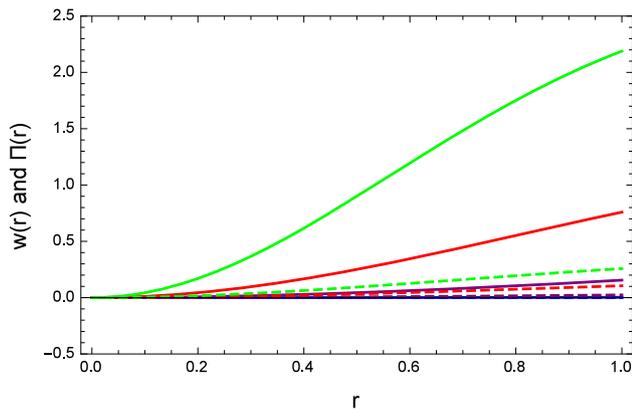


Fig. 5 $w(r)$ (solid lines) and $\Pi(r)$ (dashed lines) vs the radial coordinate r for fixed $\alpha = 1$ and $K = -2$ (Blue), $K = -20$ (Purple), $K = -50$ (Red), $K = -100$ (Green)

5 Concluding remarks

We have employed the idea of complexity in self-gravitating systems as defined by Herrera et al. [8,21], to investigate the effect of spheroidicity on pressure anisotropy and density inhomogeneity within the compact object. We employed the Vaidya–Tikekar superdense star to exploit the effect of departure from spherical symmetry on the complexity of the system. The results we have achieved firmly indicated that spheroidicity and the EoS parameter are inherently linked to pressure anisotropy and density inhomogeneity which drives complexity. We believe that our work sheds new light on the nature of matter and its link to complexity within an astrophysical setting. In a recent paper by Bogadi and Govender [34], the evolution of the complexity factor was investigated in a dynamical scenario in which an initial static stellar object described by Vaidya–Tikekar model used here undergoes dissipative gravitational collapse. It was shown that the complexity factor is affected by the spheroidal parameter, K particularly close to the time of formation of the horizon.

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