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A novel definition of complexity in torsion based theory

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Abstract Despite coming across quite effective definitions of complexity in terms of many modified theories of gravity, it still has a question about its existence in $f(T)$ gravity, where the torsion scalar T is accountable for gravitational impacts. The emergence of complexity factor is due to division of intrinsic curvature in an orthogonal way as described by Herrera (Phys Rev D 97:044010, 2018). To initiate the analysis, we reckon the interior region is like a spherically symmetric static configuration filled by the locally anisotropic fluid and exteriorly associated with a spherical hypersurface. In this framework, we acquire the $f(T)$ field equations and utilize the already formulated relationship between the intrinsic curvature and the conformal tensor to perform our analysis. We bring into action the definitions of the two frequently availed masses (Tolman and Misner–Sharp) for spherical composition and investigate the appealing correlation between them and the conformal tensor. The impact of the local anisotropy and the homogeneity and inhomogeneity of energy density has substantial importance in this regard. We build up some relation in terms of already defined variables and interpret the complexity as a single scalar Y_{TF}. It deduce that this factor vanished when the fluid content is homogenous and also when the impact of two anisotropic terms cancel out in the case of inhomogeneous fluid content. We determine a few definite interior solutions which fulfill the criterion of vanishing scalar Y_{TF}. Certain defined ideas in fulfillment of the vanishing complexity factor constraint, are applied for $f(T)$ gravity.

1 Introduction

Even though the general relativity (GR) is appraised as a successful theory of gravity, still, it encounters challenges both observationally, e.g., dark energy (DE), dark matter (DM), etc, and theoretically, e.g., quantization, singularity, etc. Numerous modified gravity theories (MGT's) as an appropriate strategy to resolve these oddities have been advised [2–5]. MGT's were analyzed by holding geometry unharmed and reshaping the Einstein-Hilbert action (EHA). MGT's should permit retrieving GR at small scales with the result that certain observational limitations are matched [6]. The most simplest and remarkable theory is $f(R)$ gravity [7,8] to approach. In which the function of curvature scalar is used as the Lagrangian density. In the same fashion, the teleparallel equivalent of general relativity (TEGR) is formed, in which the Lagrangian density shows its equivalence with the torsion scalar T. It is observed that the teleparallel gravity (TG) [9] and GR is identical at the level of their field equations. The modification of TG by taking equivalence of Lagrangian density with the function of torsion scalar, results in $f(T)$ gravity [10]. In contemplation to illustrate the inflation and the late time accelerated expansion of the universe, the $f(T)$ gravity is the best substitute, where Weitzenb$\ddot{o}$ck connection (which has non-zero value of torsion scalar) are utilized. The $f(T)$ theory owes second-order field equations which makes it a more straightforward and easy approach to handle than $f(R)$ theory.

Some diverse features of $f(T)$ gravity has been analyzed in literature [11–13]. In the new version of modified gravity, named $f(T)$, Yang [14] gave a review of three different kinds of $f(T)$ gravity and showed their significance to answer the cosmic acceleration including alluring attributes of them. Bejarano et al. [15] exhibit that null tetrads work as an important tool in $f(T)$ gravity. With the aid of these tetrads, they manifest the existence of Kerr geometry as the solution of a broad range of $f(T)$ families. Paliathanasis et al. [16] evaluated the analytical solutions for homogenous and isotropic spacetime which is filled with radiation and dust fluid in the background of cosmological $f(T)$ theory. Later on, they extended their case for anisotropic Bianchi-I spacetime and utilized the method of differential equation in

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terms of movable singularities for this purpose. They deduced that the solutions for isotropic spacetime are manifested in forms of the Laurent expansion, where they obtained a class of accurate Kasner-like solutions for anisotropic spacetime. Bhatti et al. [17–19] contributed to understanding the role of $f(T)$ theory on the stability of celestial objects supported by the approach of perturbation. They build up fundamental equations like field equations, junction, and dynamical equations in the background of $f(T)$ theory. They established the fact that the character of stiffness parameter on the stability of the celestial objects in $f(T)$ theory has considerable importance. Ren et al. [20] inspected the magnification and position of lensed images by using the covariant formulation of $f(T)$ gravity. They also determined the angle of defection in the lensing background. To achieve this objective, they first attained the solutions for the symmetric sphere and then computed the effects of lensing attributes in $f(T)$ gravity. The study of the neutron star is also a hot topic in literature. In $f(T)$ gravity, the contribution about the inspection of neutron star with the aid of model $f(T) = T(1 + \alpha T)$ is carried out by Lin et al. [21]. They found that in the case of a negative value of α, the large size of the matter is held by the neutron star.

The complexity of any system is a topic of vast analysis in all branches for decades. Many factors are associated to analyze the complexity of any system. The primary concept is related to measure the information and entropy of the structure confined within a system. In the study of massive objects, the idea of the complexity of the self-gravitating system is also analyzed. While in physics, whenever, we take into account a perfect crystal (which shows the periodic behavior and is ordered symmetrically), the isolated gas (which shows the disordering and maximum amount of information) are sort of complex system with zero complexity. LopezRuiz et al. [22] initiated the idea of disequilibrium for analyzing the complexity of the system. Basically, it is the measure of the "distance" from the equally probable dispersion of the attainable form of the system. By defining complexity as a combination of both of these terms, i.e., information and disequilibrium, they deduced that the idea of complexity dissolved in the case of an ideal gas and perfect crystal. After seeing deficiency in all concepts of complexity to study the self-gravitating system, Herrera [1] developed the new concept of complexity which is based on fluid components such as energy density, pressure, etc. In short, it is linked with inclusive features of the composition of the fluid. The complexity, in this case, is built with aid of the complexity factor, which is one of the structure scalars attained from the orthogonal division of the intrinsic curvature. Herrera et al. [23] further enlarged this idea for dissipative fluid content. They not merely analyzed the complexity of the system but also determined the condition for the progression design of minimal complexity. They found that in terms of dissipation, the fluid is shearing and geodesic, and there exists a variety of solutions.

To further evaluate the role of complexity on different geometries, Herrera et al. [24] utilized the axially symmetric geometry and found three factors responsible for complexity in this case. They established the relation between complexity and symmetry. They also attained few analytical answers in this peculiar case. Additionally, the idea of complexity to investigate the evolution of spherically symmetric non-static geometry either in terms of dissipation or non-dissipation is studied by Herrera et al. [25]. They adopted the condition of quasi-homologous which is a link between areal radius velocity and areal radius. They developed certain models and calculated their suitable implications for understating evolution. Contreras and Fuenmayor [26] inspected the stability of self-gravitating spherical objects in terms of gravitational cracking by taking into account this technique. They carried out a detailed analysis to review the impacts of compactness of the source and alternation in decoupling parameters on the radial force. Herrera et al. [27] expanded the idea of the complexity factor on the hyperbolically symmetric geometry and also analyzed the role of other structure scalars which are acquired by the orthogonal division of the intrinsic curvature. They took into account the Misner–Sharp mass along with the Tolman mass on this geometry. They ended with the conclusion that the Tolman mass indicates negative nature in this scenario. Many researchers utilized this approach of complexity not only in GR, but also evaluated results for different geometries in the background of MGT's. In background of $f(R, T, Q)$ (where R is a Ricci scalar, T is trace of stress-momentum tensor and the relation of Q is defined by $Q = R_{\mu\varrho}T^{\mu\varrho}$), the idea of Herrera is generalized by Yousaf et al. [28,29]. Foremost, they developed the field equations in the context of modified theory and later on studied the impacts of this theory on the complexity factor. They considered two different geometries to perform this analysis and also developed relations among the conformal scalar, Tolman mass, and complexity factor. Whereas the impact of complexity factor on the terms of $f(G)$ theory (where G is Gauss–Bonnet term) without or with charge is investigated by [30,31].

When energy is conserved in the stellar matter, then the correspondence between the stellar composition and the metric results in a famous equation labeled Tolman–Oppenheimer–Volkoff (TOV) equation. In addition, the equation of state (Eos) also has considerable importance in the investigation of stellar objects. The Eos must be necessarily anisotropic and it has considerable importance in attaining the solutions of hydrostatic equilibrium equations. For anisotropic fluids, the study of polytropic Eos [which tells about the relationship between density and pressure and whose solution results in a Lane–Emden equation (LEEq)] has gained much significance [32,33]. Chandrasekhar [34] was the pioneer to build up the polytropes in Newtonian theory with the aid of thermodynamical laws for spheres. To attain the new class of solutions for spherical geometry corresponding to different polytropic indexes, the contribution is made by Ngubelanga and Maharaj [35]. Thirukkanesh and Ragel [36] set up a scheme to transform the Einstein field equations and attained two models in an exact form with the aid of polytropic Eos. They determined that the results in terms of energy density and mass which are attained for index $n = 2$ have correspondence with the observed experimental results. Herrera and Barreto [37] provided a general formulation of the polytropic model for relativistic compact objects equipped with anisotropic fluid. They deduced that all types of polytropic Eos turn down into the LEEq in Newtonian approximation. Furthermore, they inspected the impact of pressure anisotropy, energy density, and Tolman mass on the relativistic structure. Thirukkanesh et al. [38] determined new class of solutions for spherical geometry which satisfied the polytropic Eos $P_r = \kappa\rho^{1+\frac{1}{n}} - \beta$. They found the solution when the geometry behaves parabolic and exhibited that in this particular celestial model the radial pressure commands the tangential one. Ramos et al. [39] examined the Karmarkar or class-I solutions subsidized by anisotropic polytropes. They attained solutions of the LEEq for a distinct set of variables in the non-isothermal and isothermal patterns. They calculated the Tolman mass and analyzed the impact of the Karmarkar condition on the Tolman mass and mass function in accordance. The motivation behind the manuscript and the addressed points are as below.

1. We build up the novel definition of complexity in the framework of $f(T)$ gravity by taking inspiration from Herrera [1] work. We Implied the condition of minimal complexity in this scenario.
2. The association of $f(T)$ dark source components and implication of two models with minimal complexity constraint is investigated.

To line up our manuscript, we utilized the subsequent layout. The novel definition of complexity for the anisotropic sphere in the background of $f(T)$ theory is developed. In Sect. 2, the fundamental formulation of $f(T)$ together with the geometry and fluid basics to carry out this study is presented. We formed the $f(T)$ equations for an anisotropic sphere. Later on, we calculated the TOV equation and developed certain relations with aid of Misner–Sharp mass in $f(T)$ theory. The matching conditions in $f(T)$ theory at the hypersurface Ξ flourished in Sect. 3. We took the exterior Schwarzschild structure and combined it with an interior sphere at hypersurface Ξ. The continuity of metrics and extrinsic curvature revealed the existence of the Darmois constraints. The Sect. 4 is devoted to build up relations among mass, Tolman mass, and the conformal tensor in terms of $f(T)$ gravity. From where it is evident that all these components have substantial value in investigating the character of complexity. In Sect. 5, through the breakdown of the intrinsic curvature, the structural parameters for anisotropic fluid in the background of $f(T)$ gravity are determined. Later on, one of the parameters among all is designated as the complexity factor, which is responsible for inspecting the role of complexity. Section 6 deals with the constraint of the minimal complexity and its link with two models to calculate the solutions in $f(T)$ theory. In Sect. 7, the concluding remarks are given.

2 Anisotropic sphere in $f(T)$ gravity

The influence and importance of $f(T)$ gravity in inflation and the late-time acceleration of our universe has been widely seen in the literature. The formal scheme of $f(T)$ gravity will be presented in this section. Afterwards, a locally anisotropic sphere adjoint with few kinematical variables will be explained. In [40,41], EHA in terms of $f(T)$ gravity is interpreted as

$$S_{f(T)} = \int d^4x \left(\mathcal{L}_M + \frac{f(T)}{2\kappa^2}\right)|\mathfrak{h}|, \tag{1}$$

here $|\mathfrak{h}| = det(\mathfrak{h}^{\mu}_{\varrho})$, while $\mathfrak{h}^{\mu}_{\varrho}$ treats as the dynamic field of $f(T)$ theory. The Lagrangian for matter field is given by $\mathcal{L}_M$, where the coupling constant is given by κ and $f(T)$ is for differentiable function of torsion scalar T. The relation of metric tensor with the set of these orthonormal vectors is as follows $g_{\varrho\mu} = \theta_{ij}\mathfrak{h}^i_{\varrho}\mathfrak{h}^j_{\mu}$ adjoint $\theta_{ij} = diag(1, -1, -1, -1)$. In this scenario, the label for tangent space is used as $(\mu, \varrho, \ldots)$ and label for manifold space is used as $(i, j, \ldots)$. The role of Weitzenböck connection in $f(T)$ theory has substantial impact which follows from subsequent relation

$$T^{\sigma}_{\ \mu\varrho} = \bar{\Gamma}^{\sigma}_{\ \varrho\mu} - \bar{\Gamma}^{\sigma}_{\ \mu\varrho} = \mathfrak{h}_i^{\ \sigma}(\partial_{\varrho}\mathfrak{h}^i_{\ \mu} - \partial_{\mu}\mathfrak{h}^i_{\ \varrho}),$$

where

$$S_{\sigma}^{\ \mu\varrho} = \frac{\varrho^{\mu}_{\sigma}}{2}T^{\beta\varrho}_{\ \ \beta} - \frac{\varrho^{\varrho}_{\ \sigma}}{2}T^{\beta\mu}_{\ \ \beta} + \frac{1}{4}\left(T^{\mu\varrho}_{\sigma} + T^{\varrho\mu}_{\sigma} - T_{\sigma}^{\ \mu\varrho}\right).$$

We attain torsion scalar with the aid of the following relation

$$T = S_{\sigma}^{\ \mu\varrho}T^{\sigma}_{\ \mu\varrho},$$

where the defined property of tensor $T^{\sigma}_{\ \mu\varrho}$ is as $T^{\sigma}_{\ \mu\varrho} = -T^{\sigma}_{\ \varrho\mu}$. When we apply the variation on the EHA of Eq. (1) with aid of tetrad field, then the resultant equation takes the subsequent form

$$\mathfrak{h}_i{}^{\sigma} S_{\sigma}{}^{\mu\varrho}\partial_\mu T f_{TT} + \frac{f}{4}\mathfrak{h}_i{}^{\varrho} + \frac{f_T}{\mathfrak{h}}\partial_\mu(\mathfrak{h}\mathfrak{h}_i{}^{\sigma} S_{\sigma}{}^{\mu\varrho})$$
$$+\mathfrak{h}_i{}^{\sigma} T^{\alpha}_{\mu\sigma} S_{\alpha}{}^{\varrho\mu} f_T = \frac{\kappa^2}{2}\mathfrak{h}_i{}^{\sigma} T_{\sigma}^{\varrho(m)}, \tag{2}$$

where, $f_T \equiv \frac{\partial f}{\partial T}$, $f_{TT} \equiv \frac{\partial^2 f}{\partial T^2}$. The fluid stress-energy tensor (S-ET) has the representation of the form $T_{\sigma}^{\varrho(m)}$. In the covariant formulation of $f(T)$, the field equations based on the argument that the difference between the Ricci scalar and the torsion scalar has the same impact as the covariant form of $f(T)$ gravity are acquired as follows

$$\Upsilon_{\mu\varrho} f_{TT} - \frac{T}{2}\left(f_T - \frac{f}{T}\right) g_{\mu\varrho} + G_{\mu\varrho} f_T = \kappa^2 T^{(m)}_{\mu\varrho},$$

where $\Upsilon_{\mu\varrho} = S_{\mu\varrho}{}^{\sigma}\nabla_\sigma T$, while the value of Einstein tensor is as $G_{\mu\varrho}$. The modification of Eq. (2) provides

$$G_{\mu\varrho} = \frac{\kappa^2}{f_T}\left(T^{(T)}_{\mu\varrho} + T^{(m)}_{\mu\varrho}\right), \tag{3}$$

where correction terms in $f(T)$ gravity is

$$T^{(T)}_{\mu\varrho} = -\frac{1}{\kappa^2}\left\{\Upsilon_{\mu\varrho} f_{TT} + \frac{1}{4}\left(R f_T - \Upsilon f_{TT} + T\right) g_{\mu\varrho}\right\}. \tag{4}$$

Here we can attain the TEGR equations by implying the limit $f(T) = T$. We take into account a static spherically symmetric geometry as an interior region (Z^-) of the form

$$ds_-^2 = e^{\eta}dt^2 - e^{\xi}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{5}$$

where η and ξ depend only on r. In form of unequal stresses, the S-ET in its conventional form turn out as

$$T^{(m)}_{\mu\varrho} = u^\mu u_\varrho \rho - P h^\mu_\varrho + \Pi^\mu_\varrho,$$

where h^μ_ϱ is the orthogonal projection tensor, Π^μ_ϱ is the anisotropic tensor and P represents the isotropic tensor. The four vectors along coordinates are defined by the relation $u^\mu = (e^{-\frac{\eta}{2}}, 0, 0, 0)$; $s^\mu = (0, -e^{-\frac{\xi}{2}}, 0, 0)$; $k^\mu = (0, 0, -r, 0)$; $l^\mu = (0, 0, 0, -rsin\theta)$ and assured the subsequent features $u_\mu u^\mu = 1$, $s_\mu s^\mu = -1$, $k_\mu k^\mu = -1$, $l_\mu l^\mu = -1$, $u^\mu s_\mu = 0$. We established the two new subsidiary parameters of the form $P_\perp = k^\mu k_\varrho T_{\mu\varrho}$ and $P_r = s^\mu s_\varrho T_{\mu\varrho}$ in order to analyze the anisotropic fluid. The immediate effects of the above parameters on the conventional form of S-ET results into

$$T_{\mu\varrho} = (P_\perp + \rho)u_\mu u_\varrho - P_\perp g_{\mu\varrho} - (P_\perp - P_r)s_\mu s_\varrho, \tag{6}$$

with the subsequent supporting relations of the form

$$\Pi_{\mu\varrho} = \Pi\left(s_\mu s_\varrho + \frac{h^\mu_\varrho}{3}\right), \quad \Pi = -(P_\perp - P_r),$$

$$P = \frac{2P_\perp + P_r}{3}, \quad h^\mu_\varrho = \delta^\mu_\varrho - u^\mu u_\varrho.$$

The Eq. (3) together non-zero constituents of S-ET of the form $T^0_0 = \rho$, $T^1_1 = -P_r$, $T^2_2 = -P_\perp$ generate the subsequent field equations of $f(T)$ theory

$$-\frac{(e^{-\xi}+1)}{r^2} + \frac{\acute{\xi} e^{-\xi}}{r} = \frac{8\pi e^{\eta}}{f_T} \times\left[\rho + \frac{1}{16\pi}\left\{(T f_T - f) - f_{TT}e^{-\xi}\left(\frac{\acute{\eta}}{4} - \frac{2}{r}\right)\acute{T}\right\}\right], \tag{7}$$

$$\frac{(e^{-\xi}+1)}{r^2} + \frac{\acute{\eta} e^{-\xi}}{r} = \frac{8\pi e^{\xi}}{f_T}\left[P_r - \frac{1}{16\pi}(T f_T - f)\right], \tag{8}$$

$$\frac{e^{-\xi}}{2}\left(\eta'' + \frac{\eta'^2}{2} - \frac{\xi'\eta'}{2} + \frac{\eta'}{r} - \frac{\xi'}{r}\right) = \frac{8\pi r^2}{f_T}\left[P_\perp - \frac{1}{16\pi}\left\{(T f_T - f) + \frac{e^{-\xi} f_{TT}}{2}\left(\frac{3}{r} - \eta'\right)T'\right\}\right], \tag{9}$$

where the notions for the differential function of T and the derivative w.r.t. r are $f(T)$ and prime, respectively. We attain the hydrostatic equilibrium equation (or generalized TOV equation) of the underneath form

$$P_r' + (\rho + P_r)\frac{\eta'}{2} - \frac{2(P_r - P_\perp)}{r} + \frac{X_1^{(D)}}{8\pi} = 0, \tag{10}$$

where the notion used for extra terms of $f(T)$ theory is denoted by $X_1^{(D)}$ and presented in Appendix A. Now, to discuss the energy distribution of the geometry and gravity, we used the Misner–Sharp mass [42]. It is the quasi-local (defined on the edges of the specific domain of the spacetime) version of the mass and derived by using the mass transformation across the matter to the gravitational region in the way of collapsing process. The mathematical description of the above-stated mass for our geometry is given as

$$2m(r) = (1 + e^{-\xi})r. \tag{11}$$

The substitution of the Eq. (11) into (7) and applying process of the differentiation and integration, respectively results as

$$m(r) = 4\pi\int_0^r \frac{r^2}{f_T}\left[\rho e^{\eta} + \Phi^{(T)}_{00}\right]dr, \tag{12}$$

where $\Phi^{(T)}_{00}$ is set out in Appendix A. The combination of Eqs. (8) and (11) gives

$$\eta' = \left[2\frac{\left\{\frac{4\pi}{f_T}(P_r r^3 - (2m - r)r^2 \times \Phi^{(T)}_{11})\right\}}{(2m - r)^2} - m\right], \tag{13}$$

where $\Phi_{11}^{(T)}$ is written in Appendix A. By inserting the value of η' into Eq. (10) grant the subsequent result

$$P_r' = -(P_r+\rho)\frac{\left[2\frac{\left\{\frac{4\pi}{f_T}(p_r r^3-(2m-r)r^2\times\Phi_{11}^{(T)})\right\}}{(2m-r)^2}-m\right]}{2} - \frac{2(P_r-P_\perp)}{r}+\frac{X_1^{(D)}}{8\pi}=0. \quad (14)$$

3 Matching conditions at hypersurface Ξ

The matching hypersurface is a separation between two sections Z^- and Z^+ of the manifold Z by the $3D$ timelike hypersurface Ξ. To match the distinct geometries of the stellar objects at the hypersurface Ξ, a variety of matching conditions are available in the literature. Among them, the Darmois [43] conditions are the most appropriate ones. We take into account the Schwarzschild spacetime for exterior region (Z^+) as

$$ds_+^2 = YdT^2 - \frac{1}{Y}dR^2 - R^2(d\theta^2+sin^2\theta d\phi^2), \quad (15)$$

where $Y=(1-\frac{2M}{R})$. The conditions are given as

- The metrics must hold continuity over the hypersurface Ξ, i.e.,

$$[ds_+^2]_\Xi = [ds^2]_\Xi = [ds_-^2]_\Xi. \quad (16)$$

- Also, the extrinsic curvature (EC) must possess the continuity over the hypersurface Ξ, i.e.,

$$[K_{ab}]_\Xi = [K_{ab}]_- = [K_{ab}]_+, \qquad (a,b=0,2,3). \quad (17)$$

The value of EC is as

$$K_{ab}^\pm = -n_\beta^\pm\left(\frac{\partial^2 x_\pm^\beta}{\partial\chi^a\partial\chi^b}+\Gamma^\beta_{\sigma\gamma}\frac{\partial x_\pm^\sigma\partial x_\pm^\gamma}{\partial\chi^a\partial\chi^b}\right), \quad (\beta,\sigma,\gamma=0,1,2,3), \quad (18)$$

where χ^a, $n_\beta^\pm$ and $x_\pm^\beta$ are the hypersurface components, outward unit vector for both regions, and components of the Z^- and Z^+ sections, respectively. By implying Eqs. (16), (17) and (18) at $r_\Xi = r = constant$, we get

$$e^{-\xi}\overset{\Xi}{=}\left(1-\frac{2M}{r}\right), \quad e^{\eta}\overset{\Xi}{=}\left(1-\frac{2M}{r}\right), \quad P_r^{(eff)}\overset{\Xi}{=}0, \quad M\overset{\Xi}{=}m, \quad (19)$$

where $P_r^{(eff)}\overset{\Xi}{=}\frac{8\pi}{f_T}(P_r+\Theta_{11}^{(T)})$, here $\Theta_{11}^{(T)}=\frac{-1}{16\pi}\times(Tf_T-f)$ are dark source terms of $f(T)$ theory.

4 Relation of conformal tensor, mass and Tolman mass

The intrinsic curvature, the Ricci scalar and the conformal tensor is defined by the subsequent expression

$$R^\tau_{\mu\varrho\sigma} = C^\tau_{\mu\varrho\sigma}+\frac{1}{2}R^\tau_\varrho g_{\mu\sigma}-\frac{1}{2}R_{\mu\varrho}\delta^\tau_\sigma+\frac{1}{2}R_{\mu\sigma}\delta^\tau_\varrho -\frac{1}{2}R^\tau_\sigma g_{\mu\varrho}-\frac{1}{6}\mathfrak{R}(\delta^\tau_\varrho g_{\mu\sigma}-g_{\mu\varrho}\delta^\tau_\sigma). \quad (20)$$

The conformal tensor deduces information relating to the tidal forces, when any kind of object passes over the geodesic. It can be measured by looking at the distance variation of the adjacent geodesic. The conformal tensor is the only component of intrinsic curvature that deals with gravitational wave transmission employing regions lacking matter. One another feature is that it is trace-free. It consists of 256 components and among them 10 components are expressed independently in 4D. By involving a four-velocity vector, we can split 10 components into two tensors of rank two. The conformal tensor is a composition of two parts, one is electric and another one is magnetic. When we deal with spherical geometry, the effect of the magnetic part is not considered. Because during the analysis of the flow, the behavior of expansion of lines has no dependency on each other. So, in this way, the collapse revolves around the fluid components. The conformal tensor along with its scalar has subsequent expression

$$C_{\sigma\xi\nu\tau} = (g_{\sigma\xi\mu\varrho}g_{\nu\tau\pi\theta}-\eta_{\sigma\xi\mu\varrho}\eta_{\nu\tau\pi\theta})V^\mu V^\mu E^{\varrho\theta},$$

where $g_{\sigma\xi\mu\varrho}=g_{\sigma\mu}g_{\xi\varrho}-g_{\sigma\varrho}g_{\xi\mu}$ and the expression for the Levi-Civita tensor is given as $\eta_{\sigma\xi\mu\varrho}$. The conformal tensor can be re-arranged as below $E_{\mu\varrho}=\psi(s_\mu s_\varrho+\frac{1}{3}h_{\mu\varrho})$, along with corresponding value of conformal scalar ψ

$$\psi = \frac{e^{-\xi}}{4}\left(\frac{\xi'\eta'}{2}-\eta''-\frac{\eta'^2}{2}\right)+\frac{e^{-\xi}}{2r}\left(\frac{\eta'}{2}-\frac{1}{r}\right) -\frac{1}{2r}\left(\frac{e^{-\xi}\xi'}{2}+\frac{1}{r}\right). \quad (21)$$

The relation of the mass using Eqs. (7)–(9), (11) and (21) yield

$$\frac{3m(r)}{r^3} = \frac{4\pi}{f_T}\left[(\rho e^\eta+P_r e^\xi-r^2P_\perp) -(\Phi_{00}^{(T)}-\Phi_{11}^{(T)}+\Phi_{22}^{(T)})\right]-\psi. \quad (22)$$

Here, $\Phi_{22}^{(T)}$ is set out in Appendix A. It is evident that the mass function has a dependence on the conformal scalar ψ, fluid constituents, and dark source components of f_T theory. After implying the process of differentiation w.r.t. r on the above result and utilizing Eq. (11), it turns out to be

$$\psi = \frac{4\pi}{f_T}\Big[(\rho e^{\eta} + P_r e^{\xi} - r^2 P_\perp) - \int_0^r (2\Phi_{00}^{(T)} - \Phi_{11}^{(T)} + \Phi_{22}^{(T)})dr\Big] + \frac{4\pi}{r^3}\Big[\int_0^r (\frac{e^{\eta}\rho}{f_T})' + (\frac{\Phi_{00}^{(T)\prime}}{f_T})\Big]\tilde{r}^3 d\tilde{r}. \quad (23)$$

This equation indicates that the conformal scalar ψ resides only on the dark source components of $f(T)$ theory and fluid components like density inhomogeneity and local anisotropy. It reveals that the density inhomogeneity and anisotropy have substantial influence in the analysis of stellar bodies. On substituting back Eq. (23) into (22) bring out following result

$$m(r) = \frac{4\pi r^3}{3f_T}\Big[\int_0^r (2\Phi_{00}^{(T)} - \Phi_{11}^{(T)} + \Phi_{22}^{(T)})dr - (\Phi_{00}^{(T)} - \Phi_{11}^{(T)} + \Phi_{22}^{(T)})\Big] - \frac{4\pi}{r^3}\left[\int_0^r \left(\frac{e^{\eta}\rho}{f_T}\right)' - \left(\frac{\Phi_{00}^{(T)\prime}}{f_T}\right)\right]\tilde{r}^3 d\tilde{r}. \quad (24)$$

The above result shows that the mass has correspondence with dark source components of $f(T)$ theory, density inhomogeneity, and a homogenous form of density. An additional version of the mass, i.e., TolmanWhittaker mass (TW-M) is employed to proceed with our analysis. It was built to calculate the total energy accumulation of the system, and that is why referred to as "active gravitational mass". This mass formula is an appropriate version of the mass formula to carry out the analysis of anisotropic fluid distributions. To deal with the slow progression or staticity of the system, the subsequent formation of the TW-M inside the sphere with radius r is used

$$m_T = 4\pi\int_0^r e^{\frac{(\eta+\xi)}{2}}\tilde{r}^2(\rho + P_r + 2P_\perp)d\tilde{r}. \quad (25)$$

When we take into account the Eqs. (7)–(9) into (25) and after implying the technique of integration, it brings out

$$m_T = \frac{f_T}{2}e^{\frac{(\eta-\xi)}{2}}r^2\eta' - 4\pi\left[X_2^{(D)}(r) + \int_0^r\left\{X_2^{(D)}(r)d\tilde{r}\right\}_{,1}\right], \quad (26)$$

where value of $X_2^{(D)}$ is set out in Appendix A. The use of value η' from Eq. (13) and e^{ξ} from (19) in above equation bring out the result of the following form

$$m_T = \frac{f_T}{2}e^{\frac{(\eta+3\xi)}{2}}\left\{\frac{4\pi}{f_T}(P_r r^3 - (2m-r)r^2\Phi_{11}^{(T)}) - m\right\} - 4\pi\left[X_2^{(D)} + \int_0^r\left\{X_2^{(D)}(r)d\tilde{r}\right\}_{,1}\right]. \quad (27)$$

It is prominent from the above equation that the TW-M has true understating as "active gravitational mass". Now, we imply the derivative of Eq. (25) with regard to r and utilized the Eq. (26) to furnish the following result

$$\acute{m}_T - \frac{3}{r}m_T = -\frac{f_T}{2}r^2\left\{\psi + 4\pi(P_r - P_\perp) - 4\pi\Phi_{00}^{(T)} - 4\pi\Phi_{22}^{(T)}\right\} - \frac{12\pi}{r}\left\{X_2^{(D)}(r) + \int_0^r\left\{X_2^{(D)}(r)d\tilde{r}\right\}_{,1}\right\}.$$

While after integrating, we attain

$$m_T = (m_T)_\Xi\left(\frac{r}{r_\Xi}\right)^3 + r^3\int_r^{r_\Xi e}\left[\frac{e^{\frac{\eta+\xi}{2}}}{\tilde{r}}f_T(4\pi(P_r - P_\perp) + \psi - 4\pi\Phi_{00}^{(T)} - 4\pi\Phi_{22}^{(T)}) - \frac{1}{4\pi r^4}\left\{X_2^{(D)}(r) + \int_0^r\left\{X_2^{(D)}(r)d\tilde{r}\right\}_{,1}\right\}\right]. \quad (28)$$

The above equation exhibit that the TW-M has correspondence with the fluid components, dark source components of $f(T)$ and the conformal scalar ψ.

5 Correlation of structure scalars and the conformal scalar within f_T theory

To understand the behavior of the anisotropic fluids appropriately, we use a few structural parameters. These parameters are attained by dividing the Riemann tensor in an orthogonal direction. Herrera et al. [44] took the initiative to build up the strategy to evaluate these structural parameters. These are trace and trace-free components of the three tensors and each one corresponds to a unique significance. To carry our analysis within $f(T)$ gravity, we utilized the expression of the three tensors $X_{\mu\varrho}, Y_{\mu\varrho}$ and $Z_{\mu\varrho}$ of the form already given in [1]. The division of Eq. (20) by making use of $f(T)$ field equations is given by

$$R^{\mu\sigma}_{\ \ \varrho\rho} = C^{\mu\sigma}_{\ \ \varrho\rho} + 16\pi(T^{(T)} + T^{(m)})^{[\mu}_{[\varrho}\delta^{\sigma]}_{\nu]} + 8\pi(T^{(T)} + T^{(m)})(\frac{1}{3}\delta^{\mu}_{[\varrho}\delta^{\sigma}_{\rho]} - \delta^{[\mu}_{\ [\varrho}\delta^{\sigma]}_{\rho]}). \quad (29)$$

When we utilize Eqs. (4) and (6) into (29), we obtain

$$R^{\mu\sigma}_{\ \ \varrho\rho} = R^{\mu\sigma}_{(I)\varrho\rho} + R^{\mu\sigma}_{(II)\varrho\rho} + R^{\mu\sigma}_{(III)\varrho\rho},$$

here

$$R^{\mu\sigma}_{(I)\varrho\rho} = 16\pi\rho V^{[\mu}V_{[\varrho}\delta^{\sigma]}_{\rho]} - 16\pi Ph^{[\mu}_{[\varrho}\delta^{\sigma]}_{\rho]} + 8\pi\left(\rho - 3P + \frac{(Tf_T - f)}{16\pi}\delta^{\mu\varrho}\right.$$

$$-\frac{f_{TT}}{8\pi}S^{\mu\tau}\nabla_\tau T\Big) + 8\pi\left(\frac{1}{3}\delta^{\mu}_{[\varrho}\delta^{\sigma}_{\rho]} - \delta^{[\mu}_{[\varrho}\delta^{\sigma]}_{\rho]}\right),$$

$$R^{\mu\sigma}_{(II)\varrho\rho} = 16\Pi^{[\mu}_{[\varrho}\delta^{\sigma]}_{\rho]} + \frac{1}{8\pi}\left(\frac{(Tf_T - f)}{2}\delta^{[\mu}_{[\varrho}\delta^{\sigma]}_{\rho]} - f_{TT}S^{[\mu\tau}_{[\varrho}\delta^{\sigma]}_{\rho]}\nabla_\tau T\right),$$

$$R^{\mu\sigma}_{(III)\varrho\rho} = 4V^{[\mu}V_{[\varrho}E^{\sigma]}_{\rho]} - \psi^{\mu\sigma}_{\alpha}\psi_{\varrho\rho\beta}E^{\alpha\beta},$$

with

$$\psi_{\mu\sigma\varrho} = V^{\alpha}\eta_{\alpha\mu\sigma\varrho}, \quad \psi_{\mu\sigma\varrho}V^{\varrho} = 0,$$

with the help of the above stated results, we can derive the values of three tensors $X_{\mu\varrho}$, $Y_{\mu\varrho}$ and $Z_{\mu\varrho}$ in form of the physical variables as

$$X_{\mu\varrho} = -\frac{8\pi}{3}\rho h_{\mu\varrho} + 4\pi\Pi_{\mu\varrho} - E_{\mu\varrho} + X^{(D)}_3, \tag{30}$$

$$Y_{\mu\varrho} = \frac{4\pi}{3}(\rho + 3P)h_{\mu\varrho} + 4\pi\Pi_{\mu\varrho} + E_{\mu\varrho} + X^{(D)}_4, \tag{31}$$

$$Z_{\mu\varrho} = 0, \tag{32}$$

where, the value of $X^{(D)}_3$ and $X^{(D)}_4$ is set out in Appendix A. We can divide them in their trace-free and trace parts as follows

$$X_{\mu\varrho} = X_T\frac{h_{\mu\varrho}}{3} + X_{TF}\left(s_\mu s_\varrho + \frac{h_{\mu\varrho}}{3}\right);$$

$$Y_{\mu\varrho} = Y_T\frac{h_{\mu\varrho}}{3} + Y_{TF}\left(s_\mu s_\varrho + \frac{h_{\mu\varrho}}{3}\right),$$

where expressions for X_T, X_{TF}, Y_T and Y_{TF} are as given

$$X_T = 8\pi\rho + X^{(D)}_5, \quad X_{TF} = 4\pi\Pi - \psi + X^{(D)}_6,$$

$$Y_T = 4\pi(\rho + 2P_\perp + P_r) + X^{(D)}_7,$$

$$Y_{TF} = 4\pi\Pi + \psi + X^{(D)}_8, \tag{33}$$

where the values of $X^{(D)}_5$, $X^{(D)}_6$, $X^{(D)}_7$ and $X^{(D)}_8$ are set out in Appendix A. Using the value of scalar Y_{TF} from Eq. (33) and value of ψ from Eq. (23), it leads to

$$Y_{TF} = 4\pi\Pi + X^{(D)}_8 + \frac{4\pi}{f_T}\times\left[(\rho e^{\eta} + P_r e^{\xi} - r^2P_\perp) - \int_0^r(2\Phi^{(T)}_{00} - \Phi^{(T)}_{11} + \Phi^{(T)}_{22})dr\right] - \frac{4\pi}{r^3}\left[\int_0^r\left(\frac{e^{\eta}\rho}{f_T}\right)' + \left(\frac{\Phi^{(T)\prime}_{00}}{f_T}\right)\right]\tilde{r}^3d\tilde{r}. \tag{34}$$

The local anisotropy of fluid can be seen from this result

$$Y_{TF} + X_{TF} = 8\pi\Pi + X^{(D)}_8 + X^{(D)}_6.$$

To understand the importance of the scalars Y_T and Y_{TF}, we utilize Eqs. (28) and (33), to attain

$$m_T = (m_T)_{\Xi e}\left(\frac{r}{r_\Xi}\right)^3 + \int_r^{r_{\Xi e}}\frac{e^{\eta+\xi}}{\tilde{r}}f_T(Y_{TF} + X^{(D)}_9)d\tilde{r} \tag{35}$$

$$m_T = \int_0^r e^{\frac{\eta+\xi}{2}}\tilde{r}^2(Y_T + X^{(D)}_7)d\tilde{r}. \tag{36}$$

Where $X^{(D)}_9 = -X^{(D)}_8 + X^{(D)}_{8I}$, and value of $X^{(D)}_{8I}$ is given in Appendix A. It is evident from the above results that the Y_T and Y_{TF} has correspondence with TW-M.

6 Vanishing complexity factor with few models for sphere

In this section, two models admitting the vanishing of complexity factor will be presented. Since the inspection of complexity is widely carried out in the literature. Here, in our strategy, one of the scalars attained through orthogonal division of the Riemann tensor is associated to describe the complexity of the system. The scalar Y_{TF} is designated as a complexity factor because of its significant features. It involves the impact of TW-M on the anisotropy and density inhomogeneity of the fluid content. This system of $f(T)$ modified equations consists of the five parameters like $(\eta, \xi, P_r, \rho, P_\perp)$. After implying the vanishing complexity factor constraint $Y_{TF} = 0$, the need to get one more condition is still there. The vanishing complexity constraint from Eq. (34) turns out as

$$\Pi = \frac{1}{f_T}\left[(\rho e^{\eta} + P_r e^{\xi} - r^2P_\perp) - \int_0^r(2\Phi^{(T)}_{00} - \Phi^{(T)}_{11} + \Phi^{(T)}_{22})dr\right] - \frac{1}{r^3}\left[\int_0^r\left(\frac{e^{\eta}\rho}{f_T}\right)' + \left(\frac{\Phi^{(T)\prime}_{00}}{f_T}\right)\right]\tilde{r}^3d\tilde{r} + \frac{X^{(D)}_8}{4\pi}. \tag{37}$$

The above equation reveals the correspondence of the vanishing complexity constraint to the anisotropic fluid with density inhomogeneity and dark source components of $f(T)$ theory. Now we shall present two models for an anisotropic sphere.

6.1 The Gokhroo and Mehra constraint-satisfactory model for anisotropic sphere

Gokhroo and Mehra [45] inspected the interior solutions of a sphere with anisotropic pressure. The state of energy density is not constant but it is variable. To analyze the behavior of stellar bodies in terms of energy density, we utilized the supposition made by Gokhroo and Mehra given as

$$\rho = \left(1 - \frac{r^2 K}{r_\Xi^2}\right)\rho_0, \tag{38}$$

where ρ_0 treated as constant and K set ranges between 0 and 1. Making use of Eq. (38) into (12), we attain

$$m = \frac{4\pi e^{\eta}}{f_T} \times \frac{r^3 \rho_0}{3}\left(1 - \frac{3K_r^2}{5r_\Xi^2}\right) + 4\pi\int_0^r \frac{r^2}{f_T} \times \Phi_{00}^{(T)} dr - X_{10}^{(D)}.$$

When we substitute this equation into Eq. (11), it turn out as

$$e^{-\xi} = 1 - \frac{16\pi e^{\eta} r^2 \sigma}{f_T} + \frac{48\pi K e^{\eta} r^4 \sigma}{5r_\Xi^2 f_T} - \frac{8\pi}{r}\int_0^r \frac{r^2}{f_T} \times \Phi_{00}^{(T)} dr + \frac{2}{r} X_{10}^{(D)},$$

where $\sigma = \frac{\rho_0}{3}$. From the and Eq. (9), we acquire the subsequent form

$$\frac{8\pi}{f_T}(P_r - P_\perp) = \left(\frac{e^{-\xi}+1}{r^2}\right) - \frac{e^{-\xi}}{2}\left(\eta'' + \frac{\eta'^2}{2} - \frac{\xi'\eta'}{2} - \frac{\eta'}{r} - \frac{\xi'}{r}\right) - \frac{8\pi}{f_T}\left(\Phi_{11}^{(T)} - \Phi_{22}^{(T)}\right), \tag{39}$$

we initiate the subsequent variables

$$e^{\eta(r)} = \frac{1}{e^{\int(\frac{2}{r} - 2y)dr}}; \quad e^{-\xi} = z,$$

with aid of the above variables, Eq. (39) brings out

$$\frac{2}{y}\left(\frac{1}{r^2} - \frac{8\pi\Pi}{f_T}\right) = z' + z\left[\frac{2y'}{y} + 2y - \frac{6}{r} + \frac{4}{yr^2}\right] + \frac{2}{y} \times \frac{8\pi}{f_T}(\Phi_{11}^{(T)}(z) - \Phi_{22}^{(T)}(z)). \tag{40}$$

Which has solution of the subsequent form in terms of Π and y as

$$ds^2 = \frac{1}{e^{\int(\frac{2}{r} - 2y)dr}} dt^2 - \frac{y^2 e^{\int 2y + frac4yr^2} dr}{r^6(2\int(\frac{1-8\pi\Pi r^2}{r^2 f_T}) - \frac{8\pi}{f_T}(\Phi_{11}^{(T)}(z) - \Phi_{22}^{(T)}(z)) e^{\int 2y + \frac{4}{yr^2}} dr + c_1} \times dr^2 - r^2(d\theta^2 + sin^2 d\phi^2)$$

where c_1 is constant of integration. The above equation indicates the importance of generating functions Π and y in term of $f(T)$ gravity. The physical variables in this framework turn out as

$$m'(r) = \frac{4\pi r^2}{f_T}\left[e^{\eta}\rho - \Phi_{00}^{(T)}\right],$$

$$\frac{4\pi}{f_T}\frac{r}{(2m-r)}\left[P_r - \frac{1}{16\pi}(Tf_T - f)\right] = \frac{y(2m - r - r^2) - m + r}{r^3},$$

$$8\pi P_\perp = \frac{f_T}{r^2}\left[\left(\frac{2m}{r} - 1\right)\left(y' + y^2 - \frac{y}{r} + \frac{1}{r^2}\right) + y\left(\frac{m'}{r} - \frac{m}{r^2}\right) + \frac{r^2}{2f_T} X_{11}^{(D)}\right],$$

where value of $X_{11}^{(D)}$ set out in Appendix A.

6.2 Polytropic Eos-satisfactory model for anisotropic sphere

This section is fixed for investigating the role of polytropic Eos in framework of $f(T)$ theory. Foremost, the following supposition are made

$$P_r = \mathbf{K}\rho^{\theta} = \mathbf{K}\rho^{1+\frac{1}{n}}, \tag{41}$$

where $\mathbf{K}$, ρ, n, θ are polytropic constant, energy density, index and exponent respectively. We utilized the subsequent dimensionless parameters to transform the TOV equation into dimensionless form

$$\beta = \frac{P_{rc}}{\rho_c}, \quad r = \frac{\zeta}{C}, \quad C^2 = \frac{4\pi\rho_c}{\beta(n+1)}, \quad \theta^n = \frac{\rho}{\rho_c}, \quad \upsilon(\zeta) = \frac{m(r)C^3}{4\pi\rho_c}, \tag{42}$$

where the term c refers that the calculations are done at the center. Taking account these values into TOV, we acquire the subsequent result

$$2\zeta^2\frac{d\theta}{d\zeta}N^2\rho + \frac{4\pi\rho_c^2\beta\zeta\theta}{f_T(n+1)}L - \frac{\theta^n\beta\zeta}{(1+n)} \times N^2 L\Phi_{11}^{(T)}$$

$$+\frac{2\pi\rho_c^2\theta^n\upsilon L}{\beta C^2(n+1)}-\frac{2\pi\zeta}{\beta(1+n)}N^2+\frac{X_1^{(D)}C}{8\pi\beta(1+n)}N^2=0, \tag{43}$$

where $L=(1+\beta\theta)$ and $N=(1-\frac{2\upsilon r(n+1)}{\zeta})$. Now putting the values of variables from Eq. (42) into Eq. (11), we acquire the subsequent result for the mass function

$$\frac{d\upsilon}{d\zeta}=\frac{\zeta^2}{f_T}\left(\theta^n e^\eta+\frac{\Phi_{00}^{(T)}}{\rho_c}\right). \tag{44}$$

The combination of the above Eqs. (43)–(44) result in the LEEq. It is the description of the polytropic objects in state of hydrostatic equilibrium. We get the value for diminishing complexity factor by inserting parameters from Eq. (42) into (37) as

$$\begin{aligned}\Pi=&\frac{1}{f_T}\left[\left(\theta^n\rho_c e^\eta+\mathbf{K}\theta^{1+n}\rho_c^{1+\frac{1}{n}}e^\xi-\frac{C^2}{\zeta^2}P_\perp\right)\right.\\&\left.-\int_0^r(2\Phi_{00}^{(T)}-\Phi_{11}^{(T)}+\Phi_{22}^{(T)})dr\right]\\&-\frac{C^3}{\zeta^3}\left[\int_0^r\left(\frac{e^\eta\theta^n\rho_c}{f_T}\right)'+\left(\frac{\Phi_{00}^{(T)\prime}}{f_T}\right)\right]\\&\times\frac{\tilde{\zeta}^3}{C^3}\times d\left(\frac{\tilde{\zeta}}{C}\right)+\frac{X_8^{(D)}}{4\pi}.\end{aligned} \tag{45}$$

The complexity of the above Eqs. (43)–(45) is evident from their expressions. It is not essay to attain accurate analytical solutions. Their numerical values can be acquired by taking different values of the above-described parameters. The extension of the Newtonian to relativistic polytropes accepts two feasibilities, one along the different parameters is explained in Eq. (41) and other one is $P_r=\mathbf{K}\rho_a^\theta=\mathbf{K}\rho_a^{1+\frac{1}{n}}$. The values of the TOV equation and Π, in this case, would turn out as

$$\begin{aligned}&2\zeta^2\frac{d\theta_a}{d\zeta}N^2\rho+\frac{4\pi\rho_c^2\beta\zeta\theta_a}{f_T(n+1)}L\\&-\frac{\theta_a^n\beta\zeta}{(1+n)}\times N^2L\Phi_{11}^{(T)}+\frac{2\pi\rho_c^2\theta_a^n\upsilon L}{\beta C^2(n+1)}\\&-\frac{2\pi\zeta}{\beta(1+n)}N^2+\frac{X_1^{(D)}C}{8\pi\beta(1+n)}N^2=0,\end{aligned} \tag{46}$$

$$\begin{aligned}\Pi=&\frac{1}{f_T}\left[\left(\theta_a^n\rho_c e^\eta+\mathbf{K}\theta_a^{1+n}\rho_c^{1+\frac{1}{n}}e^\xi-\frac{C^2}{\zeta^2}P_\perp\right)\right.\\&\left.-\int_0^r(2\Phi_{00}^{(T)}-\Phi_{11}^{(T)}+\Phi_{22}^{(T)})dr\right]\\&-\frac{C^3}{\zeta^3}\left[\int_0^r\left(\frac{e^\eta\theta_a^n\rho_c}{f_T}\right)'+\left(\frac{\Phi_{00}^{(T)\prime}}{f_T}\right)\right]\frac{\tilde{\zeta}^3}{C^3}\\&\times d\left(\frac{\tilde{\zeta}}{C}\right)+\frac{X_8^{(D)}}{4\pi},\end{aligned} \tag{47}$$

where $\theta_a^n=\frac{\rho_a}{\rho_{ac}}$. The Eqs. (44), (46) and (47) deals with the polytropic Eos with zero complexity of stellar objects in framework of $f(T)$ gravity.

7 Concluding comments

This paper attempts to build up the definition of complexity based on the Herrera [1] strategy in the background of $f(T)$ gravity. This theory has been widely analyzed as an alternative way to explain the accelerated expansion of the universe. To carry out our inspection, first of all, we provided the fundamental formulation of theory and expressions for a few basic variables. Then, we established the $f(T)$ field equations in the presence of an anisotropic sphere. The generalized TOV equation is formed with the help of conservation law to enhance our analysis at a higher range. We derived the matching constraint for the static anisotropic sphere in the framework of $f(T)$ gravity to combine the interior and exterior regions. We utilized the two masses in the case of an anisotropic sphere and developed certain relations in terms of physical variables and the conformal tensor (which satisfies $E_{\mu\sigma}V^\sigma;\quad E_\varrho^\varrho=0$). The effect of the Misner–Sharp mass, the conformal tensor, and the active gravitational mass on the sphere coupled with anisotropic fluid content correspond to the density inhomogeneity, anisotropy, and dark source components of $f(T)$ theory is studied. To attain the complexity factor labeled as Y_{TF}, we carried out the division of the Riemann tensor in an orthogonal direction. We acquired the structural scalars from the procedure and build up the constraint of the diminishing complexity factor. We found few models to attain the solutions in this scenario. We close the discussion with subsequent comments.

The motivation to build a novel definition of $f(T)$ gravity is that complexity is responsible to understand the system excellently. Many factors are responsible for complexity, including fluid anisotropy and density inhomogeneity. The factor Y_{TF} includes the impact of density inhomogeneity, $f(T)$ dark source components, and local anisotropy on the total energy budget of the system. If we add up the charge, then the impact of the charge will also be contributed in Y_{TF}. We implied the process of orthogonal division of the Riemann tensor to acquire the structure parameters introduced by Herrera et al. [44]. They established five structure parameters and their corresponding features of them. They found solutions in the case of these structure parameters. The idea is widely extended in the case of dissipation or non-dissipation of the non-static anisotropic sphere in [46]. They have done their inspection in presence of an electric charge and determined that the heat fluxes are dealt with scalar Z. To understand the progression of the shear and expansion tensor, the main candidates are Y_{TF} and Y_T. Recently, this idea is extended in the case of hyperbolic symmetry by the different contributors [47–49].

In matching the diminishing complexity constraint, many models can be applied as an extra constraint to analyze the system. Our particular choice is concerned with "Gokhroo and Mehra" and "polytropic Eos". With the help of these models, we acquired four unknowns from the five possible unknowns $(\eta,\ \xi,\ P_r,\ \rho,\ P_\perp)$ of the system. In the "Gokhroo and Mehra" model, we build up the expressions for physical parameters in the background of $f(T)$ theory. For this concern, two unknown functions are utilized and deduced that expressions for physical parameters can be found in form of generating functions Π and y. While in "polytropic Eos", we formed the system of equations for TOV, mass, and diminishing complexity factor. We utilized the dimensionless variables in this scheme. The motivation to investigate the polytropic models for polytropic Eos has increased among researchers. Within modified gravity, Henttunen et al. [50] inspected the stellar structures in $f(R)$ theory along a range of polytropic Eos. They also determined effective solutions within star's core. While in Palatini $f(R)$ theory, the contribution made by Wojnar [51]. The dark source components of $f(T)$ theory contributed in complexity factor as shown in Eq. (34) and for $f_T=0$, the results are compatible with Herrera et al. [1].

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data generated or analysed during this study are included in this accepted manuscript.]

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8 Appendix A

The value of $X_1^{(D)}$, $\Phi_{00}^{(T)}$, $\Phi_{11}^{(T)}$ and $\Phi_{22}^{(T)}$ is listed below

$$
\begin{aligned}
X_1^{(D)} &= -\frac{\eta' e^{-2\xi}}{32\pi}\left(\frac{\eta'}{4}-\frac{2}{r}\right)T' \\
&\quad +\frac{\xi' e^{-\xi}}{16\pi} left[\{Tf_T-f\}_{,1}-(Tf_T-f)\Big] \\
&\quad +\frac{e^{-2\xi}}{r}\left[\frac{f_{TT}}{16\pi}\left(\frac{3}{r}-\eta'\right)\right], \\
\Phi_{00}^{(T)} &= \frac{e^{\eta}}{16\pi}\left\{(Tf_T-f)-f_{TT}e^{-\xi}\left(\frac{\eta'}{4}-\frac{2}{r}\right)T'\right\}; \\
\Phi_{11}^{(T)} &= -\frac{e^{\xi}}{8\pi}\left(\frac{Tf_T-f}{2}\right), \\
\Phi_{22}^{(T)} &= \frac{r^2}{16\pi}\left\{-(Tf_T-f)+\frac{f_{TT}}{2}\left(\eta' e^{\xi}-\frac{3}{r}e^{-\xi}\right)T'\right\};
\end{aligned}
$$

where the value of $X_2^{(D)}$, $X_3^{(D)}$, $X_4^{(D)}$, $X_5^{(D)}$, $X_6^{(D)}$, $X_7^{(D)}$, $X_8^{(D)}$, $X_{8I}^{(D)}$, $X_9^{(D)}$, $X_{10}^{(D)}$ and $X_{11}^{(D)}$ is as under

$$
\begin{aligned}
X_2^{(D)} &= e^{\frac{(\eta+\xi)}{2}}\left(\Phi_{11}^{(T)}+\Phi_{00}^{(T)}+\Phi_{22}^{(T)}\right)d\tilde{r}^3, \\
X_3^{(D)} &= \frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)\delta_{\mu\varrho}+f_{TT}S_{\mu\varrho\tau}\nabla_\tau T\right\} \\
&\quad -\frac{1}{3}\left\{f_{TT}S^{\mu\tau}\nabla_\tau T-\left(\frac{Tf_T-f}{2}\right)\right\}h_{\mu\varrho}, \\
X_4^{(D)} &= \frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)g_{\mu\varrho}-f_{TT}S^{\tau}_{\varrho\mu}\nabla_\tau T\right. \\
&\quad \left.+\frac{f_{TT}}{2}(V_\varrho+V_\phi-g_{\mu\varrho}V^\sigma)S^\tau_\mu\nabla_\tau T\right\} \\
&\quad -\frac{h_{\mu\varrho}}{2}\left\{\left(\frac{Tf_T-f}{2}\right)-f_{TT}S^{\mu\tau}\nabla_\tau T\right\}, \\
X_5^{(D)} &= -\frac{1}{9}\left\{f_{TT}S^{\mu\tau}\nabla_\tau T-\left(\frac{Tf_T-f}{2}\right)\right\}; \\
X_6^{(D)} &= \frac{1}{2}\left\{f_{TT}S_{\mu\varrho\tau}\nabla_\tau T+\left(\frac{Tf_T-f}{2}\right)\delta_{\mu\varrho}\right\}, \\
X_7^{(D)} &= \frac{1}{6}\left\{f_{TT}S^{\mu\tau}\nabla_\tau T-\left(\frac{Tf_T-f}{2}\right)\right\}; \\
X_8^{(D)} &= \frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)g_{\mu\varrho}-f_{TT}S^\tau_{\varrho\mu}\nabla_\tau T\right. \\
&\quad \left.+\frac{f_{TT}}{2}(V_\varrho+V_\mu-g_{\mu\varrho}V^\sigma)S^\tau_\mu\nabla_\tau T\right\}, \\
X_{8I}^{(D)} &= -4\pi\Phi_{22}^{(T)}-4\pi\Phi_{00}^{(T)}-\frac{1}{4\pi r^4} \\
&\quad \times\left\{+\int_0^r X_2^{(D)}(r)d\tilde{r}+X_2^{(D)}(r)\right\}, \\
X_{10}^{(D)} &= 4\pi\int\left(\frac{e^\eta}{f_T}\right)'\frac{r^3\rho_0}{3}\left(1-\frac{3K_r^2}{5r_\Xi^2}\right)dr, \\
X_{11}^{(D)} &= \left\{\left(Tf_T-f\right)+\frac{f_{TT}}{2}\left(3-2(2y-1\right)\right. \\
&\quad \left.\times\left(\frac{2m}{r}-1\right)^2T'\right\},
\end{aligned}
$$

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