



The low-lying hidden- and double-charm tetraquark states in a constituent quark model with instanton-induced interaction

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Abstract Spectrum of the low-lying hidden- and double-charm tetraquark states are investigated in a nonrelativistic quark potential model, where the instanton-induced interaction is taken as the residual spin-dependent hyperfine interaction between quarks. The model parameters are fixed by fitting the spectrum of the ground hadron states. Our numerical results show that masses of several presently studied tetraquark states are close to those of the experimentally observed candidates of exotic meson, which indicates that the corresponding compact tetraquark components may take considerable probabilities in those observed exotic states.

1 Introduction

More and more exotic hadron candidates, which exhibit different properties from mesons and baryons in the traditional quark model, have been observed experimentally [1] since the discovery of $X(3872)$ [2]. The typical mesonic exotic states, such as the charmonium-like state $Z_c(3900)^\pm$ observed in the $\pi^\pm J/\psi$ invariant mass spectra of the process $e^+e^- \rightarrow \pi^\pm \pi^- J/\psi$ [3] and the $Z_{cs}(3985)$ state observed in the $D_s^- D^{*0}$ and $D_s^{*-} D^0$ final states of the $e^+e^- \rightarrow K^+ D_s^- D^{*0} (D_s^{*-} D^0)$ reaction [4], may contain at least four $c\bar{c}q\bar{q}$ quarks. Theoretically, the exotic meson states can be explained as compact tetraquark states in quark potential model [5–11], QCD sum rule [12–15], and diquark–diquark picture [16–18]. While, they could be also described as the molecular states in effective field theory [19–24]. See also Refs. [25–29] for recent reviews.

Very recently, an exotic state T_{cc}^+ was reported in the $D^0 D^0 \pi^+$ mass spectrum by LHCb collaboration [30,31], which has a mass very close to the $D^0 D^{*+}$ threshold and its width is extremely narrow. In fact, taking into account the heavy quark symmetry between Ξ_{cc} and T_{cc} , the existence of tetraquark states which contain two heavy quarks has been predicted theoretically in Refs. [32,33], after the discovery of double-charm baryon Ξ_{cc}^{++} [34]. Especially, the binding energy of DD^* molecule predicted by one boson exchange model is in agreement with the experimental value. Within the DD^* molecule nature, the strong and radiative decay widths of T_{cc}^+ are studied in Refs. [24,35]. The successful prediction and the discovery of T_{cc}^+ are enormously helpful to our understanding of the hadron structure.

The constituent quark model (CQM), in which different versions of the explicit hyperfine interactions between quarks have been proposed, is one of the most successful phenomenological method to study the hadron structure. For instance, in the Godfrey–Isgur model [36], all the effects of one gluon exchange interactions (OGE) are included, and it gives a good description for meson spectra from π to Υ meson. The chiral constituent quark model [37–42] is another widely used version to investigate hadron spectrum, which usually contains one boson exchange (OBE) interactions, characterizing chiral symmetry spontaneously broken of QCD at low energy. Tentatively, different versions of CQM have been compared in the estimations on spectrum of baryon excitations in a multiquark picture recently [43–48].

In addition to the widely employed OGE- and OBE-CQM, the instanton-induced interaction between quarks has also been used in Refs. [43–45] to investigate the spectrum of baryon excitations as pentaquark states. This kind of interaction was firstly introduced by 't Hooft to solve the $U(1)$ -

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problem [49], and the instanton vacuum of QCD could provide a mechanism of spontaneous chiral symmetry breaking of QCD [50, 51]. Phenomenologically, it has been shown that one can successfully reproduce the spectrum of light hadrons [52–55] and the single charm baryons [56] employing the instanton-induced interaction. Furthermore, the instanton-induced interaction has also been employed to study the tetraquark states [57, 58]. Consequently, here we use it to investigate the spectra of low-lying hidden- and double-charm tetraquark states.

The present manuscript is organized as follows. In Sect. 2, we present the formalism of the CQM with instanton-induced interaction between quarks, including effective Hamiltonian, wave functions for tetraquark configurations and model parameters. The masses of hidden- and double-charm tetraquark states in our model and discussions are given in Sect. 3. Finally, Sect. 4 is a brief summary of present manuscript.

2 Framework

2.1 Effective Hamiltonian

In present work, a non-relativistic quark potential model is employed for calculations on the spectra of hidden- and double-charm tetraquark states. The dynamics of the model are governed by an effective Hamiltonian as follow:

$$H_{eff.} = \sum_{i=1}^4 (m_i + T_i) - T_{C.M.} + V_{Conf.} + V_{Ins.}, \quad (1)$$

where m_i and T_i are the constituent mass and kinetic energy of i -th quark, $T_{C.M.}$ denotes center of mass kinetic energy. $V_{Conf.}$ stands for the quark confinement potential, which is taken to be the widely accepted Cornell potential in present work [59]:

$$V_{Conf.} = \sum_{i < j} -\frac{3}{16} \left(\tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \right) \left(b r_{ij} - \frac{4}{3} \frac{\alpha_{ij}}{r_{ij}} + C_0 \right), \quad (2)$$

where $\tilde{\lambda}_{i(j)}^c$ is Gell–Mann matrix in $SU(3)$ color space acting on the $i(j)$ -th quark, b , α_{ij} and C_0 are strength of quark confinement, QCD effective coupling constant between two quarks and zero point energy, respectively.

For the residual spin-dependent interaction $V_{Ins.}$ in Eq. (1), we employ a phenomenologically extended version of 't Hooft's instanton-induced interaction [56]:

$$V_{Ins.} = V_{Ins.}^{qq} + V_{Ins.}^{q\bar{q}}, \quad (3)$$

with

$$V_{Ins.}^{qq} = \sum_{i < j} -\hat{g}_{ij}^{qq} \left(P_{ij}^{S=1} P_{ij}^{C,6} + 2 P_{ij}^{S=0} P_{ij}^{C,\bar{3}} \right) \delta^3(\vec{r}_{ij}),$$

$$V_{Ins.}^{q\bar{q}} = \sum_{i < j} \hat{g}_{ij}^{q\bar{q}} \left[\frac{3}{2} P_{ij}^{S=1} P_{ij}^{C,8} + P_{ij}^{S=0} \left(\frac{1}{2} P_{ij}^{C,8} + 8 P_{ij}^{C,1} \right) \right] \delta^3(\vec{r}_{ij}), \quad (4)$$

where $V_{Ins.}^{qq}$ is for the interaction between a quark–quark or antiquark–antiquark pair, and $V_{Ins.}^{q\bar{q}}$ is for that between a quark–antiquark pair. $\hat{g}_{ij}^{qq}$ and $\hat{g}_{ij}^{q\bar{q}}$ are flavor-dependent coupling strength operators, whose explicit matrix elements are shown in Tables 1 and 2. $P_{ij}^{S=0}$ and $P_{ij}^{S=1}$ are spin projector operators onto spin-singlet and spin-triplet states, respectively; $P_{ij}^{C,\bar{3}}$, $P_{ij}^{C,6}$, $P_{ij}^{C,1}$ and $P_{ij}^{C,8}$ are color projector operators onto color anti-triplet $\bar{3}_c$, color sextet 6_c , color singlet 1_c and color octet 8_c , respectively.

One should note that instanton-induced interaction is a pure contact interaction, and $\delta^3(\vec{r}_{ij})$ will lead to an unbound Hamiltonian if we don't treat it perturbatively. Here we regularize it as in Refs. [36, 39, 57],

$$\delta^3(\vec{r}_{ij}) \rightarrow \left(\frac{\sigma}{\sqrt{\pi}} \right)^3 \exp \left(-\sigma^2 r_{ij}^2 \right). \quad (5)$$

The regularization parameter σ should correlate to the finite size of constituent quarks and therefore flavor dependent [39], but here we just make σ to be a unified parameter for different quark flavors and absorb the flavor dependence into $\hat{g}_{ij}^{qq}$ and $\hat{g}_{ij}^{q\bar{q}}$.

2.2 Configurations of four quark systems

Considering the Pauli principle for two-quark (-antiquark) subsystems, there are eight possible flavor-spin-color symmetry configurations for $qq\bar{q}\bar{q}$ systems, as follows:

$$\begin{aligned} |1\rangle &= \{qq\}_{6_c} \{\bar{q}\bar{q}\}_{\bar{6}_c}, & |2\rangle &= [qq]_{\bar{3}_c} [\bar{q}\bar{q}]_{3_c}, \\ |3\rangle &= \{qq\}_{\bar{3}_c}^* [\bar{q}\bar{q}]_{3_c}, & |4\rangle &= [qq]_{\bar{3}_c} \{\bar{q}\bar{q}\}_{3_c}^*, \\ |5\rangle &= [qq]_{6_c}^* \{\bar{q}\bar{q}\}_{\bar{6}_c}, & |6\rangle &= \{qq\}_{6_c} [\bar{q}\bar{q}]_{\bar{6}_c}^*, \\ |7\rangle &= \{qq\}_{\bar{3}_c}^* \{\bar{q}\bar{q}\}_{3_c}^*, & |8\rangle &= [qq]_{6_c}^* [\bar{q}\bar{q}]_{\bar{6}_c}^*, \end{aligned} \quad (6)$$

where $\{\dots\}$ denotes a permutation symmetric flavor wave function and $[\dots]$ denotes a permutation antisymmetric flavor wave function of two-quark (-antiquark) subsystem. The superscript $*$ means diquark or antidiquark forms a spin-triplet, while the configurations without superscript are spin-singlet. The blackened number in the subscript denotes color wave function of two-quark (-antiquark) subsystem. As we can see, there are two possible color neutral structures of tetraquark systems, namely, $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$. Note that here we only consider the S -wave tetraquark systems, so the spatial wave functions are always completely permutation symmetric.

Table 1 Matrix elements of flavor-dependent coupling strength operators $\hat{g}_{ij}^{qq}$. $g_{f_1 f_2}$ is flavor-dependent model parameter, $n = u, d$ stands for light flavor

	ud	du	us	ds	sd	su	uc	dc	cd	cu	sc	cs	uu	dd	ss	cc
ud	g_{nn}	$-g_{nn}$	0				0				0		0			
du	$-g_{nn}$	g_{nn}														
us	0		g_{ns}	0	0	$-g_{ns}$	0				0		0			
ds			0	g_{ns}	$-g_{ns}$	0										
sd			0	$-g_{ns}$	g_{ns}	0										
su			$-g_{ns}$	0	0	g_{ns}										
uc	0		0				g_{nc}	0	0	$-g_{nc}$	0		0			
dc							0	g_{nc}	$-g_{nc}$	0						
cd							0	$-g_{nc}$	g_{nc}	0						
cu							$-g_{nc}$	0	0	g_{nc}						
sc	0		0				0				g_{sc}	$-g_{sc}$	0			
cs											$-g_{sc}$	g_{sc}				
uu	0		0				0				0		0			
dd																
ss																
cc																

Table 2 Matrix elements of flavor-dependent coupling strength operator $\hat{g}_{ij}^{q\bar{q}}$. $g_{f_1 f_2}$ is flavor-dependent model parameter, $n = u, d$ stands for light flavor

	$u\bar{d}$	$d\bar{u}$	$u\bar{s}$	$d\bar{s}$	$s\bar{d}$	$s\bar{u}$	$u\bar{c}$	$d\bar{c}$	$c\bar{d}$	$c\bar{u}$	$s\bar{c}$	$c\bar{s}$	$u\bar{u}$	$d\bar{d}$	$s\bar{s}$	$c\bar{c}$
$u\bar{d}$	$-g_{nn}$	0	0				0				0		0			
$d\bar{u}$	0	$-g_{nn}$														
$u\bar{s}$	0		$-g_{ns}$	0	0	0	0				0		0			
$d\bar{s}$			0	$-g_{ns}$	0	0										
$s\bar{d}$			0	0	$-g_{ns}$	0										
$s\bar{u}$			0	0	0	$-g_{ns}$										
$u\bar{l}$	0		0				$-g_{nc}$	0	0	0	0		0			
$d\bar{l}$							0	$-g_{nc}$	0	0						
$c\bar{d}$							0	0	$-g_{nc}$	0						
$c\bar{u}$							0	0	0	$-g_{nc}$						
$s\bar{l}$	0		0				0				$-g_{sc}$	0	0			
$c\bar{s}$											0	$-g_{sc}$				
$u\bar{u}$	0		0				0				0		0	g_{nn}	g_{ns}	g_{nc}
$d\bar{d}$													g_{nn}	0	g_{ns}	g_{nc}
$s\bar{s}$													g_{ns}	g_{ns}	0	g_{sc}
$c\bar{l}$													g_{nc}	g_{nc}	g_{sc}	0

Explicit flavor wave functions of the studied hidden- and double-charm tetraquark systems are presented in the following subsections, respectively.

2.2.1 Hidden-charm systems

There are nine flavor configurations for $q\bar{q}c\bar{c}$ with q the light quarks u, d , and s , which are: $uc\bar{u}\bar{c}, uc\bar{d}\bar{c}, uc\bar{s}\bar{c}, dc\bar{d}\bar{c}, dc\bar{s}\bar{c},$

$sc\bar{s}\bar{c}, dc\bar{u}\bar{c}, sc\bar{u}\bar{c},$ and $sc\bar{d}\bar{c}$. Compositions of these configurations could lead to the tetraquark states with quantum numbers isospin I and strangeness S , as below:

- $I = 0, S = 0.$

$$X_{n\bar{n}} = \frac{1}{\sqrt{2}} (uc\bar{u}\bar{c} + dc\bar{d}\bar{c}), \quad (7)$$

Table 3 The symmetry configurations of hidden-charm systems

For Z_{cs} states		For X and Z_c states	
J^P	Configuration	J^{PC}	Configuration
0^+	1)	0^{++}	1)
	2)		2)
	7)		7)
	8)		8)
1^+	3)	1^{++}	$ 3'\rangle = \frac{1}{\sqrt{2}}(3\rangle + 4\rangle)$
	4)		$ 5'\rangle = \frac{1}{\sqrt{2}}(5\rangle + 6\rangle)$
	5)		$ 4'\rangle = \frac{1}{\sqrt{2}}(3\rangle - 4\rangle)$
	6)	1^{+-}	$ 6'\rangle = \frac{1}{\sqrt{2}}(5\rangle - 6\rangle)$
	7)		7)
	8)		8)
	7)		7)
2^+	7)	2^{++}	7)
	8)		8)

$$X_{s\bar{s}} = sc\bar{s}\bar{c}. \quad (8)$$

- $I = \frac{1}{2}, S = \pm 1.$

$$Z_{cs}^+ = uc\bar{s}\bar{c}, \quad \bar{Z}_{cs}^0 = -sc\bar{d}\bar{c}, \quad (9)$$

$$Z_{cs}^0 = dc\bar{s}\bar{c}, \quad Z_{cs}^- = sc\bar{u}\bar{c}. \quad (10)$$

- $I = 1, S = 0.$

$$Z_c^+ = -uc\bar{d}\bar{c}, \quad (11)$$

$$Z_c^0 = \frac{1}{\sqrt{2}}(uc\bar{u}\bar{c} - dc\bar{d}\bar{c}), \quad (12)$$

$$Z_c^- = dc\bar{u}\bar{c}. \quad (13)$$

X and Z_c^0 are pure neutral systems, thus they could have C -parity. Note that $\bar{Z}_{cs}^0$ and Z_{cs}^- are the conjugate states of Z_{cs}^0 and Z_{cs}^+ , respectively. Thus, we only consider the latter set in following, since a state should share the same energy with its conjugate state in present model.

Accordingly, the symmetry configurations of hidden-charm systems in Eq. (6) could be classified by quantum numbers $J^{P(C)}$, which are listed in Table 3.

2.2.2 Double-charm systems

Similarly as in the previous section, the flavor wave functions of $cc\bar{q}\bar{q}$ systems with proper quantum numbers can be decomposed as follows:

- $I = 0, S = 0.$

$$(T_{cc}^{0,0})^+ = \frac{1}{\sqrt{2}}cc(-\bar{d}\bar{u} + \bar{u}\bar{d}), \quad (14)$$

Table 4 The symmetry configurations of double-charm systems

$T_{cc}^{I,S}$	J^P	Configuration
$T_{cc}^{0,0}$	1^+	3)
		6)
$T_{cc}^{0,2}$	0^+	1)
		7)
	1^+	7)
		7)
$T_{cc}^{\frac{1}{2},1}$	0^+	1)
		7)
	1^+	3)
		6)
		7)
		7)
$T_{cc}^{1,0}$	2^+	7)
		7)
	0^+	1)
		7)
	1^+	7)
		7)

- $I = 0, S = 2.$

$$(T_{cc}^{0,2})^{++} = cc\bar{s}\bar{s}, \quad (15)$$

- $I = \frac{1}{2}, S = 1.$

$$(T_{cc}^{\frac{1}{2},1})^+ = -\frac{1}{\sqrt{2}}cc(\bar{d}\bar{s} \pm \bar{s}\bar{d}), \quad (16)$$

$$(T_{cc}^{\frac{1}{2},1})^0 = \frac{1}{\sqrt{2}}cc(\bar{u}\bar{s} \pm \bar{s}\bar{u}), \quad (17)$$

- $I = 1, S = 0.$

$$(T_{cc}^{1,0})^{++} = cc\bar{d}\bar{d}, \quad (18)$$

$$(T_{cc}^{1,0})^+ = -\frac{1}{\sqrt{2}}cc(\bar{d}\bar{u} + \bar{u}\bar{d}), \quad (19)$$

$$(T_{cc}^{1,0})^0 = cc\bar{u}\bar{u}, \quad (20)$$

here we use $(T_{cc}^{I,S})^Q$ to denote $cc\bar{q}\bar{q}$ systems, where I, S and Q are isospin, strangeness, and electric charge of the state, respectively.

For the double-charm states, the flavor wave function of diquark must be symmetric since it is cc , and the symmetry of antidiquark flavor wave functions is determined by Eqs. (14–20). All the configurations of double-charm tetraquark systems and their corresponding quantum numbers are listed in Table 4.

Table 5 Parameters used in this work

$m_n = 340 \text{ MeV}$	$m_s = 511 \text{ MeV}$
$m_c = 1674 \text{ MeV}$	
$b = 0.39 \text{ GeV} \cdot \text{fm}^{-1}$	$C_0 = -296 \text{ MeV}$
$\alpha_{nn} = 0.600$	$\alpha_{ns} = 0.600$
$\alpha_{ss} = 0.562$	$\alpha_{nc} = 0.509$
$\alpha_{sc} = 0.480$	$\alpha_{cc} = 0.450$
$g_{nn} = 0.169 \times 10^{-4} \text{ MeV}^{-2}$	$g_{ns} = 0.107 \times 10^{-4} \text{ MeV}^{-2}$
$g_{nc} = 0.040 \times 10^{-4} \text{ MeV}^{-2}$	$g_{sc} = 0.031 \times 10^{-4} \text{ MeV}^{-2}$
$\sigma = 485 \text{ MeV}$	

Table 6 Ground state hadron spectrum. The columns denoted by “M” are model calculations while the columns denoted by “PDG” are experimental data for the masses of these states in PDG [1]. The units are in MeV

States	M	PDG	States	M	PDG
π	139	138	J/ψ	3097	3097
ρ	775	775	Σ_c	2447	2454
ω	775	783	Σ_c^*	2518	2518
ϕ	1019	1019	Ξ_c	2507	2469
K	499	496	Ξ_c'	2554	2577
K^*	894	895	Ξ_c^*	2627	2646
D	1866	1867	Ω_c	2664	2695
D^*	2009	2009	Ω_c^*	2737	2766
D_s	1969	1968	Ξ_{cc}	3607	3621
D_s^*	2111	2112			

2.3 Wave functions

For the calculation of Hamiltonian matrices, a Gaussian functions expansion method [60–63] is used here to solve the Schrödinger equation of Hamiltonian Eq. (1), where the orbital wave function of the ground S -wave tetraquark system can be expanded by a series of Gaussian functions,

$$\Psi(\{\vec{r}_i\}) = \prod_{i=1}^4 \sum_{\ell} C_{i\ell} \left(\frac{1}{\pi b_{i\ell}^2} \right)^{3/4} \exp \left[-\frac{1}{2b_{i\ell}^2} r_i^2 \right], \quad (21)$$

here $\{b_{i\ell}\}$ are harmonic oscillator length parameters and can be related to the frequencies $\{\omega_{\ell}\}$ with $1/b_{i\ell}^2 = m_i \omega_{\ell}$, where we have taken the ansatz that ω_{ℓ} is independent to the quark mass as in Ref. [62]. Then the orbital wave function can be simplified to be

$$\begin{aligned} \Psi(\{\vec{r}_i\}) &= \sum_{\ell} C_{\ell} \prod_{i=1}^4 \left(\frac{m_i \omega_{\ell}}{\pi} \right)^{3/4} \exp \left[-\frac{m_i \omega_{\ell}}{2} r_i^2 \right] \\ &= \sum_{\ell} C_{\ell} \psi(\omega_{\ell}, \{\vec{r}_i\}), \end{aligned} \quad (22)$$

which is often adopted for the calculations of multi-quark systems [60, 61].

On the other hand, here we define the Jacobi coordinates by $\{\vec{r}_i\}$ as

$$\vec{\xi}_1 = \vec{r}_1 - \vec{r}_2, \quad (23)$$

$$\vec{\xi}_2 = \vec{r}_3 - \vec{r}_4, \quad (24)$$

$$\vec{\xi}_3 = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} - \frac{m_3 \vec{r}_3 + m_4 \vec{r}_4}{m_3 + m_4}, \quad (25)$$

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + m_4 \vec{r}_4}{m_1 + m_2 + m_3 + m_4}, \quad (26)$$

to remove contributions of the motion of the center of mass directly. Where we have enumerated the two quarks by 1, 2 and the two antiquarks by 3, 4, respectively.

In the calculations we define $1/b_{\ell}^2 \equiv m_q \omega_{\ell}$ (m_q denotes the constituent mass of the corresponding quark). Following the work of Ref. [63], the parameters $\{b_{\ell}\}$ are set to be geometric series,

$$b_{\ell} = b_1 a^{\ell-1} \quad (\ell = 1, 2, \dots, n), \quad (27)$$

where n is the number of Gaussian functions and a the ratio coefficient, then there are only three parameters $\{b_1, b_n, n\}$ need to be determined.

Using the orbital wave function in Eq. (22), we compute the diagonal elements of the Hamiltonian matrices by solving the generalized matrix eigenvalue problem:

$$\sum_{\ell} \sum_{\ell'} C_{\ell}^i \left(H_{\ell\ell'}^d - E_i^d N_{\ell\ell'} \right) C_{\ell'}^i = 0, \quad (28)$$

where $i = 1-n$ and

$$H_{\ell\ell'}^d = \langle \psi(\omega_{\ell}) (FSC) | H_{eff} | \psi(\omega_{\ell'}) (FSC) \rangle, \quad (29)$$

$$N_{\ell\ell'} = \langle \psi(\omega_{\ell}) (FSC) | \psi(\omega_{\ell'}) (FSC) \rangle, \quad (30)$$

here (FSC) stands for flavor $\otimes$ spin $\otimes$ color wave function. One can choose a set of $\{b_1, b_n, n\}$ tentatively, then extend and densify the harmonic oscillator length parameters to get a minimum energy E_m^d , which should be correlative to the energy of physical state according to the Rayleigh-Ritz variational principle. In present work we take $\{b_1, b_n, n\} = \{0.02 \text{ fm}, 6 \text{ fm}, 40\}$ for the light quarks, and one can directly obtain the ranges for the strange and charm quarks by replacing the light quark mass by strange and charm quark masses, respectively. Besides, the nondiagonal elements of Hamiltonian matrices can be easily obtained using $\{C_{\ell}^m\}$.

2.4 Model parameters

There are sixteen parameters in our model, namely, constituent quark masses m_n , m_s and m_c ; quark confinement strength b , QCD effective coupling constants α_{ij} (i and j are quark flavor), and zero point energy C_0 for Cornell potential;

Table 7 The numerical results of ground S -wave X states with $I^G = 0^+$

Quark content	J^{PC}	Single configuration		Configurations mixing	
		Config.	Energies (MeV)	Energies (MeV)	Mixing coefficients
$c\bar{c}n\bar{n}$	0^{++}	[1]	4161.55	4039.34	(−0.64, −0.43, 0.00, −0.64)
		[2]	4131.90	4115.93	(0.00, −0.75, −0.43, 0.50)
		[7]	4212.69	4231.32	(0.28, −0.46, 0.84, 0.02)
		[8]	4131.28	4250.83	(0.71, −0.20, −0.34, −0.59)
$c\bar{c}n\bar{n}$	1^{++}	[3′]	4151.06	4052.78	(0.51, −0.86)
		[5′]	4088.23	4186.52	(−0.86, −0.51)
$c\bar{c}n\bar{n}$	1^{+-}	[4′]	4193.21	4053.08	(−0.46, 0.51, −0.25, 0.68)
		[6′]	4190.93	4186.87	(−0.26, 0.41, −0.55, −0.68)
		[7]	4211.40	4228.55	(0.75, 0.05, −0.61, 0.24)
		[8]	4127.50	4254.52	(0.40, 0.76, 0.51, −0.11)
$c\bar{c}n\bar{n}$	2^{++}	[7]	4208.78	4055.73	(−0.54, −0.84)
		[8]	4119.53	4272.57	(−0.84, 0.54)
$c\bar{c}s\bar{s}$	0^{++}	[1]	4341.92	4203.76	(−0.24, −0.37, −0.14, 0.89)
		[2]	4293.15	4270.15	(0.61, −0.77, 0.10, −0.14)
		[7]	4361.49	4340.17	(0.34, 0.32, 0.81, 0.35)
		[8]	4237.31	4419.79	(−0.67, −0.41, 0.56, −0.27)
$c\bar{c}s\bar{s}$	1^{++}	[3′]	4328.42	4284.16	(−0.39, −0.92)
		[5′]	4292.14	4336.39	(−0.92, 0.39)
$c\bar{c}s\bar{s}$	1^{+-}	[4′]	4350.70	4265.06	(0.00, 0.00, −0.17, 0.98)
		[6′]	4347.16	4283.64	(0.70, −0.72, 0.00, 0.00)
		[7]	4372.05	4375.42	(0.00, 0.00, 0.98, 0.17)
		[8]	4268.42	4414.23	(0.72, 0.70, 0.00, 0.00)
$c\bar{c}s\bar{s}$	2^{++}	[7]	4392.52	4286.55	(0.52, −0.85)
		[8]	4325.86	4431.83	(−0.85, −0.52)

First and second columns are the quark content of each state and the corresponding J^{PC} quantum numbers, the fourth column are the results of single configuration calculations, fifth and sixth columns are the model results after considering configuration mixing, but not considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing.

Table 8 The results of the X states with quantum numbers $J^{PC} = 1^{++}$ by considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing

Model values			Experimental values	
M (MeV)	$c\bar{c}n\bar{n}$	$c\bar{c}s\bar{s}$	States	M (MeV)
4052.72	99.98%	0.02%	—	—
4176.07	93.70%	6.30%	$X(4140)$	4146.8 ± 2.4
4284.76	0.46%	99.54%	$X(4274)$	4274^{+8}_{-6}
4346.29	5.86%	94.14%	—	—

g_{nn} , g_{ns} , g_{nc} , g_{sc} , and σ for instanton-induced interaction. All these above model parameters are collected in Table 5.

Since none of the exotic states is identified as a pure tetraquark state, one cannot take the masses of these exotic states as input to evaluate the model parameters. In our model, the interactions between quarks and antiquarks in tetraquark states are simple superposition of two-body confinements and two-body residual interactions, which is the same as the description of traditional baryons and mesons in CQM.

Therefore, we just extract model parameters from traditional hadron spectra. In this work, eleven mesons and eight baryons are taken as inputs to determine these model parameters.

It is worthy to mention that, if the pseudoscalar mesons whose dominant contents are considered to be pure $q_{f_1}\bar{q}_{f_2}$ with $f_1 = f_2$, as shown in Table 2, the instanton-induced interaction effects will vanish, this should lead to the mass relation $m_{\eta_c} = m_{J/\psi}$, it's obviously unreasonable. In fact, as discussed in [64], the instanton-induced interaction should cause mixing between this kind of states with quantum number 0^{-+} naturally, although the mixing between the $c\bar{c}$ pair with the light quark–antiquark pairs should be very small, it will obviously influence on the mass of η_c . Consequently, to get the correct mass relations, one has to consider the flavor-configuration mixing for η , η' and η_c . However, the mixing angles are unknown and difficult to determine [64]. Thus, these states η , η' and η_c are not taken into account for determining the model parameters shown in Table 5. In addition, the effective strong coupling constant between two charm quarks is determined by fitting the mass of J/ψ .

Table 9 As in Table 7 but for the case of Z_c states

Quark content	$I^G(J^{PC})$	Single configuration		Configurations mixing	
		Config.	Energies (MeV)	Energies (MeV)	Mixing coefficients
$c\bar{c}n\bar{n}$	$1^-(0^{++})$	1)	4079.46	3777.23	(0.23, 0.42, 0.30, 0.82)
		2)	4048.33	4018.55	(−0.76, −0.17, −0.42, 0.46)
		7)	4093.05	4076.61	(−0.04, −0.78, 0.60, 0.19)
		8)	3872.81	4221.26	(−0.60, 0.43, 0.61, −0.28)
$c\bar{c}n\bar{n}$	$1^-(1^{++})$	3')	4099.70	4052.82	(−0.40, −0.91)
		5')	4061.95	4108.84	(−0.91, 0.40)
$c\bar{c}n\bar{n}$	$1^+(1^{+-})$	4')	4095.30	3814.57	(−0.43, −0.53, −0.31, 0.66)
		6')	4050.89	4050.16	(0.48, 0.54, −0.14, 0.68)
		7)	4118.37	4149.29	(−0.28, 0.09, 0.90, 0.32)
		8)	3959.10	4209.65	(0.71, −0.65, 0.26, 0.07)
$c\bar{c}n\bar{n}$	$1^-(2^{++})$	7)	4164.70	4052.98	(−0.52, −0.85)
		8)	4095.25	4206.97	(−0.85, 0.52)

Table 10 As in Table 7 but for the case of Z_{cs} states

Quark content	$I(J^P)$	Single configuration		Configurations mixing	
		Config.	Energies (MeV)	Energies (MeV)	Mixing coefficients
$c\bar{c}n\bar{s}$	$\frac{1}{2}(0^+)$	1)	4198.04	3951.72	(0.17, 0.45, −0.24, 0.84)
		2)	4158.90	4129.62	(−0.76, −0.23, 0.45, 0.41)
		7)	4210.36	4186.38	(0.00, −0.76, −0.61, 0.23)
		8)	4019.17	4318.75	(−0.62, 0.41, −0.61, −0.26)
$c\bar{c}n\bar{s}$	$\frac{1}{2}(1^+)$	3)	4205.97	3988.72	(−0.31, −0.31, 0.37, −0.37, 0.29, 0.68)
		4)	4209.70	4159.09	(0.34, 0.34, −0.28, 0.48, 0.10, 0.67)
		5)	4176.11	4165.97	(0.31, −0.21, −0.70, −0.60, 0.03, 0.09)
		6)	4174.25	4212.73	(0.64, −0.67, 0.28, 0.23, −0.05, −0.02)
		7)	4231.97	4253.47	(−0.20, −0.16, −0.07, 0.01, −0.92, 0.28)
		8)	4089.37	4307.39	(−0.49, −0.52, −0.47, 0.47, 0.22, −0.05)
		7)	4272.18	4167.21	(0.52, −0.85)
$c\bar{c}n\bar{s}$	$\frac{1}{2}(2^+)$	8)	4205.90	4310.87	(−0.85, −0.52)

With these model parameters shown in Table 5, the model calculations for the hadron spectra are shown in Table 6.

3 Results and discussions

With the model parameters in Table 5, one can calculate the mass spectra of the ground S -wave hidden- and double-charm tetraquark systems. The numerical results are shown in Tables 7, 8, 9, 10, 11, 12, 13.

As discussed in the Sect. 2, there are two possible color structures of tetraquark configurations, namely, $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ and $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$. The results of color factor $\langle \bar{\lambda}_i^c \cdot \bar{\lambda}_j^c \rangle$ show that, the Cornell potentials are attractive between each of quarks and antiquarks in tetraquark states with $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ color structure (configurations |2), |3), |4), |7)). However, they are repul-

sive for those states with $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ color structure (configurations |1), |5), |6), |8)). Intuitively, one might think that the tetraquark states with $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ structure could be more bound than those states with the $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$, and thus have lower energies.

However, this is not always true. Because, on one hand, a tighter binding is accompanied by a higher quark kinetic energy, on the other hand a color dependent residual interaction, Instanton-induced interaction, is considered in the model, which has a much more complicated interaction structure in tetraquark states compared with traditional meson states.

Consequently, a physical tetraquark state should be composed of the two kinds of components with $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ and $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ color structures. In Tables 7, 9, 10 and 13, the numerical results including mixing of the two kinds of components are

Table 11 The comparison of model results with experimental masses for Z_c states

Present results		Experimental data		
$I^G(J^{PC})$	M(MeV)	States	$I^G(J^{PC})$	M(MeV)
$1^+(1^{+-})$	3814.57	$Z_c(3900)$	$1^+(1^{+-})$	3887.1 ± 2.6
$1^+(1^{+-})$	4050.16	$X(4020)$	$1^+(?^{2-})$	4024.1 ± 1.9
$1^-(1^{++})$	4052.82	$X(4050)$	$1^-(?^{2+})$	$4051 \pm 14^{+20}_{-41}$
$1^-(2^{++})$	4052.98			
$1^+(1^{+-})$	4050.16	$X(4055)$	$1^+(?^{2-})$	$4054 \pm 3 \pm 1$
$1^-(0^{++})$	4076.61	$X(4100)$	$1^-(?^{2+})$	$4096 \pm 20^{+18}_{-22}$
$1^-(0^{++})$	4221.26	$X(4250)$	$1^-(?^{2+})$	$4248^{+44+180}_{-29-35}$
$1^-(2^{++})$	4206.97			

Fourth, fifth and sixth columns collect the experimental data of Z_c states near 4.1 GeV, and first two columns collect the model results in which model masses are close to the experimental masses.

Table 12 Comparison of model results with experimental masses for Z_{cs} states

Model values		Experiment values		
$I(J^P)$	M(MeV)	States	$I(J^P)$	M(MeV)
$1/2(0^+)$	3951.72	$Z_{cs}(3985)$	$1/2(?^?)$	$3982.5^{+1.8}_{-2.6} \pm 2.1$
$1/2(1^+)$	3988.72			
$1/2(1^+)$	3988.72	$Z_{cs}(4000)$	$1/2(1^+)$	$4003 \pm 6^{+4}_{-14}$
$1/2(1^+)$	4165.97	$Z_{cs}(4220)$	$1/2(1^?)$	$4216 \pm 24^{+43}_{-30}$
$1/2(1^+)$	4212.73			
$1/2(1^+)$	4253.47			

Fourth, fifth and sixth columns collect the experimental data of Z_{cs} states, and first two columns collect the model results in which model masses are close to the experimental masses.

presented in the last two columns. Obviously, effects of configurations mixing are important.

In the following subsections, we discuss the results of X , Z_c , Z_{cs} and double-charm tetraquark states, respectively.

3.1 X states

In this subsection, we present the numerical results for spectrum of the ground S -wave $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ states, those we call X states.

Similar to η - η' mixing, there are transitions between $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ quark configurations, which can be naturally caused by the instanton-induced interaction. Consequently, we will discuss the results within two models: one takes into account only the single configurations $cn\bar{c}\bar{n}$ or $cs\bar{c}\bar{s}$, respectively; while the other one includes the mixing of these configurations. The numerical results of energies for the $I^G = 0^+$ X states are given in Tables 7 and 8, respectively, where the masses and configurations mixing coefficients for the obtained tetraquark states are shown explicitly.

Meanwhile, we depict the obtained mass spectrum of X states in Fig. 1 comparing with the experimental measurements, where red solid lines denote $cn\bar{c}\bar{n}$ system and blue lines denote $cs\bar{c}\bar{s}$ system, hollow circles with crosses denote results after considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing, gray dotted lines are corresponding S -wave charm meson pair thresholds, finally, the green rectangles represent experimental masses of these X states taken from PDG [1], the rectangle widths stand their masses uncertainties.

Experimentally, in 2003, the $X(3872)$ state was observed by Belle collaboration [2] and its quantum number was determined to be $J^{PC} = 1^{++}$ [65]. Explicitly, its mass and width are: $M = 3871.65 \pm 0.06$ and $\Gamma = 1.19 \pm 0.01$ MeV [1]. Its mass is extremely close to the $D^0\bar{D}^{*0}$ mass threshold, which indicates that $X(3872)$ may be explained as a $D^0\bar{D}^{*0}$ hadronic molecule. In present work, the lowest energy of $cn\bar{c}\bar{n}$ states with quantum numbers $I^G(J^{PC}) = 0^+(1^{++})$ is about 4053 MeV, which is much higher than the mass of $X(3872)$. Consequently, our result may support that the $X(3872)$ is mostly dominated by a hadronic molecular state.

The $X(3930)$ and $X(3915)$ states are observed through two photon fusion processes by Belle in 2005 [66] and 2010 [67], respectively. They were suggested to be good candidates of the 2^3P_0 and 2^3P_2 charmonium states, $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$, respectively [66–68]. From our results shown in Table 7 and Fig. 1, it is found that our results of the states with quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$ and $I^G(J^{PC}) = 0^+(2^{++})$ are all higher than the masses of $X(3930)$ and $X(3915)$. Thus $X(3930)$ and $X(3915)$ cannot be described by the compact tetraquark states.

The CDF collaboration reported the discoveries of $X(4140)$ and $X(4274)$ in the $J/\psi\phi$ invariant mass distribution of the process $B^+ \rightarrow J/\psi\phi K^+$ in 2009 [69] and 2011 [70], respectively. Later, LHCb collaboration confirmed their existences and determined their spin-parity quantum numbers to be $J^P = 1^+$ [71–73]. Masses and widths of $X(4140)$ and $X(4274)$ are [1]

$$(M_{X(4140)} = 4146.8 \pm 2.4, \Gamma_{X(4140)} = 22^{+8}_{-7}) \text{ MeV},$$

$$(M_{X(4274)} = 4274^{+8}_{-6}, \Gamma_{X(4274)} = 49 \pm 12) \text{ MeV},$$

respectively. The next lowest energy of the $cn\bar{c}\bar{n}$ states with quantum number $I^G(J^{PC}) = 0^+(1^{++})$ in present model is around 4187 MeV, very close to the mass of $X(4140)$ state, while the lowest energy of the $cs\bar{c}\bar{s}$ states with quantum number $I^G(J^{PC}) = 0^+(1^{++})$ in present model is about 4284 MeV, very close to the mass of $X(4274)$ state. Thus $X(4140)$ and $X(4274)$ can be interpreted as compact $c\bar{c}n\bar{n}$ and $c\bar{c}s\bar{s}$ tetraquark states with quantum number $I^G(J^{PC}) = 0^+(1^{++})$, respectively.

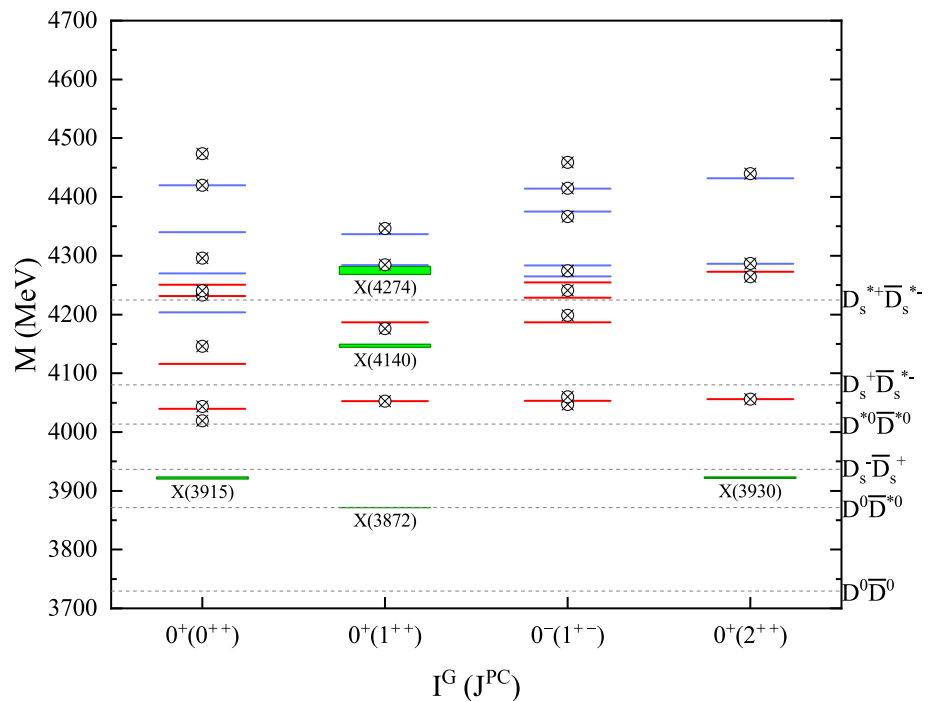
The results by considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing are also given in Fig. 1, which are denoted by hollow circles with

Table 13 The model results of ground S -wave double-charm tetraquark states.

States	Quark content	$(I, S)(J^P)$	Single configuration		Configurations mixing	
			Config.	M (MeV)	M (MeV)	Mixing coefficients
$T_{cc}^{0,0}$	$cc\bar{n}\bar{n}$	$(0, 0)(1^+)$	$ 3\rangle$	3998.90	3982.12	$(-0.95, -0.30)$
			$ 6\rangle$	4152.40	4169.18	$(-0.30, 0.95)$
$T_{cc}^{1,0}$	$cc\bar{n}\bar{n}$	$(1, 0)(0^+)$	$ 1\rangle$	4204.41	4032.30	$(0.42, -0.91)$
			$ 7\rangle$	4069.69	4241.80	$(-0.91, -0.42)$
$T_{cc}^{1,0}$	$cc\bar{n}\bar{n}$	$(1, 0)(1^+)$	$ 7\rangle$	4092.34	—	—
$T_{cc}^{1,0}$	$cc\bar{n}\bar{n}$	$(1, 0)(2^+)$	$ 7\rangle$	4134.59	—	—
$T_{cc}^{\frac{1}{2},1}$	$cc\bar{n}\bar{s}$	$(\frac{1}{2}, 1)(0^+)$	$ 1\rangle$	4296.64	4132.95	$(-0.45, -0.89)$
			$ 7\rangle$	4174.01	4337.71	$(-0.89, 0.45)$
$T_{cc}^{\frac{1}{2},1}$	$cc\bar{n}\bar{s}$	$(\frac{1}{2}, 1)(1^+)$	$ 3\rangle$	4134.14	4115.10	$(-0.94, -0.34, 0.00)$
			$ 6\rangle$	4257.37	4197.18	$(0.00, 0.00, 1.00)$
			$ 7\rangle$	4197.18	4276.41	$(0.34, -0.94, 0.00)$
$T_{cc}^{\frac{1}{2},1}$	$cc\bar{n}\bar{s}$	$(\frac{1}{2}, 1)(2^+)$	$ 7\rangle$	4240.55	—	—
$T_{cc}^{0,2}$	$cc\bar{s}\bar{s}$	$(0, 2)(0^+)$	$ 1\rangle$	4387.45	4234.84	$(0.48, -0.88)$
			$ 7\rangle$	4280.69	4433.30	$(-0.88, -0.48)$
$T_{cc}^{0,2}$	$cc\bar{s}\bar{s}$	$(0, 2)(1^+)$	$ 7\rangle$	4304.25	—	—
$T_{cc}^{0,2}$	$cc\bar{s}\bar{s}$	$(0, 2)(2^+)$	$ 7\rangle$	4348.45	—	—

First three columns are the labels of these states in the model, quark content and quantum number of each state respectively, the fourth, fifth columns give the results of single configuration calculations, and the sixth, seventh columns give the results after considering configuration mixing.

Fig. 1 Mass spectrum of ground S -wave X states. Red solid lines denote $cn\bar{c}\bar{n}$ system and blue lines denote $cs\bar{c}\bar{s}$ system, hollow circles with crosses denote results after considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing, gray dotted lines are corresponding S -wave charm meson pair thresholds, the green rectangles represent X states experimental masses taken from PDG [1], the rectangle width stand their masses uncertainties



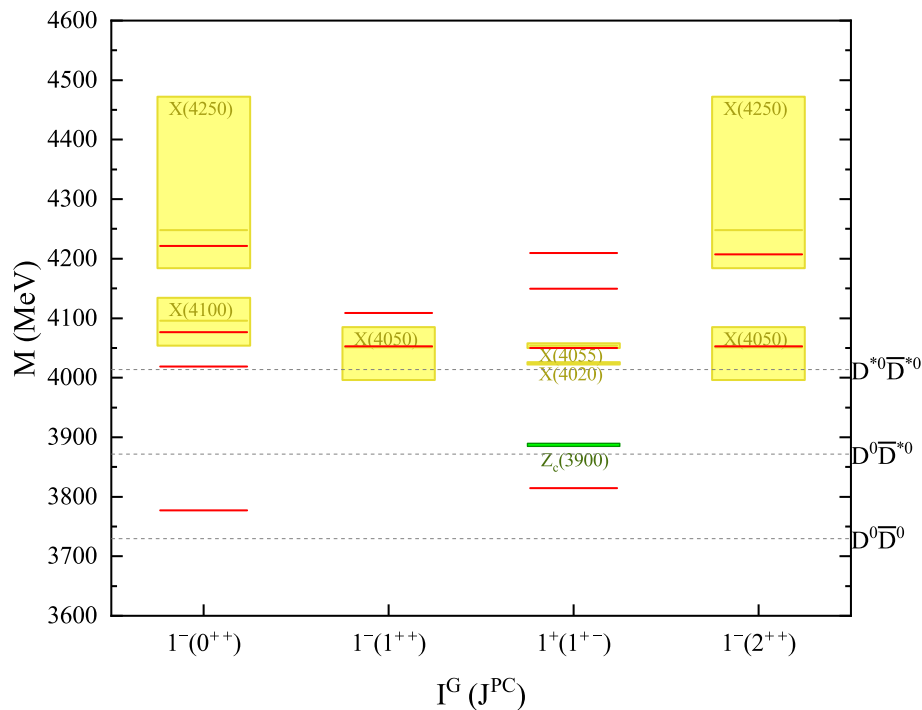


Fig. 2 Mass spectra of ground S -wave Z_c states. Red solid lines denote the model results. Gray dotted lines are corresponding S -wave charm meson pair thresholds. The green rectangle represents experimental mass of $Z_c(3900)$ with definite quantum number, the rectangle width is its mass uncertainty. The yellow rectangles represent the electric charmonium-like states, observed in experiments near 4.1 GeV, whose

crosses. It can be found that configurations mixing effects in the tetraquark states with quantum numbers $J^{PC} = 0^{++}$ are significant, while those in the $J^{PC} = 1^{++}$ tetraquark states are tiny. The results of four of the obtained X states with quantum number $J^{PC} = 1^{++}$, considering $cn\bar{c}\bar{n}$ and $cs\bar{c}\bar{s}$ mixing, are shown in Table 8 with explicit probabilities of the admixtures. One can find that every state is dominated by a single configuration.

Finally, as shown in Fig. 1, one may note that most of the energies for the obtained X states in present work are near but higher than thresholds of corresponding S -wave meson-meson channels, with only a few exceptions including the states shown in Table 8. This may indicate that most of the presently obtained X states with compact tetraquark structure cannot form considerable components in the physical meson exotics.

3.2 Z_c and Z_{cs} states

The numerical results for S -wave $cn\bar{c}\bar{n}$ and $cn\bar{c}\bar{s}$ systems are collected in Tables 9 and 10, respectively. In these tables, the first and second columns give the quark content of each state and their corresponding quantum numbers, the fourth column shows the numerical results considering only single configurations, while numbers in the fifth column are

spin-parity quantum numbers have not been pinned down yet, the corresponding quantum numbers in figure are suggested by model calculation, the rectangle width stand their masses uncertainties, while the deep yellow solid lines in rectangles are central values of their experimental masses. All the experimental values shown in this figure are taken from PDG [1]

the results obtained by including the configurations mixing effects, and the explicit probability amplitudes for the mixed configurations are shown in the last column.

In addition, we also depict the mass spectra of S -wave $cn\bar{c}\bar{n}$ and $cn\bar{c}\bar{s}$ tetraquark states in Figs. 2 and 3 comparing with the experimental measurements [1, 4, 73], where the red solid lines are results obtained in present work, while the green and yellow rectangles represent the states with and without definite quantum numbers in experiments, respectively, with the rectangle widths indicating their masses uncertainties, and the gray dotted lines denote the corresponding S -wave charm meson pair thresholds.

In 2013, BESIII collaboration reported the electric charmoniumlike state $Z_c(3900)^\pm$ in the $\pi^\pm J/\psi$ invariant mass spectrum of the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ [3]. And the same structure was also observed in the same process by Belle collaboration [74]. Later on, $Z_c(3885)^\pm$ was discovered in the $(D\bar{D}^*)^\pm$ invariant mass spectrum of the process $e^+e^- \rightarrow \pi^\mp (D\bar{D}^*)^\pm$ by BESIII collaboration, which should have a very similar mass with the $Z_c(3900)^\pm$ state [75]. Since they have the same spin-parity $J^P = 1^+$, similar mass and width, $Z_c(3900)$ and $Z_c(3885)$ are probably the same state, and mainly couple dominantly to the $D\bar{D}^*$ channel [75]. Thus we will not distinguish the $Z_c(3900)$ and $Z_c(3885)$ in the following discussions.

The average mass and width of $Z_c(3900)$ from PDG [1] are: $M_{Z_c(3900)} = 3887.1 \pm 2.6$ and $\Gamma_{Z_c(3900)} = 28.4 \pm 2.6$ MeV. In present model, the lowest mass of ground S -wave $cn\bar{c}\bar{n}$ states with quantum numbers $I^G(J^{PC}) = 1^+(1^{+-})$, considering configuration mixing, is about 3815 MeV. This value is lower than, but not far from, the experimental mass of $Z_c(3900)$. This may indicate that the compact $c\bar{c}n\bar{n}$ tetraquark components may take notable probability in the $Z_c(3900)$ state.

The $Z_c(4025)$ state was observed in $D\bar{D}^*$ final state in Ref. [76] and, meanwhile, a similar structure $Z_c(4020)$ was observed in πh_c final state by BESIII collaboration [77]. At present, the $Z_c(4020)$ and $Z_c(4025)$ states are denoted as an identical state $X(4020)$ in PDG [1]. Its average mass and width are $M_{Z_c(4020)} = 4024.1 \pm 1.9$, $\Gamma_{Z_c(4020)} = 13 \pm 5$ MeV, and its quantum number are probably $J^{PC} = 1^{+-}$ [21]. In present model, the next-lowest mass of ground S -wave $cn\bar{c}\bar{n}$ state with quantum number $I^G(J^{PC}) = 1^+(1^{+-})$, considering configuration mixing, is about 4050 MeV, which is very close to the mass of $X(4020)$. Therefore, one may expect that the $c\bar{c}n\bar{n}$ tetraquark configuration obtained here could be a component in the $X(4020)$ state.

Besides, there are several other electric charmonium-like particles observed experimentally, which need to be further confirmed. Here we also compare the present numerical results with these experimental data, as shown in the Table 11, the last three columns collect the experimental data of Z_c states near 4.1 GeV, and the first two columns are the presently obtained numerical results those are close to the experimental data.

The $X(4050)$ and $X(4250)$ states were reported by Belle collaboration in the $\pi^+\chi_{c1}$ invariant mass distribution of the process $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ [78]. In present model, we get two states with masses very close two $X(4050)$, whose quantum numbers are $I^G(J^{PC}) = 1^-(1^{++})$ and $I^G(J^{PC}) = 1^-(2^{++})$, respectively. Meanwhile, masses of two obtained states with the quantum numbers $I^G(J^{PC}) = 1^-(0^{++})$ and $I^G(J^{PC}) = 1^-(2^{++})$, respectively, are a little bit lower than $X(4250)$. Finally, two obtained states, whose quantum numbers are $I^G(J^{PC}) = 1^+(1^{+-})$ and $1^-(0^{++})$, respectively, are very close to the $X(4055)^\pm$ observed in $\pi^\pm\psi$ final state through process $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ by Belle [79], and $X(4100)$ observed in the $\eta_c\pi\pi^-$ invariant mass spectrum in $B^0 \rightarrow \eta_c K^+\pi^-$ decay by LHCb collaboration [80]. We look forward to further examinations on these results by future experiments.

Now we turn to the Z_{cs} states. The $Z_{cs}(3985)$ was the first observed charmonium-like state containing a strange quark, it was discovered in the $D_s^- D^{*0}(D_s^{*-} D^0)$ channel by BESIII very recently [4]. The mass and width of $Z_{cs}(3985)$ are

$M_{Z_{cs}(3985)} = 3982.5_{-2.6}^{+1.8} \pm 2.1$ and $\Gamma_{Z_{cs}(3985)} = 12.8_{-4.4}^{+5.3} \pm 3.0$ MeV. Its mass is very close to the $D_s^- D^{*0}(D_s^{*-} D^0)$ mass threshold. Recently, an theoretical analysis shows that the $Z_{cs}(3985)$ cannot be a pure $D_s^- D^{*0}(D_s^{*-} D^0)$ molecule [23]. And in Ref. [81], it has been shown that the two charm-meson molecular components make up about 40 percent of the physical $Z_{cs}(3985)$ state.

In addition, the $Z_{cs}(3985)$ state has also been studied within constituent quark models [9,10]. For example, in Ref. [9], the diquark-antidiquark $[cs][\bar{c}\bar{n}]$ configurations have been investigated, and it's found that there would be resonance states falling in the mass range of 3916.5 ~ 3964.6 MeV with quantum numbers $J^P = 0^+$, and 4008.8 ~ 4091.2 MeV with $J^P = 1^+$. Thus, one may conclude that a compact tetraquark state with $J^P = 0^+$ or $J^P = 1^+$ may take considerable probability in $Z_{cs}(3985)$. And in a very recent paper [82], it's shown that the Z_{cs} state may correspond to a cusp structure.

In present model, the lowest mass of ground S -wave $cn\bar{c}\bar{s}$ states with quantum number $I(J^P) = 1/2(0^+)$ and $I(J^P) = 1/2(1^+)$ are about 3952 MeV and 3989 MeV, respectively. Both of them are close to the experimental mass of $Z_{cs}(3985)$. Therefore, based on the above discussions, our results also support that the $Z_{cs}(3985)$ may be not a pure hadronic molecule, and the compact tetraquark component should take notable probability in it.

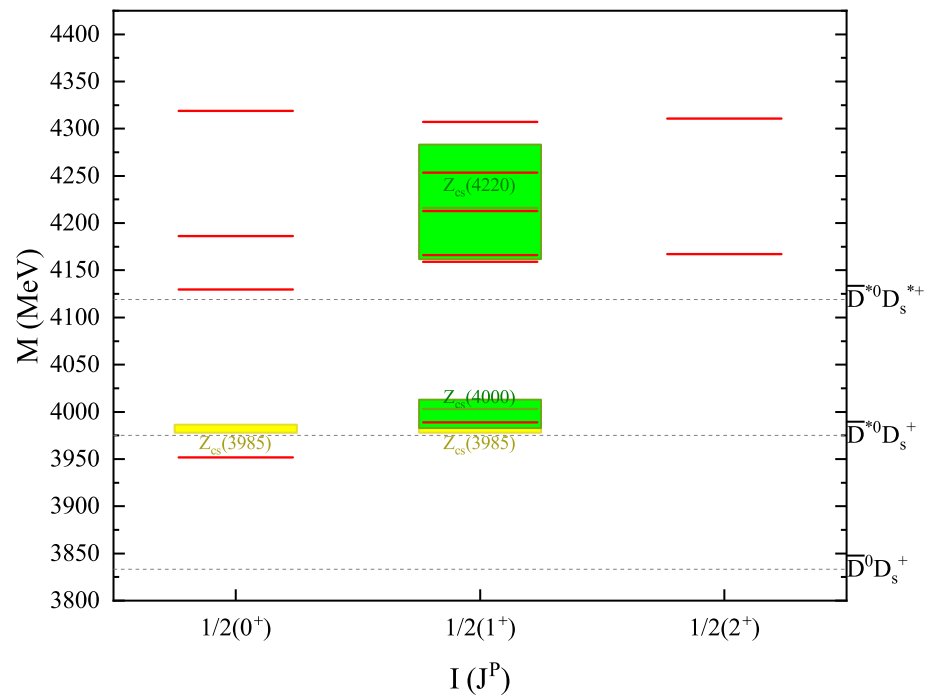
Besides, LHCb collaboration performed an analysis of the $B^+ \rightarrow J/\psi\phi K^+$ decay and reported two Z_{cs} particles, $Z_{cs}(4000)$ and $Z_{cs}(4220)$ in the invariant $J/\psi K^+$ mass distributions [73]. The spin-parity of $Z_{cs}(4000)$ is $J^P = 1^+$ and $Z_{cs}(4220)$ is $J^P = 1^+$ or $J^P = 1^-$. Their fitting masses and widths are

$$\begin{aligned} M_{Z_{cs}(4000)} &= 4003 \pm 6_{-14}^{+4} \text{ MeV}, \\ \Gamma_{Z_{cs}(4000)} &= 131 \pm 15 \pm 26 \text{ MeV}, \\ M_{Z_{cs}(4220)} &= 4216 \pm 24_{-30}^{+43} \text{ MeV}, \\ \Gamma_{Z_{cs}(4220)} &= 233 \pm 52_{-73}^{+97} \text{ MeV}, \end{aligned}$$

respectively. Although the mass of $Z_{cs}(4000)$ is very close to that of $Z_{cs}(3985)$, there is no evidence that $Z_{cs}(4000)$ and $Z_{cs}(3985)$ are the same state [73]. In present model, the lowest ground S -wave $cn\bar{c}\bar{s}$ state with quantum numbers $I(J^P) = 1/2(1^+)$ falls at ~ 3989 MeV, which thus could be interpreted as component of $Z_{cs}(4000)$. Yet, in the mass range 4.16 ~ 4.26 GeV for $I(J^P) = 1/2(1^+)$, there are three states whose masses are consistent with the experimental mass of $Z_{cs}(4220)$.

Accordingly, the experimental data of Z_{cs} states, and presently obtained results those are close to the experimental data, are collected in Table 12.

Fig. 3 Mass spectra of ground S -wave Z_{cs} states. Red solid lines denote the model results. Gray dotted lines are corresponding S -wave charm meson pair thresholds. The green rectangles represent the experimental masses of $Z_{cs}(4000)$ and $Z_{cs}(4220)$ with definite quantum numbers [73], the rectangle width stand their masses uncertainties and the deep green solid lines in the rectangles are central values of their masses. The yellow rectangles represent the experimental mass of $Z_{cs}(3985)$, the rectangle width stand its mass uncertainty. Spin-parity quantum numbers of $Z_{cs}(3985)$ in figure are suggested by model calculation



3.3 Double-charm T_{cc} states

Very recently, the first double-charm exotic state T_{cc}^+ with $I(J^P) = 0(1^+)$, whose minimal quark content is $cc\bar{u}\bar{d}$, was observed by LHCb [30]. Its mass is very close to and below $D^0 D^{*+}$ threshold, and its decay width is extremely narrow. A further analysis have been done by LHCb collaboration in Ref. [31]. Theoretically, the tetraquark states containing two heavy quarks have been predicted in Refs. [32,33].

In present work, the mass spectra of ground S -wave $cc\bar{n}\bar{n}$, $cc\bar{n}\bar{s}$ and $cc\bar{s}\bar{s}$ tetraquark systems are studied. The numerical results are listed in Table 13, where the first three columns are the labels of these states in the model, quark content and quantum number of each state respectively, the fourth, fifth columns show the results of single configuration calculations, and the last two columns give the results obtained by considering configurations mixing effects. The model spectrum of S -wave double-charm tetraquark states are also shown in Fig. 4, where red solid lines denote $T_{cc}^{0,0}$ states, blue solid lines denote $T_{cc}^{1,0}$ states, pink solid lines denote $T_{cc}^{1/2,1}$ states, green solid line denote $T_{cc}^{0,2}$ states.

As shown in Table 13, all the presently obtained lowest compact T_{cc} states are higher than thresholds of corresponding charmed meson–meson channels. Therefore, there may be no stable bound states for S -wave double-charm tetraquark state. For instance, the lowest S -wave $cc\bar{n}\bar{n}$ states with quantum number $I(J^P) = 0(1^+)$ in present model falls at ~ 3982 MeV, which is about 100 MeV higher than the experimental data for T_{cc}^+ . Thus, T_{cc}^+ cannot be interpreted as a compact tetraquark state in our model.

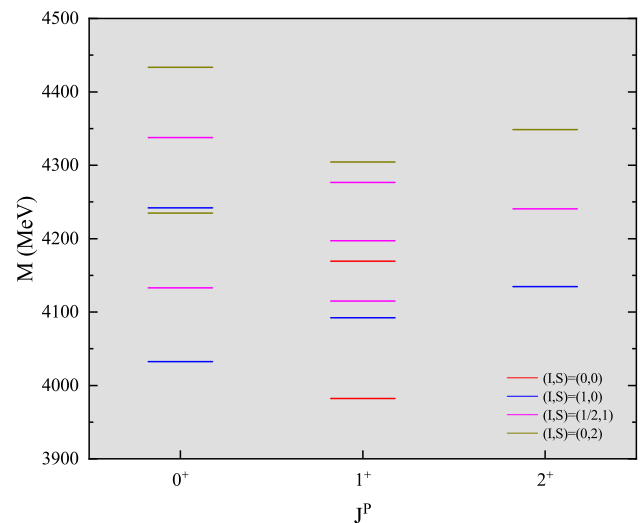


Fig. 4 The model spectrum of ground S -wave double-charm tetraquark states. Red solid lines denote $T_{cc}^{0,0}$ states, blue solid lines denote $T_{cc}^{1,0}$ states, pink solid lines denote $T_{cc}^{1/2,1}$ states, green solid line denote $T_{cc}^{0,2}$ states

In Ref. [16], the spectrum of tetraquark states containing double heavy quarks were investigated within a diquark–antidiquark picture employing the relativistic constituent quark model, the obtained numerical results are in agreements with the present ones. Very recently, double-heavy tetraquark states have been also studied using a Chiral quark model in Ref. [11], it's shown that both the meson exchange force and the coupled channel effects play pivotal roles for reproduction of the binding energy of T_{cc}^+ correlative to $D^0 D^{*+}$ threshold, revealing the molecular nature of T_{cc}^+ [11].

4 Summary

In present work, we study the ground S -wave hidden- and double-charm tetraquark states employing a nonrelativistic quark potential model, where the instanton-induced interaction is employed as the spin-dependent residual interaction between quarks. The model parameters are fixed by fitting the spectrum of the ground state hadrons. Our numerical results show that energies for several presently obtained compact tetraquark states are very close to the masses of the experimentally observed meson exotic states.

Particularly, two $c\bar{c}n\bar{n}$ and $c\bar{c}s\bar{s}$ tetraquark states with quantum numbers $I^G(J^{PC}) = 0^+(1^{++})$ fall at the energies close to $X(4140)$ and $X(4274)$, respectively. Energies of several $c\bar{c}n\bar{n}$ tetraquark states with $I^G(J^{PC}) = 1^+(1^{+-})$ are close to $Z_c(3900)$, $Z_c(4020)$ and $X(4055)$, and another obtained $c\bar{c}n\bar{n}$ state with $I^G(J^{PC}) = 1^-(0^{++})$ is close to the $X(4100)$ state. In addition, the $X(4050)$ and $X(4250)$ states, whose quantum numbers have not been determined, may be interpreted as compact $c\bar{c}n\bar{n}$ tetraquark states with quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$ or $I^G(J^{PC}) = 1^-(2^{++})$, and $I^G(J^{PC}) = 1^-(0^{++})$ or $I^G(J^{PC}) = 1^-(2^{++})$, respectively.

For the exotic meson states with one strange quark, we obtain two states very close to the experimentally observed $Z_{cs}(3985)$ state, which indicates the quantum number of $Z_{cs}(3985)$ may be $J^P = 0^+$ or $J^P = 1^+$. Furthermore, several $c\bar{c}n\bar{s}$ tetraquark states with $J^P = 1^+$ are obtained to fall in the energy range close to the $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states. Accordingly, one may conclude that the presently considered compact tetraquark states may take significant probabilities in the above referred X , Z_c and Z_{cs} states.

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