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Braneworld cosmological effect on freeze-in dark matter density and lifetime frontier

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Abstract In the 5-dimensional braneworld cosmology, the Friedmann equation of our 4-dimensional universe on a brane is modified at high temperatures while the standard Big Bang cosmology is reproduced at low temperatures. Based on two well-known scenarios, the Randall-Sundrum and Gauss-Bonnet braneworld cosmologies, we investigate the braneworld cosmological effect on the relic density of a non-thermal dark matter particle whose interactions with the Standard Model particles are so weak that its relic density is determined by the freeze-in mechanism. For dark matter production processes in the early universe, we assume a simple scenario with a light vector-boson mediator for the dark matter particle to communicate with the Standard Model particles. We find that the braneworld cosmological effect can dramatically alters the resultant dark matter relic density from the one in the standard Big Bang cosmology. As an application, we consider a right-handed neutrino dark matter in the minimal B - L extended Standard Model with a light B - Lgauge boson (Z') as a mediator. We find an impact of the braneworld cosmological effect on the search for the longlived Z' boson at the planned/proposed Lifetime Frontier experiments.

1 Introduction

Based on various cosmological and astronomical observations, including very precise measurements of the cosmic microwave background anisotropy, the so-called Λ CDM cosmological model has been established and the abundance of the (cold) dark matter (DM) is estimated as [1]

$$\Omega_{\rm DM} h^2 \simeq 0.12. \tag{1}$$

Since the Standard Model (SM) of particle physics has no suitable candidate for the cold DM particle, new physics beyond the SM is required to supplement the SM with a DM candidate. Many models of new physics have been proposed to incorporate various DM candidates. For a review, see Ref. [2].

Among several possibilities, the most popular DM scenario is the thermal DM, in which a DM particle was in thermal equilibrium in the early universe and its relic density at present is determined by the freeze-out mechanism [3]. In this scenario, we evaluate the DM relic density by solving the Boltzmann equation [4],

$$\frac{dY}{dx} = -\frac{s(T=m)}{H(T=m)} \frac{\langle \sigma v_{\rm rel} \rangle}{x^2} \left(Y^2 - Y_{EQ}^2 \right), \tag{2}$$

where x = m/T is the ratio between the DM mass (m) and the temperature of the universe (T), Y = n/s is the yield defined by the ratio of the DM number density (n(T)) to the entropy density of the universe $s(T) = (2\pi^2/45)g_*T^3$ with g_* being the effective total number of relativistic degrees of freedom, $H(T) = \sqrt{\pi^2 g_*/90} (T^2/M_P)$ is the Hubble parameter with the reduced Planck mass of $M_P = 2.43 \times$ 10^{18} GeV, $\langle \sigma v_{\rm rel} \rangle$ is the thermal-averaged DM pair annihilation cross section (σ) times DM relative velocity (v_{rel}), and Y_{EO} is the yield for the DM in thermal equilibrium. Once a DM model is fixed, we can calculate $\langle \sigma v_{rel} \rangle$ as a function of x and evaluate the yield of the thermal DM particle at present $Y(x \to \infty)$ by solving the Boltzmann equation with the initial condition of $Y = Y_{EO}$ for $x \ll 1$. In the freeze-out mechanism, we can approximate $Y(\infty)$ by $Y(\infty) \simeq Y(x_f)$ at the freeze-out temperature $x_f = m/T_f$. The relic density of the DM particle is given by

$$\Omega_{\rm DM}h^2 = \frac{m\,Y(\infty)\,s_0}{\rho_c/h^2},\tag{3}$$

where $s_0 = 2890/\text{cm}^3$ is the entropy density of the present universe, and $\rho_c/h^2 = 1.05 \times 10^{-5} \text{ GeV/cm}^3$ is the critical density.

It is well-known that the observed DM relic density of $\Omega_{\rm DM}h^2 \sim 0.1$ is obtained by $\langle \sigma v_{\rm rel} \rangle \sim 1$ pb, almost independently of the DM mass. This cross section at the electroweak



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scale implies the possibility of directly detecting a DM particle through its elastic scattering off of nucleons. Despite many experimental efforts, evidence of the DM particle has not been observed yet. For example, the most stringent upper bound of around 4×10^{-11} pb for a DM particle with a 30 GeV mass has been set by the XENON 1T experiment [5]. Although the discovery of a DM particle may be around the corner, the null result motivates us to consider the possibility that the interaction of a DM particle with SM particles is extremely weak. If this is the case, a DM particle has never been in thermal equilibrium with the plasma of the SM particles in the history of the universe. In such a case, the DM relic density is determined by the freeze-in mechanism [6,7](see Ref. [8] for a review), assuming a vanishing initial DM density at the reheating after inflation. The freeze-in DM scenario has attracted a lot of attention recently.

We also employ the Boltzmann equation to evaluate the DM relic density for the freeze-in DM particle. The difference from the thermal DM scenario is only the boundary condition, namely, $Y(x_{RH}) = 0$ instead of $Y(x \ll 1) = Y_{EQ}$, where $x_{RH} \equiv m/T_{RH} \ll 1$ with the reheating temperature T_{RH} after inflation. Since Y never reaches Y_{EQ} because of the extremely weak DM interactions with the SM particles, the Boltzmann equation has the approximate form:

$$\frac{dY}{dx} \simeq \frac{s(m)}{H(m)} \frac{\langle \sigma v_{\rm rel} \rangle}{x^2} Y_{EQ}^2 \simeq 0.698 \frac{g_{DM}^2}{g_*^{3/2}} m M_P \frac{\langle \sigma v_{\rm rel} \rangle}{x^2}.$$
(4)

In the last expression, we have used $n_{EQ} = (g_{DM}/\pi^2)T^3$ (for T > m) with g_{DM} being the DM internal degrees of freedom. Here, note that $\langle \sigma v_{rel} \rangle Y_{EQ}^2$ corresponds to the creation rate of a pair of DM particles by the thermal plasma because it balances with the DM annihilation rate when the DM particle is in thermal equilibrium. For a given $\langle \sigma v_{rel} \rangle$ as a function of x, it is easy to solve the Boltzmann equation, and we estimate $Y(\infty) \simeq Y(x = 1)$. Note that we integrate the Boltzmann equation until x = 1 since the production of DM particles from thermal plasma stops around $x \sim 1$, or equivalently, $T \sim m$ by kinematics.¹ As a simple DM scenario, we consider the case that the DM particle communicates with the SM particles through a light vector-boson mediator. In this case, we may express the DM creation/annihilation cross section as

$$\langle \sigma v_{\rm rel} \rangle = \frac{g_V^4}{128\pi} \frac{x^2}{m^2},\tag{5}$$

where g_V is a coupling of the vector-boson. Using this concrete form, one can easily solve Eq. (4) to arrived at

$$\Omega_{\rm DM} h^2 = \frac{m Y(x=\infty) s_0}{\rho_c/h^2} \simeq \frac{m Y(x=1) s_0}{\rho_c/h^2}$$

$$\simeq 1.16 \times 10^{24} \, \frac{g_{DM}^2}{g_*^{3/2}} \, g_V^4. \tag{6}$$

Interestingly, the resultant relic density is independent of the DM mass. For example, by using $g_{DM} = 2$ and $g_* = 106.75$ for the SM particle plasma, we find $g_V = 2.31 \times 10^{-6}$ to reproduce $\Omega h^2 = 0.12$.

Since the discovery of the "brane" in string theories [9] the braneworld scenarios have attracted lots of attention as phenomenological models, in which the SM particles are confined on a "3-brane" while gravity resides in the bulk space. The braneworld cosmology based on the model first proposed by Randall and Sundrum (RS) [10] (the so-called RS II model) has been intensively investigated (see Ref. [11] for a review). It has been found [12–15] that the Friedmann equation in the RS cosmology leads to a non-standard expansion law at high temperatures while the standard Big Bang cosmology is reproduced at low temperatures. The RS II model can be extended by adding the Gauss–Bonnet (GB) invariant [16–19], and the Friedmann equation for the GB braneworld cosmology has been found [20,21], which is quite different from the one in the RS braneworld cosmology.

Since the Hubble parameter is involved in the Boltzmann equation, the DM relic density depends on the expansion law of the early universe. Hence, the non-standard evolution of the universe can significantly alter the resultant DM density from that found in the standard Big Bang cosmology.

The RS braneworld cosmological effect on the thermal DM physics has been investigated in detail [22–32], and it has been shown that the resultant DM density can be considerably enhanced. On the other hand, the GB braneworld cosmological effect has been shown to considerably reduce the thermal DM density [33, 34].

In this paper, we investigate the braneworld cosmological effect on the non-thermal DM scenario in which the DM relic density is determined by the freeze-in mechanism (for a related work, see Ref. [35]). In particular, we focus on a freeze-in DM which communicates with the SM particles through a light vector-boson mediator. For the two typical scenarios, namely, the RS and the GB braneworld cosmologies, we evaluate the DM relic density under the non-standard evolution of the universe. We will find interesting RS and GB braneworld cosmological effects. As an application of our findings, we consider a right-haded neutrino (RHN) DM scenario [36] (see also Ref. [37]) in the context of the minimal B - L extended SM [38–44], where the RHN DM communicates with the SM particles through a light B - L gauge boson (Z'). Because of a small B - L gauge coupling for the freeze-in mechanism, the Z' boson can be long-lived. When the Z' boson mass lies in the range of 10 MeV $\lesssim m_{Z'} \lesssim 1$ GeV, the planned/proposed Lifetime Frontier experiments can explore such a long-lived Z' boson. We find an impact of the braneworld cosmological effect on such experiments.

¹ Here, $x \sim 1$ is for a rough estimate. In the numerical analysis presented in Sect. 4, we integrating out the Boltzmann equation until $x \gg 1$.

This paper is organized as follows: In the next section, we give a brief review on the RS and GB braneworld cosmologies. In Sect. 3, we consider a freeze-in DM scenario with a light vector-boson mediator in the braneworld cosmology and discuss how the braneworld effect alters the resultant DM density from the one in the standard Big Bang cosmology. In Sect. 4, we apply our findings in Sect. 3 to the so-called Z'-portal RHN DM scenario in the context of the minimal B - L model, and identify the parameter region to reproduce the observed DM density. We also discuss the search for a long-lived B - L gauge boson by the planned/proposed experiments at Lifetime Frontier and point out an impact of the braneworld cosmological effect on the search. Sect. 5 is devoted to conclusions.

2 Braneworld cosmologies

In this section, we give a brief review on two typical braneworld cosmologies: the RS cosmology and the GB cosmology. In both scenarios, the Standard Big Bang cosmology is reproduced at low temperatures while the evolution of the universe obeys non-standard law at high temperatures. Considering this fact, we may parametrize a modified Friedmann equation in a braneworld cosmology as

$$H = H_{\rm st}(T) \times F(T), \tag{7}$$

where $H_{\text{st}} = \sqrt{\pi^2 g_*/90} (T^2/M_P)$ is the Hubble parameter in the standard Big Bang cosmology. Since the standard Big Bang cosmology must be reproduced at low temperatures, we express F(T) as $F(T/T_t)$ with "transition temperature" (T_t) at which the modified expansion law approaches the standard expansion law, namely, $F(T/T_t) = F(x_t/x) \rightarrow 1$ for $T < T_t$, where $x_t = m/T_t$. For concreteness, let us assume the following form:

$$F(T/T_t) = \left(\frac{T}{T_t}\right)^{\gamma} = \left(\frac{x_t}{x}\right)^{\gamma}$$
(8)

for $T/T_t > 1$ with a real parameter γ . This parameterization turns out to be a very good approximation for both of the RS and GB braneworld cosmologies. As we will see, $\gamma = 2$ corresponds to the RS braneworld cosmology, while $\gamma = -2/3$ to the GB braneworld cosmology.

2.1 RS cosmology

In the RS cosmology, the Friedmann equation for a spatially flat universe is found to be [12-15]

$$H^2 = \frac{\rho}{3M_P^2} \left(1 + \frac{\rho}{\rho_{\rm RS}} \right),\tag{9}$$

where ρ is the energy density of the universe, and

$$\rho_{\rm RS} = 12 \, \frac{M_5^6}{M_p^2},\tag{10}$$

with M_5 being the 5-dimensional (5D) Planck mass. Here, we have set the model parameters to make the 4D cosmological constant and the so-called dark radiation [45] vanishing. Note that the Friedmann equation of the standard Big Bang cosmology is reproduced at low energies (temperatures) such that $\rho/\rho_{\rm RS} \ll 1$. A lower bound on $\rho_{\rm RS}^{1/4} \gtrsim 1.3$ TeV, or equivalently, $M_5 \gtrsim 1.1 \times 10^8$ GeV was obtained in Ref. [10] from the precision measurements of the gravitational law in submillimeter range.

Let us consider the radiation dominated era in the early universe, where the temperature is so high that $\rho/\rho_{RS} \gg 1$. Then, the Friedmann equation of Eq. (9) can be approximated by

$$H \simeq H_{\rm st} \sqrt{\frac{\rho}{\rho_{\rm RS}}} = H_{\rm st} \times \left(\frac{x_t}{x}\right)^2$$
 (11)

where we have used $\rho/\rho_{\rm RS} = (T/T_t)^4 = (x_t/x)^4$. Hence, we find $\gamma = 2$ in Eq. (8) for the RS cosmology $(T \gg T_t)$.

2.2 GB cosmology

The RS II model can be generalized by adding higher curvature terms [16–19]. Among various possibilities, the GB invariant is of particular interests in 5D since it is a unique nonlinear term in curvature yielding the gravitational field equations at the second order. The action of the RS II model is extended by adding the GB invariant [16–19]:

$$S = \frac{1}{2\kappa_5^2} \int d^5 x \sqrt{-g_5} \left[-2\Lambda_5 + \mathcal{R} + \alpha \left(\mathcal{R}^2 - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}_{abcd}\mathcal{R}^{abcd} \right) \right] - \int_{brane} d^4 x \sqrt{-g_4} \left(m_{\sigma}^4 + \mathcal{L}_{matter} \right), \qquad (12)$$

where indices *a*, *b*, *c*, *d* run 0 to 4, $\kappa_5^2 = 8\pi/M_5^3$, $m_{\sigma}^4 > 0$ is a brane tension, $\Lambda_5 < 0$ is the bulk cosmological constant, and a Z₂-parity across the brane in the bulk is imposed. The limit $\alpha \to 0$ recovers the RS II model.

The Friedmann equation on the spatially flat brane has been found to be [20,21]

$$\kappa_5^2(\rho + m_{\sigma}^4) = 2\mu \sqrt{1 + \frac{H^2}{\mu^2} \left(3 - \beta + 2\beta \frac{H^2}{\mu^2}\right)}, \qquad (13)$$

where $\beta = 4\alpha\mu^2 = 1 - \sqrt{1 + 4\alpha\Lambda_5/3}$. The model involves four free parameters, κ_5 , m_σ , μ and β . We impose two phenomenological requirements: (i) the Friedmann equation of the standard Big Bang cosmology must be reproduced at low energies $H^2/\mu^2 \ll 1$; (ii) 4D cosmological constant is approximately zero. These requirements lead to the following two conditions:

$$\kappa_5^2 m_\sigma^4 = 2\mu(3-\beta), \quad \frac{1}{M_P^2} = \frac{\mu}{1+\beta}\kappa_5^2.$$
 (14)

In general, the GB cosmology has three epochs in its evolution [46]: The universe obeys the standard expansion law at low energies (standard epoch). At middle energies (RS epoch), the RS cosmology is approximately realized. At high energies (GB epoch), the Friedmann equation is approximately expressed as

$$H \simeq \left(\frac{1+\beta}{4\beta} \frac{\mu}{M_P^2} \rho\right)^{1/3}.$$
 (15)

For a special value of $\beta = 0.151$ which satisfies the equation, $3\beta^3 - 12\beta^2 + 15\beta - 2 = 0$, the RS epoch collapses [46]. Since we are interested in the GB epoch, we fix $\beta = 0.151$. Hence, we can parametrize the Friedman equation in the GB epoch as

$$H \simeq \left(\frac{1+\beta}{4\beta} \frac{\mu}{M_P^2} \rho\right)^{1/3} = H_{\rm st} \times \left(\frac{\rho}{\rho_{\rm GB}}\right)^{-1/6} = H_{\rm st} \times \left(\frac{x_t}{x}\right)^{-2/3}, \tag{16}$$

where $\rho_{\text{GB}} = 3^6 \left(\frac{1+\beta}{4\beta}\mu M_P\right)^2$. Thus, taking $\gamma = -2/3$ in Eq. (8) corresponds to the GB cosmology for $T \gg T_t$.

3 Freeze-in dark matter in Braneworld cosmologies

Now we are ready to investigate the braneworld cosmological effect on the freeze-in DM scenario. Our crucial assumption is that the DM mass is larger than the transition temperature ($m > T_t$), so that DM production from the SM thermal plasma ends before the evolution of the universe transitions to the standard Big Bang cosmology. To evaluate the freeze-in DM density, we solve the Boltzmann equation. All the braneworld cosmological effect is encoded in the modification of the Hubble parameter given by Eq. (7) with Eq. (8). Therefore, we obtain a modified Boltzmann equation of the form:

$$\frac{dY}{dx} = \frac{s(m)}{H_{\rm st}(m)} \frac{\langle \sigma v_{\rm rel} \rangle}{F(x_t/x) x^2} Y_{EQ}^2$$
$$\simeq 0.698 \frac{g_{DM}^2}{g_{s*}^{3/2}} m M_P \frac{\langle \sigma v_{\rm rel} \rangle}{F(x_t/x) x^2}.$$
(17)

Mathematically, the braneworld cosmological effect is equivalent to modifying the DM creation/annihilation cross section in the standard Big Bang cosmology as

$$\langle \sigma v_{\rm rel} \rangle \rightarrow \left(\frac{\langle \sigma v_{\rm rel} \rangle}{F(x_t/x)} \right) = \langle \sigma v_{\rm rel} \rangle \left(\frac{x_t}{x} \right)^{-\gamma}$$
 (18)

for $x_t/x > 1$. This equation implies that the braneworld cosmological effect enhances (reduces) the DM relic abundance for $\gamma < 0$ ($\gamma > 0$).

In our RS and GB cosmologies, we can easily solve Eq. (17) with the cross section of Eq. (5). We then obtain

$$\Omega_{\rm DM} h^2 \simeq \frac{m Y(x=1) s_0}{\rho_c/h^2} \simeq 1.16 \times 10^{24} \frac{g_{DM}^2}{g_*^{3/2}} g_V^4 \times \frac{x_t^{-\gamma}}{\gamma+1}.$$
(19)

The resultant DM density is modified by a factor of $R(\gamma) = \frac{x_t^{-\gamma}}{\gamma+1}$ from the one in the standard Big Bang cosmology. Therefore, in order to reproduce the observed DM density of $\Omega_{\text{DM}}h^2 = 0.12$, the coupling g_V is fixed to be

$$g_V = 2.31 \times 10^{-6} R(\gamma)^{-1/4}.$$
 (20)

For the RS and the GB cosmologies, we find $R(2) \simeq 1.32 \sqrt{x_t}$ and $R(-2/3) \simeq 0.76 x_t^{-1/6}$, respectively. When $x_t \gg 1$, or equivalently, $m \gg T_t$, the braneworld cosmological effect requires a different coupling value to reproduce the observed DM density.

4 Application to Z'-portal RHN DM with a light Z' boson

In the previous section, we have evaluated the freeze-in DM density with a light vector-boson mediator. We have found that the resultant relic density is independent of the DM mass, and the observed relic density is reproduced by adjusting the coupling (g_V) of the light mediator. As we have found, the resultant DM density in the RS and GB braneworld cosmologies can be significantly altered from the one in the standard Big Bang cosmology. In other words, the g_V value to reproduce $\Omega_{\text{DM}}h^2 = 0.12$ is changing in accordance with a braneworld model and the transition temperature T_t (or equivalently, x_t). As we will discuss in the following, this coupling change has an impact on the search for the vector-boson mediator at future experiments. To see this, we consider a simple "Z'-portal DM" scenario in this section.

The minimal B - L model [38–44] is a simple, wellmotivated model for the neutrino mass generation. In the model, the accidental $U(1)_{B-L}$ global symmetry of the SM is gauged, and three RHNs are introduced to keep the model free from all gauge and mixed gauge-gravitational anomalies. The $U(1)_{B-L}$ symmetry is broken by the vacuum expectation value of a B - L Higgs, which generates the B - L gauge boson (Z') mass as well as Majorana masses for the three RHNs. After the electroweak symmetry breaking, tiny neutrino masses are generated through the type-I seesaw mechanism [47–50]. A concise way to incorporate a DM candidate into the minimal B - L model has been proposed in Ref. [36], where instead of introducing a new particle, a Z_2 symmetry is introduced while keeping the minimal B - L model particle content intact. We assign an odd-parity for one RHN, while all the other particles in the model are even. In this way, the parity-odd RHN is stable and serves as the DM in our universe. It is known that only two RHNs are necessary for a realistic seesaw scenario to reproduce the neutrino oscillation data. This setup is called "Minimal Seesaw" [51,52]. Therefore, through the Z_2 -parity, the three RHNs are categorized into 2 + 1 with two RHNs for the seesaw mechanism and one RHN for the DM candidate. The particle content of the model is listed in Table 1. Except for the Z_2 -parity assignment, the particle content is exactly the same as the one of the minimal B - L model.

The RHN DM can communicate with the SM particles through the Higgs boson exchange (Higgs-portal) and/or the Z' boson exchange (Z'-portal). For a RHN DM as a thermal DM particle, the Higgs-portal case [36, 53, 54] and the Z' portal case [55-58] (see [59] for a review) have been extensibly studied. In the study for the Z'-portal case, it has been pointed out that the DM physics and the search for a Z' boson resonance at the Large Hadron Collider (LHC) are complementary to narrow down the allowed model-parameter space. For a similar study of a Z'-portal Dirac fermion DM in the context of the minimal B-L model, see Refs. [60,61]. The RHN DM as a freeze-in DM particle has also been studied [62,63] (For a similar study of the Dirac fermion case, see Refs. [64,65]). In particular, the Z'-portal RHN DM with a light Z' boson has been studied in Ref. [62] and it has been pointed out that the parameter region motivated from the DM physics can be explored by the planned/proposed experiments at the Lifetime Frontier. In the following, we extend the analysis in Ref. [62] to the RS and GB braneworld cosmologies and investigate an impact of the braneworld cosmological effect on the Lifetime Frontier experiments.

In order to evaluate the freeze-in RHN DM relic density, we first evaluate a thermal-averaged cross section for the RHN pair creation from thermal plasma. The main process² is $f\bar{f} \rightarrow Z' \rightarrow NN$ and its explicit form is given by

$$\sigma(s) = \frac{13}{48\pi} g_{BL}^4 \frac{\sqrt{s(s-4m^2)}}{s^2},$$
(21)

where g_{BL} is the B - L gauge coupling, and we have neglected Z' boson mass $(m_{Z'} \ll m)$.³ The thermal average

Table 1 The particle content of the minimal B-L model with the RHN DM [36]. In addition to the SM particle content (i = 1, 2, 3 for three generations), the three right-handed neutrinos categorized into 2+1 (N_R^j (j = 1, 2) and N_R) and the B - L Higgs field (Φ) are introduced. The Z_2 -odd N_R is a unique DM candidate in the model

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$	Z_2
q_L^i	3	2	1/6	1/3	+
u_R^i	3	1	2/3	1/3	+
d_R^i	3	1	-1/3	1/3	+
l_L^i	1	2	-1/2	-1	+
N_R^j	1	1	0	-1	+
N_R	1	1	0	-1	_
e_R^i	1	1	-1	-1	+
Η	1	2	-1/2	0	+
Φ	1	1	0	2	+

of the pair creation/annihilation cross section is calculated as

$$\begin{aligned} \langle \sigma v_{\rm rel} \rangle &= (s Y_{EQ})^{-2} g_{DM}^2 \frac{m}{64\pi^4 x} \\ &\times \int_{4m^2}^{\infty} ds \, 2(s - 4m^2) \, \sigma(s) \, \sqrt{s} \, K_1\left(\frac{x\sqrt{s}}{m}\right), \end{aligned}$$
(22)

where

$$sY_{EQ} = \frac{g_{DM}}{2\pi^2} \frac{m^3}{x} K_2(x),$$
(23)

 $g_{DM} = 2$, and K_i is the modified Bessel function of the *i*-th kind. For fixed values of g_{BL} and *m*, we obtain $\langle \sigma v_{rel} \rangle$ as a function of *x*, and numerically solve the Boltzmann equation to evaluate the relic density. As we have found in the previous section, the braneworld cosmological effects are encoded in Eq. (18) and we simply scale the thermal-averaged cross section by $(x/x_t)^{\gamma}$.

For example values of the input parameters (m = 10 TeV, $T_t = 1$ TeV, $m_{Z'} = 1$ GeV, and $g_{BL} = 1.54 \times 10^{-6}$), we show in Fig. 1 the numerical solutions of the Boltzmann equation in the standard Big Bang (black solid line), RS (red dashed line) and GB (blue dotted line) cosmologies. We find that as long as $m \gg m_{Z'}$, the results are independent of m and $m_{Z'}$. As we see in the figure, the resultant DM density is enhanced (reduced) in the GB (RS) cosmology, compared to the one obtained in the standard cosmology. Corresponding DM relic densities are $\Omega_{\text{DM}}h^2 = 7.2 \times 10^{-3}$ for the standard cosmology while $\Omega_{\text{DM}}h^2 = 0.12$ and 4.5×10^{-5} for the GB and RS cosmologies, respectively. We can see that Eq. (19) is satisfactory as a rough estimate. The discrepancy

² If Z' boson is in thermal equilibrium with the SM particle plasma, the process $Z'Z' \rightarrow NN$ should be considered. We have checked that the cross section of this process is an order of magnitude smaller than the cross section of the process $f\bar{f} \rightarrow Z' \rightarrow NN$ [65].

³ For more precise analysis, we may take into account thermal corrections for the Z' boson mass, $\Delta m_{Z'} \sim g_{BL}T$, which dominates over $m_{Z'}$ for $T \gtrsim m_{Z'}/g_{BL}$. If the thermal mass dominates, our cross section formula should be corrected (we thank the referee for bringing up this point). Considering that the resultant DM relic density is mainly

Footnote 3 continued

determined by $T \sim m$, we can derive an upper bound on $m \gtrsim m_{Z'}/g_{BL}$ to ignore the thermal mass corrections. Our parameter choice for Figs. 1 and 2 is consistent with this bound.



Fig. 1 Numerical solutions of the Boltzmann equation in the GB braneworld cosmology (dotted blue line), the Standard Big Bang cosmology (solid black line), and the RS braneworld cosmology (dashed red line). Here, we set m = 10 TeV, $T_t = 1$ TeV, $m_{Z'} = 1$ GeV, and $g_{BL} = 1.54 \times 10^{-6}$

between the formula of Eq. (19) and the actual numerical solution originates from the fact that the thermal-averaged cross section is not exactly proportional to x^2 around $x \sim 1$ while it is a very good approximation for $x \ll 1$.

Let us now discuss an impact of the braneworld cosmological effect on the future experiments at the Lifetime Frontier. In order to reproduce the observed relic density for the RHN DM via the light Z'-portal interaction, the B - L gauge coupling is found to be very small. Hence, the Z' boson becomes long-lived. Such a long-lived B - L gauge boson can be explored at Lifetime Frontier experiments. The recently approved ForwArd Search Experiment (FASER) [66-68] has a physics run planned at the LHC Run-3 and its upgraded version (FASER 2) at the High-Luminosity LHC. The prospect of the B - L gauge boson search at FASER is summarized in Ref. [67]. FASER 2 can search for a long-lived Z' boson with its mass in the range of 10 MeV $\lesssim m_{Z'} \lesssim 1$ GeV for the B-L gauge coupling in the range of $10^{-8} \lesssim g_{BL} \lesssim 10^{-4.5}$. The planned/proposed experiments, such as Belle II [69], LHCb [70,71], SHiP [72] and LDMX [73], which will also search for a long-lived Z' boson, will cover a parameter region complementary to FASER.

In Fig. 2, we show our results for the B - L gauge coupling as a function of $m_{Z'}$ to reproduce the observed DM relic density in the standard Big Bang (horizontal black line), RS (horizontal red line) and GB (horizontal blue line) cosmologies, along with the search reach of various planned/proposed experiments at the Lifetime Frontier and the current excluded region (gray shaded) [74]. Here, we have fixed $x_t = 100$ ($x_t = 10^6$) for the RS (GB) cosmology. As expected, we find that the results are independent of $m_{Z'} \ll m$. The braneworld effects shift the resultant g_{BL} value upwards in the RS cosmology and downwards in the GB cosmology from the one in the standard cosmology. Therefore, if a long-lived Z' boson is observed in the future, we can measure not only g_{BL} and $m_{Z'}$



Fig. 2 Gauge coupling values as a function of $m_{Z'} \ll m$ to reproduce the observed DM relic density in the standard Big Bang (horizontal black line), RS (horizontal red line) and GB (horizontal blue line) cosmologies, along with the search reach of various planned/proposed experiments at the Lifetime Frontier and the current excluded region (gray shaded). We have fixed $x_t = 100 (x_t = 10^6)$ for the RS (GB) cosmology. the gauge couplings are found to be $g_{BL} = 3.50 \times 10^{-5}$, 3.11×10^{-6} and 2.27×10^{-7} for the RS, standard Big Bang and GB cosmologies, respectively

but also obtain the information about possible non-standard evolution of the early universe.

5 Conclusions

In the 5D braneworld cosmology, the Friedmann equation of our 4D universe on a brane is modified at high temperatures while the standard Big Bang cosmology is recovered at low temperatures. Based on two well-known scenarios, the Randall-Sundrum and Gauss-Bonnet braneworld cosmologies, we have investigated the braneworld cosmological effect on the relic density of a non-thermal DM particle whose interactions with the Standard Model particles are so weak that its relic density is determined by the freeze-in mechanism. For DM production processes in the early universe, we have considered a simple DM scenario with a light mediator for the DM particle to communicate with the Standard Model particles. We have found that the braneworld cosmological effect can dramatically alter the resultant DM density from the one in the standard Big Bang cosmology. As an application, we consider the Z'-portal RHN DM in the minimal B - L extended Standard Model with a light Z' boson as a mediator. We have found an impact of the braneworld cosmological effect on the search for the long-lived Z' boson at the planned/proposed Lifetime Frontier experiments.

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