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Addendum

Addendum to: Dynamical torsion suppression in Brans–Dicke inflation and Lorentz violation (Eur Phys J C (2022)): Einstein–Cartan–Brans–Dicke–Maxwell universe with a chiral dynamo?

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Abstract We have recently shown that in Einstein–Cartan–Brans–Dicke (ECBD) gravity, torsion effects are not present in today's universe because they are suppressed in a four-dimensional system, as in the bulk of five-dimensional braneworld endowed with torsion. In this addendum, we naturally introduce new features on the ECBD Maxwell Lagrangian dynamics deriving the chiral dynamo equation, and studying its physical properties. Magnetogenesis results are derived in this ECBDM universe. In particular we show, in the present universe, torsional magnetic fields of the second-order in the ohmic resistivity behave as $B_{\text{ECBD}} = 10^{-13}$ Gauss, which is exactly the estimate made by Miniati et al. (Phys Rev Lett, 2018) for the axion QCD magnetic field in the present universe. We found this at $1pc$ coherent length, whereas they found it at $20pc$. To obtain this result, we consider a strong magnetic field of the order of the Biermann battery $10^{30}G$. This seems to show that our model of ECBDM gravity can be used with success in inflationary magnetogenesis with torsion. Inflation or deflation are shown to depend upon torsion chirality. Torsion is shown to depend upon the BD parameter ω and decays in time as in the reference to which this addendum refers (Garcia de Andrade in Eur Phys J C, 2002).

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1 The extended model

Recently, we show, the author showed that Einstein–Cartan–Brans–Dicke (ECBD) gravity can suppress the dynamical torsion in inflationary phases [1]; in the same way, Paul et al. [2] have shown that this result is valid in the case of dynamical torsion in the bulk of a five-dimensional (5D) spacetime. As we have done in most of my papers [6], in the present paper, we investigate the dynamics of torsion in the case of chiral dynamos in ECBD. To this end we might consider the same Lagrangian

$$\mathcal{L}_{\text{ECBDM}} = \int d^4x\sqrt{-g}\left(-\phi R + \omega\frac{\partial^{\mu}\phi\partial_{\mu}\phi}{\phi} - \frac{\phi^3 F^2}{4} + J^{\mu}A_{\mu}\right) \tag{1}$$

where $\mu = 0, 1, 2, 3$ and $F^2 = F_{\mu\nu}F^{\mu\nu}$ is the electromagnetic field tensor squared, and ECBDM is the Einstein–Cartan–Brans–Dicke–Maxwell, where we add the electromagnetic Lagrangian without the axial anomaly term. Note that, torsion is not coupled with the EM fields. In this section we couple scalar fields to torsion via non-minimal coupling [10]. R in this Lagrangian is the Ricci–Cartan scalar. The new EM terms introduced here now allow us to introduce the chiral dynamo problem. The chiral total current is

$$\mathbf{J} = \sigma[\mathbf{E} + \mathbf{v}\times\mathbf{B}] + \mu_5\mathbf{B} \tag{2}$$

The chiral current is superposed on the electric current, where $\sigma = \eta^{-1}$, where σ is the electrical conductivity and η is the electrical resistivity. Now,

$$\ddot{\phi} + 3H\dot{\phi} = \frac{4\pi}{\omega}\left(\rho - 3p - \frac{8\pi\sigma^2}{\phi}\right) \tag{3}$$

where ϕ is the inflaton field and $H = \frac{\dot{a}}{a}$ is the Hubble expansion, while a is the cosmic scale factor. The Friedmann–

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Robertson–Walker (FRW) metric is given by

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right] \tag{4}$$

Units are used where, the gravitational constant $G = 1$, and the Ricci–Cartan scalar R is given by

$$R = [\dot{S}_0 - 6{S_0}^2 + 12H^2] \tag{5}$$

Now let us obtain the first field equations from the Euler-Lagrange (EL)

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}_{\mathrm{ECBDM}}}{\partial\dot{\phi}} - \frac{\partial\sqrt{g}\mathcal{L}_{\mathrm{ECBDM}}}{\partial\phi} = 0 \tag{6}$$

Here, the interaction term between the scalar inflatons and EM invariant F^2 is of the Ratra type [11]. Thus, by computing the partial derivatives of the Lagrangian (1) and substituting them into the EL equation, one obtains

$$R = \frac{3\phi^2}{4}F^2 + 2\omega^2\beta^2 + 3\omega\left(3H\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi}\right) \tag{7}$$

This is the well-known Rainich already unified field equation [13], as can be seen by taking the inflaton as constant. Let us now obtain the remaining equations in the form of EL equations as

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}_{\mathrm{ECBDM}}}{\partial\dot{S}_0} - \frac{\partial\sqrt{g}\mathcal{L}_{\mathrm{ECBDM}}}{\partial S_0} = 0 \tag{8}$$

where a is the cosmic scale and $H = \frac{\dot{a}}{a}$ is the Hubble function. Then, this equation yields

$$\dot{\phi} + 3H\phi = -2S_0 \tag{9}$$

where S_0 is the zero component of the axial torsion vector. This equation tells us that if torsion is constant, its solution yields a decaying inflaton field in cosmic time t, which depends upon torsion. The stronger the torsion, the faster the decay of the inflaton field. Now we consider the case of the EL equation for the cosmic scale a. This yields

$$\mathcal{L}_{\mathrm{ECBDM}} + 8H^2(1-\dot{\phi}) - 8\phi\left[H^2 + \frac{\dot{a}}{a}\right] = 0 \tag{10}$$

But it is easy to show that

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} \tag{11}$$

Substitution of expression (11) into (10) yields

$$\phi^{-1}\mathcal{L}_{\mathrm{ECBDM}} + 8H^2\left(1 - \frac{\phi}{\phi}\right) - [2H^2 + \dot{H}] = 0 \tag{12}$$

Making use of expression (9) and only considering Hubble parameter H up to $\mathcal{O}(H^2)$, we obtain

$$\phi^{-1}\mathcal{L}_{\mathrm{ECBDM}} + 8H^2(1+2S_0) - [2H^2 + \dot{H}] = 0 \tag{13}$$

Taking the ECBD Lagrangian in the form

$$\phi^{-1}\mathcal{L}_{\mathrm{ECBDM}} \approx 12H^2 - \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{14}$$

Note that from this equation that if we constrain ECBDM cosmology to the case where expansion rebounds, universe expansion stops, ($H = 0$) and torsion is constant, this assumption can be used as a boundary condition to determine the BD parameter ω. The last expression reduces to

$$\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 + {S^2}_0 = 0 \tag{15}$$

Assuming this specific boundary condition where inflation turns into deflation, one obtains from the last expression that

$$S^0 = \sqrt{\omega}t^{-1} \tag{16}$$

from previous equations. Then, torsion is non-constant, and substitution of the last expression squared into (14) yields the result

$$\left(\frac{\dot{\phi}}{\phi}\right) = S_0 = \sqrt{\omega}t^{-1} \tag{17}$$

This expression shall be very useful to solve chiral dynamo equation in the next section. Note that expression (16) above is quite important, due to the fact that torsion is determined in terms of the BD parameter ω. This agrees with the status quo that general relativity is a torsionless equation, since GR is characterized by ($\omega = 0$).

2 Helical chiral dynamos in Einstein–Cartan–Brans–Dicke–Maxwell gravity

In this section we shall finally derive the chiral dynamo equation in the case of the ECBDM model of the previous section. We recall that though we do not start from the coupling of torsion to EM fields; the magnetic field may be computed in terms of torsion, as we shall see in this section. Here in differential forms notation $F = dA$ where $A = (A_i dx^i)$ is the magnetic potential four-dimensional one form, where no torsion is present. Variation of the four-dimensional potential in the above Lagrangian is

$$\partial_i(a^3\phi^3F^{ij}) = a^3\phi^3{J_{5+c}}^j \tag{18}$$

where electric current J_c is given by

$$\mathbf{J}_{(5+c)} = \mu_5 \mathbf{B} + \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{19}$$

First current is the chiral, and the second is the normal electric current. Substitution of this current into the expression (19) yields

$$(\lambda^2 - \lambda\mu_5)\left[3\left(\frac{\dot{\phi}}{\phi} + H\right) - \sigma\right]\nabla\times\mathbf{E} = \sigma\nabla\times[\mathbf{v}\times\mathbf{B}] \tag{20}$$

which from from the Bianchi identity

$$\partial_{[i}F_{jk]} = 0 \tag{21}$$

we obtain the Faraday effect

$$\nabla\times\mathbf{E} = -\partial_t\mathbf{B} \tag{22}$$

After some algebra we obtain the chiral dynamo equation

$$\partial_t\mathbf{B} = -[\eta(\lambda^2 - \lambda\mu_5)[1 + 3\eta(H + \omega t^{-1}) - i\mathbf{v}.\mathbf{k}]\mathbf{B} \tag{23}$$

where we have used Eq. (17) of the previous section. Since in the early universe the conductivity is very high and the ohmic resistivity is quite low, we may keep only terms up to first order in resistivity and drop terms like $\mathcal{O}(\eta^2)$ and compute second-order contributions of the ohmic resistivity or diffusivity by the end of this section, where we compare both results. Under this assumption chiral dynamo equation reduces to

$$\partial_t\mathbf{B} = -\eta(\lambda^2 - \lambda\mu_5)\mathbf{B} \tag{24}$$

We now assume that chirality dominates over the relation between the magnetic helicity parameter and the chiral chemical potential, such as $\mu_5 \geq \lambda$. This allows us to express the last chiral dynamo equation in the form

$$\partial_t\mathbf{B} = \eta\lambda\mu_5\mathbf{B} \tag{25}$$

and then now it is much easier to find the solution of this simple dynamo equation. This solution can be approximated by

$$\mathbf{B} \approx \mathbf{B}_{\text{seed}}(\eta\lambda\mu_5 t) \tag{26}$$

By making use of the $\lambda = L^{-1}{}_B$, where L_B is the coherent length of the magnetic field and taking $L_B = 1pc$ as in the Googol et al. primordial magnetogenesis torsionless paper [15], since $t = 10^{18}s$ and the $B^{\text{seed}}{}_{\text{ECBDM}} = 10^{13}G$ at today's axion field, we would need a magnetic field in the present universe of the order of $B_{\text{ECBDM}} = 10^{-34}G$. We now use the expression for the dynamo solution of the order of $\mathcal{O}(\eta^2)$, which yields a solution as

$$\mathbf{B} \approx \mathbf{B}_{\text{seed}}(\eta^2\lambda\mu_5 t) \tag{27}$$

This immediately yields $B_{\text{ECBDM}}chiral = 10^{-13}\omega G$ as a helical magnetic field in the present universe. The ω factor which is sometimes used in BD gravity as $\omega = \frac{1}{6}$ will not appreciably change our results here; since we would obtain a $10^{-14}G$ for the axion magnetic field. We have used the following data in this computation: a seed field from the Biermann battery mechanism as $B_{Bierm} = 10^{30}G$, a chiral chemical potential as $\mu_5 = 10$, as a value used by Shober et al. [18] to investigate chiral dynamos in torsionless spacetime. By substituting these data into expression (27), one obtains the estimate $B^{\text{inflaton}}{}_{\text{ECBDM}} = 10^{-13}$ Gauss, which is exactly the result obtained by Miniati et al. [23] from $20pc$ coherent length. Though we apparently do not see the presence of torsion in this chiral dynamo equation, it is actually present in the relation between H and S_0 above.

3 Conclusions

In this addendum we have investigated the magnetogenesis in the case of the ECBDM model for cosmology. One obtains in the present universe a magnetic field of 10^{-13} Gauss from a second order in the ohmic diffusivity in chiral dynamo equation. With GUT seed fields of the order of $10^{41}G$ obtained by Berera in the early universe, one would be able to obtain, instead an axionic magnetic field in the present universe of 10^{-13}. Using our solution, a magnetic field of $B_{\text{ECBD}}(\eta^2) = 10^{-3}$ Gauss, which is a field found at the core of some galaxies is found. We note that the ECBDM universe seems to be a promising model to improved results for magnetic fields from torsion, which are able to seed galactic dynamos at a reasonable coherence length. It is also important to note that, here we have not address back reaction into the metric, which would require a metric of non-Friedmannian nature, because we are not using the axial anomaly term $E.B$, and most of the magnetic fields obtained are very weak to back-react on the isotropic homogeneous universe considered. Of course, solutions of several Lagrangians can be used in the near future to test the model dependence of the electrodynamics which we were using to further investigate magnetogenesis in Riemann–Cartan spacetime [24]. We were told very recently that Bamba et al. [25] have investigated helical magnetogenesis with a reheating phase and high-order curvature baryogenesis without torsion. It would be interesting if we tried to add torsion couplings even non-minimally with magnetogenesis along their lines. This work may appear in the near future. More on GR magnetogenesis may be found in the last reference [26]. Actually, Bamba et al. [28] have investigated the non-helical magnetic fields in the reheating phase in higher-order curvature coupling. This is an interesting subject to work and to review in order to perform extensions to modified gravity magnetogenesis.

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Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: No new data have been produced during this study. All used data are available upon reasoable request from L C Garcia de Andrade.]

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References

1. L. Garcia de Andrade, Eur. Phys. J. C (2022)
2. T. Paul, S. SenGupta, Dynamical suppression of spacetime torsion. Eur. Phys. J. C **79**, 591 (2019)
3. E. Elizalde, S.D. Odintsov, T. Paul, D. Gomez, Phys Rev D **99**, 063506 (2019)
4. T. Paul, Antisymmetric tensor fields in modified gravity: A summary, arXiv:2009.07732
5. K.-W. Ng, Damping of tensor modes in Inflation. arXiv:1111.0295v3
6. L. Garcia, de Andrade. Eur. Phys. J. C **77**, 401 (2017)
7. L.C. Garcia de Andrade, Einstein–Cartan magnetogenesis and chiral dynamos. Akademic Publishers Republic of Moldavia, (2021)
8. L.C. Garcia de Andrade, Ann. Phys. **431**, 168558 (2021)
9. L. Garcia, de Andrade. Eur. Phys. J. Plus **136**, 146 (2021)
10. I. Shapiro, Phys Aspects of spacetime Torsion, Phys. Rep. (2002)
11. A. Berera, Phys. Rev. Lett. (1995)
12. H.Q. Lu, K.S. Cheng, Class. Quantum Grav. **12**, 2755 (1995)
13. J.A. Wheeler, Geometrodynamics, Princeton Univ Press (1969)
14. L. Garcia de Andrade, Eur. Phys. J. C (2022)
15. V. Demozzi, V. Mukhanov, H. Rubstein, Magnetic fields from inflation. arXiv:0907.1030v1
16. J. Shober, I. Rogachevskii, A. Brandenburg, Production of a chiral magnetic anomaly with emerging turbulence and mean-field dynamo action. Phys. Rev. Lett. **128**, 065053 (2022)
17. J. Schober, I. Rogachevskii, A. Brandenburg, Dynamo instabilities in plasmas with inhomogemeous chiral chemical potential. Phys. Rev. D **105**, 043507 (2022)
18. J. Shober, I. Rogachevskii, A. Brandenburg, Production of a chiral magnetic anomaly with emerging turbulence and mean-field dynamo action. Phys. Rev. Lett. **128**, 065053 (2022)
19. J. Schober, I. Rogachevskii, A. Brandenburg, Dynamo instabilities in plasmas with inhomogemeous chiral chemical potential. Phys. Rev. D **105**, 043507 (2022)
20. L. Garcia de Andrade, Eur. Phys. J. C **77**, 401 (2017)
21. L.C. Garcia de Andrade, Ann. Phys. **431**, 168558 (2021)
22. L. Garcia, de Andrade. Eur. Phys. J. Plus **136**, 146 (2021)
23. F. Miniati, G. Gregori, B. Reville, S. Sarkar, Phys. Rev. Lett. **101**, 021301 (2018)
24. L. C. Garcia de Andrade, Topology in Einstein–Cartan magnetogenesis and dynamo effects (2021). Academic Publishers
25. K. Bamba, S. D. Odintsov, T. Paul, D. Maity, Helical magnetogenesis with reheating phase of high order curvature coupled with baryogenesis. Phys. Dark Universe **36**, 101025 (2022). arXiv:2107.11524 [gr-qc]
26. K. Bamba, M. Sasaki, Large scale magnetic field in the inflationary universe. JCAP **0702**, 030 (2007)
27. K. Bamba, J. Yokoyama, Large scale magnetic field from inflation in dilaton electromagnetism. Phys. Rev. D, **69**, 043507 (2004) and JCAP **10**, 004 (2013)
28. K. Bamba, E. Elizalde, S.D. Odintsov, T. Paul, Inflationary magnetogenesis with reheating phase from higher-order curvature. JCAP **4**, 009 (2021). arXiv:2012.12742