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On gauge-invariant deformation of reducible gauge theories

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Abstract New method for construction of gauge-invariant deformed theory from an initial gauge theory proposed in our previous papers (Buchbinder and Lavrov in JHEP 06:854, 2021; Buchbinder and Lavrov in Eur Phys J C 81:856, 2021) for closed/open gauge algebras is extended to the case of reducible gauge algebras. The deformation procedure is explicitly described with the help of generating functions of anticanonical transformations depending on fields of the initial gauge action only. The deformed gauge-invariant action and the deformed gauge generators are described with the help of the generating functions in a closed and simple form. As an example of reducible gauge systems we consider the free fermionic p-form fields or, in another words, the antisymmetric tensor-spinor fields. It is proved that gauge-invariant deformation of fermionic p-form fields leads always to non-local deformed theory which does not contain a closed local sector. In its turn the model based on two fermionic 2-form fields and a real massive scalar field admits local interactions between these fields in local sector of the deformed action.

1 Introduction

Recently, a new approach to gauge-invariant deformation of gauge theories has been proposed in our papers [1,2]. This approach is closely related with the Batalin-Vilkovisky (BV) formalism [3–5] which is the most powerful method for covariant quantization of general gauge theories. The central role in the BV formalism belongs to the classical master equation formulated in terms of the antibracket. It is a remarkable fact that the antibracket is invariant under anticanonical transformations that helps in studying different properties of gauge theories [6–10]. In this connection, it seems useful to remind the standard approach to the problem of gauge-invariant deformation for systems with gauge invariance.

Construction of consistent interactions among fields with a gauge freedom or gauge-invariant deformations of a free gauge system is formulated as follows [11]. Starting point of deformation procedure is a given theory described by an action $S_0 = S_0[A]$ of field $A = \{A^i\}$ which is supposed to be invariant under gauge transformations,

$$S_{0,i} R^i_{0\alpha} = 0, \quad \delta A^i = R^i_{0\alpha} \xi^\alpha, \tag{1}$$

where ξ^α are arbitrary functions of space-time coordinates. It is required to construct a final (deformed) action S as

$$S = S_0 + g S_1 + g^2 S_2 + \cdots , \tag{2}$$

where g is a deformation parameter, in such a way that initial gauge generators $R^i_{0\alpha}$ are deformed,

$$R^i_{0\alpha} \quad \rightarrow \quad R^i_\alpha = R^i_{0\alpha} + g R^i_{1\alpha} + g^2 R^i_{2\alpha} + \cdots , \tag{3}$$

to final gauge generators R^i_α [11,12] so that the deformed action S is invariant under the deformed gauge symmetry,

$$S_{,i} R^i_\alpha = 0. \tag{4}$$

To arrive these results it has been proposed [11,12] to embed the deformation procedure in the BV formalism as a part of solutions to the classical master equation for an action $\mathcal{S}$,

$$(\mathcal{S}, \mathcal{S}) = 0, \tag{5}$$

with the boundary condition

$$\mathcal{S}\big|_{g=0} = S_0. \tag{6}$$

The bridge connecting solutions $\mathcal{S}$ to the classical master equation with the deformed action S is established with the help of the relation

$$\mathcal{S}\big|_{\text{antifields} = 0} = S. \tag{7}$$

Solutions to the classical master equation are searched in the form of Taylor expansion with respect to parameter g,

$$\mathcal{S} = S_0 + g\mathcal{S}_1 + g^2\mathcal{S}_2 + \cdots . \tag{8}$$

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Then, the classical master equation for action $\mathcal{S}$ generates the infinite set of equations

$$(S_0, S_0) = 0, \quad (S_0, \mathcal{S}_1) = 0, \quad 2(S_0, \mathcal{S}_2) + (\mathcal{S}_1, \mathcal{S}_1) = 0, \quad \cdots . \tag{9}$$

Usually, this system of equations is analyzed with the help of the cohomological approach [11,12] (see also recent applications [13,14]). In general, this approach to the deformation procedure does not give a possibility to present the deformed action and the deformed gauge generators in an explicit and closed form. In fact, it was a reason for us to reconsider the gauge-invariant deformations of gauge systems within the BV formalism using the invariance of antibracket under anticanonical transformations [1,2].

The anticanonical transformations by itself can be described in two ways, namely, in terms of generating functionals or with the help of generators. Being equivalent on theoretical level, they might be distinguished in practical applications. It happened really in our reformulation of the deformation procedure. The description of anticanonical transformations with the help of generating functionals is more preferred. Moreover, it was realized [2] that the deformation problem can be solved using the so-called minimal anticanonical transformations in the minimal antisymplectic space when the corresponding functionals are described with the help of generating functions depending on fields of initial theory in the number equals to the number of initial fields and having the same transformation properties as initial fields. The gauge-invariant deformations in papers [1,2] have been solved for initial gauge theories with closed/open algebras. Main goal of present paper is to extend the new approach for reducible gauge theories.

The paper is organized as follows. In Sect. 2, we review the basic notions of reducible gauge theories and corresponding gauge algebras underlying such gauge systems. Section 3 is devoted to presentation of the deformed gauge action and corresponding deformed gauge symmetry in terms of a single generating function depending on initial fields only. In Sect. 4, we consider the free fermionic p-form fields as an example of reducible gauge system subjected to gauge-invariant deformations. In Sect. 5, local interactions of fermionic 2-form fields and a real massive scalar field as the result of suitable deformation of the initial free model of these fields are constructed. In Sect. 6, we summarize the results.

In the paper, we systematically use the DeWitt's condensed notations [15] and employ the symbols $\varepsilon(A)$ for the Grassmann parity and $\mathrm{gh}(A)$ for the ghost number, respectively. The right and left functional derivatives are marked by special symbols "$\leftarrow$" and "$\rightarrow$" respectively. Arguments of any functional are enclosed in square brackets [ ], and arguments of any function are enclosed in parentheses, ( ). The symbol $F_{,i}(A)$ is used for right partial derivative of function $F(A)$ with respect to A^i.

2 Reducible gauge theories

We consider a gauge theory of the fields $A = \{A^i\}$ with Grassmann parities $\varepsilon(A^i) = \varepsilon_i$ and ghost numbers $\mathrm{gh}(A^i) = 0$. The theory is described by the initial action $S_0[A]$ and gauge generators $R^i_\alpha(A)$ $(\varepsilon(R^i_\alpha(A)) = \varepsilon_i + \varepsilon_\alpha,\ \mathrm{gh}(R^i_\alpha(A)) = 0)$. The action is invariant under the gauge transformations

$$\delta A^i = R^i_\alpha(A)\xi^\alpha, \tag{10}$$

where the gauge parameters ξ^α $(\varepsilon(\xi^\alpha) = \varepsilon_\alpha)$ are the arbitrary functions of space-time coordinates. Condition of gauge invariance is written in the standard form[1]

$$S_{0,i}[A]R^i_\alpha(A) = 0, \quad \alpha = 1, 2, ..., m. \tag{11}$$

It is assumed that the fields $A = \{A^i\}$ are linear independent with respect to the index i however, in general, these generators may be linear dependent with respect to index α. Linear dependence of $R^i_\alpha(A)$ implies that the matrix $R^i_\alpha(A)$ has at the extremals $S_{0,j}[A] = 0$ zero-eigenvalue eigenvectors $Z^\alpha_{\alpha_1} = Z^\alpha_{\alpha_1}(A)$, such that

$$R^i_\alpha(A)Z^\alpha_{\alpha_1}(A) = S_{0,j}[A]K^{ji}_{\alpha_1}(A), \quad \alpha_1 = 1, ..., m_1, \tag{12}$$

and the number $\varepsilon_{\alpha_1} = 0, 1$ can be found in such a way that $\varepsilon(Z^\alpha_{\alpha_1}) = \varepsilon_\alpha + \varepsilon_{\alpha_1}$. Matrices $K^{ij}_{\alpha_1} = K^{ij}_{\alpha_1}(A)$ in (12) can be chosen to possess the properties:

$$K^{ij}_{\alpha_1} = -(-1)^{\varepsilon_i\varepsilon_j}K^{ji}_{\alpha_1}, \quad \varepsilon(K^{ji}_{\alpha_1}) = \varepsilon_i + \varepsilon_j + \varepsilon_{\alpha_1}.$$

The generators $R^i_\alpha(A)$ satisfy the following relations

$$\begin{aligned} &R^i_{\alpha,j}(A)R^j_\beta(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta}R^i_{\beta,j}(A)R^j_\alpha(A) \\ &\quad = -R^i_\gamma(A)F^\gamma_{\alpha\beta}(A) - S_{0,j}[A]M^{ji}_{\alpha\beta}(A), \end{aligned} \tag{13}$$

where $F^\gamma_{\alpha\beta}(A) = F^\gamma_{\alpha\beta}$ $(\varepsilon(F^\gamma_{\alpha\beta}) = \varepsilon_\alpha + \varepsilon_\beta + \varepsilon_\gamma,\ \mathrm{gh}(F^\gamma_{\alpha\beta}) = 0)$ are the structure coefficients depending, in general, on the fields A^i with the following symmetry properties $F^\gamma_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}F^\gamma_{\beta\alpha}$, and $M^{ij}_{\alpha\beta}(A) = M^{ij}_{\alpha\beta}$ satisfy the conditions

$$M^{ij}_{\alpha\beta} = -(-1)^{\varepsilon_i\varepsilon_j}M^{ji}_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}M^{ij}_{\beta\alpha}. \tag{14}$$

In its turn, the set $Z^\alpha_{\alpha_1}$ may be linearly dependent as itself, so that at the extremals $S_{0,i} = 0$ there exists the set of zero-eigenvalue eigenvectors $Z^{\alpha_1}_{\alpha_2} = Z^{\alpha_1}_{\alpha_2}(A)$

$$Z^\alpha_{\alpha_1}Z^{\alpha_1}_{\alpha_2} = S_{0,j}L^{j\alpha}_{\alpha_2}, \quad \alpha_2 = 1, ..., m_2 \tag{15}$$

and numbers $\varepsilon_{\alpha_2} = 0, 1$ such that $\varepsilon(Z^{\alpha_1}_{\alpha_2}) = \varepsilon_{\alpha_1} + \varepsilon_{\alpha_2}$. In the general case the set $Z^{\alpha_1}_{\alpha_2}$ can be redundant and so on. In such

[1] To simplify presentation of all relations containing the right functional derivative of functional $S_0[A]$ with respect to field A^i we will use the symbol $S_{0,i}[A] = S_{0,i}$.

a way the sequence of reducibility equations arises:

$$Z^{\alpha_{s-2}}_{\alpha_{s-1}} Z^{\alpha_{s-1}}_{\alpha_s} = S_{0,j} L^{j\alpha_{s-2}}_{\alpha_s}, \quad \alpha_s = 1, ..., m_s; s = 1, .., L, \tag{16}$$

where the following notations are introduced:

$$Z^{\alpha_1}_{\alpha_0} \equiv R^i_\alpha, \quad L^{j\alpha_1}_{\alpha_0} \equiv K^{ji}_\alpha, \quad \varepsilon(Z^{\alpha_{s-1}}_{\alpha_s}) = \varepsilon_{\alpha_{s-1}} + \varepsilon_{\alpha_{\alpha_s}}. \tag{17}$$

If the set $\{Z^{\alpha_{L-1}}_{\alpha_L}\}$ is linear independent then one meets with a gauge theory of L-stage reducibility.

The set of gauge generators $\{R^i_\alpha\}$, eigenvectors $\{Z^{\alpha_{s-1}}_{\alpha_s}\}$ and structure functions $\{L^{j\alpha_{s-2}}_{\alpha_s}\}$ defines the structure of gauge algebra on the first level. For irreducible theories, the structure of gauge algebra on the second level is defined by the set of structure functions $\{F^\gamma_{\alpha\beta}\}$ and matrices $\{M^{ij}_{\alpha\beta}\}$ in Eq. (13). For reducible theories, the existence of relations among the $Z^{\alpha_{s-1}}_{\alpha_s}$ (16) leads to the appearance of new structure functions. Let us demonstrate this point for a first-stage reducible gauge theory. To this end, let us multiply the relation (13) by the eigenvector $Z^\beta_{\alpha_1}$. We obtain

$$\left(R^i_{\alpha,j} R^j_\beta - (-1)^{\varepsilon_\alpha\varepsilon_\beta} R^i_{\beta,j} R^j_\alpha + R^i_\gamma F^\gamma_{\alpha\beta} + S_{0,j} M^{ji}_{\alpha\beta}\right) Z^\beta_{\alpha_1} = 0. \tag{18}$$

First, note that relations (12) allows us to express $R^j_\beta Z^\beta_{\alpha_1}$ as a term proportional to the equations of motion. Second, by differentiating Eqs. (12) and (11) with respect to A one obtains that

$$R^i_{\beta,j} Z^\beta_{\alpha_1} (-1)^{\varepsilon_j(\varepsilon_\beta+\varepsilon_{\alpha_1})} + R^i_\beta Z^\beta_{\alpha_1,j} = S_{0,jl} K^{li}_{\alpha_1} (-1)^{\varepsilon_j(\varepsilon_i+\varepsilon_{\alpha_1})} + S_{0,l} K^{li}_{\alpha_1,j}, \tag{19}$$

$$S_{0,ji} R^j_\alpha (-1)^{\varepsilon_l\varepsilon_\alpha} + S_{0,i} R^i_{\alpha,j} = 0. \tag{20}$$

Then, multiplying Eqs. (19) by R^j_α, using the Noether identities (11) and relations (20), we find

$$-(-1)^{\varepsilon_\alpha\varepsilon_\beta} R^i_{\beta,j} R^j_\alpha Z^\beta_{\alpha_1} = (-1)^{\varepsilon_\alpha\varepsilon_{\alpha_1}} R^i_\beta Z^\beta_{\alpha_1,j} R^j_\alpha + S_{0,j}(R^j_{\alpha,l} K^{il}_{\alpha_1} (-1)^{\varepsilon_\alpha\varepsilon_i} - K^{ij}_{\alpha_1,l} R^l_\alpha (-1)^{\varepsilon_\alpha\varepsilon_{\alpha_1}}).$$

Returning with this result into (18), one can obtained the relations

$$R^i_\beta\big((-1)^{\varepsilon_\alpha\varepsilon_{\alpha_1}} Z^\beta_{\alpha_1,j} R^j_\alpha - F^\beta_{\alpha\gamma} Z^\gamma_{\alpha_1}\big) = S_{0,j} Y^{ji}_{\alpha_1\alpha}$$

where all terms proportional to the equation of motion have been collected into $Y^{ji}_{\alpha\alpha_1}$. Taking into account the completeness of the set of eigenvectors $Z^\alpha_{\alpha_1}$, the general solution to this equation,

$$(-1)^{\varepsilon_\alpha\varepsilon_{\alpha_1}} Z^\beta_{\alpha_1,j} R^j_\alpha - F^\beta_{\alpha\gamma} Z^\gamma_{\alpha_1} = -Z^\beta_{\beta_1} P^{\beta_1}_{\alpha_1\alpha} - S_{0,j} Q^{j\beta}_{\alpha_1\alpha}, \tag{21}$$

defines a new gauge-structure relation similar to Eq. (13). Therefore, two new structure functions $P^{\beta_1}_{\alpha\alpha_1}$ and $Q^{j\beta}_{\alpha\alpha_1}$ arise to complete definition of the structure of gauge algebra for the first-stage reducible theory on the second level. To define the structure of gauge algebra on the third level, one has to consider the Jacobi identity for gauge transformations and some consequences from gauge-structure relations of previous levels.

In principal, there is no problem in deriving the corresponding gauge algebra for reducible gauge theories of any stage of reducibility but here we omit further calculations. Let us remark only that, in general, the structure of gauge algebra looks like a set of infinite number of structure functions which define infinite number of gauge-structure relations. It is remarkable fact that all these relations can be collected within the BV method in a solution to the classical master equation.

Within the BV formalism, studies of classical aspects of gauge-invariant deformations can be performed in the minimal antisymplectic space of fields ϕ^A and antifields ϕ^*_A as it was pointed out in [2]. For reducible L-stage gauge theory of fields A^i, it contains main chains of the ghost $C^{\alpha_s}_s$, and pyramids of the ghost for ghost $C^{\alpha_s}_{s(n_s)}$,

$$\phi^A = \left(A^i;\ C^{\alpha_s}_s, s = 0, 1, ..., L, C^{\alpha_s}_{s(n_s)}, s = 1, ..., L, n_s = 1, ..., s\right) \tag{22}$$

with the properties

$$\begin{aligned} \varepsilon(C^{\alpha_s}_s) &= (\varepsilon_{\alpha_s} + s + 1) \text{ mod}2, \quad s = 0, 1, ..., L, \\ \varepsilon(C^{\alpha_s}_{s(n_s)}) &= (\varepsilon_{\alpha_s} + s + 1) \text{ mod}2, \quad s = 1, ..., L, \quad n_s = 1, ..., s, \\ gh(C^{\alpha_s}_s) &= (s+1), \ s = 0, 1, ..., L \\ gh(C^{\alpha_s}_{s(n_s)}) &= s + 1 - 2n_s, \ s = 1, ..., L, \ n_s = 1, ..., s, \end{aligned} \tag{23}$$

and the corresponding set of antifields

$$\phi^*_A = \left(A^*_i, C^*_{s\alpha_s}, s = 0, 1, ..., L, \quad C^*_{s(n_s)\alpha_s}, s = 1, ..., L, n_s = 1, ..., s\right). \tag{24}$$

The statistics of ϕ^*_A is opposite to the statistics of the corresponding fields ϕ^A

$$\varepsilon(\phi^*_A) = \varepsilon_A + 1,$$

and the ghost numbers of fields and corresponding antifields are connected by the rule

$$gh(\phi^*_A) = -1 - gh(\phi^A).$$

In comparison with original proposal of Ref. [5], we have slightly (for simplicity and uniformity) changed notation of pyramids of fields. As an example, for a second-stage reducible theory, the following identification for the pyramids of fields exists:

$$C^{'\alpha_1}_1 \equiv C^{\alpha_1}_{1(1)}, \quad C^{'\alpha_2}_2 \equiv C^{\alpha_2}_{2(1)}.$$

The basic object of the BV formalism is the extended action $S = S[\phi, \phi^*]$ satisfying the classical master equation,

$$(S, S) = 0, \tag{25}$$

and the boundary condition,

$$S[\phi,\phi^*]\Big|_{\phi^*=0}=S_0[A]. \tag{26}$$

The classical master equation (25) is written in terms of antibracket which is defined for any functionals $F[\phi,\phi^*]$ and $H[\phi,\phi^*]$ in the form

$$(G,H)=G\left(\overleftarrow{\partial}_{\phi^A}\overrightarrow{\partial}_{\phi^*_A}-\overleftarrow{\partial}_{\phi^*_A}\overrightarrow{\partial}_{\phi^A}\right)H. \tag{27}$$

The gauge invariance of the initial action $S_0[A]$ leads to invariance of the action $S[\phi,\phi^*]$,

$$\delta_B S=0, \tag{28}$$

under the global supersymmetry transformations (BRST transformations [16,17])

$$\delta_B\phi^A=(\phi^A,S)\mu=\overrightarrow{\partial}_{\phi^*_A}S\,\mu,\quad \delta_B\phi^*_A=0, \tag{29}$$

as a consequence that S satisfies the classical master equation. Here, μ is a constant Grassmann parameter. We emphasize that the antibracket is a key element of compact description of the classical gauge theories within the BV formalism. An important property of the antibracket (27), is its invariance with respect to anticanonical transformations of fields and anti-fields [3,4]. It leads to statement that any two solutions of classical master equation (25) are related one to another by some anticanonical transformation.

3 Deformed action

New approach to gauge-invariant deformation of a gauge theory was proposed in our papers [1,2] for theories with the closed/open gauge algebras. Here, we are going to generalize the results for theories when the gauge algebra is reducible.

Classical aspects of the gauge-invariant deformation of initial theory can be studied in the minimal antisymplectic space using the minimal anticanonical transformations as it was proved in [2]. It means that the anticanonical transformations

$$\phi^*_A=Y[\phi,\Phi^*]\overleftarrow{\partial}_{\phi^A},\quad \Phi^A=\overrightarrow{\partial}_{\Phi^*_A}Y[\phi,\Phi^*], \tag{30}$$

where $Y=Y[\phi,\Phi^*]$ $(\varepsilon(Y)=1,\ \mathrm{gh}(Y)=-1)$ is the generating functional are non-trivial in the sector of minimal antisymplectic space only

$$Y[\phi,\Phi^*]=\Phi^*_A\phi^A+\mathcal{A}^*_i h^i(A). \tag{31}$$

Here, $h^i(A)=h^i$ $(\varepsilon(h^i)=\varepsilon_i,\ \mathrm{gh}(h^i)=0)$ are arbitrary functions of fields A^i having the same transformation laws as for A^i.

For simplicity of presentation and notations without loss of generality of all conclusions and statements, we restrict ourselves to the case of first-stage reducibility of the initial action when in (30) and (31) $\phi^A=(A^i,C^\alpha,C^{\alpha_1})$, $\phi^*_A=(A^*_i,C^*_\alpha,C^*_{\alpha_1})$ and $\Phi^A=(\mathcal{A}^i,\mathcal{C}^\alpha,\mathcal{C}^{\alpha_1})$, $\Phi^*_A=(\mathcal{A}^*_i,\mathcal{C}^*_\alpha,\mathcal{C}^*_{\alpha_1})$. Taking into account the gauge invariance of the initial action (10) and the boundary condition (26), one can write the action $S=S[\phi,\phi^*]$ up to the terms linear in antifields in the form

$$S=S_0[A]+A^*_iR^i_\alpha(A)C^\alpha+C^*_\gamma\Big(Z^\gamma_{\alpha_1}(A)C^{\alpha_1}_1 -\frac{1}{2}F^\gamma_{\alpha\beta}(A)C^\beta C^\alpha(-1)^{\varepsilon_\alpha}\Big)+O(\phi^{*\,2}). \tag{32}$$

Making use of the anticanonical transformations (31) in the action (32), we obtain the functional $\widetilde{S}=\widetilde{S}[\phi,\phi^*]=S[\Phi(\phi,\phi^*),\Phi^*(\phi,\phi^*)]$ which satisfies the classical master equation

$$(\widetilde{S},\widetilde{S})=0, \tag{33}$$

and has the following form up to the terms linear in antifields

$$\widetilde{S}=\widetilde{S}_0[A]+A^*_i\widetilde{R}^i_\alpha(A)C^\alpha+C^*_\gamma\Big(\widetilde{Z}^\gamma_{\alpha_1}(A)C^{\alpha_1}_1 -\frac{1}{2}\widetilde{F}^\gamma_{\alpha\beta}(A)C^\beta C^\alpha(-1)^{\varepsilon_\alpha}\Big)+O(\phi^{*\,2}), \tag{34}$$

where the quantities

$$\widetilde{S}_0[A]=S_0[A+h(A)],\quad \widetilde{R}^i_\alpha(A)=(M^{-1}(A))^i{}_jR^j_\alpha(A+h(A)),$$
$$\widetilde{F}^\gamma_{\alpha\beta}(A)=F^\gamma_{\alpha\beta}(A+h(A)),\quad \widetilde{Z}^\alpha_{\alpha_1}(A)=Z^\alpha_{\alpha_1}(A+h(A)) \tag{35}$$

present the deformed initial action, $\widetilde{S}_0[A]$, the deformed gauge generators, $\widetilde{R}^i_\alpha(A)$, the deformed structure coefficients, $\widetilde{F}^\gamma_{\alpha\beta}(A)$, and the deformed eigenvectors, $\widetilde{Z}^\alpha_{\alpha_1}(A)$. The matrix $(M^{-1}(A))^i{}_j$ is inverse to

$$M^i{}_j(A)=\delta^i{}_j+h^i{}_{,j}(A), \tag{36}$$

The action $\widetilde{S}_0[A]$ is invariant under the gauge transformations $\delta A^i=\widetilde{R}^i_\alpha(A)\xi^\alpha$,

$$\widetilde{S}_{0,i}[A]\widetilde{R}^i_\alpha(A)=0. \tag{37}$$

Therefore, the main problem of gauge-invariant deformation of a given gauge system has the explicit closed solution as for the deformed action as well as for deformed gauge generators. Such solutions is described in terms of generating function $h(A)$ only. The first relations in deformed gauge algebra for first-stage reducible theories read

$$\widetilde{R}^i_{\alpha,j}(A)\widetilde{R}^j_\beta(A)-(-1)^{\varepsilon_\alpha\varepsilon_\beta}\widetilde{R}^i_{\beta,j}(A)\widetilde{R}^j_\alpha(A) =-\widetilde{R}^i_\gamma(A)\widetilde{F}^\gamma_{\alpha\beta}(A)-\widetilde{S}_{0,j}[A]\widetilde{M}^{ji}_{\alpha\beta}(A),$$
$$\widetilde{R}^i_\alpha(A)\widetilde{Z}^\alpha_{\alpha_1}(A)=\widetilde{S}_{0,j}[A]\widetilde{K}^{ji}_{\alpha_1}(A), \tag{38}$$

where the functions $\widetilde{M}^{ji}_{\alpha\beta}(A)$ and $\widetilde{K}^{ji}_{\alpha_1}(A)$ are

$$\widetilde{M}^{ji}_{\alpha\beta}(A)=-(M^{-1}(A))^j{}_l(M^{-1}(A))^i{}_kM^{kl}_{\alpha\beta}(A+h(A))(-1)^{\varepsilon_l\varepsilon_i}, \tag{39}$$

$$\widetilde{K}^{ji}_{\alpha_1}(A)=-(M^{-1}(A))^j{}_l(M^{-1}(A))^i{}_kZ^{kl}_{\alpha_1}(A+h(A))(-1)^{\varepsilon_l\varepsilon_i}, \tag{40}$$

In the same manner, we deduce the relations which define the structure functions of deformed algebra on the second level

$$(-1)^{\varepsilon_\alpha\varepsilon_{\alpha_1}}\widetilde{Z}^{\beta}_{\alpha_1,j}(A)\widetilde{R}^{j}_{\alpha}(A)-\widetilde{F}^{\beta}_{\alpha\gamma}(A)\widetilde{Z}^{\gamma}_{\alpha_1}(A)$$
$$=-\widetilde{Z}^{\beta}_{\beta_1}(A)\widetilde{P}^{\beta_1}_{\alpha_1\alpha}(A)-\widetilde{S}_{0,j}[A]\widetilde{Q}^{j\beta}_{\alpha_1\alpha}(A), \quad (41)$$

where

$$\widetilde{P}^{\beta_1}_{\alpha_1\alpha}(A)=P^{\beta_1}_{\alpha_1\alpha}(A+h(A)),$$
$$\widetilde{Q}^{j\beta}_{\alpha_1\alpha}(A)=(M^{-1}(A))^{j}_{\ k}Q^{k\beta}_{\alpha_1\alpha}(A+h(A)). \quad (42)$$

The action (34) is invariant under the BRST transformations,

$$\delta_B\widetilde{S}=0,\quad \delta_B\phi^A=(\phi^A,\widetilde{S})\mu=\overrightarrow{\partial}_{\phi^*_A}\widetilde{S}\,\mu,\quad \delta_B\phi^*_A=0. \quad (43)$$

and, therefore, the deformed theory repeats all basic properties of the initial system on quantum level.

From the analysis carried out, the following conclusions can be drawn: (1) for any reducible theory with an gauge-invariant action $S_0[A]$ the deformed gauge-invariant action, $\widetilde{S}_0[A]$, is described by the formula $\widetilde{S}_0[A]=S_0[A+h(A)]$ where $h(A)$ is a generating function of the anticanonical transformation, (2) the deformed gauge generators, $\widetilde{R}^i_\alpha(A)$, are defined through the initial ones, $R^i_\alpha(A)$, by the relations (35), (3) the chain of deformed eigenvectors, $\widetilde{Z}^{\alpha_{s-1}}_{\alpha_s}(A), s=1,2,...,L$, is expressed in the form $\widetilde{Z}^{\alpha_{s-1}}_{\alpha_s}(A)=Z^{\alpha_{s-1}}_{\alpha_s}(A+h(A))$, (4) the structure quantities appearing in relations (16) are deformed by the rule $\widetilde{L}^{j\alpha_{s-2}}_{\alpha_s}(A)=(M^{-1}(A))^{j}_{\ k}L^{k\alpha_{s-2}}_{\alpha_s}(A+h(A))$, (5) the deformed gauge algebra looks like as initial gauge algebra in which all structure coefficients are replaced by the deformed ones, (6) the same conclusion is valid for relation between actions S and $\widetilde{S}$ satisfying the classical master equation.

4 On deformation of fermionic p-form fields

As an example of reducible theories, we consider antisymmetric tensor-spinor fields or, in another words, fermionic p-form fields, $\psi^a_{\mu_1\mu_2...\mu_p}$, where a is a spinor index and the μ_i are space-time indices. The fields $\psi^a_{\mu_1\mu_2...\mu_p}$ are totally antisymmetric in their space-time indices:

$$\psi^a_{\mu_1\mu_2...\mu_p}=\psi^a_{[\mu_1\mu_2...\mu_p]}. \quad (44)$$

Anti-symmetrization of tensor $A_{\mu_1\mu_2...\mu_p}$ is understood in standard sense

$$A_{[\mu_1\mu_2...\mu_p]}=\frac{1}{p!}\sum_{\sigma(\mu_1\mu_2...\mu_p)}\mathrm{sgn}\sigma\, A_{\sigma(\mu_1)\sigma(\mu_2)...\sigma(\mu_p)} \quad (45)$$

where summation is over all permutations of indices $\mu_1\mu_2...\mu_p$ and the symbol sgnσ is the sign of given permutation.

The free action for such a field in flat space-time is described by the functional [18–20] [2]

$$S_0[\psi]=-(-1)^{\frac{p(p-1)}{2}}\int d^nx\overline{\psi}_{\mu_1\mu_2...\mu_p}\Gamma^{\mu_1\mu_2...\mu_p\nu\nu_1\nu_2...\nu_p}\partial_\nu\psi_{\nu_1\nu_2...\nu_p}, \quad (46)$$

where $\overline{\psi}=\psi^\dagger\gamma^0$ and the notation

$$\Gamma^{\mu_1\mu_2...\mu_p\nu\nu_1\nu_2...\nu_p}=\gamma^{[\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_p}\gamma^{\nu}\gamma^{\nu_1}\gamma^{\nu_2}\cdots\gamma^{\nu_p]} \quad (47)$$

is used. γ-matrices satisfy the standard relations

$$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}. \quad (48)$$

The action (46) can be considered as a direct generalization of the Rarita-Schwinger action for a fermionic one-form ψ^a_μ

$$S_0[\psi]=-\int d^nx\overline{\psi}_\mu\Gamma^{\mu\nu\sigma}\partial_\nu\psi_\sigma, \quad (49)$$

which is invariant under the gauge transformations

$$\delta\psi^a_\mu=\partial_\mu\Lambda^a. \quad (50)$$

The gauge generators

$$R^a_{\mu b}=\partial_\mu\delta^a_b,\quad \delta\psi^a_\mu=R^a_{\mu b}\Lambda^b \quad (51)$$

do not depend on fields ψ^a_μ, and this simple model belongs to the class of gauge theories with Abelian irreducible gauge algebra. The free theory of fermionic 2-form tensor-spinor fields, $\psi^a_{\mu\nu}$, presents a model of first-stage reducible gauge theory with action

$$S_0[\psi]=\int d^nx\overline{\psi}_{\mu\nu}\Gamma^{\mu\nu\rho\sigma\delta}\partial_\rho\psi_{\sigma\delta}, \quad (52)$$

being invariant under the following gauge transformations

$$\delta\psi^a_{\mu\nu}=2\partial_{[\mu}\Lambda^a_{\nu]},\qquad \delta\Lambda^a_\mu=\partial_\mu\Lambda^a. \quad (53)$$

The gauge generators

$$R^{a\sigma}_{\mu\nu b}=2\partial_{[\mu}\delta^\sigma_{\nu]}\delta^a_b,\qquad \delta\psi^a_{\mu\nu}=R^{a\sigma}_{\mu\nu b}\Lambda^b_\sigma, \quad (54)$$

have the zero-eigenvalue eigenvectors

$$Z^a_{\mu b}=\partial_\mu\delta^a_b,\qquad R^{a\sigma}_{\mu\nu b}Z^b_{\sigma c}=0. \quad (55)$$

[2] Note that in Ref. [18], for the first time, an action for antisymmetric tensor-spinor fields has been constructed as well in AdS space of arbitrary dimensions.

It is clear that the action (46) is disappeared if the dimension of space-time satisfies the conditions $n \leq 2p$. The action is invariant under reducible gauge transformations. They are

$$\begin{aligned}
\delta\psi^{a}_{\mu_1\mu_2\ldots\mu_p} &= p\partial_{[\mu_1}\Lambda^{(p-1)a}_{\mu_2\cdots\mu_p]},\\
\delta\Lambda^{(p-1)a}_{\mu_2\cdots\mu_p} &= (p-1)\partial_{[\mu_2}\Lambda^{(p-2)a}_{\mu_3\cdots\mu_p]},\\
\delta\Lambda^{(p-2)a}_{\mu_3\cdots\mu_p} &= (p-2)\partial_{[\mu_3}\Lambda^{(p-3)a}_{\mu_4\cdots\mu_p]}, \ldots, \quad \delta\Lambda^{(1)a}_{\mu} = \partial_\mu\Lambda^{(0)a}.
\end{aligned} \tag{56}$$

where $\Lambda^{(k)a}_{\mu_1\cdots\mu_k}$, $k = 0, 1, \ldots, p-1$, is a rank-k antisymmetric tensor-spinor. From (56), it follows the identification for gauge generators

$$R^{a\nu_2\cdots\nu_p}_{\mu_1\mu_2\cdots\mu_p b} = p\partial_{[\mu_1}\delta^{\nu_2}_{\mu_2}\cdots\delta^{\nu_p}_{\mu_p]}\delta^a_b, \tag{57}$$

and for the set of zero-eigenvalue eigenvectors

$$Z^{a\nu_2\cdots\nu_{s-1}}_{\mu_1\mu_2\cdots\mu_{s-1}b} = (s-1)\partial_{[\mu_1}\delta^{\nu_2}_{\mu_2}\cdots\delta^{\nu_{s-1}}_{\mu_{s-1}]}\delta^a_b, \quad s = 2, 3, \ldots, p, \tag{58}$$

in such a way that the relations (12), (15) in general setting are reading now as

$$R^{a\sigma_2\cdots\sigma_p}_{\mu_1\mu_2\cdots\mu_p c}Z^{c\nu_2\cdots\nu_{p-1}}_{\sigma_2\cdots\sigma_p b} = 0, \qquad Z^{a\sigma_2\cdots\sigma_{s-1}}_{\mu_1\mu_2\cdots\mu_{s-1}c}Z^{c\nu_2\cdots\nu_{s-2}}_{\sigma_2\cdots\sigma_{s-1}b} = 0, \quad s = 3, \ldots, p. \tag{59}$$

Therefore, we have the free $(p-1)$-stage reducible gauge theory.

We are going to study consistent deformations of the action (46) in a way describing above. To do this correctly, we give the table of "quantum" numbers of quantities entering in presentation of action (46) and used later:

Quantity	$\psi, \overline{\psi}$	$d^n x$	∂_ν	Γ	$\Box$	φ	m
ε	1	0	0	0	0	0	0
gh	0	0	0	0	0	0	0
dim	(n-1)/2	-n	1	0	2	(n-2)/2	1
ε_f	1,-1	0	0	0	0	0	0

where "ε" describes the Grassmann parity, the symbol "gh" is used to denote the ghost number, "dim" means the canonical dimension and "ε_f" is the fermionic number. Using the table of "quantum" numbers, it is easy to establish the quantum numbers of any quantities met in this section.

Deformation of initial theory is described by the generating function $h^a_{\mu_1\mu_2\ldots\mu_p}(\psi)$ having the same "quantum" numbers as $\psi^a_{\mu_1\mu_2\ldots\mu_p}$. Due to $\varepsilon_f(\psi) = 1$ the generating function $h_{\mu_1\mu_2\ldots\mu_p}(\psi)$ should be a polynomial containing in each their term even number, say $2k$, of fields ψ and, therefore, odd number $2k-1$ of fields $\overline{\psi}$. Such structure of terms $(\overline{\psi})^{2k-1}(\psi)^{2k}$ leads automatically to the relation $\varepsilon(h) = 1$. Canonical dimension of product of fields is equal to

$$\dim((\overline{\psi})^{2k-1}(\psi)^{2k}) = (4k-1)\frac{(n-1)}{2}. \tag{60}$$

To arrive at the needed relation $\dim(h) = (n-1)/2$, we have to use the dimensional quantities ∂ and $\Box = \partial^\nu\partial_\nu$ in the term under consideration. If the term contains l partial derivatives, then one needs to introduce the operator $\Box$ in the negative power $(2k-1)(n-1)+l/2$. Moreover, the function $h_{\mu_1\mu_2\ldots\mu_p}$ should be an antisymmetric tensor-spinor field. The simple example of generating function $h_{\mu_1\mu_2\ldots\mu_p}(\psi)$ satisfying all listed requirements and corresponding to the case $k = 1$ and $l = 1$ reads

$$h_{\mu_1\mu_2\ldots\mu_p}(\psi) = \frac{1}{\Box^{\frac{n}{2}}}\psi_{\mu_1\mu_2\ldots\mu_p}\overline{\psi}_{\nu_1\nu_2\ldots\nu_p} \times\Gamma^{\nu_1\nu_2\ldots\nu_p\nu\rho_1\rho_2\ldots\rho_p}\partial_\nu\psi_{\rho_1\rho_2\ldots\rho_p}, \qquad n > 2p. \tag{61}$$

In the case of the Rarita–Schwinger action (49) it means

$$h_\mu(\psi) = \frac{1}{\Box^{\frac{n}{2}}}\psi_\mu\overline{\psi}_\nu\Gamma^{\nu\sigma\rho}\partial_\sigma\psi_\rho, \tag{62}$$

as well as for the fermionic 2-form tensor-spinor fields (52) the generating function has the form

$$h_{\mu\nu}(\psi) = \frac{1}{\Box^{\frac{n}{2}}}\psi_{\mu\nu}\overline{\psi}_{\alpha\beta}\Gamma^{\alpha\beta\delta\sigma\rho}\partial_\delta\psi_{\sigma\rho}. \tag{63}$$

Minimal dimension of space-time in the Rarita–Schwinger model is equal to 3. Therefore, we can conclude that the non-locality of deformed action comes from the fourth order vertex due to presence of operator $1/\Box$ and from the sixth order vertex because of $(1/\Box)^2$. As the dimension of space - time grows, so does the degree of the operator $1/\Box$ responsible for the non-locality of the deformed action. Analogous statement about the non-locality is valid for the deformed model of fermionic 2-form tensor-spinor fields. In general, any consistent gauge-invariant deformation of antisymmetric tensor-spinor fields creates a non-local deformed action which has no some closed local sector. Let us stress once again that appearance of the operator $(1/\Box)$ in the generating function $h_{\mu_1\mu_2\ldots\mu_p}(\psi)$ is dictated by the strong motivations, namely: (1) non-triviality of the deformed gauge action requires to use a non-local generating functions because generating local functions with higher derivatives are forbidden by dimension reasons, (2) generating functions must be non-linear in fields to reproduce vertexes of interactions, (3) preservation of the fermionic number restricts possible non-linearity in fields of the generating functions which must contain odd orders of fields in its Taylor expansion, (4) compensation for the growing positive dimension of the terms in the generating function containing fields can be achieved using the corresponding positive powers of the operator $(1/\Box)$. The situation differs from the case of Abelian vector field or massless bosonic higher spin fields [1] when the non-locality of generating functions due to the operator $(1/\Box)$ is responsible for existence of the local gauge sectors of deformed actions which are gauge invariant under local pieces of the deformed gauge generators.

5 Interactions of fermionic 2-form fields and scalar field

Now we are going to demonstrate how the introduction of new degree of freedom in the form of a scalar field may change the conclusion given in the previous section about non-local nature of interactions of fermionic p-form fields.

We start with the free action of fermionic 2-form fields, $\psi^a_{\mu\nu}$, and a real massive scalar field, φ,

$$S_0[\psi,\varphi] = \int d^n x \overline{\psi}_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2\nu\nu_1\nu_2}\partial_\nu\psi_{\nu_1\nu_2} + \frac{1}{2}\int d^n x\big(\partial^\mu\varphi\partial_\mu\varphi - m^2\varphi^2\big), \quad n>4. \tag{64}$$

The action is invariant under the gauge transformations

$$\delta\psi_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}, \quad \delta\varphi = 0, \tag{65}$$

and belongs to gauge fields of first-stage reducibility with zero-eigenvalue eigenvectors which do not depend on fields and, therefore, do not transform under deformations described in section 3. In this case, the identification with general notations begins with fields $A^i = (\psi_{\mu\nu},\varphi)$ and generating functions of anticanonical transformations $h^i(A) = (h_{\mu\nu}(\psi,\varphi), h(\psi,\varphi))$.

The deformation of initial classical system (64) is determined by arbitrary choice of generating functions with only restrictions concerning "quantum numbers" and transformation rules which should coincide with properties of corresponding fields so that

$$\dim(h_{\mu\nu}) = \frac{n-1}{2}, \quad \mathrm{gh}(h_{\mu\nu}) = 0, \quad \varepsilon(h_{\mu\nu}) = 1, \quad \varepsilon_f(h_{\mu\nu}) = 1, \tag{66}$$

$$\dim(h) = \frac{n-2}{2}, \quad \mathrm{gh}(h) = 0, \quad \varepsilon(h) = 0, \quad \varepsilon_f(h) = 0, \tag{67}$$

and h must be a real scalar function while $h_{\mu\nu}$ must be a fermionic 2-form fields.

It is not difficult to propose the generating functions of anticanonical transformations which will be responsible to generate cubic vertexes in lower order of the deformation procedure,

$$h_{\mu\nu}(\psi,\varphi) = g(m)^{\frac{4-n}{2}}\frac{1}{\Box}\partial_\alpha\big(\gamma^\alpha\psi_{\mu\nu}\varphi\big), \qquad h(\psi,\varphi) = 0. \tag{68}$$

The deformed action, $\widetilde{S}_0[\psi,\varphi]$, can be presented in the form

$$\widetilde{S}_0[\psi,\varphi] = S_0[\psi,\varphi] + 2g(m)^{\frac{4-n}{2}}\int d^n x\overline{\psi}_{\mu\nu}\Gamma^{\mu\nu\alpha\beta}\psi_{\alpha\beta}\varphi + (non-local\ \ interaction\ \ terms). \tag{69}$$

In deriving (69), the relations

$$\Gamma^{\mu_1\mu_2\nu\nu_1\nu_2}\gamma^\mu + \Gamma^{\mu_1\mu_2\mu\nu_1\nu_2}\gamma^\nu = 2\Gamma^{\mu_1\mu_2\nu_1\nu_2}g^{\mu\nu} + (terms\ \ responsible\ \ for\ \ non-local\ \ contributions), \tag{70}$$

were used. The action

$$S_1[\psi,\varphi] = S_0[\psi,\varphi] + S_{int}[\psi,\varphi],$$
$$S_{int}[\psi,\varphi] = 2g(m)^{\frac{4-n}{2}}\int d^4 x\overline{\psi}_{\mu\nu}\Gamma^{\mu\nu\alpha\beta}\psi_{\alpha\beta}\varphi \tag{71}$$

describes the local sector of the deformed initial system. In its turn, the deformed gauge transformations of fields $\psi_{\mu\nu}$ read

$$\widetilde{\delta}\psi_{\mu\nu} = \delta\psi_{\mu\nu} + \delta_1\psi_{\mu\nu} + O(g^2), \tag{72}$$

where

$$\delta_1\psi_{\mu\nu} = -g(m)^{\frac{4-n}{2}}\frac{1}{\Box}\gamma^\sigma\partial_\sigma(\varphi\delta\psi_{\mu\nu}). \tag{73}$$

Let us consider the variation of action $S_1[\psi,\varphi]$ under the gauge transformations $\bar{\delta}\psi_{\mu\nu} = \delta\psi_{\mu\nu} + \delta_1\psi_{\mu\nu}$,

$$\bar{\delta}S_1[\psi,\varphi] = \delta_1 S_0[\psi,\varphi] + \delta S_{int}[\psi,\varphi] + O(g^2). \tag{74}$$

We have

$$\delta_1 S_0[\psi,\varphi] = -g(m)^{\frac{4-n}{2}}\int d^4 x\overline{\psi}_{\mu\nu}\Gamma^{\mu\nu\alpha\beta}\delta\psi_{\alpha\beta}\varphi + (non-local\ \ terms), \tag{75}$$

$$\delta S_{int}[\psi,\varphi] = g(m)^{\frac{4-n}{2}}\int d^4 x\overline{\psi}_{\mu\nu}\Gamma^{\mu\nu\alpha\beta}\delta\psi_{\alpha\beta}\varphi. \tag{76}$$

From Eqs. (75), (76), it follows that the local action $S_1[\psi,\varphi]$ describes interactions between fermionic 2-form fields and real massive scalar field and is invariant in the first order of deformation parameter under the gauge transformations $\bar{\delta}\psi_{\mu\nu}$ up to non-local terms. We see that construction of a local gauge theory of interacting antisymmetric spin-tensor fermionic fields meets with certain difficulties even if one introduces new degrees of freedom in the form of massive scalar field: although, in contrast with case which was studied in the previous section, the deformed action contains a local part but it is not invariant under local gauge symmetries. We can conclude that up to now, description of interactions of fermionic p-form fields in terms of a local gauge theory remains open problem.

6 Conclusion

In the present paper, we have extended the new approach proposed in [1,2] for gauge theories with closed/open algebras to the procedure of gauge-invariant deformations of classical reducible gauge theories. The deformation procedure of an initial theory with gauge freedom can be embedded into solutions to the classical master equation of the BV formalism [11,12]. Instead of using the cohomological approach to find solutions to the classical master equation in the form of Taylor expansion with respect to a deformation parameter [11,12], it was proposed to take into account the invariance of the classical master equation under anticanonical transformations. It allows to convert a given initial gauge theory presented in the BV formalism through an action satisfying the classical master equation and the boundary condition involving the initial gauge action into solutions to the classical master equation containing full information about deformed gauge-invariant theory [1,2].

Using analysis of anticanonical transformations in process of gauge-invariant deformation of solutions to the classical master equation in the minimal antisymplectic space [1,2], we have made use of a single generating function $h(A)$ depending on fields of initial theory A only. It means that non-trivial part of the generating functional Y has the form $A^*h(A)$. In general, the generating functional Y of anticanonical transformations may contain terms of higher order in antifields $(A^*)^m(C^*)^n(C_1^*)^k H_{m,n,k}(A)$. Using $H_{m,n,k}(A)$ when at least the index $m > 1$ does not change the structure of deformation in the initial configuration space but leads only to redefinition of structure functions in the deformed gauge algebra. It is well-known fact that the structure functions of any gauge algebra are not define uniquely [3,4]. In fact, this arbitrariness has been fixed by special type of anticanonical transformations in our method. The deformation of initial action has the form of replacement in the initial action the gauge field A by the field $A + h(A)$. In particular, it means that the generating function $h(A)$ should be a non-local one or/and should contain higher derivatives because otherwise one meets with trivial deformation when the deformed theory is classically equivalent with the initial gauge system. In general, the deformed gauge theory is non-local but sometimes it may happen that there exists a local gauge-invariant sector as a part of full theory. At the present, we have two important examples of such situation, namely, the deformation of free Abelian gauge theory allows to reproduce the Yang-Mills theory as well as the suitable non-local deformation of free theory of massless bosonic higher spin fields [21] leads to generation of all local cubic vertexes known in the literature [22–25] (for discussions of the non-locality of higher order vertexes, see [26–31]).

The deformation of gauge generators is described by the same function $h(A)$ in the form of shift $A \rightarrow A + h(A)$ of the argument of initial gauge generators followed by rotation defining by the inverse matrix to the $M^i{}_j(A) = \delta^i{}_j + h^i{}_{,j}(A)$. The deformation of zero-eigenvalue eigenvectors is described as the shift $A \rightarrow A + h(A)$ in their arguments. We have calculated some lower relations in the deformed reducible algebra with deformed structure coefficients in the case of first-stage reducibility. Generalization to arbitrary L-stage reducible gauge algebra looks like as a technical task. We emphasize that the deformed gauge algebra belongs to the same class of reducible algebras as for the initial gauge algebra.

We have studied the free fermionic p-form fields as an example of reducible gauge theory subjected to suitable gauge-invariant deformation. We have proved that consistent self-interactions of these fields are always described by non-local vertexes. Therefore, if we deal with fermionic p-form fields only then there is no possibility to construct a local gauge theory of interactions between these fields. In principle, adding new degree of freedom to a given dynamical system may change some properties of deformed theories. In fact, it was a reason for us to consider the model of free fermionic 2-form fields and a massive scalar field subjected to a non-local gauge-invariant deformation leading to existence of cubic vertexes in the deformed action. It was shown that in the first order with respect of the deformation parameter the deformed action contains a local part with cubic interactions of fields but, unfortunately, it is not invariant under local gauge symmetries. So, construction of a local gauge theory, containing interactions between completely antisymmetric spin-tensor fields, remains unsolved.

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References

1. I.L. Buchbinder, P.M. Lavrov, On a gauge-invariant deformation of a classical gauge-invariant theory. JHEP **06**, 854 (2021). arXiv:2104.11930 [hep-th]

2. I.L. Buchbinder, P.M. Lavrov, On classical and quantum deformations of gauge theories. Eur. Phys. J. C **81**, 856 (2021). arXiv:2108.09968 [hep-th]
3. I.A. Batalin, G.A. Vilkovisky, Gauge algebra and quantization. Phys. Lett. B **102**, 27–31 (1981)
4. I.A. Batalin, G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators. Phys. Rev. D **28**, 2567–2582 (1983)
5. I.A. Batalin, G.A. Vilkovisky, Closure of the gauge algebra, generalized Lie algebra equations and Feynman rules. Nucl. Phys. B **234**, 106 (1984)
6. B.L. Voronov, P.M. Lavrov, I.V. Tyutin, Canonical transformations and gauge dependence in general gauge theories. Sov. J. Nucl. Phys. **36**, 292 (1982)
7. I.A. Batalin, P.M. Lavrov, I.V. Tyutin, Finite anticanonical transformations in field-antifield formalism. Eur. Phys. J. C **75**, 270 (2015). arXiv:1501.07334 [hep-th]
8. I.A. Batalin, P.M. Lavrov, Closed description of arbitrariness in resolving quantum master equation. Phys. Lett. B **758**, 54–58 (2016). arXiv:1604.01888 [hep-th]
9. I.A. Batalin, P.M. Lavrov, I.V. Tyutin, Anticanonical transformations and Grand Jacobian. Russ. Phys. J. **64**, 688–694 (2021). arXiv:2011.06429 [hep-th]
10. A. Andrasi, J.C. Taylor, Generating functions for anti-canonical transformations in the Zinn–Justin and Batalin and Vilkoviski formalisms. arXiv:2201.02106 [hep-th]
11. G. Barnich, M. Henneaux, Consistent coupling between fields with gauge freedom and deformation of master equation. Phys. Lett. B **311**, 123–129 (1993). arXiv:hep-th/9304057
12. M. Henneaux, Consistent interactions between gauge fields: the cohomological approach. Contemp. Math. **219**, 93–110 (1998). arXiv:hep-th/9712226
13. A. Danehkar, On the cohomological derivation of Yang-Mills theory in the antifield formalism. JHEP Gravit. Cosmol. **03**, 368–387 (2017). arXiv:0707.4025 [physics.gen-ph]
14. M. Sakaguchi, H. Suzuki, On the interacting higher spin bosonic gauge fields in BRST-antifield formalism, Prog. Theor. Exp. Phys. **4**, 043B01 (2021). arXiv:2011.02689 [hep-th]
15. B.S. DeWitt, *Dynamical theory of groups and fields*, (Gordon and Breach, 1965)
16. C. Becchi, A. Rouet, R. Stora, The abelian Higgs Kibble Model, unitarity of the S-operator. Phys. Lett. B **52**, 344–346 (1974)
17. I.V. Tyutin, Gauge invariance in field theory and statistical physics in operator formalism, Lebedev Institute preprint No. 39 (1975). arXiv:0812.0580 [hep-th]
18. I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina, Lagrangian formulation of massive fermionic totally antisymmetric tensor field theory in AdS_d space. Nucl. Phys. B **819**, 453–477 (2009). arXiv:0902.1471 [hep-th]
19. Yu.M. Zinoviev, Note on antisymmetric spin-tensors. JHEP **04**, 035 (2009). arXiv:0903.0262 [hep-th]
20. V. Lekeu, Y. Zhang, On the quantisation and anomalies of antisymmetric tensor-spinors. JHEP **11**, 078 (2021). arXiv:2109.03963 [hep-th]
21. C. Fronsdal, Massless field with integer spin. Phys. Rev. D **18**, 3624 (1978)
22. R.R. Metsaev, Cubic interaction vertices of massive and massless higher spin fields. Nucl. Phys. B **759**, 141–201 (2006). arXiv:hep-th/0512342
23. R. Manvelyan, K. Mkrtchyan, W. Ruehl, A generating function for the cubic interactions of higher spin fields. Phys. Lett. B **696**, 410–415 (2011). arXiv:1009.1054 [hep-th]
24. R.R. Metsaev, BRST-BV approach to cubic interaction for massive and massless higher spin fields. Phys. Lett. B **720**, 237–243 (2013). arXiv:1205.3131 [hep-th]
25. M.V. Khabarov, Yu.M. Zinoviev, Massless higher spin cubic vertices in flat four dimensional space. JHEP **08**, 112 (2020). arXiv:2005.09851 [hep-th]
26. M. Taronna, Higher-spin interactions: four-point functions and beyond. JHEP **04**, 029 (2012). arXiv:1107.5843 [hep-th]
27. P. Dempster, M. Tsulaia, On the structure of quartic vertices for massless higher spin fields on Minkowski background. Nucl. Phys. B **865**, 353–375 (2012). arXiv:1203.5597 [hep-th]
28. M. Taronna, On the non-local obstruction to interacting higher spins in flat space. JHEP **05**, 026 (2017). arXiv:1701.05772 [hep-th]
29. R. Roiban, A.A. Tseytlin, On four-point interactions in massless higher spin theory in flat space. JHEP **04**, 139 (2017). arXiv:1701.05773 [hep-th]
30. D. Ponomarev, A note on (non)-locality in holographic higher spin theories. Universe **4**, 2 (2018). arXiv:1710.00403 [hep-th]
31. O.A. Gelfond, M.A. Vasiliev, Spin-locality of higher-spin theories and star-product functional classes. JHEP **03**, 002 (2020). arXiv:1910.00487 [hep-th]