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Canonical analysis of non-relativistic string with non-relativistic world-sheet

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Abstract We perform canonical analysis of non-relativistic string theory with non-relativistic world-sheet gravity. We determine structure of constraints and symplectic structure of canonical variables.

1 Introduction and summary

AdS/CFT correspondence is the most known example of holographic duality [1]. This correspondence, in its strongest form, claims that $SU(N)$ $\mathcal{N} = 4$ SYM theory in four dimensions is equivalent to type IIB theory on $AdS_5 \times S^5$ at any values of N and 'tHooft coupling λ. On the other hand understanding this duality at the strongest form is still lacking and hence we should restrict to some limits of this correspondence.

Recently such an interesting limit was suggested in [2] and it is known as Spin Matrix Theory (SMT) and describes near BPS limit of AdS/CFT. It is quantum mechanical theory with Hamiltonian given as sum of harmonic oscillator operators that transform both in adjoin representation of $SU(N)$ and in a particular spin subgroup G_s of the global superconformal $PSU(2, 2|4)$ symmetries of $\mathcal{N} = 4$.

One can ask the question what is the dual description of this quantum mechanical model. It was suggested [3] and further studied in [4–6] that dual theory in the bulk corresponds to non-relativistic string theory with non-relativistic world-sheet known as SMT string. These special non-relativistic theories should be considered in the broader context of non-relativistic string theories that were studied recently in [3–6] and also [7–21]. This development is related to the generalization of Newton–Cartan geometry [22] to the stringy Newton–Cartan geometry [7] and torsional Newton–Cartan geometry. Moreover, SMT string was derived in [3–5] by specific non-relativistic limit on the world-sheet of non-relativistic string in torsional NC background. Recently this

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SMT string was very intensively studied in [6] where particular class of backgrounds for SMT string, known as flat-fluxed backgrounds, was analysed. In these backgrounds SMT string reduces to a free theory. These world-sheet theories are analogues of the Polyakov action on Minkowski target space-time.

The next step would be to analyse properties of SMT string in general background. In order to do this we should certainly study classical dynamics as for example its Hamiltonian form. The aim of this paper is to find such a formulation in the most general case.

Let us be more explicit. We start with the action for SMT string that was found in [5] and perform canonical analysis of this theory. As opposite to Polyakov form of the relativistic string now the action is formulated using vierbein $e_\alpha^{\ a}$ where $\alpha = 0, 1$ correspond to world-sheet coordinates while $a = 0, 1$ correspond to tangent space coordinates. Note that $e_\alpha^{\ a}$ is invertible matrix with inverse $\theta_a^{\ \alpha}$. Now it is crucial that the quadratic term with $\partial_\alpha x^\mu \partial_\beta x^\nu h_{\mu\nu}$ is multiplied with $\theta_1^\alpha \theta_1^\beta$ as opposite to the relativistic case when this term has the form $\theta_a^\alpha \theta_b^\beta \eta^{ab}$. Then it is necessary to distinguish two cases. In the first case we presume that $\theta_1^0 \neq 0$. Then the relation between momenta and time derivative of x^μ is invertible. As a result we obtain Hamiltonian together with set of the primary constraints that follow from the structure of the theory. Careful analysis of the preservation of the primary constraints gives two secondary constraints that are first class constraints that reflect the fact that the theory is invariant under world-sheet diffeomorphism. We also identify four additional second class constraints and Poisson brackets between them. Finally we determine symplectic structure for canonical variables which is given in terms of the Dirac brackets. We identify that in this case the Dirac brackets coincide with Poisson brackets.

The situation is different when $\theta_1^0 = 0$. In this case it is not possible to express time derivative of x^μ using canonical variables. Instead we get new d-constraints where d-is

number of dimensions labelled with x^μ. Then the canonical analysis is slightly more complicated than in previous case. However we again find two first class constraints that reflect invariance of the action under reparameterization. We further identify second class constraints and Poisson brackets between them. The presence of these constraints then imply non-trivial symplectic structure between canonical variables x^μ which confirms analysis presented in [6].

Let us outline our results and suggest further directions of research. We found Hamiltonian formulation of SMT string and we identified structure of constraints. We discussed two cases when in the first one we were able to invert relation between time derivative of x^μ and canonical momenta. In fact, this is the most general situation where all components of θ_1^α are non-zero. On the other hand the second case when $\theta_1^0 = 0$ deserves separate treatment. This fact suggests that the spatial gauge as was used in [6] cannot be reached from the general Hamiltonian. It is instructive to compare this situation with the standard relativistic Lagrangian where the relation between momenta and ∂x^μ contains expression $\theta_a^0 \eta_b^\alpha \eta^{ab} \partial_\alpha x^\mu$ that can be certainly inverted even if we impose condition $\theta_1^0 = 0$. On the other hand when we studied the situation when $\theta_1^0 = 0$ separately we found theory with non-trivial symplectic structure as in [6].

Certainly this work can be extended in many directions. It would be nice to study the most general form of the string with the non-relativistic world-sheet and study its consistency from canonical point of view. It would be also extremely interesting to study supersymmetric generalization of this two dimensional theory. However the most important issue is to study relation between string theory with non-relativistic world sheet and spin matrix models. In other words we should apply these general calculations to the concrete background as was found in [6]. Then we could analyse spectrum of SMT string. This is the place where the canonical form of the SMT string derived in this paper could be useful, for example, when we try to fix two first class constraints that were identified in this paper by specific gauge fixing known as uniform light cone gauge [24–26], for review see [23]. Then we could study solutions of the equations of motion of gauge fixed SMT string and compare it with the specific configurations of spin matrix models. Another possibility would be to study SMT string with the help of BRST formalism as for example in [27] and then quantum consistency of this theory. We hope to return to these problems in future.

This paper is organized as follows. In the next Sect. 2 we review basic properties of non-relativistic string and we perform canonical analysis it the most general case. We also determine symplectic structure of given theory. In Sect. 3

we separately discuss the case $\theta_1^0 = 0$ and we determine corresponding Hamiltonian and symplectic structure.

2 Hamiltonian analysis of SMT string

We begin with the Polyakov form of the action for SMT string that was introduced in [5] and that has the form

$$S = -\frac{T}{2} \int d^2\sigma (2\epsilon^{\alpha\beta} m_\alpha \partial_\beta \eta + e\theta_1^\alpha \theta_1^\beta h_{\alpha\beta} + \omega \epsilon^{\alpha\beta} e_\alpha^0 \tau_\beta$$
$$+ \psi \epsilon^{\alpha\beta} (e_\alpha^0 \partial_\beta \eta + e_\alpha^1 \tau_\beta)). \qquad (1)$$

Let us explain meaning of various symbols that appear in (1). The world-sheet is labelled by $\sigma^0, \sigma^1 \equiv \sigma$ and T is string tension. Further, m_μ, $h_{\mu\nu}$ and τ_μ are target space-time Newton-Cartan fields that obey conditions

$$\tau_\mu h^{\mu\nu} = 0 , \quad v^\mu h_{\mu\nu} = 0 , \quad \tau_\mu v^\mu = -1 ,$$
$$h_{\mu\nu} h^{\nu\rho} - \tau_\mu v^\rho = \delta_\mu^\rho. \qquad (2)$$

The world-sheet metric is defined with the help of zwiebein e_α^a, $a = 0, 1$ with inverse θ_a^α that obey

$$e_\alpha^a \theta_b^\alpha = \delta_b^a, \quad e_\alpha^a \theta_a^\beta = \delta_\alpha^\beta. \qquad (3)$$

As was argued in [5] the world-sheet theory is non-relativistic since e_α^a play different role in the action. This can be already seen from (1) since zweibein inverse θ_a^α does not appear in Lorentz invariant way $\theta_a^\alpha \theta_b^\beta \eta^{ab}$ but instead there is an expression $\theta_1^\alpha \theta_1^\beta$. This fact has an important consequence for the structure of this theory. Note also that

$$e = \det e_\alpha^a \qquad (4)$$

and

$$m_\alpha = m_\mu \partial_\alpha x^\mu , \quad h_{\alpha\beta} = h_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu , \quad \tau_\alpha = \tau_\mu \partial_\alpha x^\mu, \qquad (5)$$

where x^μ label embedding of the string into target space-time. Finally η is scalar field defined on world-sheet that corresponds to the embedding of the string into periodic target space direction.

We should stress that the theory is manifestly invariant under world-sheet diffeomorphism $\sigma'^\alpha = f^\alpha(\sigma)$ where world-volume fields transform as

$$x'^\mu(\sigma') = x^\mu(\sigma), \quad \eta'(\sigma') = \eta(\sigma) , \quad e_\beta'^b(\sigma') = e_\alpha^b(\sigma) \frac{\partial \sigma^\alpha}{\partial \sigma'^\beta}. \qquad (6)$$

Our goal is to find Hamiltonian formulation of this theory in order to investigate possible non-relativistic nature of it. First of all we start with the definition of conjugate momenta. From (1) we obtain

$$\pi^\alpha_b = \frac{\partial \mathcal{L}}{\partial(\partial_0 e^a_\alpha)} \approx 0 \,, \quad p_\psi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} \approx 0$$

$$\pi_\omega = \frac{\partial \mathcal{L}}{\partial(\partial_0 \omega)} \approx 0,$$

$$p_\eta = \frac{\partial \mathcal{L}}{\partial(\partial_0 \eta)} = T m_1 + \frac{T}{2} \psi e^0_1,$$

$$p_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 x^\mu)} = -T m_\mu \partial_1 \eta - T e \theta^0_1 \theta^\beta_1 h_{\mu\nu} \partial_\beta x^\nu$$
$$+ \frac{T}{2} \omega e^0_1 \tau_\mu + \frac{T}{2} \psi e^1_1 \tau_\mu. \tag{7}$$

It is clear that definition of p_η implies following primary constraint

$$\Sigma_1 \equiv p_\eta - T m_1 - \frac{T}{2} \psi e^0_1 \approx 0. \tag{8}$$

In this section we will presume that θ^0_1 is non-zero and hence we can express time derivative of x^μ as function of p_μ. On the other hand there is another primary constraint that follows from the definition of p_μ given in (7)

$$\Sigma_2 \equiv v^\mu p_\mu + T v^\mu m_\mu \partial_1 \eta + \frac{T}{2} \omega e^0_1 + \frac{T}{2} \psi e^1_1 \approx 0 \tag{9}$$

using $v^\mu h_{\mu\nu} = 0, \quad v^\mu \tau_\mu = -1$.

Returning to (7) we obtain bare Hamiltonian density in the form

$$\mathcal{H}_B = p_\mu \partial_0 x^\mu + p_\eta \partial_0 \eta - \mathcal{L}$$
$$= -\frac{1}{2 T e \theta^0_1 \theta^0_1} [p_\mu h^{\mu\nu} p_\nu + 2 T p_\mu h^{\mu\nu} m_\nu \partial_1 \eta$$
$$+ T^2 \partial_1 \eta m_\mu h^{\mu\nu} m_\nu \partial_1 \eta]$$
$$- \frac{\theta^0_1 \theta^1_1}{\theta^0_1 \theta^0_1} (p_\mu \partial_1 x^\mu + p_\mu v^\mu \tau_1 + \partial_1 \eta m_1 + \partial_1 \eta m_\mu v^\mu \tau_1)$$
$$+ \frac{T}{2} \omega e^0_0 \tau_1 + \frac{T}{2} \psi (e^1_0 \tau_1 + e^0_0 \partial_1 \eta). \tag{10}$$

As is well known from the theory of systems with constraints the time evolution is governed by extended Hamiltonian that incorporates bare Hamiltonian together with set of all primary constraints. Explicitly we have

$$\mathcal{H}_E = \mathcal{H}_B + \Omega^1 \Sigma_1 + \Omega^2 \Sigma_2 + \Omega^a_\alpha \pi^\alpha_a + \Omega_\psi p_\psi + \Omega_\omega p_\omega, \tag{11}$$

where Ω^1, Ω^2, Ω^a_α, Ω_ψ and Ω_ω are Lagrange multipliers.

Now we should analyse condition of the preservation of all primary constraints $\pi^\alpha_a \approx 0$, $p_\omega \approx 0$, $p_\psi \approx 0$, $\Sigma_1 \approx 0$, $\Sigma_2 \approx 0$. To do this we need following canonical Poisson brackets

$$\left\{ e^a_\alpha(\sigma), \pi^\beta_b(\sigma') \right\} = \delta^\beta_\alpha \delta^a_b \delta(\sigma - \sigma') \,,$$
$$\left\{ \psi(\sigma), p_\psi(\sigma') \right\} = \delta(\sigma - \sigma') \,,$$
$$\left\{ \omega(\sigma), p_\omega(\sigma') \right\} = \delta(\sigma - \sigma') \,. \tag{12}$$

First of all we have that $\Sigma_{1,2}$ are second class constraints together with p_ψ, p_ω as follows from Poisson brackets

$$\left\{ p_\psi(\sigma), \Sigma_1(\sigma') \right\} = \frac{T}{2} e^0_1(\sigma) \delta(\sigma - \sigma') \,,$$

$$\left\{ \pi^1_0(\sigma), \Sigma_1(\sigma') \right\} = \frac{T}{2} \psi(\sigma) \delta(\sigma - \sigma') \,,$$

$$\left\{ p_\psi(\sigma), \Sigma_2(\sigma') \right\} = -\frac{T}{2} e^1_1(\sigma) \delta(\sigma - \sigma') \,,$$

$$\left\{ p_\omega(\sigma), \Sigma_2(\sigma') \right\} = -\frac{T}{2} e^0_1(\sigma) \delta(\sigma - \sigma') \,,$$

$$\left\{ \pi^1_0(\sigma), \Sigma_2(\sigma') \right\} = -\frac{T}{2} \omega(\sigma) \delta(\sigma - \sigma') \,,$$

$$\left\{ \pi^1_1(\sigma), \Sigma_2(\sigma') \right\} = -\frac{T}{2} \psi(\sigma) \delta(\sigma - \sigma') \,.$$

$$\{ \Sigma_1(\sigma), \Sigma_2(\sigma') \} = -T v^\mu \partial_\mu m_\nu \partial_\sigma x^\nu(\sigma) \delta(\sigma - \sigma')$$
$$- T v^\mu m_\mu(\sigma') \partial_{\sigma'} \delta(\sigma - \sigma')$$
$$- T m_\mu(\sigma) \partial_\sigma \delta(\sigma - \sigma) v^\mu(\sigma')$$
$$= T v^\mu (\partial_\nu m_\mu$$
$$- \partial_\mu m_\nu) \partial_\sigma x^\nu \delta(\sigma - \sigma') \,,$$
$$\{ \Sigma_2(\sigma), \Sigma_2(\sigma) \} = \{ \Sigma_1(\sigma), \Sigma_1(\sigma') \} = 0 \tag{13}$$

using the fact that

$$f(\sigma') \partial_\sigma \delta(\sigma - \sigma') = f(\sigma) \partial_\sigma \delta(\sigma - \sigma) + \partial_\sigma f(\sigma) \delta(\sigma - \sigma') \,. \tag{14}$$

We see that there is non-zero Poisson bracket between π^1_0, π^1_1 and $\Sigma_{1,2}$ which makes analysis slightly complicated. In order to resolve this issue let us introduce $\tilde{\pi}^1_0$ as a specific linear combinations of primary constraints that has vanishing Poisson brackets with Σ_1, Σ_2. Explicitly, we have

$$\tilde{\pi}^1_0 = \pi^1_0 - \frac{1}{e^0_1} \psi p_\psi - \frac{1}{e^0_1} \omega p_\omega + \pi^1_1 \frac{e^1_1}{e^0_1} \tag{15}$$

that obeys

$$\left\{ \tilde{\pi}^1_0, \Sigma_1 \right\} = 0 \,, \quad \left\{ \tilde{\pi}^1_0, \Sigma_2 \right\} = 0. \tag{16}$$

In the same way we introduce $\tilde{\pi}^1_1$ defined as

$$\tilde{\pi}^1_1 = \pi^1_1 - \frac{1}{e^0_1} \psi p_\omega \tag{17}$$

that clearly obeys

$$\left\{ \tilde{\pi}^1_1, \Sigma_1 \right\} = 0 \,, \quad \left\{ \tilde{\pi}^1_1, \Sigma_2 \right\} = 0. \tag{18}$$

In the same way we have

$$\left\{\tilde{\pi}^1{}_0, p_\psi\right\} \approx 0, \quad \left\{\tilde{\pi}^1{}_0, p_\omega\right\} \approx 0,$$
$$\left\{\tilde{\pi}^1{}_1, p_\psi\right\} \approx 0, \quad \left\{\tilde{\pi}^1{}_1, p_\omega\right\} \approx 0,$$
$$\left\{\tilde{\pi}^1{}_0, \pi^\alpha_a\right\} \approx 0, \quad \left\{\tilde{\pi}^1{}_1, \pi^\alpha_a\right\} \approx 0. \tag{19}$$

Note that $\pi^0_1 \approx 0$, π^0_0 are unchanged. Then clearly $\tilde{\pi}^1_1 \approx 0$, $\tilde{\pi}^1_0 \approx 0$ and $\pi^0_0 \approx 0$, $\pi^0_1 \approx 0$ are first class constraints.

Now we are ready to study preservation of the primary constraints. In case of $p_\omega \approx 0$ we get

$$\partial_0 p_\omega = \{p_\omega, H_E\} = -\frac{T}{2}e_0^0\tau_1 - \Omega^2\frac{T}{2}e_1^0 = 0, \tag{20}$$

where $H_E = \int d\sigma \mathcal{H}_E$. Note that (20) can be solved for Ω^2 as

$$\Omega^2 = -\tau_1\frac{e_0^0}{e_1^0}. \tag{21}$$

Further, condition of the preservation of the constraint $p_\psi \approx 0$ implies

$$\partial_0 p_\psi = \{p_\psi, H_E\} = -\frac{T}{2}(e_0^1\tau_1 + e_0^0\partial_1\eta) + \Omega^1\frac{T}{2}e_1^0$$
$$- \Omega^2\frac{T}{2}e_1^1 = 0 \tag{22}$$

that can be solved for Ω^1 as

$$\Omega^1 = -\frac{e}{e_1^0}\tau_1 + e_0^0\partial_1\eta. \tag{23}$$

Let us finally analyse conditions of preservation of constraints $\Sigma_1 \approx 0$ and $\Sigma_2 \approx 0$. In case of $\Sigma_1 \approx 0$ we obtain

$$\partial_0\Sigma_1(\sigma) = \{\Sigma_1(\sigma), H_E\} = \int d\sigma'(\{\Sigma_1(\sigma), \mathcal{H}_B(\sigma')\}$$
$$+ \Omega_\psi\{\Sigma_1(\sigma), p_\psi(\sigma')\}$$
$$+ \Omega^2\{\Sigma_1(\sigma), \Sigma_2(\sigma')\}) = 0 \tag{24}$$

which is equation for Ω_ψ. In the same way requirement of the preservation of the constraint $\Sigma_2(\sigma) \approx 0$ implies

$$\partial_0\Sigma_2(\sigma) = \{\Sigma_2(\sigma), H_E\} = \int d\sigma'(\{\Sigma_2(\sigma), \mathcal{H}_B(\sigma')\}$$
$$+ \Omega_\psi\{\Sigma_2(\sigma), p_\psi(\sigma')\}$$
$$+ \Omega_\omega\{\Sigma_2(\sigma), p_\omega(\sigma')\}$$
$$+ \Omega^2\{\Sigma_2(\sigma), \Sigma_1(\sigma')\}) = 0 \tag{25}$$

that, using the fact that we know Ω_1 and Ω_ψ allows us to solve for Ω_ω. These results are consequence of the fact that Σ_1, Σ_2 and p_ω, p_σ are second class constraints.

As the final step we study the question of preservation of the constraints

$$\tilde{\pi}^1{}_0 \approx 0, \quad \tilde{\pi}^1{}_1 \approx 0, \quad \pi^0_1 \approx 0, \quad \pi^0_0 \approx 0. \tag{26}$$

First of all we use the fact that θ^α_a has following components

$$\theta^\alpha_a = \begin{pmatrix} \theta^0_0 & \theta^0_1 \\ \theta^1_0 & \theta^1_1 \end{pmatrix} = \frac{1}{e}\begin{pmatrix} e_1^1 & -e_1^0 \\ -e_0^1 & e_0^0 \end{pmatrix} \tag{27}$$

so that $\mathcal{H}_B$ is equal to

$$\mathcal{H}_B = -\frac{e}{2Te_1^0e_1^0}[p_\mu h^{\mu\nu}p_\nu + 2Tp_\mu h^{\mu\nu}m_\nu\partial_1\eta$$
$$+ T^2\partial_1\eta m_\mu h^{\mu\nu}m_\nu\partial_1\eta] +$$
$$+ \frac{e_0^0}{e_1^0}(p_\mu\partial_1 x^\mu + p_\mu v^\mu\tau_1 + T\partial_1\eta m_1$$
$$+ T\partial_1\eta m_\mu v^\mu\tau_1) +$$
$$+ \frac{T}{2}\omega e_0^0\tau_1 + \frac{T}{2}\psi(e_0^1\tau_1 + e_0^0\partial_1\eta) .0 \tag{28}$$

To proceed further we use the fact that

$$\left\{\pi^\alpha_a(\sigma), e(\sigma')\right\} = \left\{\pi^\alpha_a(\sigma), \det e_\beta^b(\sigma')\right\}$$
$$= -\theta^\alpha_a e(\sigma)\delta(\sigma - \sigma'). \tag{29}$$

Then we start with the requirement of the preservation of constraint π^0_0 and we obtain

$$\partial_0\pi^0_0 = \left\{\pi^0_0, H_E\right\} = \frac{e_1^1}{2Te_1^0e_1^0}[p_\mu h^{\mu\nu}p_\nu$$
$$+ 2Tp_\mu h^{\mu\nu}m_\nu\partial_1\eta + T^2\partial_1\eta m_\mu h^{\mu\nu}m_\nu\partial_1\eta]$$
$$- \frac{1}{e_1^0}(p_\mu\partial_1 x^\mu + p_\mu v^\mu\tau_1 + T\partial_1\eta m_1$$
$$+ T\partial_1\eta m_\mu v^\mu\tau_1) - \frac{T}{2}\omega\tau_1 - \frac{T}{2}\psi\partial_1\eta$$
$$= \frac{e_1^1}{2Te_1^0e_1^0}[p_\mu h^{\mu\nu}p_\nu + 2Tp_\mu h^{\mu\nu}m_\nu\partial_1\eta$$
$$+ 2Tp_\eta\tau_1 - 2T^2m_1\tau_1$$
$$+ T^2\partial_1\eta m_\mu h^{\mu\nu}m_\nu\partial_1\eta]$$
$$- \frac{1}{e_1^0}(p_\mu\partial_1 x^\mu + p_\eta\partial_1\eta)$$
$$+ \Sigma_1\left(-\frac{1}{e_1^0} + \frac{e_1^1}{e_1^0e_1^0}\right) + \frac{1}{e_1^0}\Sigma_2 \tag{30}$$

using the fact that

$$\psi = \frac{2}{Te_1^0}(-\Sigma_1 + p_\eta - Tm_1),$$
$$\omega = \frac{2}{Te_1^0}\left(\Sigma_2 + \Sigma_1\frac{e_1^1}{e_1^0} - v^\mu p_\mu - Tv^\mu m_\mu\partial_1\eta\right.$$
$$\left. - \frac{e_1^1}{e_1^0}p_\eta + \frac{e_1^1}{e_1^0}Tm_1\right) \tag{31}$$

as follows from the definition of the primary constraints Σ_1, Σ_2.

In the same way we can proceed with the time evolution of constraint π_1^0 and we get

$$\partial_0 \pi_1^0 = \left\{ \pi_1^0, H_E \right\}$$
$$= -\frac{1}{2Te_1^0}[p_\mu h^{\mu\nu} p_\nu + 2T p_\mu h^{\mu\nu} m_\nu \partial_1 \eta$$
$$+ 2T p_\eta \tau_1 - 2T^2 m_1 \tau_1 + T^2 \partial_1 \eta m_\mu h^{\mu\nu} m_\nu \partial_1 \eta]$$
$$+ \frac{1}{e_1^0} \Sigma_1 . \tag{32}$$

In case of $\tilde{\pi}_1^1 \approx 0$ we obtain

$$\partial_0 \tilde{\pi}_1^1 = \left\{ \tilde{\pi}_1^1, H_E \right\}$$
$$= \frac{e_0^0}{2Te_1^0 e_1^0}[p_\mu h^{\mu\nu} p_\nu + 2T p_\mu h^{\mu\nu} m_\nu \partial_1 \eta$$
$$+ 2T p_\eta \tau_1 - 2T^2 m_1 \tau_1$$
$$+ T^2 \partial_1 \eta m_\mu h^{\mu\nu} m_\nu \partial_1 \eta] - \frac{e_0^0}{e_1^0 e_1^0} \Sigma_1 . \tag{33}$$

In the same way we can proceed with $\tilde{\pi}_0^1$ and we obtain that all constraints (26) are preserved when we introduce two secondary constraints

$$\mathcal{H}_1 = p_\mu h^{\mu\nu} p_\nu + 2T p_\mu h^{\mu\nu} m_\nu \partial_1 \eta + 2T p_\eta \tau_1 - 2T^2 m_1 \tau_1$$
$$+ T^2 \partial_1 \eta m_\mu h^{\mu\nu} m_\nu \partial_1 \eta \approx 0 ,$$
$$\mathcal{H}_2 = p_\eta \partial_1 \eta + p_\mu \partial_1 x^\mu \approx 0 . \tag{34}$$

Note also that using these secondary constraints the Hamiltonian density $\mathcal{H}_B$ can be written as

$$\mathcal{H}_B = -\frac{e}{2Te_1^0 e_1^0} \mathcal{H}_1 + \frac{e_0^0}{e_1^0} \mathcal{H}_2 + +\frac{e_0^0}{e_1^0} \tau_1 \left(\Sigma_2 + \frac{e_1^1}{e_1^0} \Sigma_1 \right)$$
$$- \frac{\Sigma_1}{e_1^0}(e_0^1 \tau_1 + e_0^0 \partial_1 \eta) . \tag{35}$$

We see that Hamiltonian is linear combinations of constraints. As the last step we should analyse Poisson brackets between constraints $\mathcal{H}_1$ and $\mathcal{H}_2$. Since they contain spatial derivatives of x^μ it is convenient to introduce their smeared form multiplied by arbitrary functions N^1, M^1 and N^2, M^2. Explicitly, we have

$$\mathbf{T}^{1,2}(N^{1,2}) \equiv \int d\sigma N^{1,2} \mathcal{H}_{1,2} , \quad \mathbf{T}^{1,2}(M^{1,2})$$
$$= \int d\sigma M^{1,2} \mathcal{H}_{1,2} . \tag{36}$$

Then using standard Poisson brackets we obtain

$$\left\{ \mathbf{T}^1(N^1), \mathbf{T}^1(M^1) \right\} = 0 ,$$
$$\left\{ \mathbf{T}_2(N^2), \mathbf{T}_2(M^2) \right\} = \mathbf{T}_2(N^2 \partial_1 M^2 - M^2 \partial_1 N^2) . \tag{37}$$

Finally we determine Poisson bracket between generator of spatial diffeomorphism $\mathbf{T}^2(N^2)$ and $\mathcal{H}_1$ and we obtain

$$\left\{ \mathbf{T}^2(N^2), \mathcal{H}_1(\sigma) \right\} = -2\partial_1 N^2 \mathcal{H}_1 - N^2 \partial_1 \mathcal{H}_1 \approx 0 \tag{38}$$

which shows that $\mathcal{H}_1$ is tensor density. These results show that $\mathcal{H}_1 \approx 0$, $\mathcal{H}_2 \approx 0$ are correct form of diffeomorphism constraints which is consequence of the fact that action for SMT string is still diffeomorphism invariant.

Finally we should analyse conditions of the preservation of constraints $\mathcal{H}_1 \approx 0$, $\mathcal{H}_2 \approx 0$. We see that generally Poisson brackets between $\mathcal{H}_{1,2}$ and Σ_1, Σ_2 do not vanish. Instead we know that Σ_1, Σ_2 have non-zero Poisson brackets between p_ψ, ψ_ω so that they can be interpreted as second class constraints. Let us denote these second class constraints as $\Psi^A = (p_\omega, \Sigma_1, p_\psi, \Sigma_2)$ with following structure of Poisson brackets

$$\left\{ \Psi^A(\sigma), \Psi^B(\sigma') \right\} = \triangle^{AB}(\sigma, \sigma'), \tag{39}$$

where

$$\triangle^{AB} = \begin{pmatrix} 0 & \frac{1}{2}e_1^0 & 0 & -\frac{1}{2}e_1^1 \\ -\frac{1}{2}e_1^0 & 0 & 0 & v^\mu(\partial_\mu m_\nu - \partial_\mu m_\nu)\partial_1 x^\nu \\ 0 & 0 & 0 & -\frac{1}{2}e_1^0 \\ \frac{1}{2}e_1^1 & -v^\mu(\partial_\mu m_\nu - \partial_\mu m_\nu)\partial_1 x^\nu & \frac{1}{2}e_1^0 & 0 \end{pmatrix} T\delta(\sigma - \sigma') \tag{40}$$

with inverse matrix

$$\triangle_{AB} = \frac{2}{T} \begin{pmatrix} 0 & -\frac{1}{e_1^0} & -2\frac{v^\mu(\partial_\mu m_\nu - \partial_\mu m_\nu)\partial_1 x^\nu}{e_1^0 e_1^0} & 0 \\ \frac{1}{e_1^0} & 0 & -\frac{e_1^1}{e_1^0 e_1^0} & 0 \\ 2\frac{v^\mu(\partial_\mu m_\nu - \partial_\mu m_\nu)\partial_1 x^\nu}{e_1^0 e_1^0} & \frac{e_1^1}{e_1^0 e_1^0} & 0 & \frac{1}{e_1^0} \\ 0 & 0 & -\frac{1}{e_1^0} & 0 \end{pmatrix} \delta(\sigma - \sigma') . \tag{41}$$

Let us then introduce modified constraints $\tilde{\mathcal{H}}_i$, $i = 1, 2$ as

$$\tilde{\mathcal{H}}_i = \mathcal{H}_i - \Psi^A \triangle_{AB} \left\{ \Psi^B, \mathcal{H}_i \right\}, \tag{42}$$

where summation over A includes also integration over σ implicitly. Using the fact that $\{\mathcal{H}_i, \mathcal{H}_j\} \approx 0$ we easily get that

$$\left\{ \tilde{\mathcal{H}}_i, \tilde{\mathcal{H}}_j \right\} \approx 0. \tag{43}$$

Then we have

$$\left\{ \tilde{\mathcal{H}}_i, \Psi^A \right\} = \left\{ \tilde{\mathcal{H}}_i, \Psi^A \right\} + \left\{ \Psi^A, \Psi^C \right\} \triangle_{CB} \left\{ \Psi^B, \mathcal{H}_i \right\} \approx 0 \tag{44}$$

and hence $\tilde{\mathcal{H}}_i$ have vanishing Poisson brackets with all constraints. On the other hand since Ψ^A are second class constraints that vanish strongly in the end of the procedure we find that $\tilde{\mathcal{H}}_i$ coincide with $\mathcal{H}_i$. Of course, this can be done on condition that we replace ordinary Poisson brackets by Dirac brackets whose structure will be studied in the next section.

2.1 Symplectic structure

We saw above that Ψ^A are second class constraints with the matrix of Poisson brackets given in (40) and its inverse given in (41). In order to determine Dirac brackets between canonical variables we firstly calculate Poisson brackets between canonical variables and second class constraints Ψ^A

$$\left\{ x^\mu(\sigma), \Psi^A(\sigma') \right\} = (0, 0, 0, v^\mu) \delta(\sigma - \sigma'),$$
$$\left\{ p_\mu(\sigma), \Psi_A(\sigma') \right\} = (0, T \partial_\mu m_\nu \partial_1 x^\nu \delta(\sigma - \sigma') + T m_\mu(\sigma') \partial_{\sigma'} \delta(\sigma - \sigma'),$$
$$0, -\partial_\mu v^\nu p_\nu \delta(\sigma - \sigma') - T \partial_\mu (v^\nu m_\nu) \partial_1 \eta \delta(\sigma - \sigma')),$$
$$\left\{ \eta(\sigma), \Psi^A(\sigma') \right\} = (0, 0, 0, -T v^\mu m_\mu(\sigma') \partial_{\sigma'} \delta(\sigma - \sigma')),$$
$$\left\{ p_\eta(\sigma), \Psi^A(\sigma) \right\} = (0, \delta(\sigma - \sigma'), 0, 0). \tag{45}$$

Then we find following form of Dirac brackets between canonical variables

$$\left\{ \eta(\sigma), p_\eta(\sigma') \right\}_D = \left\{ \eta(\sigma), p_\eta(\sigma') \right\}$$
$$- \int d\sigma_1 d\sigma_2 \left\{ \eta(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}(\sigma_1, \sigma_2)$$
$$\left\{ \Psi^B(\sigma_2), p_\eta(\sigma') \right\} = \delta(\sigma - \sigma'),$$
$$\left\{ \eta(\sigma), \eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ \eta(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}$$
$$(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), \eta(\sigma') \right\} = 0,$$
$$\left\{ p_\eta(\sigma), p_\eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ p_\eta(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}$$

$$(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), p_\eta(\sigma') \right\} = 0$$
$$\left\{ x^\mu(\sigma), p_\nu(\sigma') \right\}_D = \left\{ x^\mu(\sigma), p_\nu(\sigma') \right\}$$
$$- \int d\sigma_1 d\sigma_2 \left\{ x^\mu(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}$$
$$(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), p_\nu(\sigma') \right\} = \delta_\nu^\mu \delta(\sigma - \sigma'),$$
$$\left\{ x^\mu(\sigma), x^\nu(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ x^\mu(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}$$
$$(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), x^\nu(\sigma') \right\} 0,$$
$$\left\{ p_\mu(\sigma), p_\nu(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ p_\mu(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}$$
$$(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), p_\nu(\sigma') \right\} = 0. \tag{46}$$

Finally we determine mixed Dirac brackets

$$\left\{ x^\mu(\sigma), \eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ x^\mu(\sigma), \Psi^A(\sigma_1) \right\}$$
$$\triangle_{AB}(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), \eta(\sigma') \right\} = 0,$$
$$\left\{ x^\mu(\sigma), p_\eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ x^\mu(\sigma), \Psi^A(\sigma_1) \right\}$$
$$\triangle_{AB}(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), p_\eta(\sigma') \right\} = 0,$$
$$\left\{ p_\mu(\sigma), \eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ p_\mu(\sigma), \Psi^A(\sigma_1) \right\}$$
$$\triangle_{AB}(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), \eta(\sigma') \right\} = 0,$$
$$\left\{ p_\mu(\sigma), p_\eta(\sigma') \right\}_D = - \int d\sigma_1 d\sigma_2 \left\{ p_\mu(\sigma), \Psi^A(\sigma_1) \right\}$$
$$\triangle_{AB}(\sigma_1, \sigma_2) \left\{ \Psi^B(\sigma_2), p_\eta(\sigma') \right\} = 0. \tag{47}$$

These results show that Dirac brackets between p_μ, x^μ, p_η, η have the same form as Poisson brackets. In the next section we consider situation when $\theta_1^0 = 0$.

3 Singular case

Canonical analysis performed in previous section was valid on condition that $\theta_1^0 \neq 0$ or equivalently $e_1^0 \neq 0$. However spatial gauge that was imposed in [5,6] is valid on condition when $e_1^0 = 0$. In other words this gauge fixing cannot be reached in previous analysis and deserves separate treatment. We call this case as singular since, as we will see below, it will not be possible to express time derivative of x^μ as function of canonical variables.

To see this explicitly we start with the action (1) from which we determine following conjugate momenta

$$\pi^\alpha_b = \frac{\partial \mathcal{L}}{\partial \partial_0 e^a_\alpha} \approx 0 , \quad p_\psi \approx 0 , \quad , \pi_\omega \approx 0 , \quad p_\eta = Tm_1 ,$$

$$p_\mu = \frac{\partial \mathcal{L}}{\partial \partial_0 x^\mu} = -Tm_\mu \partial_1 \eta + \frac{T}{2} \psi e^1_1 \tau_\mu$$

(48)

that implies an existence of primary constraints

$$\Sigma_1 \equiv p_\eta - Tm_1 , \quad \Sigma^2_\mu \equiv p_\mu + Tm_\mu \partial_1 \eta - \frac{T}{2} \psi e^1_1 \tau_\mu \approx 0 .$$

(49)

For further purposes we introduce following linear combination of constraints that we denote as $\mathcal{H}_2$:

$$\mathcal{H}_2 \equiv \partial_1 x^\mu \Sigma^2_\mu + \Sigma_1 \partial_1 \eta = p_\mu \partial_1 x^\mu + p_\eta \partial_1 \eta - \frac{T}{2} \psi e^1_1 \tau_1 \approx 0$$

(50)

that will be useful below.

As the next step we determine bare Hamiltonian density in the form

$$\mathcal{H}_B = p_\mu \partial_0 x^\mu + p_\eta \partial_0 \eta - \mathcal{L} = \frac{T}{2} e \theta^1_1 \theta^1_1 h_{11}$$

$$+ \frac{T}{2} \omega e^0_0 \tau_1 + \frac{T}{2} \psi (e^0_0 \partial_1 \eta + e^1_0 \tau_1) .$$

(51)

Let us now proceed to the analysis of preservation of primary constraints. We introduce extended Hamiltonian as

$$H_E = \int d\sigma \left(\mathcal{H}_B + \Omega^1 \Sigma_1 + \Omega^\mu_2 \Sigma^2_\mu + v^\psi p_\psi \right.$$

$$\left. + v^\omega p_\omega + \Omega^a_\alpha \pi^\alpha_a \right).$$

(52)

We observe that we can always write $\Omega^1 = \tilde{\Omega}^1 \partial_1 \eta$ so that when we use (50) we can express $\partial_1 \eta \Sigma_1$ with the help of $\mathcal{H}_2$ and hence extended Hamiltonian density $\mathcal{H}_E$ can be written in the form

$$\mathcal{H}_E = T \frac{e^0_0 e^0_0}{2e} h_{11} + \frac{T}{2} \omega e^0_0 \tau_1 + \frac{T}{2} \psi (e^0_0 \partial_1 \eta + e^1_0 \tau_1)$$

$$+ v^\psi p_\psi + v^\omega p_\omega + \tilde{\Omega}^1 \mathcal{H}_2 + \tilde{\Omega}^\mu_2 \Sigma^2_\mu + \Omega^a_\alpha \pi^\alpha_a , \quad (53)$$

where we introduced $\tilde{\Omega}^\mu_2$ as $\tilde{\Omega}^\mu_2 = \Omega^\mu_2 - \tilde{\Omega} \partial_1 x^\mu$. Then in what follows we will omit tilde on $\Omega's$.

Now we are ready to analyse requirement of the preservation of all constraints. In case of p_ω we get

$$\partial_0 p_\omega = \{p_\omega, H_E\} = -\frac{T}{2} e^0_0 \tau_1 \equiv -\frac{T}{2} e^0_0 \Sigma^{II}_\omega \approx 0 , \quad (54)$$

where $\Sigma^{II}_\omega = \tau_1 \approx 0$ is new secondary constraint. Generally this constraint would imply $\partial_1 x^\mu = 0$ however this is very strong condition. We should rather presume that the background has non-zero component τ_0 only so that this constraint is equal to $\Sigma^{II}_\omega \equiv \partial_1 x^0 \approx 0$. As a consequence $\mathcal{H}_2$ is

standard spatial diffeomorphism constraint which is the first class constraint.

Now using the fact that $\tau_0 \neq 0 , \tau_i = 0$ we have

$$\Sigma^2_0 = p_0 + Tm_0 \partial_1 \eta - \frac{T}{2} \psi e^1_1 \tau_0 , \quad \Sigma^2_i = p_i + Tm_i \partial_1 \eta .$$

(55)

For further purposes we calculate Poisson brackets between primary constraints

$$\left\{ p_\psi(\sigma), \Sigma^2_0(\sigma') \right\} = \frac{T}{2} e^1_1 \tau_0 \delta(\sigma - \sigma') ,$$

$$\left\{ \Sigma^2_i(\sigma), \Sigma^2_j(\sigma') \right\} = -T(\partial_i m_j - \partial_j m_i) \partial_1 \eta \delta(\sigma - \sigma')$$

$$\equiv -\mathcal{F}_{ij} \delta(\sigma - \sigma') . \quad (56)$$

If we calculate Poisson bracket between Σ^2_0 and Σ^2_i we find that it is non-zero and we denote its value to be equal to $-\mathcal{F}_{i0}$. Explicitly we have

$$\left\{ \Sigma^2_i(\sigma), \Sigma^2_0(\sigma') \right\} = -\mathcal{F}_{i0} \delta(\sigma - \sigma') . \quad (57)$$

Let us now study the requirement of the preservation of constraint p_ψ

$$\partial_0 p_\psi = \{p_\psi, H_E\} = -\frac{T}{2} e^0_0 \partial_1 \eta + \Omega^0_2 \frac{T}{2} e^1_1 \tau_0 = 0 \quad (58)$$

that has solution

$$\Omega^0_2 = \frac{e^0_0}{\tau_0 e^1_1} \partial_1 \eta . \quad (59)$$

In other words, $\Sigma^2_0 \approx 0, p_\psi \approx 0$ are second class constraints that can be explicitly solved for p_ψ and ψ. We return to this problem below. Instead we focus on the time evolution of constraint $\Sigma^2_0 \approx 0$ that has the form

$$\partial_0 \Sigma^2_0 = \left\{ \Sigma^2_0, H_E \right\} = \int d\sigma \left(\left\{ \Sigma^2_0, \mathcal{H}_E \right\} \right.$$

$$\left. - \frac{T}{2} e^1_1 \tau_0 \delta(\sigma - \sigma') v_\psi + \mathcal{F}_{0i} \Omega^i_2 \right) = 0 \quad (60)$$

which can be solved for v_ψ. Finally, the requirement of the preservation of constraints $\Sigma^2_i \approx 0$ has the form

$$\partial_0 \Sigma^2_i = \left\{ \Sigma^2_i, H_E \right\} = \int d\sigma' (\left\{ \Sigma^2_i, \mathcal{H}_B(\sigma') \right\}$$

$$- \mathcal{F}_{i0} \delta(\sigma - \sigma') \Omega^2_0$$

$$+ \mathcal{F}_{ij} \delta(\sigma - \sigma') \Omega^j_2) = 0 . \quad (61)$$

Since $\mathcal{F}_{ij}$ is non-singular by definition we can solve the equation above for Ω^2_i.

Let us analyse requirement of the preservation of constraints π^α_a. Following analysis presented in section (2) we replace π^1_1 with $\tilde{\pi}^1_1$ defined as

$$\tilde{\pi}^1_1 = \pi^1_1 - \frac{\psi}{e^1_1} p_\psi \quad (62)$$

that has vanishing Poisson bracket with $\Sigma_0^2 \approx 0$. Further, requirement of the preservation of π_0^0 has the form

$$\partial_0 \pi_0^0 = \left\{ \pi_0^0, H_E \right\} = -\frac{1}{e_1^{\,1}} \left[\frac{T}{2} h_{11} + \frac{1}{\tau_0} (p_0 - T m_0 \partial_1 \eta) \partial_1 \eta \right]$$
$$+ \frac{1}{e_1^{\,1} \tau_0} \Sigma_0^2 \approx 0 \qquad (63)$$

using the fact that

$$\frac{T}{2} \psi = \frac{1}{e_1^{\,1} \tau_0} (p_0 - T m_0 \partial_1 \eta - \Sigma_0^2) \qquad (64)$$

and also that e is equal to $e = \det e_\alpha^{\,a} = e_0^{\,0} e_1^{\,1}$. We see that in order to obey Eq. (63) we should introduce secondary constraint $\mathcal{H}_1$ defined as

$$\mathcal{H}_1 = \frac{T}{2} h_{11} + \frac{1}{\tau_0} (p_0 - T m_0 \partial_1 \eta) \partial_1 \eta \approx 0 . \qquad (65)$$

On the other hand requirement of the preservation of the constraint $\tilde{\pi}_1^{\,1} \approx 0$ gives

$$\partial_0 \tilde{\pi}_1^{\,1} = \left\{ \tilde{\pi}_1^{\,1}, H_E \right\} = \frac{e_0^{\,0}}{(e_1^{\,1})^2} \mathcal{H}_1 - \frac{1}{\tau_0 (e_1^{\,1})^2} \Sigma_0^2 \approx 0 . \quad (66)$$

Clearly

$$\left\{ \mathcal{H}_1(\sigma), \mathcal{H}_1(\sigma') \right\} = 0 , \quad \left\{ \mathcal{H}_1(\sigma), \mathcal{H}_2(\sigma') \right\} \approx 0 ,$$
$$\left\{ \mathcal{H}_2(\sigma), \mathcal{H}_2(\sigma') \right\} \approx 0 \qquad (67)$$

and hence they are the first class constraints reflecting invariance of the world-sheet theory under reparameterization.

3.1 Symplectic structure

In this section we study symplectic structure of the theory studied in previous section. For simplicity of our analysis we will consider partial fixed theory with fixed spatial diffeomorphism constraint $\mathcal{H}_2 \approx 0$. This can be done by introducing gauge fixing function

$$\mathcal{G} : \eta - \sigma \approx 0 \qquad (68)$$

Since $\left\{ \mathcal{G}(\sigma), \mathcal{H}_2(\sigma') \right\} = \delta(\sigma - \sigma')$, $\mathcal{H}_2$ and $\mathcal{G}$ are second class constraints that strongly vanish. From $\mathcal{H}_2 = 0$ we express p_η as

$$p_\eta = -p_\mu \partial_1 x^\mu . \qquad (69)$$

Further, as we argued in previous section, we have second class constraints $\Psi^A = (p_\psi, \Sigma_0^2, \Sigma_i^2)$ with following matrix of Poisson brackets

$$\left\{ \Psi^A(\sigma), \Psi^B(\sigma') \right\} = \begin{pmatrix} 0 & \frac{T}{2} e_1^{\,1} \tau_0 & 0 \\ -\frac{T}{2} e_1^{\,1} \tau_0 & 0 & \mathcal{F}_{0j} \\ 0 & -\mathcal{F}_{i0} & -\mathcal{F}_{ij} \end{pmatrix} \delta(\sigma - \sigma') .$$
$$(70)$$

For simplicity we will presume that $\mathcal{F}_{0j} = 0$. Then the matrix inverse to $\triangle^{AB}$ is equal to

$$\triangle_{AB} = \begin{pmatrix} 0 & -\frac{2}{T e_1^{\,1} \tau_0} & 0 \\ \frac{2}{T e_1^{\,1} \tau_0} & 0 & 0 \\ 0 & 0 & -\mathcal{F}^{ij} \end{pmatrix} , \qquad (71)$$

where $\mathcal{F}^{ij}$ is matrix inverse to $\mathcal{F}_{ij}$. Further, we have Poisson brackets

$$\left\{ x^i(\sigma), \Psi^A(\sigma') \right\} = (0, 0, \delta_j^i) \delta(\sigma - \sigma') \qquad (72)$$

and hence

$$\left\{ x^i(\sigma), x^j(\sigma') \right\}_D$$
$$= - \int d\sigma_1 d\sigma_2 \left\{ x^i(\sigma), \Psi^A(\sigma_1) \right\} \triangle_{AB}(\sigma_1, \sigma_2)$$
$$\left\{ \Psi^B(\sigma_2), x^j(\sigma') \right\} = -\mathcal{F}^{ij} \delta(\sigma - \sigma'). \qquad (73)$$

We see that there is non-trivial symplectic structure which is in agreement with the observation presented in [6]. Then the equation of motion for x^i have the form

$$\partial_0 x^i = \left\{ x^i, H \right\}_D = \mathcal{F}^{ik} \partial_1 [\lambda h_{kl} \partial_1 x^l]$$
$$- \lambda \mathcal{F}^{ik} \partial_k h_{mn} \partial_1 x^m \partial_1 x^n , \qquad (74)$$

where we used the fact that the Hamiltonian is equal to

$$\mathcal{H} = \lambda \mathcal{H}_1, \quad \mathcal{H}_1 = \frac{T}{2} h_{ij} \partial_1 x^i \partial_1 x^j + \frac{1}{\tau_0} (p_0 - T m_0) , \quad (75)$$

where λ is Lagrange multiplier and where m_0 and τ_0 do not depend on x^i.

To conclude, we derived symplectic structure for SMT string in the gauge when $e_1^{\,0} = 0$ and we showed that it is non-trivial and depend on the field m_μ.

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