Eur. Phys. J. C (2022) 82:1
https://doi.org/10.1140/epjc/s10052-021-09951-7

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

A new touch temperature of the event horizon and Rindler horizon in the Kinnersley spacetime

Jie Zhang[1,a], **Menquan Liu**[1,b], **Zhie Liu**[1], **Shuzheng Yang**[2]

[1] College of Physics and Electronic Engineering, Qilu Normal University, 2 Wenbo Road, Jinan 250300, China
[2] College of Physics and Space Science, China West Normal University, 1 Shida Road, Nanchong 637000, China

Received: 7 August 2021 / Accepted: 19 December 2021 / Published online: 3 January 2022

Abstract The Kinnersley spacetime not only describes a non-spherical symmetric, non-stationary and accelerating black hole, but also can be used to explore the characteristics of collision of two black holes because it has two horizons: the Rindler horizon and the event horizon. Previous research shows Rindler horizon and the event horizon cannot touch due to violation of the third law of thermodynamics. By solving a fermion dynamical equation including the Lorentz dispersion relation, we obtain a modified radiation temperature at the event horizon of the black hole, as well as the colliding temperature at the touch point of Rindler horizon and the event horizon. We find the temperature at the touch point is not equal to zero if $\dot{r}_H \neq 0$. This result indicates that the event horizon and Rindler horizon can collide without violation of the third law of thermodynamics when Lorentz dispersion relation is considered.

1 Introduction

Dynamical black holes are those whose mass, charge, or angular momentum evolves with time. The real black holes are, by and large, not static, owing to processes such as mass accretion or Hawking radiation. As an extension of the Vaidya metric [1] , the Kinnersley spacetime [2] describes a dynamical black hole that accelerates in recoil. The system is anisotropic as massless radiation is emitted. The metric can be viewed as a particular case of the Kerr–Schild metric. The present study is focused on a modified Hawking radiation of the Kinnersley black hole which accelerates rectilinearly in a non-uniform fashion.

For a dynamical black hole, the first law of black hole thermodynamics is no longer valid. However, the event horizon in such a case remains to be a null hyper-surface with intrinsic symmetry of space-time. Therefore, one may derive the event horizon surface from null hyper-surface equation, and subsequently, investigate the radiation characteristics of the dynamical black holes. Besides the event horizon, the Kinnersley black hole has a Rindler horizon. There is no reason to assume that the horizons' temperature is a constant or the temperatures at two horizons are always the same. But we can get their touch temperature by researching their radiation at the horizons.

Since Hawking realized that, owning to quantum effect, black holes should emit particles with a thermal distribution of energies in 1974, a series of studies on Hawking radiation for various static and stationary black holes have been carried out [3–11]. Zhao et al. found a method to determine Hawking temperature at the event horizon of a dynamical black hole in 1991. The method consists of solving the Klein-Gordon equation for the scalar field, Dirac equation for fermions, and Rarita–Schwinger equation in the dynamical curved spacetime. The vital feature of the approach resides in the introduction of a dynamic tortoise coordinate transformation, and subsequently, one can proceed to evaluate the Hawking thermal radiation [12–18].

However, for all of the studies regarding Hawking radiation before 2000, one had ignored the energy loss of black holes and the contraction of the radius of the event horizon caused by particle radiation. Consequently, the resultant radiation spectrum is strictly thermal and possesses precisely the spectrum of blackbody radiation. Parikh and Wilczek pointed out that Hawking did not use the tunneling process or give a specific location of the potential barrier in his derivation [19]. Wilczek et al. further considered the energy conservation in the study of black hole tunneling radiation. They demonstrated that the mass of the black hole would decrease as the radiation takes place. As a result, it leads to the contraction of the black hole radius, which causes the appearance of the potential barrier. Subsequently, the radiation spectrum of the

[a] e-mail: zhangjie_mail@cqu.edu.cn (corresponding author)
[b] e-mail: menquan@mail.ustc.edu.cn (corresponding author)

black hole is no longer a strict blackbody spectrum. It can be shown that the result of this modification is consistent with the unitary of quantum theory. Therefore, it is helpful to solve the information loss problem [19–24]. Based on this quantum tunneling theory, a series of studies on black hole radiation has been carried out [25–29]. Moreover, the semi-classical Hamilton–Jacobi method was proposed to study the tunneling radiation of black holes [30–32]. In particular, Lin and Yang proposed a method to study the tunneling radiation of Dirac particles, which has been employed by other authors [33,34].

Recent development in theoretical physics and black hole physics indicates the possibility of the breaking of the Lorentz relation in the limit of high energy [35]. Although a general dispersion relation has not been established, it is understood that the magnitude of the correction term is of the Plank scale. This correction may lead to a sizable influence to the tunneling radiation of black hole [36–41]. Relevant study of fermion tunneling radiation with Lorentz dispersion relationship in static and stationary black holes has been carried out recently [42–44]. However, it is more interesting to further extend the research to a more realistic scenario, including the Lorentz dispersion relation into the treatment of Hawking radiation of arbitrary spin fermions in the context of a dynamical black hole.

So far, there is no strict analytical solution for describing black holes' collision. Kinnersley black hole provides a possibility for simulating the collision process of two black holes. Kinnersley black hole is a strict solution of the Einstein field equation. Although there is only one black hole in this spacetime, there is another horizon named Rindler horizon. So we can investigate the touch process of the two horizons to analyze the characteristics of collision of two black holes. We must point out that the real collision process of black holes is different from the contraction between Rinder horizon and the event horizon. The motion of two colliding black holes is usually along geodesics, the proper acceleration is zero. But the intrinsic acceleration of Kinnersley black hole is not zero. Nevertheless, our study is still valuable for exploring characteristics of the collision of black holes. This motivates the present study, which involves an attempt to investigate if the horizons' touch can take place, how high the collision temperature at the touch point is, by considering the Lorentz dispersion relation and modified tunneling radiation.

This paper is organized as follows. In the following section, based on the Lorentz dispersion relation, the dynamical equations, namely, the Dirac equation and Rarita–Schwinger equation, for fermions with arbitrary spin are derived in the Kinnersley spacetime. Section 3 is devoted to the study of the modified tunneling rate and the radiation temperature at the event horizon. The temperature at the touch point of the event horizon and Rindler horizon, and further discussions are given in the last section.

2 Accurate dynamical equation of arbitrary spin fermions in the Kinnersley black hole

The Rarita–Schwinger equation for fermions with arbitrarily spin in the curved spacetime is [33,43–47]

$$\left(\gamma^{\mu} D_{\mu} + \frac{m}{\hbar}\right) \psi_{\alpha_1 \ldots \alpha_k} = 0. \tag{1}$$

It satisfies with the following condition:

$$\gamma^{\mu} \psi_{\mu\alpha_2 \ldots \alpha_k} = D_{\mu} \psi^{\mu}_{\alpha_2 \ldots \alpha_k} = \psi^{\mu}_{\mu\alpha_3 \ldots \alpha_k}, \tag{2}$$

where γ^{μ} is the Gamma matrix in the curved spacetime, which satisfies the following condition:

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} I, \tag{3}$$

where I is the unit matrix. D_{μ} in Eq. (1) is defined as

$$D_{\mu} = \partial_{\mu} + \Omega_{\mu} + \frac{i}{\hbar} e A_{\mu}, \tag{4}$$

where Ω_{μ} is spin-connection in the curved spacetime. In the flat spacetime, $\Omega_{\mu} = 0$. In the research of string theory and quantum gravitational theory, authors proposed a dispersion relation:

$$P_0^2 = p^2 + m^2 - (L P_0)^{\alpha} p^2. \tag{5}$$

In the natural unit, P_0 and p are the energy and momentum of particle with static mass m. L is a constant in the magnitude of Plank scale. Adopting $\alpha = 2$, the modified Rarita–Schwinger equation can be rewritten as [41]

$$\left(\gamma^{\mu} D_{\mu} + \frac{m}{\hbar} - \lambda\hbar\gamma^{t} D_{t} \gamma^{j} D_{j}\right) \psi_{\alpha_1 \ldots \alpha_k} = 0, \tag{6}$$

where $\lambda = \mathrm{i}/\hbar L$, $\lambda \ll 1$, $\lambda\hbar\gamma^{t} D_{t} \gamma^{j} D_{j}$ is a very small term, $j = 1, 2, 3$. Eq. (6) is a new modified dynamical equation for arbitrary spin fermions. The solution of this matrix equation is dependent on the specific spacetime line element. In this paper, we will extend the solution of Eq. (6) to the rectilinearly nonuniformly accelerating Kinnersley spacetime. Line element of the Kinnersley black hole in the advanced Eddington coordinate can be expressed as [11,43]

$$\begin{aligned} ds^2 &= (1 - 2ar\cos\theta - r^2 f^2 - 2Mr^{-1})\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r \\ &\quad + 2r^2 f \mathrm{d}v\mathrm{d}\theta - r^2\mathrm{d}\theta^2 - r^2 \sin^2\theta \mathrm{d}^2\varphi, \end{aligned} \tag{7}$$

where $f = -a(v)\sin\theta$, $a(v)$ is the acceleration, v is the Eddington time, θ and φ are spherical coordinate. Polar point

$\theta = 0$ is the direction of acceleration. The corresponding metric determinant is

$$g = -r^4 \sin^2 \theta. \tag{8}$$

The nonzero components of the inverse metric tensor are shown as

$$\begin{aligned} g^{01} &= g^{10} = -1, \\ g^{11} &= -(1 - 2ar\cos\theta - 2Mr^{-1}), \\ g^{21} &= g^{12} = -f, \\ g^{22} &= -r^{-2}, \\ g^{33} &= -r^{-2}\sin^{-2}\theta. \end{aligned} \tag{9}$$

The characteristic of this element line is the component of inverse metric tense $g^{00} = 0$. For fermions with arbitrarily spin, the wave function is

$$\psi_{\alpha_1 \ldots \alpha_k} = \eta_{\alpha_1 \ldots \alpha_k} e^{\frac{i}{\hbar} S}, \tag{10}$$

where $\eta_{\alpha_1 \ldots \alpha_k}$ and S are matrices and the action of the fermion, respectively. Substituting Eqs. (10) and (4) (where $A_\mu = (0, 0, 0, 0)$, $\Omega_\mu = 0$ for Kinnersley black hole) into Eq. (6), we get

$$[(i\gamma^\mu + \lambda \partial_v S \gamma^v \gamma^\mu)\partial_\mu S - \lambda \gamma^v \gamma^v (\partial_v S)^2 + m]\eta_{\alpha_1 \ldots \alpha_k} = 0. \tag{11}$$

Here t in Eq. (6) has been substituted by v. Due to $\gamma^v \gamma^v = g^{vv} = 0$ for the Kinnersley black hole, so Eq. (11) can reduce to

$$[(i\gamma^\mu + \lambda \partial_v S \gamma^v \gamma^\mu)\partial_\mu S + m]\eta_{\alpha_1 \ldots \alpha_k} = 0. \tag{12}$$

This is a matrix equation, from it we obtain two equivalent equations.

$$\begin{aligned} &[(i\gamma^v + \lambda \partial_v S \gamma^v \gamma^v)(i\gamma^\mu + \lambda \partial_v S \gamma^v \gamma^\mu)\partial_\mu S \partial_v S - m^2] \\ &\eta_{\alpha_1 \ldots \alpha_k} = 0, \\ &[(i\gamma^\mu + \lambda \partial_v S \gamma^v \gamma^\mu)(i\gamma^v + \lambda \partial_v S \gamma^v \gamma^v)\partial_\mu S \partial_v S - m^2] \\ &\eta_{\alpha_1 \ldots \alpha_k} = 0. \end{aligned} \tag{13}$$

From Eq. (13) and condition (3) we get

$$\begin{aligned} &[g^{\mu v}\partial_\mu S \partial_v S - 2i\lambda \partial_v S g^{vj}\partial_j S \gamma^\mu \partial_\mu S \\ &\quad - \lambda^2 (\partial_v S)^2 g^{v\mu}\partial_\mu S g^{vv}\partial_v S + m^2]\eta_{\alpha_1 \ldots \alpha_k} = 0, \end{aligned} \tag{14}$$

i.e.,

$$\begin{aligned} &[g^{\mu v}\partial_\mu S \partial_v S - 2i\lambda \partial_v S g^{vj}\partial_j S \gamma^\mu \partial_\mu S \\ &\quad + m^2 - \lambda^2 (\partial_v S)^2 (g^{vj}\partial_j S)^2]\eta_{\alpha_1 \ldots \alpha_k} = 0. \end{aligned} \tag{15}$$

Generally, $j = 1, 2, 3$, but from Eq. (9) one can find $j = 1$ in Eqs. (14)–(15) for the Kinnersley black hole, so Eq. (15) can be rewritten as

$$(-i\lambda \gamma^\mu \partial_\mu S + m_k)\eta_{\alpha_1 \ldots \alpha_k} = 0, \tag{16}$$

where

$$m_k = \frac{g^{\mu v}\partial_\mu S \partial_v S + m^2 - \lambda^2 (\partial_v S)^2 (g^{vj}\partial_j S)^2}{2\partial_v S g^{vj}\partial_j S}. \tag{17}$$

Multiplying $i\lambda\gamma^v \partial_v S$ on both sides of Eq. (16), we have

$$(\lambda^2 \gamma^\mu \gamma^v \partial_\mu S \partial_v S + m_k^2)\eta_{\alpha_1 \ldots \alpha_k} = 0. \tag{18}$$

From condition (3), Eq. (18) becomes

$$(\lambda^2 g^{\mu v}\partial_\mu S \partial_v S + m_k^2)\eta_{\alpha_1 \ldots \alpha_k} = 0. \tag{19}$$

Considering Hamilton–Jacobi equation, $g^{\mu v}\partial_\mu S \partial_v S + m^2 = 0$. And the condition that the matrix Eq. (19) has nontrivial solution is the value of the determinant corresponding to the matrix of equation (19) is zero, i. e.,

$$\left[\frac{g^{\mu v}\partial_\mu S \partial_v S + m^2 - \lambda^2 (\partial_v S)^2 (g^{vj}\partial_j S)^2}{2\partial_v S g^{vj}\partial_j S}\right]^2 - \lambda^2 m^2 = 0. \tag{20}$$

After simplification, it can be expressed as

$$g^{\mu v}\partial_\mu S \partial_v S + m^2 - 2\lambda m \partial_v S g^{vj}\partial_j S - \lambda^2 (\partial_v S)^2 (g^{vj}\partial_j S)^2 = 0. \tag{21}$$

This equation is derived from Eq. (12). Equation (12) and Eq. (21) are two equivalent equations considering the modified Lorentz dispersion relation. The particularity of Kinnersley black hole has been considered in the derivation process. This equation is actually a deformed Hamilton–Jacobi equation. In the derivation process, no approximation has been made. Therefore, this is an accurately modified dynamical equation of arbitrary spin fermions in the dynamic Kinnersey spacetime.

3 The correction to tunneling radiation of fermions in the Kinnersley spacetime

Substituting Eq. (9) into Eq. (21), the dynamical equation becomes

$$
\begin{aligned}
&-2\frac{\partial S}{\partial v}\frac{\partial S}{\partial r}-(1-2ar\cos\theta-2Mr^{-1})\left(\frac{\partial S}{\partial r}\right)^2-2f\frac{\partial S}{\partial r}\frac{\partial S}{\partial \theta}\\
&-\frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2-\frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2+m^2+2\lambda m\frac{\partial S}{\partial v}\frac{\partial S}{\partial r}\\
&-\lambda^2\left(\frac{\partial S}{\partial v}\right)^2\left(\frac{\partial S}{\partial r}\right)^2=0.
\end{aligned}
\tag{22}
$$

In order to solve this deformed Hamilton–Jacobi equation, we must make the following tortoise coordinate transformation:

$$
\begin{aligned}
&r_*=r+\frac{1}{2\kappa}\ln\frac{r-r_H(v,\theta)}{r_H(v_0,\theta_0)},\\
&v_*=v-v_0,\\
&\theta_*=\theta-\theta_0,
\end{aligned}
\tag{23}
$$

where r_H is the event horizon of black hole, v_0 and θ_0 are two constants corresponding to the time particle escapes from black hole, we can deduce the above equations to

$$
\begin{aligned}
&\frac{\partial}{\partial r}=\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\\
&\frac{\partial}{\partial v}=\frac{\partial}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\\
&\frac{\partial}{\partial \theta}=\frac{\partial}{\partial \theta_*}-\frac{r_H^{'}}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},
\end{aligned}
\tag{24}
$$

where κ usually denotes surface gravitation of the black hole, $\dot r_H=\partial_v r_H$, $r_H^{'}=\partial_\theta r_H$. According to null hyper-surface equation

$$
g^{\mu\nu}\frac{\partial F}{\partial x^\mu}\frac{\partial F}{\partial x^\nu}=0,
\tag{25}
$$

and substituting Eq. (9) into the above equation, the r_H equation is decided by

$$
2\dot r_H-(1-2ar_H\cos\theta-2Mr_H^{-1})+2fr_H^{'}-\left(\frac{r_H^{'}}{r_H}\right)^2=0.
\tag{26}
$$

Substituting Eq. (24) into Eq. (22), we have

$$
\begin{aligned}
&-2\left[\frac{\partial S}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\left[\frac{1+2\kappa(r-r_H)}{2\kappa(r-r_H)}\right]\frac{\partial S}{\partial r_*}\\
&-\left(1-2ar\cos\theta-2Mr^{-1}\right)\left[\frac{1+2\kappa(r-r_H)}{2\kappa(r-r_H)}\right]^2\left(\frac{\partial S}{\partial r_*}\right)^2\\
&-2f\left[\frac{1+2\kappa(r-r_H)}{2\kappa(r-r_H)}\right]\frac{\partial S}{\partial r_*}\left[\frac{\partial S}{\partial \theta_*}-\frac{r_H^{'}}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\\
&-\frac{1}{r^2}\left[\frac{\partial S}{\partial \theta_*}-\frac{r_H^{'}}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]^2-\frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2+m^2\\
&+2m\lambda\left[\frac{\partial S}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\\
&\times\left[\frac{1+2\kappa(r-r_H)}{2\kappa(r-r_H)}\right]\frac{\partial S}{\partial r_*}+o(\lambda^2)=0.
\end{aligned}
\tag{27}
$$

In order to simplify Eq. (27), when $r\to r_H$, $1+2\kappa(r-r_H)\to 1$, it yields

$$
\begin{aligned}
&\left(\frac{\partial S}{\partial r_*}\right)^2\left\{\frac{-2\dot r_H+(1-2ar\cos\theta-2Mr^{-1})[1+2\kappa(r-r_H)]}{2\kappa(r-r_H)}\right.\\
&\left.+\frac{-2fr_H^{'}+(r_H^{'}r^{-1})^2+2\lambda m\dot r_H}{2\kappa(r-r_H)}\right\}\\
&+\left(\frac{\partial S}{\partial r_*}\right)\left(2\frac{\partial S}{\partial v_*}+2fP_{\theta *}-2P_{\theta *}r_H^{'}r^{-2}-2m\lambda\frac{\partial S}{\partial v_*}\right)\\
&+2\kappa(r-r_H)\left[-m^2+\frac{1}{r^2}P_{\theta *}^2+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right]+o(\lambda^2)=0,
\end{aligned}
$$

or

$$
\begin{aligned}
&\left(\frac{\partial S}{\partial r_*}\right)^2\left\{\frac{-2\dot r_H+(1-2ar\cos\theta-2Mr^{-1})}{2\kappa(r-r_H)}\right.\\
&\left.+\frac{-2fr_H^{'}+(r_H^{'}r^{-1})^2+2\lambda m\dot r_H}{2\kappa(r-r_H)}\right\}\\
&+\left(\frac{\partial S}{\partial r_*}\right)\left(2\frac{\partial S}{\partial v_*}+2fP_{\theta *}-2P_{\theta *}r_H^{'}r^{-2}-2m\lambda\frac{\partial S}{\partial v_*}\right)=0,
\end{aligned}
\tag{28}
$$

where $P_{\theta *}=\frac{\partial S}{\partial \theta_*}$, κ is "surface gravitation of the black hole", which is related to the tortoise coordinate. The numerator in the coefficient of $(\frac{\partial S}{\partial r_*})^2$ is closely related to null hyper-surface equation (25). The coefficient at the event horizon, where $r\to r_H$, is an infinite limit of 0/0 type. Setting the coefficient of $(\frac{\partial S}{\partial r_*})^2$ is 1, we use the L'Hopital Law to get temperature parameter, i. e.,

$$\lim_{\substack{v\to v_0\\ r\to r_H\\ \theta\to\theta_0}}\left\{\frac{-2r^2\dot{r}_H+(r^2-2ar^3\cos\theta-2Mr)[1+2\kappa(r-r_H)]}{2r^2\kappa(r-r_H)}\right.$$
$$\left.+\frac{-2r^2fr_H^{'}+r_H^{'2}+2r^2\lambda m\dot{r}_H}{2r^2\kappa(r-r_H)}\right\}=1. \tag{29}$$

In Eq. (28), we can not separate the fermion action S into a sum of several parts, but we are sure that the particle energy is determined by the following formula

$$\frac{\partial S}{\partial v_*}=-\omega. \tag{30}$$

We are also sure that the action S can be expressed as $S=R(v_*,r_*,\theta_*)+n\varphi$. The component of generalized momentum in φ direction is a constant, $\frac{\partial S}{\partial\varphi}=n$. The component of generalized momentum in θ direction can be denoted as P_θ. Substituting Eqs. (29) and (30) into Eq. (28), we get

$$\left(\frac{\partial S}{\partial r_*}\right)^2-[2(1-\lambda m)\omega-2\omega_0]\left(\frac{\partial S}{\partial r_*}\right)=0,$$
i.e., (31)
$$\left(\frac{\partial S}{\partial r_*}\right)^2-2(1-\lambda m)\left(\omega-\omega_0^{'})(\frac{\partial S}{\partial r_*}\right)=0,$$

where

$$\omega_0=P_\theta(f-r_H^{'}r_H^2), \tag{32}$$
$$\omega_0^{'}=\frac{\omega_0}{1-\lambda m}=\omega_0(1+\lambda_0+\lambda_0^2+\cdots). \tag{33}$$

where $\lambda_0=\lambda m, \lambda_0\ll 1$. Eq. (31) is the equation satisfied by fermion action S at the event horizon of Kinnersley black hole. Solving Eq. (31) and using tortoise coordinate transformation Eq. (24), we get

$$\frac{\partial S}{\partial r}=\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}|_{r\to r_H}\frac{\partial S}{\partial r_*}$$
$$=\frac{1}{2\kappa(r-r_H)}[2(1-\lambda m)(\omega-\omega_0^{'})]. \tag{34}$$

Using residue theorem to solve S in the Eq. (34), it is easy to get

$$S=\frac{\mathrm{i}\pi}{\kappa}(1-\lambda m)(\omega-\omega_0^{'}). \tag{35}$$

According to the tunneling theory, we get the tunneling rate of fermions with arbitrarily spin in the rectilinearly non-uniformly accelerating Kinnersley spacetime is: [19]

$$\Gamma=\exp(-2\mathrm{Im}S)$$
$$=\exp(-\frac{\omega-\omega_0^{'}}{T_H^L}), \tag{36}$$

where radiation temperature

$$T_H^L=\frac{\kappa}{2\pi(1-\lambda m)}. \tag{37}$$

Solving Eq. (29), we get temperature parameter κ,

$$\kappa=\frac{r_H-M-3r_H^2a\cos\theta-2fr_Hr_H^{'}+2(\lambda m-1)\dot{r}_Hr_H}{2Mr_H+2ar_H^3\cos\theta}. \tag{38}$$

Note that the κ here is not surface gravitation because we are discussing a dynamical black hole. Only in the stationary space-time, κ is equal to surface gravitation. Hence the radiation temperature at the event horizon is

$$T_H^L=\frac{r_H-M-3r_H^2a\cos\theta-2fr_Hr_H^{'}+2(\lambda m-1)\dot{r}_Hr_H}{2\pi(1-\lambda m)(2Mr_H+2ar_H^3\cos\theta)}$$
$$\approx\frac{r_H-M-3r_H^2a\cos\theta-2fr_Hr_H^{'}-2\dot{r}_Hr_H}{2\pi(2Mr_H+2ar_H^3\cos\theta)}$$
$$\times(1+\lambda_0+\lambda_0^2+\cdots)$$
$$=T_0(1+\lambda_0+\lambda_0^2+\cdots), \tag{39}$$

where T_0 is the Hawking temperature at the event horizon of Kinnersley black hole before our correction. $\lambda_0=\lambda m$. Obviously, it can be seen from Eqs. (36)–(39) that the quantum tunneling rate of arbitrary spin fermions and Hawking temperature at the event horizon of Kinnersley black hole are not only related to the acceleration $a(v)$ of black hole and the azimuth θ, but also related to the change rate of the event horizon $\dot{r}_H$ and $r_H^{'}$. If $\lambda=0$ in Eqs. (38)–(39), it reduces to the classic Hawking tunneling temperature of the black hole without correction. When a=0, the metric (7) is reduced to Vaidy metric [1]. From Eqs. (37)–(38), we have

$$\kappa=\frac{1-4\dot{M}}{4M}$$
$$T_H=\frac{1-4\dot{M}}{8\pi M}. \tag{40}$$

These are just the well-known surface gravitation and Hawking temperature of the Vaidya black hole. When $\dot{M}=0$, these results can be reduced to the case of Schwarzschild black hole.

It should be further explained that Eq. (32) shows that the chemical potential is modified. It can also be seen from

Eq. (39) that the radiation temperature of the black hole change with the azimuth, i. e., the radiation temperature of each point on the black hole horizon is different. For the Kinnersley black hole with uniform acceleration and linear motion, $a(v)$ is a constant, but the corresponding expression is the same as Eq. (39).

4 The touch temperature of the event horizon and Rindler horizon

Equation (39) denotes the temperature at the event horizon of Kinnersley black hole. In the following, we will discuss a special time when the event horizon r_H is colliding Rindler horizon r_R. When θ and φ are constants in line element (7), radial light from the black hole satisfies

$$\mathrm{d}\upsilon(g_{00}\mathrm{d}\upsilon - 2\mathrm{d}r) = 0. \tag{41}$$

One of the solution of the above equation is

$$\dot{r} = \left[\frac{\mathrm{d}r}{\mathrm{d}\upsilon}\right]_{\theta,\varphi} = \frac{g_{00}}{2}. \tag{42}$$

This equation describes the null surface (horizon) generatrix, so it is also a horizon equation same to Eq. (26). Substituting g_{00} into Eq. (42), it becomes

$$2\dot{r}_H - (1 - 2ar_H\cos\theta - 2Mr_H^{-1} - r_H^2\sin^2\theta) = 0. \tag{43}$$

Comparing Eqs. (26) and (43), we have

$$r_H^{'} = ar_H^2\sin\theta. \tag{44}$$

Touch point of the event horizon and the Rindler horizon locates in the direction of $\theta = 0$, i. e., $\cos\theta = 1$, $\sin\theta = 0$. Hence $r_H^{'} = 0$ and Eq. (43) is simplified as

$$2\dot{r}_H - (1 - 2ar_H - 2Mr_H^{-1}) = 0. \tag{45}$$

When $\lambda = 0$, the temperature becomes

$$T_H = \frac{r_H - M - 3r_H^2 a - 2\dot{r}_H r_H}{2\pi(2Mr_H + 2ar_H^3)}. \tag{46}$$

The touch point of the event horizon and Rindler horizon satisfies a relationship [48]

$$a = \frac{M}{r_H^2} = \frac{M}{r_R^2}. \tag{47}$$

Using Eq. (47) into the numerator of Eq. (46),

$$T_H = \frac{r_H - 2ar_H^2 - 2M - 2\dot{r}_H r_H}{2\pi(2Mr_H + 2ar_H^3)}, \tag{48}$$

It can be found that the the numerator of Eq. (47) is equivalent to the left side of Eq. (45), so

$$T_H = 0. \tag{49}$$

According to the third law of thermodynamics, absolute zero of temperature is impossible. This means the event horizon and Rindler horizon cannot touch in the fiducial case. However, when Lorentz dispersion relation is valid, $\lambda \neq 0$, we get the temperature at the touch point

$$T_H^L = \frac{2\lambda m\dot{r}_H r_H}{2\pi(1-\lambda m)(2Mr_H + 2ar_H^3)}. \tag{50}$$

This means the event horizon and Rindler horizon can collide, and not offend against the third law of thermodynamics. It should be emphasized that this touch process is not the collision of black holes that are moving along the geodesic, but that one black hole hangs over another black hole under the action of external force, and then gradually increase the external force to gradually reduce the suspension height until they contact each other. This is different from black hole collision, but it may include some characteristics of black hole collision.

The study of quantum field theory and quantum gravitation theory shows that Lorentz dispersion relation must be modified in the field of high energy. By using the modified Lorentz dispersion relation and the extended Rarita–Schwinger equation, the modified dynamical equation of arbitrary spin fermions is derived. This equation is an accurately modified fermion dynamical equation without approximation. In the actual calculation process, one can ignore the λ^2 term during the calculation of modified fermion tunneling rate, radiation temperature and other important physical quantities of black hole. We have demonstrated that the event horizon and Rindler horizon in the Kinnersley spacetime can touch without violation of the third law of thermodynamics. This conclusion can also be demonstrated by using Lorentz-violating scalar field theory [49]. We will continue this research in the future.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant U2031121), the Science Foundation of Sichuan Science and Technology Department (Grant 2018JY0502) and the Natural Science Foundation of Shandong Province (Grant ZR2019MA059, ZR2020MA063).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Our results come from the analytical solution of the equations, so there is no data.]

Funded by SCOAP[3].

References

1. R. Balbinot, Phys. Rev. D **33**, 1611 (1986)
2. W. Kinnersley, Phys. Rev. **186**, 1335 (1969)
3. S.W. Hawking, Nature **248**, 30 (1974)
4. S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975)
5. T. Damour, R. Ruffini, Phys. Rev. D **14**, 332 (1976)
6. S. Sannan, Gen. Relativ. Gravit. **20**, 239 (1988)
7. D.C. Robinson, Phys. Rev. Lett. **34**, 905 (1975)
8. G.W. Gibbons, M. Peny, J. Proc. R. Soc. Lond. A **358**, 467 (1978)
9. Z. Zhao, Y.X. Gui, Chin. Astron. Astrophys. **7**, 201 (1983)
10. W.G. Unruh, Phys. Rev. D **14**, 870 (1976)
11. J.B. Hartle, S.W. Hawking, Phys. Rev. D **13**, 2188 (1976)
12. Z. Zhao, X.X. Dai, Chin. Phys. Lett. **8**, 548 (1991)
13. Z. Zhao, J.Y. Zhu, Int. J. Theor. Phys. **33**, 2147 (1994)
14. Z. Zhao, Z.Q. Luo, C.G. Huang, Chin. Phys. Lett. **9**, 269 (1992)
15. Z.Z. Li, Z. Zhao, Chin. Phys. Lett. **11**(1), 8 (1994)
16. M.C. Sun, R. Zhao, Z. Zhao, IL Nuovo Cimento **110**, 829 (1995)
17. S.Z. Yang, J.Y. Zhu, Z. Zhao, Acta Phys. Sin. Overseas Ed. **4**, 147 (1995)
18. P. Kraus, F. Wilczek, Nucl. Phys. B **433**(2), 403 (1995)
19. M.K. Parikh, F. Wilczek, Phys. Rev. Lett. **85**(4), 5042 (2000)
20. S. Iso, H. Umetsu, F. Wilczek, Phys. Rev. D **74**, 044017 (2006)
21. A.J.M. Medved, Phys. Rev. D **66**, 124009 (2002)
22. M.K. Parikh, *The Marcel Grossmann Meeting* (World Scientific, Singapore, 2006), pp. 1585–1590
23. J.Y. Zhang, Z. Zhao, Nucl. Phys. B **725**, 173 (2005)
24. J.Y. Zhang, Z. Zhao, J. High Energy Phys. **10**, 055 (2005)
25. D.Y. Chen, S.Z. Yang, New J. Phys. **9**(8), 252 (2007)
26. S.Z. Yang, D.Y. Chen, Chin. Phys. B **17**(3), 817 (2008)
27. M.Q. Liu, S.Z. Yang, Int. J. Theor. Phys. **46**(1), 65 (2007)
28. S.Z. Yang, H.L. Li, Q.Q. Jiang, M.Q. Liu, Sci. Sin. Phys., Mech Astron. **37**(1), 66 (2007)
29. K. Lin, S.Z. Yang, Phys. Lett. B **674**(2), 127 (2009)
30. K. Srinivasan, T. Padmanabhan, Phys. Rev. D **60**(2), 24007 (1999)
31. R. Kerner, R.B. Mann, Phys. Lett. B **665**(4), 277 (2008)
32. R. Kerner, R.B. Mann, Class. Quantum Gravity **25**(9), 09501414 (2008)
33. K. Lin, S.Z. Yang, Phys. Rev. D **79**(6), 064035 (2009)
34. S.Z. Yang, K. Lin, Sci. Sin. Phys. Mech Astron. **40**(4), 507 (2010)
35. S.I. Kruglov, Phys. Lett. B **718**, 228 (2012)
36. S.I. Kruglov, Mod. Phys. Lett. A **28**, 1350014 (2013)
37. T. Jacobson, S. Liberati, D. Mattingly, A. Erratum, Nature **424**, 1019 (2003)
38. J.R. Ellis, N.E. Mavromatos, A.S. Sakharov, Astropart. Phys. **20**, 669 (2004)
39. J. Magueijo, L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002)
40. J. Magueijo, L. Smolin, Phys. Rev. D **67**, 044017 (2003)
41. G. Amelino-Camelia, D.V. Ahluwalia, Int. J. Mod. Phys. D **11**, 35 (2002)
42. S.Z. Yang, K. Lin, Sci. Sin. Phys. Mech Astron. **49**(1), 019503 (2019)
43. J. Zhang, Z.E. Liu, B. Sha et al., Adv. High Energy Phys., 2742091 (2020)
44. J. Zhang, Z.E. Liu, B. Sha et al., Commun. Theor. Phys. **72**, 045402 (2020)
45. W. Rarita, J. Schwinger, Phys. Rev. **60**, 61 (1941)
46. G.W. Gibbons, J. Phys. A Math. Gen. **9**, 145 (1976)
47. Z.W. Feng, Q.C. Ding, S.Z. Yang, Eur. Phys. J. C **79**, 445 (2019)
48. Z. Zhao, W.Q. Liu, Y.L. Jiang, Acta Phys. Sin. **49**, 586–591 (2000)
49. J. Zhang, M.Q. Liu, Z.E. Liu et al., Gen. Relativ. Gravit. **52**, 105 (2020)