



Angular distributions for multi-body semileptonic charmed baryon decays

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Abstract We perform an analysis of angular distributions in semileptonic decays of charmed baryons $B_1^{(\prime)} \rightarrow B_2^{(\prime)} (\rightarrow B_3^{(\prime)} B_4^{(\prime)}) \ell^+ \nu_\ell$, where the $B_1 = (\Lambda_c^+, \Xi_c^{(0,+)})$ are the SU(3)-antitriplet baryons and $B'_1 = \Omega_c^-$ is an SU(3) sextet. We will firstly derive analytic expressions for angular distributions using the helicity amplitude technique. Based on the lattice quantum chromodynamics (QCD) results for $\Lambda_c^+ \rightarrow \Lambda$ and $\Xi_c^0 \rightarrow \Xi^-$ form factors and model calculation of the $\Omega_c^0 \rightarrow \Omega^-$ transition, we predict the branching fractions: $\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda (\rightarrow p\pi^-) e^+ \nu_e) = 2.48(15)\%$, $\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda (\rightarrow p\pi^-) \mu^+ \nu_\mu) = 2.50(14)\%$, $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- (\rightarrow \Lambda\pi^-) e^+ \nu_e) = 2.40(30)\%$, $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- (\rightarrow \Lambda\pi^-) \mu^+ \nu_\mu) = 2.41(30)\%$, $\mathcal{B}(\Omega_c^0 \rightarrow \Omega^- (\rightarrow \Lambda K^-) e^+ \nu_e) = 0.362(14)\%$, $\mathcal{B}(\Omega_c^0 \rightarrow \Omega^- (\rightarrow \Lambda K^-) \mu^+ \nu_\mu) = 0.350(14)\%$. We also predict the q^2 dependence and angular distributions of these processes, in particular the coefficients for the $\cos n\theta_\ell$ ($\cos n\theta_h$, $\cos n\phi$) ($n = 0, 1, 2, \dots$) terms. This work can provide a theoretical basis for the ongoing experiments at BESIII, LHCb, and Belle-II.

1 Introduction

Weak decays of heavy baryons play an important role in testing the standard model (SM), in which any significant deviation from SM expectation would indirectly provide definitive clues for new physics beyond the SM. Most previous analysis has concentrated on the heavy meson sector, such as B and D meson decays [1–10], although heavy baryon decays have recently received increasing attention by both experimental [11,13–17] and theoretical researchers [18–24]. One of the most important motivations for studying the weak decays of heavy baryons is measuring the Cabibbo–Kobayashi–

Maskawa (CKM) matrix elements which describe the quark mixing and strength of CP violation. From this viewpoint, the study of semileptonic decays of charmed baryons, which can provide an ideal way to determine the $|V_{cd}|$ and $|V_{cs}|$, and examine the CKM unitarity $\sum_{i=d,s,b} |V_{ci}|^2 = 1$, is of great value. In the singly charmed baryons with $q_1 q_2 c$, the two light quarks can be decomposed as an antitriplet and a sextet. Focusing on the ground states with $J^P = 1/2^+$, only four baryonic states, $(\Lambda_c^+, \Xi_c^0, \Xi_c^+)$, and Ω_c^0 baryons, can have measurable weak decays, while others such as Ξ'_c and Σ'_c have strong and electromagnetic decay modes [25–29].

Among various decay modes, semileptonic decays are the simplest [31–33], and in recent years, charmed baryon decays have attracted great interest from both theoretical and experimental researchers [34–45]. Semileptonic decays of Λ_c^+ have been fruitfully studied in quark models and QCD sum rules [19,46–60], and predictions for branching fractions differ substantially. A precise measurement of branching fractions of Λ_c weak decays was recently reported by the BESIII collaboration: $\mathcal{B}(\Lambda_c \rightarrow \Lambda e^+ \nu_e) = 0.0363$ and $\mathcal{B}(\Lambda_c \rightarrow \Lambda \mu^+ \nu_\mu) = 0.0363$ [37,38]. For Ξ_c , the CLEO collaboration has measured the ratio of branching fractions $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ [61,62], and the Belle collaboration recently reported [13]: $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 1.31(04 \pm 07 \pm 38)\%$, $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 1.27(06 \pm 10 \pm 37)\%$. On the theoretical side, a variety of models have been developed to analyze Ξ_c weak decays [63–67], including a recent analysis of $\Xi_c \rightarrow \Xi$ transition form factors from lattice QCD [68]. Limited by low production rates and high background levels of current experiments, measurements of Ω_c decay branching ratios are not available. In theory, branching fractions of Ω_c weak decays are predicted in the light-front quark model: $\mathcal{B}(\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e) = 5.4(\pm 0.2) \times 10^{-3}$ [69]. In this work, we will explore decay widths for semileptonic decays of charmed baryons with the

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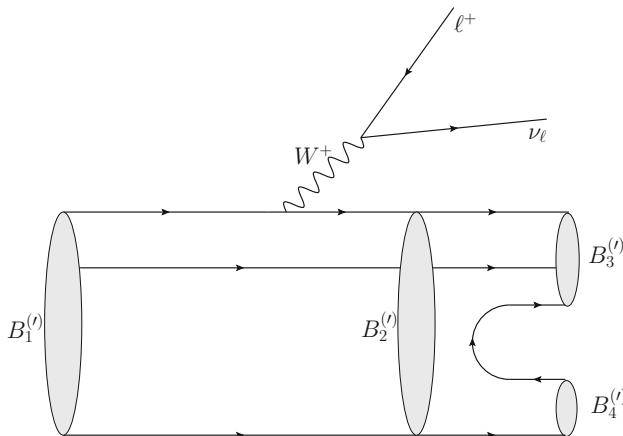


Fig. 1 Feynman diagram of charmed baryon weak decay of $B_1^{(\prime)} \rightarrow B_2^{(\prime)} (\rightarrow B_3^{(\prime)} B_4^{(\prime)}) \ell^+ \nu_\ell$

lattice QCD results for form factors, and in particular we derive, for the first time, the angular distributions for four-body weak decays of Ω_c . Feynman diagrams for these decay chains are shown in Fig. 1.

The rest of this paper is organized as follows. In Sect. 2, we present the theoretical framework for calculating the helicity amplitudes of charmed baryon decays, including the theoretical results of the Lorentz invariant leptonic and hadronic matrix elements. In Sect. 3, we list the differential decay widths of the three-body and four-body decay formulas. Integrating out the q^2 , we obtain numerical results of partial decay width, as well as the illustration of momentum transfer and angular distributions of the decay width. A brief summary will be presented in the last section.

2 Theoretical framework

2.1 Formalism

We will focus on semileptonic four-body decays of both SU(3) **3̄** and **6** ground states of singly charmed baryons, denoted as $B_1^{(\prime)} \rightarrow B_2^{(\prime)} (\rightarrow B_3^{(\prime)} B_4^{(\prime)}) \ell^+ \nu_\ell$, where $\ell = e, \mu$ and ν_ℓ are the charged and neutral leptons, respectively. For the antitriplet baryon $B_1 = (\Lambda_c^+, \Xi_c^0, \Xi_c^+)$ decay, the intermediate states B_2 are spin 1/2 baryon with an SU(3) octet [22, 70, 71], while the sextet $B'_1 = \Omega_c^0$ decays weakly via the spin-3/2 decuplet intermediate baryons $B'_2 (\Omega^-)$. $B_3^{(\prime)}$ and $B_4^{(\prime)}$ are the baryonic and mesonic final states, respectively. Examples of specific processes include:

$$\Lambda_c^+ \rightarrow \Lambda (\rightarrow p\pi^-) \ell^+ \nu_\ell, \quad (1)$$

$$\Xi_c^0 \rightarrow \Xi^- (\rightarrow \Lambda\pi^-) \ell^+ \nu_\ell, \quad (2)$$

$$\Omega_c^0 \rightarrow \Omega^- (\rightarrow \Lambda K^-) \ell^+ \nu_\ell. \quad (3)$$

The effective weak Hamiltonian for the semileptonic decays of charmed baryons can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} \left(\bar{s} \gamma^\mu (1 - \gamma_5) c \right) \left(\bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell \right), \quad (4)$$

where G_F is the Fermi constant, and V_{cs} is the CKM matrix element. Based on the above effective Hamiltonian, we can obtain the decay amplitudes of $B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs} \left\langle B_3^{(\prime)} B_4^{(\prime)} \left| \bar{s} \gamma^\mu (1 - \gamma_5) c \right| B_1^{(\prime)} \right\rangle \times \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \ell | 0 \rangle. \quad (5)$$

With the decomposition of $g_{\mu\nu}$

$$g_{\mu\nu} = - \sum_\lambda \epsilon_\mu^*(\lambda) \epsilon_\nu(\lambda) + \frac{q_\mu q_\nu}{q^2}, \quad (6)$$

the above amplitude can be decomposed into the Lorentz invariant hadronic and leptonic matrix elements:

$$\begin{aligned} \mathcal{M} &= \frac{G_F}{\sqrt{2}} V_{cs} \left\langle B_3^{(\prime)} B_4^{(\prime)} \left| \bar{s} \gamma^\mu (1 - \gamma_5) c \right| B_1^{(\prime)} \right\rangle \\ &\times \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \ell | 0 \rangle g_{\mu\nu} = \frac{G_F}{\sqrt{2}} V_{cs} \\ &\times \left(- \sum_\lambda H^\mu \epsilon_\mu^*(\lambda) \times L^\nu \epsilon_\nu(\lambda) + H^\mu \epsilon_\mu^*(t) \times L^\nu \epsilon_\nu(t) \right), \end{aligned} \quad (7)$$

where the hadronic part $H^\mu \equiv \left\langle B_3^{(\prime)} B_4^{(\prime)} \left| \bar{s} \gamma^\mu (1 - \gamma_5) c \right| B_1^{(\prime)} \right\rangle$ and leptonic part $L^\nu \equiv \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \rangle$, and ϵ is the polarization vector of the decomposed W^+ boson with helicity state λ and t , in which $\epsilon_\mu(t) \equiv q_\mu / \sqrt{q^2}$.

Focusing on the hadronic part and inserting the one-particle completeness states of the intermediate baryon

$$\begin{aligned} 1 &= \sum_{s_2} \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - m_2^2 + im_2 \Gamma_2} \\ &\times \left\langle B_2^{(\prime)}(p_2, s_2) \right\rangle \left\langle B_2^{(\prime)}(p_2, s_2) \right|, \end{aligned} \quad (8)$$

we find the H^μ given as:

$$\begin{aligned} H^\mu &= \sum_{s_2} \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - m_2^2 + im_2 \Gamma_2} \\ &\times \left\langle B_3^{(\prime)}(p_3, s_3) B_4^{(\prime)}(p_4) \left| B_2^{(\prime)}(p_2, s_2) \right. \right\rangle \\ &\times \left\langle B_2^{(\prime)}(p_2, s_2) \left| \bar{s} \gamma^\mu (1 - \gamma_5) c \right| B_1^{(\prime)}(p_1, s_1) \right\rangle. \end{aligned} \quad (9)$$

In the above, p_2 , m_2 , and Γ_2 are the four-momentum, mass, and decay rate of intermediate state $B_2^{(\prime)}$, respectively

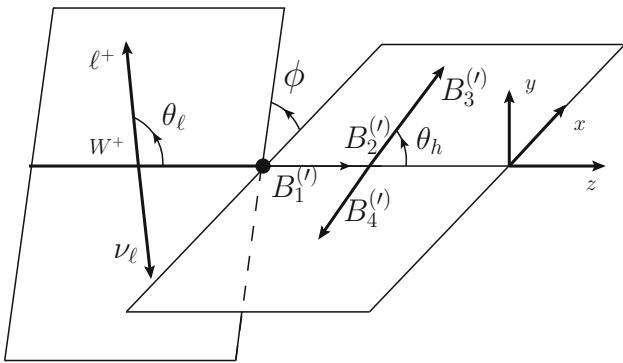


Fig. 2 Illustration of four-body decay process $B_1^{(\prime)} \rightarrow B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell$

(Fig. 2). The $B_1^{(\prime)} \rightarrow B_2^{(\prime)}$ transition H_1 can be parameterized as hadronic form factors, while the $B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}$ transition, with one final-state $B_4^{(\prime)}$ as a pseudoscalar meson, can be reduced as

$$\begin{aligned} & \left\langle B_3^{(\prime)}(p_3, s_3) B_4^{(\prime)}(p_4) \left| B_2^{(\prime)}(p_2, s_2) \right\rangle_H \right. \\ &= \left\langle B_3^{(\prime)}(p_3, s_3) B_4^{(\prime)}(p_4) \left| -i \int d^4x \mathcal{H}_{\text{int}}(x) \right| B_2^{(\prime)}(p_2, s_2) \right\rangle_I \\ &= \left\langle B_3^{(\prime)}(p_3, s_3) B_4^{(\prime)}(p_4) \left| -i \right. \right. \\ & \quad \times \int d^4x \left(i g_{H_2} \bar{\psi}_{B_3}(x) \phi_{B_4}(x) (A - B \gamma_5) \psi_{B_2}(x) \right) \\ & \left. B_2^{(\prime)}(p_2, s_2) \right\rangle = (2\pi)^4 \delta^4(p_3 + p_4 - p_2) g_{H_2} \bar{u}_{B_3} \\ & \quad \times (p_3, s_3) (A - B \gamma_5) u_{B_2}(p_2, s_2), \end{aligned} \quad (10)$$

where the subscripts “H” and “I” denote a Heisenberg and interaction representation matrix element, respectively, g_{H_2} represents the coupling constant of hadronic vertex $B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}$, and factors A and B are constants weighting the contributions from scalar and pesudoscalar density operators [62].

Therefore, the decay amplitude in Eq. (5) can be expressed as a convolution of the Lorentz invariant leptonic part $L(s_\ell, s_\nu, s_W)$ and two hadronic parts, $H_1(s_1, s_2, s_W)$, $H_2(s_2, s_3)$:

$$\begin{aligned} & i\mathcal{M}\left(B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell\right) \\ &= \sum_{s_2} i\mathcal{M}\left(B_1^{(\prime)} \rightarrow B_2^{(\prime)} \ell^+ \nu_\ell\right) \\ & \quad \times i\mathcal{M}\left(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}\right) \times \frac{i}{p_2^2 - m_2^2 + im_2\Gamma_2} \\ &= \sum_{s_2} \left(i \frac{G_F}{\sqrt{2}} V_{cs} \sum_{s_W=\{\pm,0,t\}} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \right) \end{aligned}$$

$$\begin{aligned} & \times H_2(s_2, s_3) \times \frac{i}{p_2^2 - m_2^2 + im_2\Gamma_2} \\ &= -\frac{G_F V_{cs}}{\sqrt{2}} \frac{1}{p_2^2 - m_2^2 + im_2\Gamma_2} \\ & \times \sum_{s_2} \sum_{s_W=\{\pm,0,t\}} H_1(s_1, s_2, s_W) L(s_\ell, s_\nu, s_W) \times H_2(s_2, s_3), \end{aligned} \quad (11)$$

with

$$L(s_\ell, s_\nu, s_W) \equiv \bar{u}_\nu(p_\nu, s_\nu) \gamma^\nu (1 - \gamma_5) v_\ell(p_\ell, s_\ell) \varepsilon_\nu(s_W), \quad (12)$$

$$\begin{aligned} H_1(s_1, s_2, s_W) &\equiv \left\langle B_2^{(\prime)}(p_2, s_2) \left| \bar{s} \gamma^\mu (1 - \gamma_5) c \right. \right. \\ & \quad \times \left. \left| B_1^{(\prime)}(p_1, s_1) \right\rangle \epsilon_\mu^*(s_W), \right. \end{aligned} \quad (13)$$

$$H_2(s_2, s_3) \equiv g_{H_2} \bar{u}_{B_3}(p_3, s_3) (A - B \gamma_5) u_{B_2}(p_2, s_2). \quad (14)$$

Averaging the spin of the initial state and summing over the spins of final states, we obtain the squared amplitude as

$$\begin{aligned} & \frac{1}{2} \overline{\left| i\mathcal{M}\left(B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell\right) \right|^2} \\ &= \frac{G_F^2 |V_{cs}|^2}{2} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^2 \Gamma_2^2} \frac{1}{2} \sum_{s_1} \sum_{s_3, s_l, s_\nu} \\ & \quad \times \left| \sum_{s_2} \sum_{s_W=\{\pm,0,t\}} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \right. \\ & \quad \left. \times H_2(s_2, s_3) \right|^2. \end{aligned} \quad (15)$$

2.2 Kinematics

With the abbreviations

$$s_\pm = (m_1 \pm m_2)^2 - q^2, \quad \lambda(m_1, m_2, q) = s_+ s_-, \quad (16)$$

the momentum of the initial and final states in the subprocesses can be set as follows.

1. $B_1^{(\prime)} \rightarrow B_2^{(\prime)} W^+$ subprocess: in the rest frame of initial-state $B_1^{(\prime)}$, suppose that $B_2^{(\prime)}$ moves along the positive z -direction ($\theta = 0, \phi = 0$) and W^+ boson along the negative z -direction ($\theta = \pi, \phi = \pi$), so we have

$$\begin{aligned} p_1^\mu &= (m_1, 0, 0, 0), \quad p_2^\mu = (E_2, 0, 0, |\vec{p}_2|), \\ q^\mu &= (E_W, 0, 0, -|\vec{p}_2|), \end{aligned} \quad (17)$$

with

$$\begin{aligned} E_2 &= \frac{m_1^2 + m_2^2 - q^2}{2m_1}, \\ E_W &= \frac{m_1^2 - m_2^2 + q^2}{2m_1}, \\ |\vec{p}_2| &= \frac{\sqrt{\lambda(m_1, m_2, q)}}{2m_1}. \end{aligned} \quad (18)$$

2. $B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}$ subprocess: in the rest frame of $B_2^{(\prime)}$, suppose the final-state baryon $B_3^{(\prime)}$ moves along the $(\theta = \theta_h, \phi = 0)$ direction, and therefore the meson $B_4^{(\prime)}$ moves along the $(\theta = \pi - \theta_h, \phi = \pi)$ direction. We thus have

$$\begin{aligned} p_2^\mu &= (m_2, 0, 0, 0), \\ p_3^\mu &= (E_3, |\vec{p}_3| \sin \theta_h, 0, |\vec{p}_3| \cos \theta_h), \\ p_4^\mu &= (E_4, -|\vec{p}_3| \sin \theta_h, 0, -|\vec{p}_3| \cos \theta_h), \end{aligned} \quad (19)$$

with

$$\begin{aligned} E_3 &= \frac{m_2^2 + m_3^2 - m_4^2}{2m_2}, \quad E_4 = \frac{m_2^2 - m_3^2 + m_4^2}{2m_2}, \\ |\vec{p}_3| &= \frac{\sqrt{\lambda(m_2, m_3, m_4)}}{2m_2}. \end{aligned} \quad (20)$$

3. $W^+ \rightarrow \ell^+ \nu_\ell$ subprocess: in the rest frame of W^+ , suppose the charged lepton moves along the $(\theta = \theta_l, \phi = \phi)$ direction, and thereby the neutrino along the $(\theta = \pi - \theta_l, \phi = \pi + \phi)$ direction, we have

$$\begin{aligned} q^\mu &= \left(\sqrt{q^2}, 0, 0, 0 \right), \\ p_l^\mu &= (E_l, |\vec{p}_l| \sin \theta_l \cos \phi, |\vec{p}_l| \sin \theta_l \sin \phi, |\vec{p}_l| \cos \theta_l), \\ p_\nu^\mu &= (E_\nu, -|\vec{p}_l| \sin \theta_l \cos \phi, -|\vec{p}_l| \sin \theta_l \sin \phi, \\ &\quad -|\vec{p}_l| \cos \theta_l), \end{aligned} \quad (21)$$

with

$$E_l = \frac{q^2 + m_l^2}{2\sqrt{q^2}}, \quad E_\nu = \frac{q^2 - m_l^2}{2\sqrt{q^2}}, \quad |\vec{p}_l| = \frac{q^2 - m_l^2}{2\sqrt{q^2}}. \quad (22)$$

For the phase space of n -body decays, the four-body one can be generated recursively by the two-body subprocesses. The two-body phase space can be expressed as

$$d\Phi_2(p \rightarrow p_1 p_2) = \frac{1}{(2\pi)^5} \frac{|\vec{p}_1| d \cos \theta}{4\sqrt{\hat{s}}}, \quad (23)$$

where $\sqrt{\hat{s}} = \sqrt{p^2}$ is the center-of-mass energy, and θ is the angle between the two final states. Based on this, the three-

body phase space of $B_1^{(\prime)} \rightarrow B_2^{(\prime)} \ell^+ \nu_\ell$ can be written as

$$\begin{aligned} d\Phi_3(B_1^{(\prime)} \rightarrow B_2^{(\prime)} \ell^+ \nu_\ell) &= (2\pi)^3 dq^2 \times d\Phi_2(W^+ \rightarrow \ell^+ \nu_\ell) \\ &\times d\Phi_2(B_1^{(\prime)} \rightarrow B_2^{(\prime)} W^+) \\ &= \frac{(1 - \hat{m}_l^2)\sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^7 32m_1^2} dq^2 d \cos \theta_l, \end{aligned} \quad (24)$$

where $\hat{m}_l = m_l/\sqrt{q^2}$. Similarly, the total four-body phase space is

$$\begin{aligned} d\Phi_4(B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell) &= (2\pi)^3 dp_2^2 \times d\Phi_3(B_1^{(\prime)} \rightarrow B_2^{(\prime)} \ell^+ \nu_\ell) \\ &\times d\Phi_2(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}) \\ &= \frac{(1 - \hat{m}_l^2)\sqrt{\lambda(m_1, m_2, q)}\lambda(m_2, m_3, m_4)}{(2\pi)^{10} 256m_1^2 m_2^2} \\ &\times dq^2 dp_2^2 d \cos \theta_h d \cos \theta_l d\phi. \end{aligned} \quad (25)$$

Note that the integration variable p_2^2 is artificially introduced from the insertion of intermediate state $B_2^{(\prime)}$, and in the narrow-width limit, this integration will be conducted as

$$\int dq_2^2 \frac{m_2 \Gamma_2}{\pi} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^2 \Gamma_2^2} = 1, \quad (26)$$

while

$$\begin{aligned} \int dq_2^2 \frac{m_2 \Gamma(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)})}{\pi} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^2 \Gamma_2^2} \\ = \mathcal{B}(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}), \end{aligned} \quad (27)$$

where $\Gamma(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)})$ and $\mathcal{B}(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)})$ are the total width and branching fraction of the subprocess $B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)}$, respectively.

Combining the four-body phase space in Eq. (25) and squared amplitude in Eq. (15), we can write the differential decay width of the four-body process $B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell$:

$$\begin{aligned} \frac{d\Gamma(B_1^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)} \ell^+ \nu_\ell)}{dq^2 d \cos \theta_h d \cos \theta_l d\phi} &= \frac{G_F^2 |V_{cs}|^2 (1 - \hat{m}_l^2) \sqrt{\lambda(m_1, m_2, q)} \lambda(m_2, m_3, m_4)}{(2\pi)^5 4096 m_1^3 m_2^3} \\ &\times \frac{\mathcal{B}(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)})}{\Gamma(B_2^{(\prime)} \rightarrow B_3^{(\prime)} B_4^{(\prime)})} \end{aligned}$$

$$\times \sum_{s_1} \sum_{s_3, s_l, s_v} \left| \sum_{s_2} \sum_{s_W = \{\pm, 0, t\}} H_1(s_1, s_2, s_W) \right|^2 \\ \times L(s_\ell, s_v, s_W) \times H_2(s_2, s_3) \right|. \quad (28)$$

2.3 Leptonic part

In this section, we will focus on the Lorentz invariant leptonic part defined in Eq. (12):

$$L(s_\ell, s_v, s_W) \equiv \bar{u}_v(p_v, s_v) \gamma^\mu (1 - \gamma_5) v_\ell(p_\ell, s_\ell) \epsilon_v(s_W). \quad (29)$$

Combining each component of spinors of leptons and polarization vectors of the W^+ boson, we can obtain the following nonvanishing matrix elements:

$$L \left(s_\ell = +\frac{1}{2}, s_v = -\frac{1}{2}, s_W = 0 \right) \\ = -\sqrt{2} \sin \theta_\ell N_\ell, \quad (30)$$

$$L \left(s_\ell = +\frac{1}{2}, s_v = -\frac{1}{2}, s_W = +1 \right) \\ = -e^{-i\phi} (1 - \cos \theta_\ell) N_\ell, \quad (31)$$

$$L \left(s_\ell = +\frac{1}{2}, s_v = -\frac{1}{2}, s_W = -1 \right) \\ = -e^{i\phi} (1 + \cos \theta_\ell) N_\ell, \quad (32)$$

$$L \left(s_\ell = -\frac{1}{2}, s_v = -\frac{1}{2}, s_W = 0 \right) \\ = -\sqrt{2} \hat{m}_\ell \cos \theta_\ell N_\ell, \quad (33)$$

$$L \left(s_\ell = -\frac{1}{2}, s_v = -\frac{1}{2}, s_W = +1 \right) \\ = -\hat{m}_\ell e^{-i\phi} \sin \theta_\ell N_\ell, \quad (34)$$

$$L \left(s_\ell = -\frac{1}{2}, s_v = -\frac{1}{2}, s_W = -1 \right) \\ = \hat{m}_\ell e^{i\phi} \sin \theta_\ell N_\ell, \quad (35)$$

$$L \left(s_\ell = -\frac{1}{2}, s_v = -\frac{1}{2}, s_W = t \right) \\ = \sqrt{2} \hat{m}_\ell N_\ell, \quad (36)$$

where the factor $N_\ell = i\sqrt{2(q^2 - m_l^2)}$ and $\hat{m}_l = m_l/\sqrt{q^2}$.

2.4 Hadronic part H_1 with spin-1/2 intermediate state

If B_1 is a singly charmed baryon antitriplet, the transition matrix elements with weak current $J^\mu = V^\mu - A^\mu =$

$\bar{s}\gamma^\mu(1 - \gamma_5)c$ in Eq. (13) can be parameterized as [57, 72]

$$\langle B_2(p_2, s_2) | V^\mu | B_1(p_1, s_1) \rangle = \bar{u}(p_2, s_2) \\ \times \left[\gamma^\mu f_1(q^2) + i\sigma^{\mu\nu} \frac{q_v}{m_1} f_2(q^2) + \frac{q^\mu}{m_1} f_3(q^2) \right] \\ u(p_1, s_1), \quad (37)$$

$$\times \langle B_2(p_2, s_2) | A^\mu | B_1(p_1, s_1) \rangle = \bar{u}(p_2, s_2) \\ \times \left[\gamma^\mu g_1(q^2) + i\sigma^{\mu\nu} \frac{q_v}{m_1} g_2(q^2) + \frac{q^\mu}{m_1} g_3(q^2) \right] \\ \gamma_5 u(p_1, s_1), \quad (38)$$

with the momentum transfer $q^\mu = p_1^\mu - p_2^\mu$ and $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. The form factors f_i and g_i are functions of q^2 , and the relations between these form factors and other parameterizations are collected in the Appendix.

By combining each spin component of B_1 , B_2 , and W^+ , we give the nonzero terms of $H_1(s_1, s_2, s_W)$ as

$$H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) \\ = H_{1V} \left(s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \\ = \sqrt{2s_-} \left[f_1(q^2) - \frac{m_1 + m_2}{m_1} f_2(q^2) \right], \quad (39)$$

$$H_{1V} \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right) \\ = H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \\ = \sqrt{\frac{s_-}{q^2}} \left[(m_1 + m_2) f_1(q^2) - \frac{q^2}{m_1} f_2(q^2) \right], \quad (40)$$

$$H_{1V} \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) \\ = H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \\ = \sqrt{\frac{s_+}{q^2}} \left[(m_1 - m_2) f_1(q^2) + \frac{q^2}{m_1} f_3(q^2) \right], \quad (41)$$

$$H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) \\ = -H_{1A} \left(s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \\ = \sqrt{2s_+} \left[g_1(q^2) + \frac{m_1 - m_2}{m_1} g_2(q^2) \right], \quad (42)$$

$$H_{1A} \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right) \\ = -H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \\ = \sqrt{\frac{s_+}{q^2}} \left[(m_1 - m_2) g_1(q^2) + \frac{q^2}{m_1} g_2(q^2) \right], \quad (43)$$

$$\begin{aligned} H_{1A} & \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) \\ & = -H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \\ & = \sqrt{\frac{s_-}{q^2}} \left[(m_1 + m_2) g_1(q^2) - \frac{q^2}{m_1} g_3(q^2) \right], \end{aligned} \quad (44)$$

in which the abbreviation s_{\pm} has been defined in Eq. (16).

Reversing the helicities gives the same results for the vector current, but an opposite sign exists for the axial vector current. The total amplitudes are

$$H_1(s_1, s_2, s_W) = H_{1V}(s_1, s_2, s_W) - H_{1A}(s_1, s_2, s_W). \quad (45)$$

2.5 Hadronic part H_1 with a spin-3/2 intermediate state

The charmed baryons $B'_1 = \Omega_c^0$ will weakly decay into spin-3/2 baryon decuplets Ω^- , and the transition matrix elements for this process can be parametrized as

$$\begin{aligned} \langle B'_2(p_2, s_2) | V^\mu | B'_1(p_1, s_1) \rangle \\ = \bar{U}_\alpha(p_2, s_2) \left[\gamma^\mu p_1^\alpha \frac{f_1(q^2)}{m_1} + \frac{f_2(q^2)}{m_1^2} p_1^\alpha p_1^\mu \right. \\ \left. + \frac{f_3(q^2)}{m_1 m_2} p_1^\alpha p_2^\mu + f_4(q^2) g^{\alpha\mu} \right] \gamma^5 u(p_1, s_1), \end{aligned} \quad (46)$$

$$\begin{aligned} \langle B'_2(p_2, s_2) | A^\mu | B'_1(p_1, s_1) \rangle \\ = \bar{U}_\alpha(p_2, s_2) \left[\gamma^\mu p_1^\alpha \frac{g_1(q^2)}{m_1} + \frac{g_2(q^2)}{m_1^2} p_1^\alpha p_1^\mu \right. \\ \left. + \frac{g_3(q^2)}{s_2 m_1 m_2} p_1^\alpha p_2^\mu + g_4(q^2) g^{\alpha\mu} \right] u(p_1, s_1). \end{aligned} \quad (47)$$

The definition of vectorial spinor $U_\alpha(\vec{p}, S_z)$ for the spin-3/2 baryon is shown in the Appendix. Therefore, the nonzero terms of $H_1(s_1, s_2, s_W)$ are collected as

$$\begin{aligned} H_{1V} & \left(s_1 = \frac{1}{2}, s_2 = \frac{3}{2}, s_W = 1 \right) \\ & = -H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{3}{2}, s_W = -1 \right) \\ & = \sqrt{s_-} f_4(q^2), \end{aligned} \quad (48)$$

$$\begin{aligned} H_{1V} & \left(s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) \\ & = -H_{1V} \left(s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \\ & = \sqrt{\frac{s_-}{3}} \left(\frac{s_+}{m_1 m_2} f_1(q^2) - f_4(q^2) \right), \end{aligned} \quad (49)$$

$$H_{1V} \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right)$$

$$\begin{aligned} & = -H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \\ & = \sqrt{\frac{s_-}{6q^2}} \left[\frac{(m_1 - m_2)s_+}{m_1 m_2} f_1(q^2) \right. \\ & \left. - \frac{\lambda(m_1, m_2, q)}{2m_1 m_2} \left(\frac{1}{m_1} f_2(q^2) + \frac{1}{m_2} f_3(q^2) \right) \right. \\ & \left. - \frac{m_1^2 - m_2^2 - q^2}{m_2} f_4(q^2) \right], \end{aligned} \quad (50)$$

$$\begin{aligned} H_{1V} & \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) \\ & = -H_{1V} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \\ & = \sqrt{\frac{s_+}{q^2}} \frac{s_-}{\sqrt{6}m_2} \left[\frac{m_1 + m_2}{m_1} f_1(q^2) \right. \\ & \left. - \frac{m_1^2 - m_2^2 + q^2}{2m_1^2} f_2(q^2) \right. \\ & \left. - \frac{m_1^2 - m_2^2 - q^2}{2m_1 m_2} f_3(q^2) - f_4(q^2) \right], \end{aligned} \quad (51)$$

$$\begin{aligned} H_{1A} & \left(s_1 = \frac{1}{2}, s_2 = \frac{3}{2}, s_W = 1 \right) \\ & = H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{3}{2}, s_W = -1 \right) \\ & = -\sqrt{s_+} g_4(q^2), \end{aligned} \quad (52)$$

$$\begin{aligned} H_{1A} & \left(s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) \\ & = H_{1A} \left(s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \\ & = \sqrt{\frac{s_+}{3}} \left(\frac{s_-}{m_1 m_2} g_1(q^2) - g_4(q^2) \right), \end{aligned} \quad (53)$$

$$\begin{aligned} H_{1A} & \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right) \\ & = H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \\ & = \sqrt{\frac{s_+}{6q^2}} \left[\frac{(m_1 + m_2)s_-}{m_1 m_2} g_1(q^2) \right. \\ & \left. + \frac{\lambda(m_1, m_2, q)}{2m_1 m_2} \left(\frac{1}{m_1} g_2(q^2) + \frac{1}{m_2} g_3(q^2) \right) \right. \\ & \left. + \frac{m_1^2 - m_2^2 - q^2}{m_2} g_4(q^2) \right], \end{aligned} \quad (54)$$

$$\begin{aligned} H_{1A} & \left(s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) \\ & = H_{1A} \left(s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \end{aligned}$$

$$= \sqrt{\frac{s_-}{q^2}} \frac{s_+}{\sqrt{6}m_2} \left[\frac{m_1 - m_2}{m_1} g_1(q^2) + \frac{m_1^2 - m_2^2 + q^2}{2m_1^2} g_2(q^2) + \frac{m_1^2 - m_2^2 - q^2}{2m_1 m_2} g_3(q^2) + g_4(q^2) \right], \quad (55)$$

and the total transition amplitudes give

$$H_1(s_1, s_2, s_W) = H_{1A}(s_1, s_2, s_W) - H_{1V}(s_1, s_2, s_W). \quad (56)$$

2.6 Hadronic part H_2 with spin-1/2 intermediate state

The amplitude for a spin-1/2 charm baryon B_2 decaying into a spin-1/2 baryon B_3 , and a spin-0 meson B_4 can be written in the form [29]

$$i\mathcal{M} = iG_F m_4^2 \bar{u}_{B_3}(p_3, s_3)(A - B\gamma_5)u_{B_2}(p_2, s_2), \quad (57)$$

which corresponds to the hadronic coupling constant $g_H = G_F m_4^2$ in Eq. (14) with low-energy effective theory. Here, G_F is a standard model electroweak coupling constant. A and B are factors weighting the contributions from scalar and pesudoscalar operators [30]. It is convenient to introduce the asymmetry parameter

$$\alpha = \frac{2 \operatorname{Re}[s^* r]}{|s|^2 + |r|^2}, \quad (58)$$

with $s = A$ and $r = B \times |\vec{p}_3|/(E_3 + m_3)$, and follow $(s \pm r)^2 = (|s|^2 + |r|^2)(1 \pm \alpha)$.

Therefore, combining each spin component of B_2 and B_3 , we get the following hadronic matrix elements $H_2(s_2, s_3)$:

$$H_2\left(s_2 = \frac{1}{2}, s_3 = \frac{1}{2}\right) = N_h(s+r) \cos(\theta_h/2), \quad (59)$$

$$H_2\left(s_2 = \frac{1}{2}, s_3 = -\frac{1}{2}\right) = -N_h(s-r) \sin(\theta_h/2), \quad (60)$$

$$H_2\left(s_2 = -\frac{1}{2}, s_3 = \frac{1}{2}\right) = N_h(s+r) \sin(\theta_h/2), \quad (61)$$

$$H_2\left(s_2 = -\frac{1}{2}, s_3 = -\frac{1}{2}\right) = N_h(s-r) \cos(\theta_h/2), \quad (62)$$

with $N_h = G_F m_\pi^2 \sqrt{2m_2(E_3 + m_3)}$. Using the two-body phase space and squared matrix elements, we obtain the spin-averaged decay width

$$\Gamma(B_2 \rightarrow B_3 B_4) = \int_{-1}^1 \frac{1}{2m_2} (2\pi)^4 \frac{1}{(2\pi)^5} \frac{|\vec{p}_3| d \cos \theta_h}{4m_2} \times \frac{1}{2} |\mathcal{M}(B_2 \rightarrow B_3 B_4)|^2$$

$$= \frac{N_h^2 \sqrt{\lambda(m_2, m_3, m_4)}}{16\pi m_2^3} (|s|^2 + |r|^2). \quad (63)$$

2.7 Hadronic part H_2 with spin-3/2 intermediate state

The transition for a spin-3/2 baryon Ω^- decaying into a spin-1/2 baryon Λ and a spin-0 pseudoscalar meson K^- is parametrized as [73]

$$i\mathcal{M} = i \frac{g_h}{m_4} \bar{u}_{B'_3}(p_3, s_3)(A - B\gamma_5)U_{B'_2}^\alpha(p_2, s_2)p_{4\alpha}, \quad (64)$$

where g_h denotes the coupling constant of the hadronic vertex $\Omega \Lambda K$. This nonperturbative parameter is usually determined from the experimental partial decay width of the relevant process, but here we can absorb it into the spin-averaged decay width in Eq. (73) to avoid the uncertainties and model dependence of nonperturbative methods.

Using the spinors and vectorial spinors, we can obtain the following terms:

$$H_2\left(s_2 = \frac{3}{2}, s_3 = \frac{1}{2}\right) = N'_h |\vec{p}_3| (s+r) \sin \theta_h \cos \frac{\theta_h}{2}, \quad (65)$$

$$H_2\left(s_2 = \frac{3}{2}, s_3 = -\frac{1}{2}\right) = -N'_h |\vec{p}_3| (s-r) \sin \theta_h \sin \frac{\theta_h}{2}, \quad (66)$$

$$H_2\left(s_2 = -\frac{3}{2}, s_3 = \frac{1}{2}\right) = -N'_h |\vec{p}_3| (s+r) \sin \theta_h \sin \frac{\theta_h}{2}, \quad (67)$$

$$H_2\left(s_2 = -\frac{3}{2}, s_3 = -\frac{1}{2}\right) = -N'_h |\vec{p}_3| (s-r) \sin \theta_h \cos \frac{\theta_h}{2}, \quad (68)$$

$$H_2\left(s_2 = \frac{1}{2}, s_3 = \frac{1}{2}\right) = \frac{\sqrt{3}}{3} N'_h |\vec{p}_3| (s+r) \cos \frac{\theta_h}{2} (1 - 3 \cos \theta_h), \quad (69)$$

$$H_2\left(s_2 = \frac{1}{2}, s_3 = -\frac{1}{2}\right) = \frac{\sqrt{3}}{3} N'_h |\vec{p}_3| (s-r) \sin \frac{\theta_h}{2} (1 + 3 \cos \theta_h), \quad (70)$$

$$H_2\left(s_2 = -\frac{1}{2}, s_3 = \frac{1}{2}\right) = -\frac{\sqrt{3}}{3} N'_h |\vec{p}_3| (s+r) \sin \frac{\theta_h}{2} (1 + 3 \cos \theta_h), \quad (71)$$

$$\begin{aligned} H_2 & \left(s_2 = -\frac{1}{2}, s_3 = -\frac{1}{2} \right) \\ & = \frac{\sqrt{3}}{3} N'_h |\vec{p}_3| (s - r) \cos \frac{\theta_h}{2} (1 - 3 \cos \theta_h), \end{aligned} \quad (72)$$

where $N'_h = g_h \sqrt{(E_3 + m_3)m_2}/m_4$. The spin-averaged decay width of the process is given as

$$\begin{aligned} \Gamma(B'_2 \rightarrow B'_3 B'_4) & = \int_{-1}^1 \frac{1}{2m_2} (2\pi)^4 \\ & \times \frac{1}{(2\pi)^5} \frac{|\vec{p}_3| d \cos \theta_h}{4m_2} \\ & \frac{1}{4} |\mathcal{M}(B'_2 \rightarrow B'_3 B'_4)|^2 \\ & = \frac{N'^2_h |\vec{p}_3|^3}{12\pi m_2^2} (|s|^2 + |r|^2). \end{aligned} \quad (73)$$

3 Theoretical results of differential decay width

3.1 Differential decay width of (Λ_c^+, Ξ_c^0)

Combining the leptonic matrix elements in Eqs. (30)–(36) and hadronic ones H_1 in Eqs. (39)–(45), together with the three-body phase space in Eq. (24), we can obtain the three-body decay width of $B_1 \rightarrow B_2 \ell^+ \nu_\ell$ processes, in which $B_1 = (\Lambda_c^+, \Xi_c^0)$ and corresponding $B_2 = (\Lambda, \Xi^-)$:

$$\begin{aligned} \frac{d\Gamma(B_1 \rightarrow B_2 \ell^+ \nu_\ell)}{d \cos \theta_\ell d q^2} & = \frac{q^2 \sqrt{\lambda(m_1, m_2, q)}}{1024\pi^3 m_1^3} \\ & \times (1 - \hat{m}_\ell)^2 \frac{G_F^2}{2} |V_{cs}|^2 \\ & \times \left\{ \left| H_{\frac{1}{2},1} \right|^2 [(1 - \cos \theta_\ell)^2 + \hat{m}_\ell^2 \sin^2 \theta_\ell] \right. \\ & + \left| H_{-\frac{1}{2},-1} \right|^2 [(1 + \cos \theta_\ell)^2 + \hat{m}_\ell^2 \sin^2 \theta_\ell] \\ & + \left(\left| H_{\frac{1}{2},0} \right|^2 + \left| H_{-\frac{1}{2},0} \right|^2 \right) 2 (\sin^2 \theta_\ell + \hat{m}_\ell \cos^2 \theta_\ell) \\ & + \left(\left| H_{\frac{1}{2},t} \right|^2 + \left| H_{-\frac{1}{2},t} \right|^2 \right) 2 \hat{m}_\ell^2 \\ & \left. + 2 \left(\left| H_{\frac{1}{2},0} H_{\frac{1}{2},t}^* \right| + \left| H_{-\frac{1}{2},0} H_{-\frac{1}{2},t}^* \right| \right) 2 \hat{m}_\ell^2 \cos \theta_\ell \right\}, \end{aligned} \quad (74)$$

in which H_{s_2, s_W} are short for the hadronic amplitudes $H(s_1, s_2, s_W)$ defined in Eqs. (39)–(44).

Additionally, taking the hadronic matrix elements H_2 from Eqs. (59)–(62) into account, we can obtain the four-body differential decay width for $B_1 \rightarrow B_3 B_4 \ell^+ \nu_\ell$ defined

in Eq. (28):

$$\begin{aligned} \frac{d\Gamma(B_1 \rightarrow B_3 B_4 \ell^+ \nu_\ell)}{d \cos \theta_\ell d \cos \theta_h d \phi d q^2} & = \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2) \sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^4 256 m_1^3} \mathcal{B}(B_2 \rightarrow B_3 B_4) \\ & \times \left\{ \left| H_{\frac{1}{2},1} \right|^2 (1 + \alpha \cos \theta_h) [\hat{m}_\ell^2 \sin^2 \theta_\ell + (1 - \cos \theta_\ell)^2] \right. \\ & + \left| H_{-\frac{1}{2},-1} \right|^2 (1 - \alpha \cos \theta_h) [\hat{m}_\ell^2 \sin^2 \theta_\ell + (1 + \cos \theta_\ell)^2] \\ & + 2 \left| H_{\frac{1}{2},0} \right|^2 (1 + \alpha \cos \theta_h) [\hat{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell] \\ & + 2 \left| H_{-\frac{1}{2},0} \right|^2 (1 - \alpha \cos \theta_h) [\hat{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell] \\ & + 2 \left| H_{\frac{1}{2},t} \right|^2 (1 + \alpha \cos \theta_h) \hat{m}_\ell^2 \\ & + 2 \left| H_{-\frac{1}{2},t} \right|^2 (1 - \alpha \cos \theta_h) \hat{m}_\ell^2 \\ & + 2\sqrt{2}\alpha \sin \theta_h \sin \theta_\ell \cos \phi \left| H_{\frac{1}{2},1} H_{-\frac{1}{2},0}^* \right| \\ & \times (1 - \cos \theta_\ell + \hat{m}_\ell^2 \cos \theta_\ell) \\ & + 2\sqrt{2}\alpha \sin \theta_h \sin \theta_\ell \cos \phi \left| H_{-\frac{1}{2},-1} H_{\frac{1}{2},0}^* \right| \\ & \times (1 + \cos \theta_\ell - \hat{m}_\ell^2 \cos \theta_\ell) \\ & + 2\sqrt{2}\alpha \sin \theta_h \sin \theta_\ell \cos \phi \left| H_{\frac{1}{2},1} H_{-\frac{1}{2},t}^* \right| \hat{m}_\ell^2 \\ & - 2\sqrt{2}\alpha \sin \theta_h \sin \theta_\ell \cos \phi \left| H_{-\frac{1}{2},-1} H_{\frac{1}{2},t}^* \right| \hat{m}_\ell^2 \\ & + 4\hat{m}_\ell^2 \cos \theta_\ell \left| H_{-\frac{1}{2},0} H_{-\frac{1}{2},t}^* \right| (1 - \alpha \cos \theta_h) \\ & \left. + 4\hat{m}_\ell^2 \cos \theta_\ell \left| H_{\frac{1}{2},0} H_{\frac{1}{2},t}^* \right| (1 + \alpha \cos \theta_h) \right\}. \end{aligned} \quad (75)$$

By integrating out the parameters step by step, we can get the results of q^2 -, θ_l -, θ_h - and ϕ -dependence forms of total decay width, respectively:

- q^2 -dependence:

$$\begin{aligned} \frac{d\Gamma}{dq^2} & = \frac{\pi G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2) \sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^4 48 m_1^3} \\ & \times \mathcal{B}(B_2 \rightarrow B_3 B_4) \\ & \times \left[\left(2 + \hat{m}_\ell^2 \right) \left(\left| H_{-\frac{1}{2},-1} \right|^2 + \left| H_{\frac{1}{2},1} \right|^2 + \left| H_{-\frac{1}{2},0} \right|^2 \right. \right. \\ & \left. \left. + \left| H_{\frac{1}{2},0} \right|^2 \right) + 3\hat{m}_\ell^2 \left(\left| H_{-\frac{1}{2},t} \right|^2 + \left| H_{\frac{1}{2},t} \right|^2 \right) \right], \end{aligned} \quad (76)$$

- θ_l -dependence:

$$\frac{d\Gamma}{d \cos \theta_\ell}$$

$$= \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2) \sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^4 256 m_1^3} \\ \times \mathcal{B}(B_2 \rightarrow B_3 B_4) (A_\ell + B_\ell \cos \theta_\ell + C_\ell \cos 2\theta_\ell), \quad (77)$$

with

$$A_\ell = 2\pi \left[(3 + \hat{m}_\ell^2) \left(|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \right) \right. \\ \left. + 2(1 + \hat{m}_\ell^2) \left(|H_{-\frac{1}{2}, 0}|^2 + |H_{\frac{1}{2}, 0}|^2 \right) \right. \\ \left. + 4\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, t}|^2 + |H_{\frac{1}{2}, t}|^2 \right) \right], \quad (78)$$

$$B_\ell = 8\pi \left[|H_{-\frac{1}{2}, -1}|^2 - |H_{\frac{1}{2}, 1}|^2 \right. \\ \left. + 2\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, 0} H_{-\frac{1}{2}, t}^*| + |H_{\frac{1}{2}, 0} H_{\frac{1}{2}, t}^*| \right) \right], \quad (79)$$

$$C_\ell = 2\pi (1 - \hat{m}_\ell^2) \left[|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \right. \\ \left. - 2 \left(|H_{-\frac{1}{2}, 0}|^2 + |H_{\frac{1}{2}, 0}|^2 \right) \right], \quad (80)$$

- θ_h -dependence:

$$\frac{d\Gamma}{d\cos\theta_h} \\ = \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2) \sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^4 256 m_1^3} \\ \times \mathcal{B}(B_2 \rightarrow B_3 B_4) (A_h + B_h \cos \theta_h), \quad (81)$$

with

$$A_h = \frac{8\pi}{3} \left[(2 + \hat{m}_\ell^2) \left(|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right. \right. \\ \left. \left. + |H_{\frac{1}{2}, 0}|^2 \right) + 3\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, t}|^2 + |H_{\frac{1}{2}, t}|^2 \right) \right], \quad (82)$$

$$B_h = \frac{8\pi}{3} \alpha \left[(2 + \hat{m}_\ell^2) \left(-|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \right. \right. \\ \left. \left. - |H_{-\frac{1}{2}, 0}|^2 + |H_{\frac{1}{2}, 0}|^2 \right) + 3\hat{m}_\ell^2 \left(-|H_{-\frac{1}{2}, t}|^2 + |H_{\frac{1}{2}, t}|^2 \right) \right], \quad (83)$$

- ϕ -dependence:

$$\frac{d\Gamma}{d\phi} = \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2) \sqrt{\lambda(m_1, m_2, q)}}{(2\pi)^4 256 m_1^3} \\ \times \mathcal{B}(B_2 \rightarrow B_3 B_4) (A_\phi + B_\phi \cos \phi \theta_h), \quad (84)$$

with

$$A_\phi = \frac{8}{3} \left[(2 + \hat{m}_\ell^2) \left(|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right. \right.$$

$$+ |H_{\frac{1}{2}, 0}|^2 \right) + 3\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, t}|^2 + |H_{\frac{1}{2}, t}|^2 \right) \right], \quad (85)$$

$$B_\phi = \frac{\pi^2}{\sqrt{2}} \alpha \left[|H_{\frac{1}{2}, 1} H_{-\frac{1}{2}, 0}^*| + |H_{-\frac{1}{2}, -1} H_{\frac{1}{2}, 0}^*| \right. \\ \left. + \hat{m}_\ell^2 \left(|H_{\frac{1}{2}, 1} H_{-\frac{1}{2}, t}^*| - |H_{-\frac{1}{2}, -1} H_{\frac{1}{2}, t}^*| \right) \right]. \quad (86)$$

3.2 Differential decay width of Ω_c^0

Using the leptonic matrix elements in Eqs. (30)–(36) and hadron ones H_1 in Eqs. (48)–(55), together with the three-body phase space in Eq. (24), we can obtain the three-body decay width from the spin-1/2 charmed baryon Ω_c^0 to spin-3/2 baryon Ω^- and leptons:

$$\frac{d\Gamma(\Omega_c^0 \rightarrow \Omega^- l^+ \nu_\ell)}{d\cos\theta_\ell dq^2} \\ = \frac{q^2 \sqrt{\lambda(m_{\Omega_c}, m_\Omega, q)}}{1024\pi^3 m_{\Omega_c}^3} (1 - \hat{m}_\ell^2)^2 \frac{G_F^2}{2} |V_{cs}|^2 \\ \times \left\{ \left(|H_{\frac{3}{2}, 1}|^2 + |H_{\frac{1}{2}, 1}|^2 \right) [(1 - \cos\theta_\ell)^2 + \hat{m}_\ell^2 \sin^2\theta_\ell] \right. \\ \left. + \left(|H_{-\frac{3}{2}, -1}|^2 + |H_{-\frac{1}{2}, -1}|^2 \right) [(1 + \cos\theta_\ell)^2 + \hat{m}_\ell^2 \sin^2\theta_\ell] \right. \\ \left. + \left(|H_{\frac{1}{2}, 0}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right) 2(\sin^2\theta_\ell + \hat{m}_\ell \cos^2\theta_\ell) \right. \\ \left. + \left(|H_{\frac{1}{2}, t}|^2 + |H_{-\frac{1}{2}, t}|^2 \right) 2\hat{m}_\ell^2 \right. \\ \left. + 2 \left(|H_{\frac{1}{2}, 0} H_{\frac{1}{2}, t}^*|^* + |H_{-\frac{1}{2}, 0} H_{-\frac{1}{2}, t}^*|^* \right) (2\hat{m}_\ell^2 \cos\theta_\ell) \right\}, \quad (87)$$

in which H_{s_2, s_W} are short for the hadronic parts in Eqs. (48)–(55).

Bringing the results of the leptonic part in Eqs. (30)–(36), hadronic part H_1 in Eqs. (48)–(55), and H_2 in Eqs. (65)–(72) together, the differential decay width of the four-body process $\Omega_c^0 \rightarrow \Lambda K^- \ell^+ \nu_\ell$ can be expressed as

$$\frac{d\Gamma(\Omega_c^0 \rightarrow \Lambda K^- \ell^+ \nu_\ell)}{dq^2 d\cos\theta_\ell d\cos\theta_t d\phi} \\ = \frac{3G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2)^2 \sqrt{\lambda(m_{\Omega_c}, m_\Omega, q)} \mathcal{B}(\Omega^- \rightarrow \Lambda K^-)}{512(2\pi)^4 m_{\Omega_c}^3} \\ \times \left\{ \left| H_{-\frac{3}{2}, -1} \right|^2 [\hat{m}_\ell^2 \sin^2\theta_\ell + (1 + \cos\theta_\ell)^2] \right. \\ \times \sin^2\theta_h (1 - \alpha \cos\theta_h) \\ \left. + \left| H_{\frac{3}{2}, 1} \right|^2 [\hat{m}_\ell^2 \sin^2\theta_\ell + (1 - \cos\theta_\ell)^2] \right. \\ \times \sin^2\theta_h (1 + \alpha \cos\theta_h) \\ \left. + \frac{1}{12} \left| H_{\frac{1}{2}, 1} \right|^2 [\hat{m}_\ell^2 \sin^2\theta_\ell + (1 - \cos\theta_\ell)^2] \right\}$$

$$\begin{aligned}
& \times [2(5 + 3 \cos 2\theta_h) + \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{12} |H_{-\frac{1}{2}, -1}|^2 [\hat{m}_\ell^2 \sin^2 \theta_\ell + (1 + \cos \theta_\ell)^2] \\
& \times [2(5 + 3 \cos 2\theta_h) - \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{6} |H_{-\frac{1}{2}, 0}|^2 [\hat{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell] \\
& \times [2(5 + 3 \cos 2\theta_h) - \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{6} |H_{\frac{1}{2}, 0}|^2 [\hat{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell] \\
& \times [2(5 + 3 \cos 2\theta_h) + \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{6} |H_{-\frac{1}{2}, t}|^2 \hat{m}_\ell^2 \\
& \times [2(5 + 3 \cos 2\theta_h) - \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{6} |H_{\frac{1}{2}, t}|^2 \hat{m}_\ell^2 \\
& \times [2(5 + 3 \cos 2\theta_h) + \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{3} |H_{-\frac{1}{2}, 0} H_{-\frac{1}{2}, t}^* \hat{m}_\ell^2 \cos \theta_\ell| \\
& \times [2(5 + 3 \cos 2\theta_h) - \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{1}{3} |H_{\frac{1}{2}, 0} H_{\frac{1}{2}, t}^* \hat{m}_\ell^2 \cos \theta_\ell| \\
& \times [2(5 + 3 \cos 2\theta_h) + \alpha(7 \cos \theta_h + 9 \cos 3\theta_h)] \\
& + \frac{2\sqrt{3}}{3} |H_{-\frac{3}{2}, -1} H_{\frac{1}{2}, 1}^* (\hat{m}_\ell^2 - 1)| \\
& \times \sin^2 \theta_\ell \cos 2\phi \sin^2 \theta_h (1 - 3\alpha \cos \theta_h) \\
& + \frac{2\sqrt{3}}{3} |H_{\frac{3}{2}, 1} H_{-\frac{1}{2}, -1}^* (\hat{m}_\ell^2 - 1)| \\
& \times \sin^2 \theta_\ell \cos 2\phi \sin^2 \theta_h (1 + 3\alpha \cos \theta_h) \\
& + \frac{\sqrt{6}}{6} |H_{-\frac{3}{2}, -1} H_{-\frac{1}{2}, 0}^*| \sin \theta_\ell \cos \phi \{(\sin \theta_h - 3 \sin 3\theta_h) \\
& \times [\hat{m}_\ell^2 \cos \theta_\ell + \alpha(1 + \cos \theta_\ell)] \\
& + 4 \sin 2\theta_h (\alpha \hat{m}_\ell^2 \cos \theta_\ell + 1 + \cos \theta_\ell)\} \\
& - \frac{\sqrt{6}}{6} |H_{\frac{3}{2}, 1} H_{\frac{1}{2}, 0}^*| [\hat{m}_\ell^2 \cos \theta_\ell + (1 - \cos \theta_\ell)] \\
& \times \sin \theta_\ell \cos \phi [4 \sin 2\theta_h - \alpha(\sin \theta_h - 3 \cos 3\theta_h)] \\
& + \frac{\sqrt{6}}{6} |H_{-\frac{3}{2}, -1} H_{-\frac{1}{2}, t}^*| \hat{m}_\ell^2 \\
& \times \sin \theta_\ell \cos \phi [(\sin \theta_h - 3 \sin 3\theta_h) + 4\alpha \sin 2\theta_h] \\
& - \frac{\sqrt{6}}{6} |H_{\frac{3}{2}, 1} H_{\frac{1}{2}, t}^*| \hat{m}_\ell^2 \\
& \times \sin \theta_\ell \cos \phi [4 \sin 2\theta_h - \alpha(\sin \theta_h - 3 \cos 3\theta_h)] \\
& - \frac{\sqrt{2}}{6} |H_{\frac{1}{2}, 1} H_{-\frac{1}{2}, 0}^*| [\cos \theta_\ell \hat{m}_\ell^2 - \alpha(1 - \cos \theta_\ell)] \\
& \times \sin \theta_\ell \cos \phi (5 \sin \theta_h + 9 \sin 3\theta_h) \\
& + \frac{\sqrt{2}}{6} |H_{-\frac{1}{2}, -1} H_{\frac{1}{2}, 0}^*| [(1 + \cos \theta_\ell) - \alpha \hat{m}_\ell^2 \cos \theta_\ell] \\
& \times \sin \theta_\ell \cos \phi (5 \sin \theta_h + 9 \sin 3\theta_h) \\
& - \frac{\sqrt{2}}{6} |H_{\frac{1}{2}, 1} H_{-\frac{1}{2}, t}^*| \hat{m}_\ell^2 \sin \theta_\ell \cos \phi
\end{aligned}$$

$$\begin{aligned}
& \times (5 \sin \theta_h + 9 \sin 3\theta_h) - \frac{\sqrt{2}}{6} |H_{-\frac{1}{2}, -1} H_{\frac{1}{2}, t}^*| \\
& \times \hat{m}_\ell^2 \sin \theta_\ell \cos \phi \alpha (5 \sin \theta_h + 9 \sin 3\theta_h) \Big\}. \quad (88)
\end{aligned}$$

By integrating out the parameters step by step, we can get the results of q^2 -, θ_l -, θ_h - and ϕ -dependence form of total decay width, respectively:

- q^2 -dependence:

$$\begin{aligned}
\frac{d\Gamma}{dq^2} = & \frac{32\pi}{9} \left[\left(2 + \hat{m}_\ell^2 \right) \left(|H_{-\frac{3}{2}, -1}|^2 + |H_{\frac{3}{2}, 1}|^2 \right. \right. \\
& + |H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, 0}|^2 + |H_{\frac{1}{2}, 0}|^2 \\
& \left. \left. + 3\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, t}|^2 + |H_{\frac{1}{2}, t}|^2 \right) \right) \right] \quad (89)
\end{aligned}$$

- θ_l -dependence:

$$\begin{aligned}
\frac{d\Gamma}{d \cos \theta_\ell} = & \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{3G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2)^2 \sqrt{\lambda(m_{\Omega_c}, m_\Omega, q)}}{512(2\pi)^4 m_{\Omega_c}^3} \\
& \times \mathcal{B}(B'_2 \rightarrow B'_3 B'_4) (A'_\ell + B'_\ell \cos \theta_\ell + C'_\ell \cos 2\theta_\ell), \quad (90)
\end{aligned}$$

with

$$\begin{aligned}
A'_\ell = & \frac{4\pi}{3} \left[\left(3 + \hat{m}_\ell^2 \right) \left(|H_{\frac{3}{2}, 1}|^2 \right. \right. \\
& + |H_{-\frac{3}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, -1}|^2 \\
& \left. \left. + 2(1 + \hat{m}_\ell^2) \left(|H_{\frac{1}{2}, 0}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right) \right. \right. \\
& + 4\hat{m}_\ell^2 \left(|H_{\frac{1}{2}, t}|^2 + |H_{-\frac{1}{2}, t}|^2 \right) \Big], \quad (91)
\end{aligned}$$

$$\begin{aligned}
B'_\ell = & \frac{16\pi}{3} \left[\left(|H_{-\frac{3}{2}, -1}|^2 - |H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{1}{2}, -1}|^2 - |H_{\frac{1}{2}, 1}|^2 \right) \right. \\
& \left. + 2\hat{m}_\ell^2 \left(|H_{-\frac{1}{2}, 0} H_{-\frac{1}{2}, t}^*| + |H_{\frac{1}{2}, 0} H_{\frac{1}{2}, t}^*| \right) \right], \quad (92)
\end{aligned}$$

$$\begin{aligned}
C'_\ell = & \frac{4\pi}{3} (1 - \hat{m}_\ell^2) \left[\left(|H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{3}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \right. \right. \\
& \left. \left. + |H_{-\frac{1}{2}, -1}|^2 \right) - 2 \left(|H_{\frac{1}{2}, 0}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right) \right], \quad (93)
\end{aligned}$$

- θ_h -dependence:

$$\begin{aligned}
\frac{d\Gamma}{d \cos \theta_h} = & \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{3G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2)^2 \sqrt{\lambda(m_{\Omega_c}, m_\Omega, q)}}{512(2\pi)^4 m_{\Omega_c}^3} \\
& \times \mathcal{B}(B'_2 \rightarrow B'_3 B'_4) \\
& \times (A'_h + B'_h \cos \theta_h + C'_h \cos 2\theta_h + D'_h \cos 3\theta_h), \quad (94)
\end{aligned}$$

with

$$\begin{aligned} A'_h &= \frac{4\pi}{9} \left\{ (2 + \hat{m}_\ell^2) \left[3 \left(|H_{\frac{3}{2},1}|^2 + |H_{-\frac{3}{2},-1}|^2 \right) \right. \right. \\ &\quad \left. \left. + 5 \left(|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right) \right] \right. \\ &\quad \left. + 15 \hat{m}_\ell^2 \left(|H_{\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2 \right) \right\}, \end{aligned} \quad (95)$$

$$\begin{aligned} B'_h &= \frac{2\pi\alpha}{9} \left\{ (2 + \hat{m}_\ell^2) \left[3 \left(|H_{\frac{3}{2},1}|^2 - |H_{-\frac{3}{2},-1}|^2 \right) \right. \right. \\ &\quad \left. \left. + 7 \left(|H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right) \right] \right. \\ &\quad \left. + 21 \hat{m}_\ell^2 \left(|H_{\frac{1}{2},t}|^2 - |H_{-\frac{1}{2},t}|^2 \right) \right\}, \end{aligned} \quad (96)$$

$$\begin{aligned} C'_h &= \frac{4\pi}{3} \left[(2 + \hat{m}_\ell^2) \left(-|H_{\frac{3}{2},1}|^2 - |H_{-\frac{3}{2},-1}|^2 \right. \right. \\ &\quad \left. \left. + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right) \right. \\ &\quad \left. + 3 \hat{m}_\ell^2 \left(|H_{\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2 \right) \right], \end{aligned} \quad (97)$$

$$\begin{aligned} D'_h &= \frac{2\pi\alpha}{3} \left\{ (2 + \hat{m}_\ell^2) \left[\left(|H_{-\frac{3}{2},-1}|^2 - |H_{\frac{3}{2},1}|^2 \right) \right. \right. \\ &\quad \left. \left. + 3 \left(|H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right) \right] \right. \\ &\quad \left. - 9 \hat{m}_\ell^2 \left(|H_{-\frac{1}{2},t}|^2 - |H_{\frac{1}{2},t}|^2 \right) \right\}, \end{aligned} \quad (98)$$

- ϕ -dependence:

$$\begin{aligned} \frac{d\Gamma}{d\phi} &= \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{3G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_\ell^2)^2 \sqrt{\lambda(m_{\Omega_c}, m_\Omega, q)}}{512(2\pi)^4 m_{\Omega_c}^3} \\ &\times \mathcal{B}(B'_2 \rightarrow B'_3 B'_4) (A'_\phi + B'_\phi \cos\phi + C'_\phi \cos 2\phi), \end{aligned} \quad (99)$$

with

$$\begin{aligned} A'_\phi &= \frac{16}{9} \left[(2 + \hat{m}_\ell^2) \left(|H_{-\frac{3}{2},-1}|^2 + |H_{\frac{3}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right. \right. \\ &\quad \left. \left. + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) \right. \\ &\quad \left. + 3 \hat{m}_\ell^2 \left(|H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right], \end{aligned} \quad (100)$$

$$\begin{aligned} B'_\phi &= \frac{\pi^2}{12\sqrt{2}} \left[\sqrt{3}\alpha \left(|H_{-\frac{3}{2},-1} H_{-\frac{1}{2},0}^*| + |H_{\frac{3}{2},1} H_{\frac{1}{2},0}^*| \right) \right. \\ &\quad \left. + \sqrt{3} \hat{m}_\ell^2 \left(|H_{-\frac{3}{2},-1} H_{-\frac{1}{2},t}^*| + \alpha |H_{\frac{3}{2},1} H_{\frac{1}{2},t}^*| \right) \right. \\ &\quad \left. + 5 \left(|H_{-\frac{1}{2},-1} H_{\frac{1}{2},0}^*| + \alpha |H_{\frac{1}{2},1} H_{-\frac{1}{2},0}^*| \right) \right. \\ &\quad \left. - 5 \hat{m}_\ell^2 \left(\alpha |H_{-\frac{1}{2},-1} H_{\frac{1}{2},t}^*| + |H_{\frac{1}{2},1} H_{-\frac{1}{2},t}^*| \right) \right], \end{aligned} \quad (101)$$

$$C'_\phi = -\frac{32(1 - \hat{m}_\ell)^2}{9\sqrt{3}} \left(|H_{-\frac{3}{2},-1} H_{\frac{1}{2},1}^*| + |H_{\frac{3}{2},1} H_{-\frac{1}{2},-1}^*| \right). \quad (102)$$

4 Numerical analysis

4.1 Input

In this section, all parameters used in the calculations will be collected, including the baryon masses and CKM matrix (Table 1). In addition, the lepton mass $m_e = 0.005$ GeV, $m_\mu = 0.1134$ GeV, and Fermi constant $G_F = 1.166 \times 10^{-5}$ GeV $^{-2}$. The CKM matrix element $V_{cs} = 0.973$. The decay widths of the baryons can be obtained from the reciprocal of lifetimes: $\Gamma(\text{MeV}) = 1/\tau \times 6.582 \times 10^{-22}$. In the calculation of the heavy baryon four-body decays, the asymmetry parameters and branch ratios are collected as [29]

$$\begin{aligned} \alpha(\Lambda^0 \rightarrow p\pi^-) &= 0.732, \quad \alpha(\Xi^- \rightarrow \Lambda^0\pi^-) \\ &= -0.401, \quad \alpha(\Omega^- \rightarrow \Lambda^0 K^-) = 0.0157, \\ \mathcal{B}(\Lambda^0 \rightarrow p\pi^-) &= 63.900\%, \quad \mathcal{B}(\Xi^- \rightarrow \Lambda^0\pi^-) \\ &= 99.887\%, \quad \mathcal{B}(\Omega^- \rightarrow \Lambda^0 K^-) = 67.800\%. \end{aligned} \quad (103)$$

4.2 Numerical results and discussions

In the hadronic part with a spin-1/2 intermediate state, we use the lattice QCD calculation of the $\Lambda_c^+ \rightarrow \Lambda$ [74] and $\Xi_c^0 \rightarrow \Xi^-$ [68] for the decay processes of $\Lambda_c^+ \rightarrow \Lambda\ell^+\nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^-\ell^+\nu_\ell$, respectively. In order to access the q^2 distribution, we use the modified z expansions in the physical limits [75], and the fit functions are shown as

$$f(q^2) = \frac{1}{1 - q^2/m_{\text{pole}}^2} \sum_{n=0}^{n_{\max}} a_n^f [z(q^2)]^n. \quad (104)$$

The expansion variable is defined as

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (105)$$

with $t_0 = q_{\max}^2 = (m_{\Xi_c} - m_\Xi)^2$ for $\Xi_c \rightarrow \Xi$, $t_0 = q_{\max}^2 = (m_{\Lambda_c} - m_\Lambda)^2$ for $\Lambda_c \rightarrow \Lambda$, and $t_+ = (m_D + m_K)^2$. The pole masses in the form factors are used as $m_{\text{pole}}^{f+, f_\perp} = 2.12$ GeV, $m_{\text{pole}}^{f_0} = 2.318$ GeV, $m_{\text{pole}}^{g+, g_\perp} = 2.460$ GeV, and $m_{\text{pole}}^{g_0} = 1.968$ GeV.

- Λ_c decays.

We collect the fitted form factor parameters from [74] in Table 2. The resulting SM predictions for the $\Lambda_c^+ \rightarrow \Lambda\ell^+\nu_\ell$ and $\Lambda_c^+ \rightarrow p\pi^-\ell^+\nu_\ell$ decay widths and branching fractions with corresponding error estimates are listed as

$$\begin{aligned} \Lambda_c^+ \rightarrow \Lambda e^+\nu_e : \quad \Gamma &= 1.262(75) \times 10^{-13} \text{s}^{-1}, \\ \mathcal{B} &= 3.88(23)\%, \end{aligned} \quad (106)$$

Table 1 The masses (m) and lifetimes (τ) of baryons [29]

Baryons	Λ_c^+	Ξ_c^0	Ω_c^0	Λ	Ξ^-	Ω^-
m (GeV)	2.2860	2.4709	2.6952	1.1156	1.3217	1.6724
τ (10^{-13} s)	2.024	1.530	2.680	2632	1639	821

Table 2 The $\Lambda_c \rightarrow \Lambda$ form factors calculated on the lattice [74]

	$a_0^{f_\perp}$	$a_1^{f_\perp}$	$a_2^{f_\perp}$	$a_0^{f_+}$	$a_1^{f_+}$	$a_2^{f_+}$	$a_0^{f_0}$	$a_1^{f_0}$	$a_2^{f_0}$
Nominal	1.30 ± 0.06	-3.27 ± 1.18	7.16 ± 11.6	0.81 ± 0.03	-2.89 ± 0.52	7.82 ± 4.53	0.77 ± 0.02	-2.24 ± 0.51	5.38 ± 4.80
	$a_0^{g_\perp}$	$a_1^{g_\perp}$	$a_2^{g_\perp}$	$a_0^{g_+}$	$a_1^{g_+}$	$a_2^{g_+}$	$a_0^{g_0}$	$a_1^{g_0}$	$a_2^{g_0}$
Nominal	0.68 ± 0.02	-1.19 ± 0.35	6.24 ± 4.89	0.68 ± 0.02	-2.44 ± 0.25	13.7 ± 2.15	0.71 ± 0.03	-2.86 ± 0.44	11.8 ± 2.47

$$\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu : \Gamma = 1.272(72) \times 10^{-13} s^{-1}, \\ \mathcal{B} = 3.91(22)\%, \quad (107)$$

$$\Lambda_c^+ \rightarrow p \pi^- e^+ \nu_e : \Gamma = 8.061(482) \times 10^{-14} s^{-1}, \\ \mathcal{B} = 2.48(15)\%, \quad (108)$$

$$\Lambda_c^+ \rightarrow p \pi^- \mu^+ \nu_\mu : \Gamma = 8.126(462) \times 10^{-14} s^{-1}, \\ \mathcal{B} = 2.50(14)\%. \quad (109)$$

The experimental measurements of the $\Lambda_c^+ \rightarrow \Lambda \ell^+ \nu_\ell$ branching fractions have been reported by the BESIII collaboration as $\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = 3.63(38)(20)$ and $\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu) = 3.49(46)(27)$ [37,38], which are highly consistent with our results in Eqs. (106)–(107). For the processes of four-body decays, based on the form factors, we predict the differential decay widths for $\Lambda_c^+ \rightarrow p \pi^- \ell^+ \nu_\ell$ as a function of q^2 in Fig. 3a. Note that the increasing errors in the small- q^2 region come from the uncertainties of form factors at large momentum transfer in the lattice calculations (Fig. 4). In Eqs. (76–84), we show the theoretical results of θ_l , θ_h - and ϕ -dependence of the total four-body decay width with different final-state leptons, in which the coefficients are functions of q^2 only. After we integrate out q^2 , the angular distributions with $\cos \theta_l$, $\cos \theta_h$, and ϕ are shown as

$$1. \quad \Lambda_c^+ \rightarrow p \pi^- e^+ \nu_e$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.4448(254) + 0.1992(172) \cos \theta_l \\ - 0.1657(204) \cos 2\theta_l, \quad (110)$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_h} = 0.5000(299) - 0.3188(210) \cos \theta_h, \quad (111)$$

$$\frac{d\Gamma}{\Gamma d \phi} = 0.1592(95) - 0.0276(35) \cos \theta_\phi. \quad (112)$$

$$2. \quad \Lambda_c^+ \rightarrow p \pi^- \mu^+ \nu_\mu$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.455(57) + 0.265(28) \cos \theta_l \\ - 0.135(24) \cos 2\theta_l, \quad (113)$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_h} = 0.500(63) + 0.162(19) \cos \theta_h, \quad (114)$$

$$\frac{d\Gamma}{\Gamma d \phi} = 0.159(20) + 0.012(3) \cos \theta_\phi. \quad (115)$$

- Ξ_c^0 decays

Based on the recently announced $\Xi_c \rightarrow \Xi$ form factors calculated by lattice QCD [68], where the z -expansion parameters of helicity-based form factors are collected in Table 3, we perform the predictions for $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Lambda \pi^- \ell^+ \nu_\ell$ decay widths and branching fractions with corresponding uncertainties

$$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e : \Gamma = 1.032(129) \times 10^{-13} s^{-1}, \\ \mathcal{B} = 2.40(30)\%, \quad (116)$$

$$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu : \Gamma = 1.038(131) \times 10^{-13} s^{-1}, \\ \mathcal{B} = 2.41(30)\%, \quad (117)$$

$$\Xi_c^0 \rightarrow \Lambda \pi^- e^+ \nu_e : \Gamma = 1.031(129) \times 10^{-13} s^{-1}, \\ \mathcal{B} = 2.40(30)\%, \quad (118)$$

$$\Xi_c^0 \rightarrow \Lambda \pi^- \mu^+ \nu_\nu : \Gamma = 1.037(131) \times 10^{-13} s^{-1}, \\ \mathcal{B} = 2.41(30)\%. \quad (119)$$

Very recently, the ALICE [12] and Belle [13] collaborations reported their measurements of branching fractions for $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$ respectively: $\mathcal{B}_{\text{ALICE}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 2.43(25)(35)(72)\%$, $\mathcal{B}_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 1.31(4)(7)(38)\%$ and $\mathcal{B}_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 1.27(6)(10)(37)\%$.

We also predict the differential decay widths for $\Xi_c^0 \rightarrow \Lambda \pi^- \ell^+ \nu_\ell$ as a function of q^2 in Fig. 5; the errors in plots mainly come from the uncertainties of form fac-

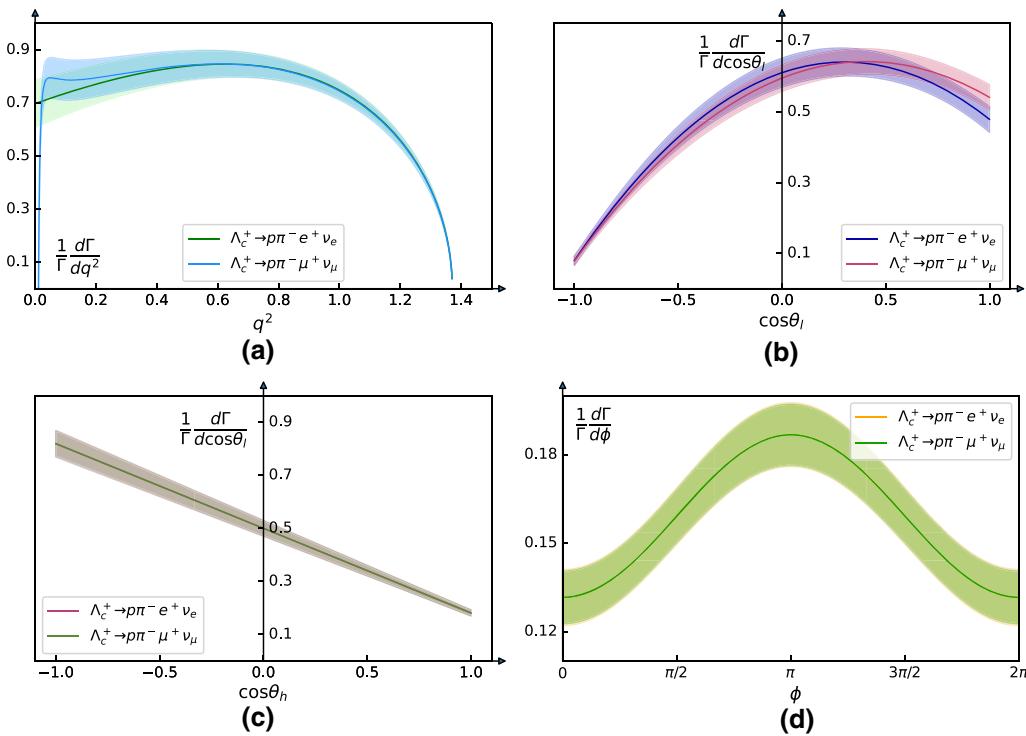


Fig. 3 Differential decay width for $\Lambda_c^+ \rightarrow p\pi^- \ell^+ \nu_\ell$ as a function of q^2 (a), $\cos \theta_l$ (b), $\cos \theta_h$ c, and ϕ d, respectively. It should be noted that in b and c, the errors increase with θ_l from -1 to 1 and θ_h from 1 to -1, as a result of the remarkable uncertainties of form factors at the small- q^2 region

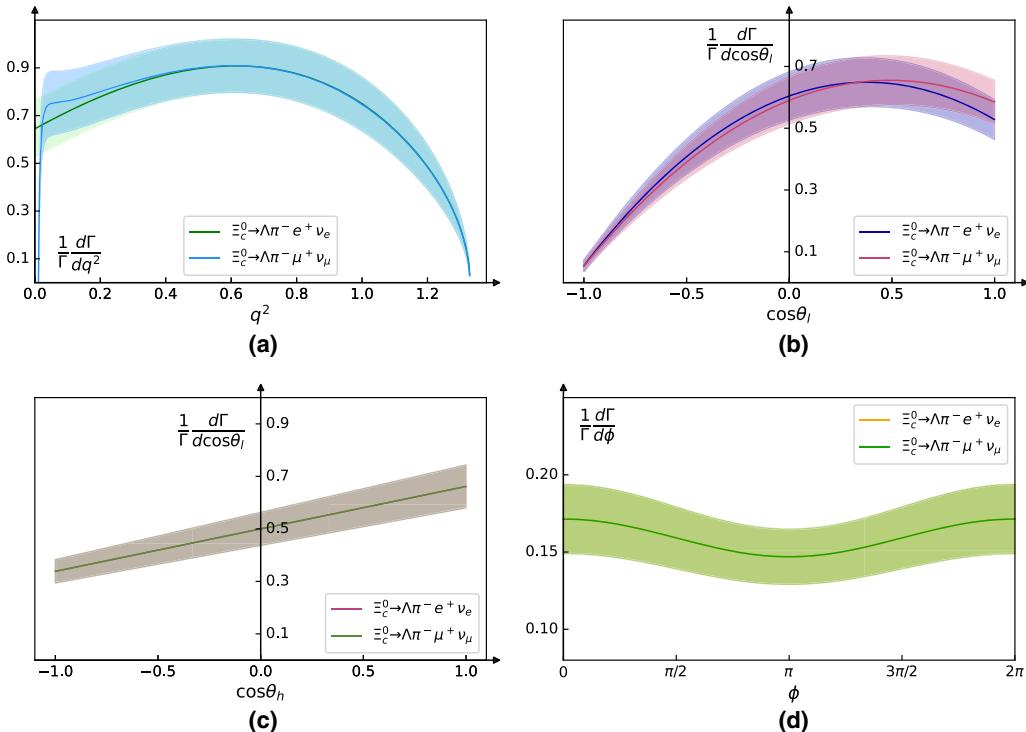
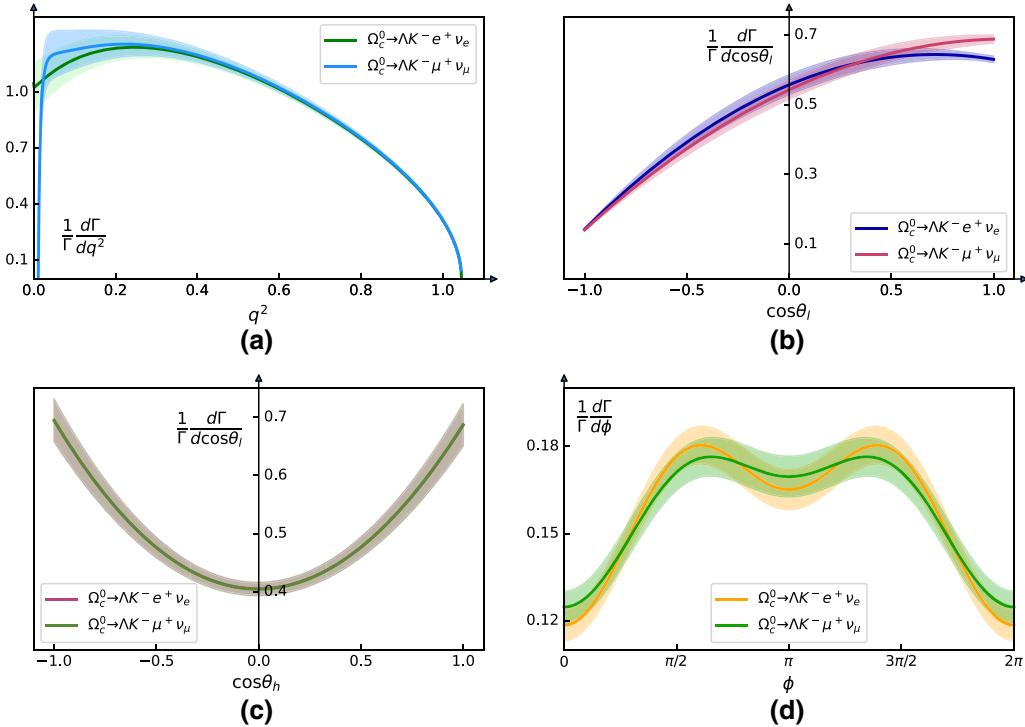


Fig. 4 Differential decay width for $\Xi_c^0 \rightarrow \Lambda\pi^- \ell^+ \nu_\ell$ as a function of q^2 (a), $\cos \theta_l$ (b), $\cos \theta_h$ (c), and ϕ (d). Note that in b and c, the errors increase with θ_l from -1 to 1 and θ_h from 1 to -1, as a result of the remarkable uncertainties of form factors at the small- q^2 region

Table 3 Results for the z-expansion parameters describing $\Xi_c \rightarrow \Xi$ form factors with statistical errors

	f_{\perp}	f_0	f_+	g_{\perp}	g_0	g_+
a_0	1.51 ± 0.09	0.64 ± 0.09	0.77 ± 0.07	0.56 ± 0.07	0.63 ± 0.07	0.56 ± 0.08
a_1	-1.88 ± 1.21	-1.83 ± 1.22	-4.09 ± 1.18	-0.35 ± 1.26	-1.63 ± 1.36	0.00 ± 1.38
a_2	1.71 ± 0.49	0.56 ± 0.51	0.35 ± 0.49	0.15 ± 0.29	0.15 ± 0.29	0.14 ± 0.29

**Fig. 5** Differential decay width for $\Omega_c \rightarrow \Lambda K^- \ell^+ \nu_{\ell}$ as a function of q^2 (a), $\cos \theta_l$ (b), $\cos \theta_h$ (c), and ϕ (d). Note that in b and c, the errors increase with θ_l from -1 to 1 and θ_h from 1 to -1 , as a result of the remarkable uncertainties of form factors at the small- q^2 region.

tors extracted from lattice QCD, and its behavior is the same as $\Lambda_c^+ \rightarrow p \pi^- \ell^+ \nu_{\ell}$. The results of the θ_l -, θ_h - and ϕ -dependence of differential decay widths with different leptonic final states are listed as follows

$$1. \quad \Xi_c^0 \rightarrow \Lambda \pi^- e^+ \nu_e$$

$$\begin{aligned} \frac{d\Gamma}{\Gamma d \cos \theta_l} &= 0.4478(557) + 0.2363(249) \cos \theta_l \\ &\quad - 0.1565(273) \cos 2\theta_l, \\ \frac{d\Gamma}{\Gamma d \cos \theta_h} &= 0.5000(626) + 0.1616(191) \cos \theta_h, \\ \frac{d\Gamma}{\Gamma d \phi} &= 0.1592(199) + 0.0123(30) \cos \theta_{\phi}. \end{aligned} \quad (120)$$

$$2. \quad \Xi_c^0 \rightarrow \Lambda \pi^- \mu^+ \nu_{\mu}$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.4549(569) + 0.2650(280) \cos \theta_l$$

$$\begin{aligned} &- 0.1354(240) \cos 2\theta_l, \\ \frac{d\Gamma}{\Gamma d \cos \theta_h} &= 0.500(630) + 0.1617(193) \cos \theta_h, \\ \frac{d\Gamma}{\Gamma d \phi} &= 0.1592(200) + 0.0122(29) \cos \theta_{\phi}, \end{aligned} \quad (121)$$

and the corresponding plots of angular distributions are collected in Fig. 5.

- Ω_c^0 decays

Due to the absence of the lattice QCD calculation, we use $\Omega_c^0 \rightarrow \Omega^-$ transition form factors calculated by the light-front quark model [69]. The form factors can be expressed as the following double-pole form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_F^2) + b(q^4/m_F^4)}, \quad (122)$$

Table 4 $\Omega_c^0 \rightarrow \Omega^-$ transition form factors with $F(0)$ at $q^2 = 0$

	f_1	f_2	f_3	f_4	g_1	g_2	g_3	g_4
$F(0)$	$0.54 + 0.13\delta$	$0.35 - 0.36\delta$	$0.33 + 0.59\delta$	$0.97 + 0.22\delta$	$2.05 + 1.38\delta$	$-0.06 + 0.33\delta$	$-1.32 - 0.32\delta$	$-0.44 + 0.11\delta$
a	-0.27	-30.0	0.96	-0.53	-3.66	-1.15	-4.01	-1.29
b	1.65	96.82	9.25	1.41	1.41	71.66	5.68	-0.58

where $F(0)$ is the value of the form factors at $q^2 = 0$ with $m_F = 1.86$ GeV, $\delta \equiv \delta m_c/m_c = \pm 0.04$. The numerical results from the light-front quark model are collected in Table 4.

Through the decay width of Ω_c in Eq. (88) and integrating out all variables, we can obtain the numerical results of decay widths and branching fractions with errors:

$$\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e : \Gamma = 1.311(51) \times 10^{-14} s^{-1}, \\ \mathcal{B} = 0.534(21)\%, \quad (123)$$

$$\Omega_c^0 \rightarrow \Omega^- \mu^+ \nu_\mu : \Gamma = 1.294(51) \times 10^{-14} s^{-1}, \\ \mathcal{B} = 0.516(21)\%, \quad (124)$$

$$\Omega_c^0 \rightarrow \Lambda K^- e^+ \nu_e : \Gamma = 8.889(344) \times 10^{-15} s^{-1}, \\ \mathcal{B} = 0.362(14)\%, \quad (125)$$

$$\Omega_c^0 \rightarrow \Lambda K^- \mu^+ \nu_\nu : \Gamma = 8.771(343) \times 10^{-15} s^{-1}, \\ \mathcal{B} = 0.350(14)\%. \quad (126)$$

Figure 5a shows the differential decay widths of $\Omega_c^0 \rightarrow \Lambda K^- \ell^+ \nu_\ell$ as a function of q^2 . We also give the results of angular distributions of total decay width; the numerical results are listed in Eqs. (127, 128), and in Fig. 5, the illustration of the angular distributions with ϕ , $\cos \theta_l$ and $\cos \theta_h$ are displayed.

1. $\Omega_c^0 \rightarrow \Lambda K^- e^+ \nu_e$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.472(2) + 0.244(3) \cos \theta_l \\ - 0.085(14) \cos 2\theta_l, \\ \frac{d\Gamma}{\Gamma d \cos \theta_h} = 0.548(24) - 0.002(0) \cos \theta_h \\ + 0.143(13) \cos 2\theta_h - 0.001(0) \cos 3\theta_h, \\ \frac{d\Gamma}{\Gamma d \phi} = 0.159(6) - 0.0233(8) \cos \theta_\phi \\ - 0.0173(4) \cos 2\theta_\phi. \quad (127)$$

2. $\Omega_c^0 \rightarrow \Lambda K^- \mu^+ \nu_\mu$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.479(16) \\ + 0.273(5) \cos \theta_l - 0.064(11) \cos 2\theta_l,$$

$$\begin{aligned} \frac{d\Gamma}{\Gamma d \cos \theta_h} &= 0.5479(238) - 0.0021(6) \cos \theta_h \\ &+ 0.1436(131) \cos 2\theta_h \\ &- 0.0014(0) \cos 3\theta_h, \\ \frac{d\Gamma}{\Gamma d \phi} &= 0.1596(62) - 0.0224(8) \cos \theta_\phi \\ &- 0.0120(2) \cos 2\theta_\phi. \end{aligned} \quad (128)$$

5 Summary

In summary, we have studied charmed baryon four-body semileptonic decays including the antitriplet charmed baryons (Λ_c^+ , Ξ_c^0) and sextet Ω_c^0 . With the form factors of the $\Lambda_c^+ \rightarrow \Lambda$ and $\Xi_c^0 \rightarrow \Xi^-$ calculated by lattice QCD and the light-front quark model calculation of $\Omega_c^0 \rightarrow \Omega^-$ form factors, we have predicted the decay widths and branching fractions using the helicity amplitude technique for three-body and four-body decays of the charmed baryons:

$$\begin{aligned} \Gamma(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) &= 1.262(75) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) &= 3.88(23)\%, \\ \Gamma(\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu) &= 1.272(72) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu) &= 3.91(22)\%, \\ \Gamma(\Xi_c \rightarrow \Xi^- e^+ \nu_e) &= 1.032(129) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Xi_c \rightarrow \Xi^- e^+ \nu_e) &= 2.40(30)\%, \\ \Gamma(\Xi_c \rightarrow \Xi^- \mu^+ \nu_\mu) &= 1.038(131) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Xi_c \rightarrow \Xi^- \mu^+ \nu_\mu) &= 2.41(30)\%, \\ \Gamma(\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e) &= 1.311(51) \times 10^{-14} s^{-1}, \\ \mathcal{B}(\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e) &= 0.534(21)\%, \\ \Gamma(\Omega_c^0 \rightarrow \Omega^- \mu^+ \nu_\mu) &= 1.294(51) \times 10^{-14} s^{-1}, \\ \mathcal{B}(\Omega_c^0 \rightarrow \Omega^- \mu^+ \nu_\mu) &= 0.516(21)\%, \\ \Gamma(\Lambda_c^+ \rightarrow p \pi^- e^+ \nu_e) &= 8.061(482) \times 10^{-14} s^{-1}, \\ \mathcal{B}(\Lambda_c^+ \rightarrow p \pi^- e^+ \nu_e) &= 2.48(15)\%, \\ \Gamma(\Lambda_c^+ \rightarrow p \pi^- \mu^+ \nu_\mu) &= 8.126(462) \times 10^{-14} s^{-1}, \\ \mathcal{B}(\Lambda_c^+ \rightarrow p \pi^- \mu^+ \nu_\mu) &= 2.50(14)\%, \\ \Gamma(\Xi_c \rightarrow \Lambda \pi^- e^+ \nu_e) &= 1.031(129) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Xi_c \rightarrow \Lambda \pi^- e^+ \nu_e) &= 2.40(30)\%, \end{aligned}$$

$$\begin{aligned}\Gamma(\Xi_c \rightarrow \Lambda\pi^-\mu^+\nu_\nu) &= 1.037(131) \times 10^{-13} s^{-1}, \\ \mathcal{B}(\Xi_c \rightarrow \Lambda\pi^-\mu^+\nu_\nu) &= 2.41(30)\%, \\ \Gamma(\Omega_c \rightarrow \Lambda K^-e^+\nu_e) &= 8.889(344) \times 10^{-15} s^{-1}, \\ \mathcal{B}(\Omega_c \rightarrow \Lambda K^-e^+\nu_e) &= 0.362(14)\%, \\ \Gamma(\Omega_c \rightarrow \Lambda K^-\mu^+\nu_\nu) &= 8.771(343) \times 10^{-15} s^{-1}, \\ \mathcal{B}(\Omega_c \rightarrow \Lambda K^-\mu^+\nu_\nu) &= 0.350(14)\%.\end{aligned}$$

In addition, we give the angular distributions of all the processes with different angular $\cos\theta_l, \cos\theta_h$ and ϕ . In the future, we expect to study the more general cases of semileptonic charmed baryon decays by calculating form factors such as $\Xi_c \rightarrow \Lambda$ and so on. This work can provide a theoretical basis for the ongoing experiments at BESIII, LHCb, and Belle-II.

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Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: All data analysed in this manuscript are available from the corresponding author on reasonable request.]

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Appendix

A. Definitions of spinors, polarization vectors, and vectorial spinors

In this part, we will introduce the spinors and polarization vectors of fermions and the vector boson [76]. The spinors for spin-1/2 fermions are

$$u\left(\vec{p}, +\frac{1}{2}\right) = N_p \begin{pmatrix} e^{-i\phi/2} \cos(\theta/2) \\ e^{i\phi/2} \sin(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{-i\phi/2} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{i\phi/2} \sin(\theta/2) \end{pmatrix}, \quad (129)$$

$$u\left(\vec{p}, -\frac{1}{2}\right) = N_p \begin{pmatrix} -e^{-i\phi/2} \sin(\theta/2) \\ e^{i\phi/2} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{-i\phi/2} \sin(\theta/2) \\ -\frac{|\vec{p}|}{E+m} e^{i\phi/2} \cos(\theta/2) \end{pmatrix}, \quad (130)$$

where $N_p = \sqrt{E+m}$ and $p^\mu = (p^0, \vec{p} \sin\theta \cos\phi, \vec{p} \sin\theta \sin\phi, \vec{p} \cos\theta)$. The spinors for anti-fermions can be obtained by $\bar{v}(\vec{p}, s) = -i\gamma^2 u^*(\vec{p}, s)$. The polarization vectors can be expressed as

$$\begin{aligned}\epsilon^\mu(+1) &= \frac{1}{\sqrt{2}}(0, -\cos\theta \cos\phi + i \sin\phi, -\cos\theta \sin\phi \\ &\quad - i \cos\phi, \sin\theta), \\ \epsilon^\mu(-1) &= \frac{1}{\sqrt{2}}(0, \cos\theta \cos\phi + i \sin\phi, \cos\theta \sin\phi \\ &\quad - i \cos\phi, -\sin\theta), \\ \epsilon^\mu(0) &= \frac{1}{\sqrt{m}}(|\vec{p}|, p^0 \sin\theta \cos\phi, \\ &\quad p^0 \sin\theta \sin\phi, p^0 \cos\theta).\end{aligned} \quad (131)$$

The vectorial spinor $U_\alpha(\vec{p}, S_z)$ for the spin-3/2 baryon is given as

$$U_\alpha(\vec{p}, S_z) = \sum_{s_z, s'_z} \left\langle s_z = 1, s_z; s'_z = \frac{1}{2}, s'_z \mid s_z = 1, s'_z = \frac{1}{2}; \right. \\ \left. S_z = \frac{3}{2}, S_z \right\rangle \epsilon_\alpha(\vec{p}, s_z) u(\vec{p}, s'_z), \quad (132)$$

where $u(\vec{p}, s'_z)$ is a wave function for spin-1/2, and $\epsilon_\alpha(\vec{p}, s_z)$ for spin-1. The Clebsch–Gordan coefficients

$$S_z = \frac{3}{2}: \quad \left\langle s_z = 1, s'_z = \frac{1}{2} \mid S_z = \frac{3}{2} \right\rangle = 1, \quad (133)$$

$$S_z = \frac{1}{2}: \quad \left\langle s_z = 1, s'_z = -\frac{1}{2} \mid S_z = \frac{1}{2} \right\rangle \\ = \frac{1}{\sqrt{3}}, \quad \left\langle s_z = 0, s'_z = \frac{1}{2} \mid S_z = \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}}, \quad (134)$$

$$S_z = -\frac{1}{2}: \quad \left\langle s_z = 0, s'_z = -\frac{1}{2} \mid S_z = -\frac{1}{2} \right\rangle \\ = \sqrt{\frac{2}{3}}, \quad \left\langle s_z = -1, s'_z = \frac{1}{2} \mid S_z = -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}}, \quad (135)$$

$$S_z = -\frac{3}{2}: \quad \left\langle s_z = -1, s'_z = -\frac{1}{2} \mid S_z = -\frac{3}{2} \right\rangle = 1. \quad (136)$$

B. Transformations of various-based form factors

Apart from the generalized parametrization in Eqs. (37, 38) for the spin-1/2-to-spin-1/2 case, another common treatment

is called helicity-based form factors,

$$\begin{aligned} & \langle B_2(p_2, s_2) | V^\mu | B_1(p_1, s_1) \rangle \\ &= \bar{u}(p_2, s_2) \left[(m_1 - m_2) \frac{q^\mu}{q^2} f_0(q^2) \right. \\ &+ \frac{m_1 + m_2}{s_+} \left(p_1^\mu + p_2^\mu - (m_1^2 - m_2^2) \frac{q^\mu}{q^2} \right) f_+(q^2) \\ &+ \left. \left(\gamma^\mu - \frac{2m_2}{s_+} p_1^\mu - \frac{2m_1}{s_+} p_2^\mu \right) f_\perp(q^2) \right] u(p_1, s_1), \end{aligned} \quad (137)$$

$$\begin{aligned} & \langle B_2(p_2, s_2) | A^\mu | B_1(p_1, s_1) \rangle \\ &= -\bar{u}(p_2, s_2) \gamma_5 \left[(m_1 + m_2) \frac{q^\mu}{q^2} g_0(q^2) \right. \\ &+ \frac{m_1 - m_2}{s_-} \left(p_1^\mu + p_2^\mu - (m_1^2 - m_2^2) \frac{q^\mu}{q^2} \right) g_+(q^2) \\ &+ \left. \left(\gamma^\mu - \frac{2m_2}{s_-} p_1^\mu - \frac{2m_1}{s_-} p_2^\mu \right) g_\perp(q^2) \right] u(p_1, s_1), \end{aligned} \quad (138)$$

where one can easily obtain the helicity-based form factors from the generalized ones under the following transformation:

$$\begin{pmatrix} f_\perp(q^2) \\ f_0(q^2) \\ f_+(q^2) \end{pmatrix} = \begin{pmatrix} 1 & -\frac{m_1+m_2}{m_1} & 0 \\ 1 & 0 & \frac{q^2}{m_1(m_1-m_2)} \\ 1 - \frac{q^2}{m_1(m_1+m_2)} & 0 \end{pmatrix} \begin{pmatrix} f_1(q^2) \\ f_2(q^2) \\ f_3(q^2) \end{pmatrix}, \quad (139)$$

$$\begin{pmatrix} g_\perp(q^2) \\ g_0(q^2) \\ g_+(q^2) \end{pmatrix} = \begin{pmatrix} 1 & \frac{m_1-m_2}{m_1} & 0 \\ 1 & 0 & -\frac{q^2}{m_1(m_1+m_2)} \\ 1 & \frac{q^2}{m_1(m_1-m_2)} & 0 \end{pmatrix} \begin{pmatrix} g_1(q^2) \\ g_2(q^2) \\ g_3(q^2) \end{pmatrix}. \quad (140)$$

The helicity-based parametrized form factors of $\Omega_c^0 \rightarrow \Omega^-$ are given as

$$\begin{aligned} & \langle B'_2(p_2, s_2) | \bar{s} \gamma^\mu c | B'_1(p_1, s_1) \rangle \\ &= -\bar{u}(p_2, s_2) \gamma_5 \left[f_0 \frac{m_2}{s_-} \frac{(m_2 + m_1) p_1^\alpha q^\mu}{q^2} \right. \\ &+ f_+ \frac{m_1}{s_+} \frac{(m_1 - m_2) p^\alpha (q^2(p_1^\mu + p_2^\mu) - (m_2^2 - m_1^2) q^\mu)}{q^2 s_+} \\ &+ f_\perp \frac{m_1}{s_+} \left(p^\alpha \gamma^\mu - \frac{2p^\alpha (m_2 p_2^\mu - m_1 p_1^\mu)}{s_+} \right) \\ &+ f_{\perp'} \frac{m_1}{s_+} \left(p^\alpha \gamma^\mu + \frac{2p_1^\alpha p_2^\mu}{m_1} + \frac{2p_1^\alpha (m_1 p_2^\mu - m_2 p_1^\mu)}{s_+} \right) \\ &\left. - \frac{s_+ g^{\alpha\mu}}{m_2} \right] u(p_1, s_1), \\ & \langle B'_2(p_2, s_2) | \bar{s} \gamma^\mu \gamma_5 c | B'_1(p_1, s_1) \rangle \\ &= \bar{u}(p_2, s_2) \left[g_0 \frac{m_2}{s_+} \frac{(m_2 - m_1) p_1^\alpha q^\mu}{q^2} \right. \\ &+ g_+ \frac{m_2}{s_-} \frac{(m_2 + m_1) p^\alpha (q^2(p_1^\mu + p_2^\mu) - (m_1^2 - m_2^2) q^\mu)}{q^2 s_-} \\ &+ g_\perp \frac{m_2}{s_-} \left(p^\alpha \gamma^\mu - \frac{2p^\alpha (m_1 p_2^\mu + m_2 p_1^\mu)}{s_+} \right) \\ &+ g_{\perp'} \frac{m_2}{s_-} \left(p^\alpha \gamma^\mu - \frac{2p_1^\alpha p_2^\mu}{m_2} \right. \\ &\left. \left. + \frac{2p_1^\alpha (m_1 p_2^\mu + m_2 p_1^\mu)}{s_-} \right) + \frac{s_- g^{\alpha\mu}}{m_2} \right] u(p_1, s_1). \end{aligned} \quad (141)$$

Similarly, we can obtain the helicity-based form factors from the generalized ones in Eqs. (46, 47) under the following transformation

$$\begin{pmatrix} f_\perp(q^2) \\ f'_\perp(q^2) \\ f_0(q^2) \\ f_+(q^2) \end{pmatrix} = \begin{pmatrix} \frac{s_+}{m_1 m_2} & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{(m_1 - m_2)^2 - q^2}{m_1 m_2} & -\frac{(m_1^2 - m_2^2 + q^2)s_-}{2m_1^2 m_2(m_1 + m_2)} & \frac{(-m_1^2 + m_2^2 + q^2)s_-}{2m_1 m_2^2(m_1 + m_2)} & \frac{-2(m_1 - m_2)^2 + 2q^2 + \frac{(-m_1^2 + m_2^2 + q^2)s_-}{m_2(m_1 + m_2)}}{s_+} \\ \frac{s_+}{m_1 m_2} & -\frac{s_- s_+}{2m_1^3 m_2 - 2m_1^2 m_2^2} & -\frac{s_- s_+}{2m_1^2 m_2^2 - 2m_1 m_2^3} & -2 - \frac{s_-}{(m_1 - m_2)m_2} \end{pmatrix} \begin{pmatrix} f_1(q^2) \\ f_2(q^2) \\ f_3(q^2) \\ f_4(q^2) \end{pmatrix}, \quad (142)$$

$$\begin{pmatrix} g_\perp(q^2) \\ g'_\perp(q^2) \\ g_0(q^2) \\ g_+(q^2) \end{pmatrix} = \begin{pmatrix} \frac{s_-}{m_1 m_2} & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{(m_1 + m_2)^2 - q^2}{m_1 m_2} & \frac{(m_1^2 - m_2^2 + q^2)s_+}{2m_1^2 m_2(m_1 - m_2)} & \frac{(m_1^2 - m_2^2 - q^2)s_-}{2m_1 m_2^2(m_1 - m_2)} & \frac{-2(m_1 + m_2)^2 + 2q^2 + \frac{(m_1^2 - m_2^2 - q^2)s_+}{m_2(m_1 - m_2)}}{s_-} \\ \frac{s_-}{m_1 m_2} & \frac{s_- s_+}{2m_1^3 m_2 + 2m_1^2 m_2^2} & \frac{s_- s_+}{2m_1^2 m_2^2 + 2m_1 m_2^3} & -2 + \frac{s_+}{(m_1 + m_2)m_2} \end{pmatrix} \begin{pmatrix} g_1(q^2) \\ g_2(q^2) \\ g_3(q^2) \\ g_4(q^2) \end{pmatrix}. \quad (143)$$

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