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The scalar $f_0(500)$, $f_0(980)$, and $a_0(980)$ resonances and vector mesons in the single Cabibbo-suppressed decays $\Lambda_c \to pK^+K^-$ and $p\pi^+\pi^-$

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Abstract In the chiral unitary approach, we have studied the single Cabibbo-suppressed decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ by taking into account the s-wave meson–meson interaction as well as the contributions from the intermediate vectors ϕ and ρ^0. Our theoretical results for the ratios of the branching fractions of $\Lambda_c \to p\bar{K}^{*0}$ and $\Lambda_c \to p\omega$ with respect to the one of $\Lambda_c \to p\phi$ are in agreement with the experimental data. Within the picture that the scalar resonances $f_0(500)$, $f_0(980)$, and $a_0(980)$ are dynamically generated from the pseudoscalar-pseudoscalar interactions in s-wave, we have calculated the K^+K^- and $\pi^+\pi^-$ invariant mass distributions respectively for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$. One can find a broad bump structure for the $f_0(500)$ and a narrow peak for the $f_0(980)$ in the $\pi^+\pi^-$ invariant mass distribution of the decay $\Lambda_c \to p\pi^+\pi^-$. For the K^+K^- invariant mass distribution, in addition to the narrow peak for the ϕ meson, there is an enhancement structure near the K^+K^- threshold mainly due to the contribution from the $f_0(980)$. Both the $\pi^+\pi^-$ and K^+K^- invariant mass distributions are compatible with the BESIII measurement. We encourage our experimental colleagues to measure these two decays, which would be helpful to understand the nature of the $f_0(500)$, $f_0(980)$, and $a_0(980)$.

1 Introduction

The non-leptonic decays of the lightest charmed baryon Λ_c play an important role in the study of strong and weak interactions [1–6]. In the last decades, lots of the information about the Λ_c decays has been accumulated [7–11], which provides a good platform to investigate the possible final state interference effects where some resonances can be dynamically generated [12–17].

Recently, the BESIII Collaboration has reported the branching fractions of the $\Lambda_c \to pK^+K^-$, $p\pi^+\pi^-$,

$$\frac{\mathcal{B}(\Lambda_c \to p\phi)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = (1.81 \pm 0.33 \pm 0.13)\%, \quad (1)$$

$$\frac{\mathcal{B}(\Lambda_c \to pK^+K^-)_{\mathrm{non}-\phi}}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = (9.36 \pm 2.22 \pm 0.71)\%, \quad (2)$$

$$\frac{\mathcal{B}(\Lambda_c \to p\pi^+\pi^-)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = (6.70 \pm 0.48 \pm 0.25)\%, \quad (3)$$

and also measured the $\pi^+\pi^-$ and K^+K^- invariant mass distributions, respectively [18], where one can find a broad bump around 500 MeV for the scalar resonance $f_0(500)$ and a narrow peak around 980 MeV for the scalar resonance $f_0(980)$ in the $\pi^+\pi^-$ invariant mass distribution, in addition to the peak for the ρ^0 meson. Later, the LHCb Collaboration has also reported these ratios using the proton-proton collision data [11],

$$\frac{\mathcal{B}(\Lambda_c \to pK^+K^-)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = (1.70 \pm 0.03 \pm 0.03)\%, \quad (4)$$

$$\frac{\mathcal{B}(\Lambda_c \to p\pi^+\pi^-)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = (7.44 \pm 0.08 \pm 0.18)\%. \quad (5)$$

Before the BESIII and LHCb results, the above two decay modes have also been observed by the NA32 [19], E687 [20], CLEO [21], and Belle Collaborations [7].

Within the chiral unitary approach, the scalar resonances $f_0(500)$, $f_0(980)$, $a_0(980)$, and $K_0^*(700)$ [known as $\kappa(800)$] appear as composite states of meson-meson, automatically

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dynamically generated by the interaction of pseudoscalar-pseudoscalar where the kernel for the Bethe-Salpter equation is taken from the chiral Lagrangians [22–27]. The productions of $f_0(500)$, $f_0(980)$, and $a_0(980)$ have been recently studied with the chiral unitary approach and the final state interactions in the decays of the D^0 [28], D_s^+ [29], $\bar{B}$ and $\bar{B}_s$ [30–33], χ_{c1} [34,35], τ^- [36], and J/ψ [37].

In this work, we perform the calculations for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ taking into account the meson-meson interaction in coupled channels and also the contributions from the intermediate vector mesons ϕ and ρ^0. The final states interaction of the pseudoscalar-pseudoscalar in the decay $\Lambda_c \to p\pi^+\pi^-$ can propagate in s-wave, which will generate the $f_0(500)$ and $f_0(980)$ resonances, and for the decay $\Lambda_c \to pK^+K^-$, the $f_0(980)$ and $a_0(980)$ resonances dynamically generated from the s-wave final state interaction will result in an enhancement structure close to the K^+K^- threshold.

The paper is organized as follows. In Sect. 2, we present the formalism and ingredients for the decays of the $\Lambda_c \to pK^+K^-$ and $p\pi^+\pi^-$ decays. Numerical results for the K^+K^- and $\pi^+\pi^-$ invariant mass distributions and discussions are given in Sect. 3, followed by a short summary in the last section.

2 Formalism

In this section, we will present the formalism for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$. For the three-body decays of Λ_c, the s-wave final state interactions of $\pi^+\pi^-$ or K^+K^- will dynamically generate the scalar resonances $f_0(500)$, $f_0(980)$, and $a_0(980)$. In addition, the three-body decays can happen via the intermediate vector mesons ρ^0 or ϕ. We first introduce the formalism for the mechanism of final state interactions of $\pi^+\pi^-$ or K^+K^- in s-wave in Sect. 2.1, then we show the details for the mechanism of the Λ_c decays via the intermediate vector mesons ρ^0 and ϕ in Sect 2.2.

2.1 s-wave final state interactions of K^+K^- and $\pi^+\pi^-$

Following Refs. [38–41], we take the decay mechanism of the internal W emission mechanism for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ as depicted in Fig. 1a, b. For the weak decays of Λ_c, the c quark decays into a W^+ boson and an s (or d) quark, then the W^+ boson decays into an $\bar{s}u$ (or $\bar{d}u$) pair. In order to give rise to the final states of pK^+K^- (or $p\pi^+\pi^-$), the $s\bar{s}$ (or $d\bar{d}$) quark pair need to hadronize together with the $\bar{q}q$ ($= \bar{u}u + \bar{d}d + \bar{s}s$) produced in the vacuum, which are given by,

$$\begin{aligned} H^{(a)} &= V^{(a)} s(\bar{u}u + \bar{d}d \\ &\quad + \bar{s}s)\bar{s}u \frac{1}{\sqrt{2}}(ud - du) = V^{(a)}\left(M^2\right)_{33} p, \end{aligned} \tag{6}$$

$$\begin{aligned} H^{(b)} &= V^{(b)} d(\bar{u}u + \bar{d}d \\ &\quad + \bar{s}s)\bar{d}u \frac{1}{\sqrt{2}}(ud - du) = V^{(b)}\left(M^2\right)_{22} p, \end{aligned} \tag{7}$$

where $V^{(a)}$ and $V^{(b)}$ are the weak interaction strengths. We use $|p\rangle = \frac{1}{\sqrt{2}}|u(ud - du)\rangle$, and $|\Lambda_c\rangle = \frac{1}{\sqrt{2}}|c(ud - du)\rangle$. M is the $q\bar{q}$ matrix,

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}.$$

The matrix M in terms of pseudoscalar mesons can be written as,

$$M \Rightarrow P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}. \tag{8}$$

Then, we have,

$$\begin{aligned} H^{(a)} &= V^{(a)}\left(M^2\right)_{33} p \\ &= V_P V_{cs} V_{us}\left(K^-K^+ + K^0\bar{K}^0 + \frac{1}{3}\eta\eta\right) p, \end{aligned} \tag{9}$$

$$\begin{aligned} H^{(b)} &= V^{(b)}\left(M^2\right)_{22} p \\ &= V_P V_{cd} V_{ud}\left(\pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 \right. \\ &\quad \left. + \frac{1}{3}\eta\eta - \frac{2}{\sqrt{6}}\pi^0\eta + K^0\bar{K}^0\right) p, \end{aligned} \tag{10}$$

where we neglect the η' because of its large mass. V_P is the strength of the production vertex, and contains all dynamical factors, which is assumed to be same for Fig. 1a, b within the SU(3) flavor symmetry. In the following, we will see that this hypothesis is also reasonable by comparing the predicted ratios of the two body decays of Λ_c with the experimental measurements. In this work we take $V_{cd} = V_{us} = -0.22534$, $V_{cs} = V_{ud} = 0.97427$ [42].

On the other hand, the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ can also proceed with the color favored external W emission mechanism: (i) the charmed quark turns into W^+ and the s (or d) quark, with the K^+ or π^+ emission from the W^+; (ii) the remaining quarks s (or d) and ud in the Λ_c, together with the $u\bar{u}$ pair created from the vacuum, hadronize to the K^-p (or π^-p), as depicted in Fig. 1c, d respectively. Thus, we have,

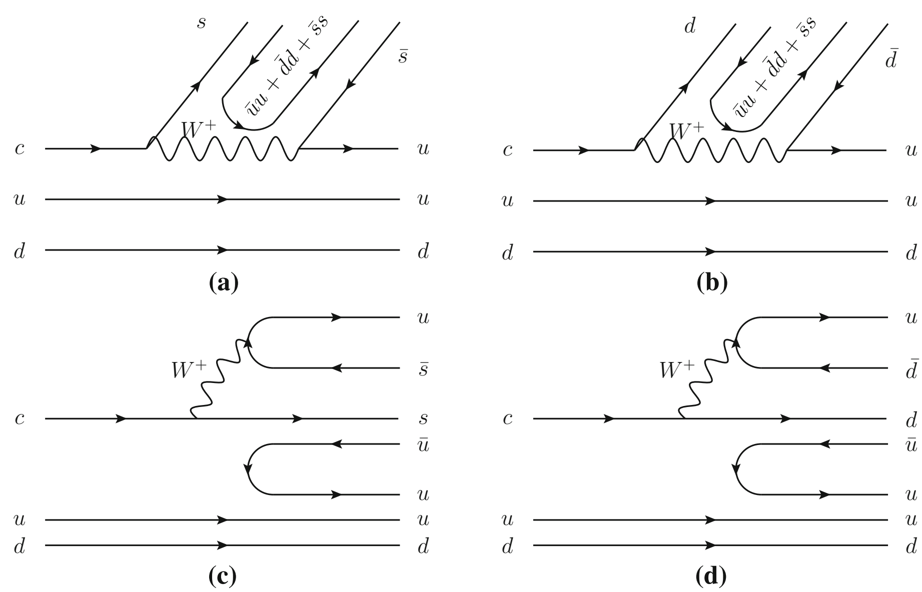

Fig. 1 The diagrams of the decays $\Lambda_c \to pK^+K^-$ and $p\pi^+\pi^-$, **a** the internal W emission for $\Lambda_c \to pK^+K^-$, **b** the internal W emission for $\Lambda_c \to p\pi^+\pi^-$, **c** the external W emission for $\Lambda_c \to pK^+K^-$, and **d** the external W emission for $\Lambda_c \to p\pi^+\pi^-$

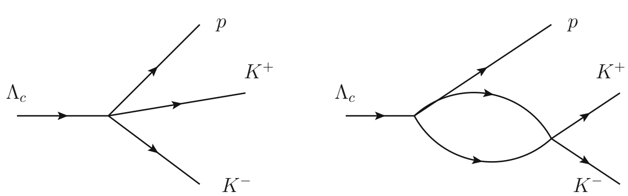

Fig. 2 The mechanisms of the decay $\Lambda_c \to pK^+K^-$, left) tree diagram, right) the s-wave final state interactions

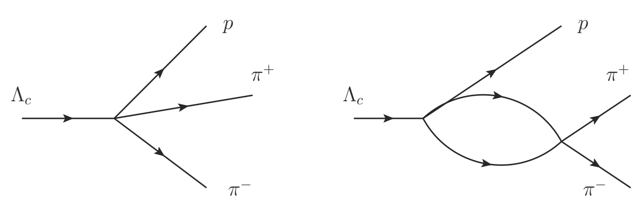

Fig. 3 The mechanisms of the decay $\Lambda_c \to p\pi^+\pi^-$, left) tree diagram, right) the s-wave final state interactions

$$H^{(c)} = V^{(c)}(u\bar{s})s\bar{u}u\frac{1}{\sqrt{2}}(ud - du) = C \times V_P V_{cs} V_{us} K^+K^- p, \tag{11}$$

$$H^{(d)} = V^{(d)}(u\bar{d})d\bar{u}u\frac{1}{\sqrt{2}}(ud - du) = C \times V_P V_{cd} V_{ud} \pi^+\pi^- p, \tag{12}$$

where we take the same normalization factor V_P as Eqs. (9) and (10), the color factor C accounts for the relative weight of the external emission mechanism with respect to the one of the internal emission mechanism [41,43]. The value of C should be around 3, because the quarks from the W decay in the external emission diagram (for example, the u and $\bar{s}$ of Fig. 1c) have three choices of the colors (we take $N_c = 3$), while the quarks from the W decay in the internal emission diagram (for example, the u and $\bar{s}$ of Fig. 1a) have the fixed colors. We will keep this factor in the following formalism, and present our results by varying its value in Sect. 3.

After the production of a meson-meson pair, the final state interaction in the s-wave between the mesons takes place, as shown in Figs. 2 and 3 for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$. Since the isospin of $\pi^+\pi^-$ system is $I = 0$, we will take into account the contributions from all the mechanisms of Fig. 1, except the $\pi^0\eta p$ states of Eq. (10) because of the isospin violation, and the amplitude is given by,

$$\begin{aligned}
t^{s-\text{wave}}_{\Lambda_c\to p\pi^+\pi^-} &= t^{(a)}_{\Lambda_c\to p\pi^+\pi^-} + t^{(b)}_{\Lambda_c\to p\pi^+\pi^-} \\
&\quad + t^{(c)}_{\Lambda_c\to p\pi^+\pi^-} + t^{(d)}_{\Lambda_c\to p\pi^+\pi^-} \\
&= V_P V_{cs} V_{us} \Big[G_{K^+K^-} t_{K^+K^-\to\pi^+\pi^-} \\
&\quad + G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to\pi^+\pi^-} + \frac{1}{3} \times 2 \times \frac{1}{2} G_{\eta\eta} \tilde{t}_{\eta\eta\to\pi^+\pi^-} \Big] \\
&\quad + V_P V_{cs} V_{us} \Big[1 + G_{\pi^+\pi^-} t_{\pi^+\pi^-\to\pi^+\pi^-} \\
&\quad + \frac{1}{2} \times 2 \times \frac{1}{2} G_{\pi^0\pi^0} \tilde{t}_{\pi^0\pi^0\to\pi^+\pi^-} \\
&\quad + \frac{1}{3} \times 2 \times \frac{1}{2} G_{\eta\eta} \tilde{t}_{\eta\eta\to\pi^+\pi^-} + G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to\pi^+\pi^-} \Big] \\
&\quad + C \times V_P V_{cs} V_{us} \left[G_{K^+K^-} t_{K^+K^-\to\pi^+\pi^-} \right] \\
&\quad + C \times V_p V_{cs} V_{us} \left[1 + G_{\pi^+\pi^-} t_{\pi^+\pi^-\to\pi^+\pi^-} \right] \\
&= V_P V_{cs} V_{us} \Big[(1 + C) + (1 + C) G_{K^+K^-} t_{K^+K^-\to\pi^+\pi^-} \\
&\quad + 2 G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to\pi^+\pi^-} + (1 + C) G_{\pi^+\pi^-} t_{\pi^+\pi^-\to\pi^+\pi^-} \\
&\quad + \frac{1}{2} G_{\pi^0\pi^0} \tilde{t}_{\pi^0\pi^0\to\pi^+\pi^-} + \frac{2}{3} G_{\eta\eta} \tilde{t}_{\eta\eta\to\pi^+\pi^-} \Big].
\end{aligned} \tag{13}$$

For the decay $\Lambda_c \to pK^+K^-$, the amplitude is given by,

$$
\begin{aligned}
t^{s-\text{wave}}_{\Lambda_c\to pK^+K^-} &= t^{(a)}_{\Lambda_c\to pK^+K^-} \\
&+t^{(b)}_{\Lambda_c\to pK^+K^-} + t^{(c)}_{\Lambda_c\to pK^+K^-} + t^{(d)}_{\Lambda_c\to pK^+K^-} \\
&= V_P V_{cs} V_{us}\Big[1 + G_{K^+K^-} t_{K^+K^-\to K^+K^-} \\
&+G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to K^+K^-} \\
&+\frac{1}{3}\times 2\times\frac{1}{2} G_{\eta\eta}\tilde{t}_{\eta\eta\to K^+K^-}\Big] \\
&+V_P V_{cs} V_{us}\Big[G_{\pi^+\pi^-} t_{\pi^+\pi^-\to K^+K^-} \\
&+\frac{1}{2}\times 2\times\frac{1}{2} G_{\pi^0\pi^0}\tilde{t}_{\pi^0\pi^0\to K^+K^-} \\
&+\frac{1}{3}\times 2\times\frac{1}{2} G_{\eta\eta}\tilde{t}_{\eta\eta\to K^+K^-} \\
&-\frac{2}{\sqrt{6}} G_{\pi^0\eta} t_{\pi^0\eta\to K^+K^-} + G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to K^+K^-}\Big] \\
&+C\times V_P V_{cs} V_{us}\Big[1 + G_{K^+K^-} t_{K^+K^-\to K^+K^-}\Big] \\
&+C\times V_p V_{cs} V_{us}\Big[G_{\pi^+\pi^-} t_{\pi^+\pi^-\to K^+K^-}\Big] \\
&= V_P V_{cs} V_{us}\Big[(1+C) + G_{K^+K^-} t_{K^+K^-\to K^+K^-} \\
&+G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to K^+K^-} + (1+C) G_{\pi^+\pi^-} t_{\pi^+\pi^-\to K^+K^-} \\
&+\frac{1}{2} G_{\pi^0\pi^0}\tilde{t}_{\pi^0\pi^0\to K^+K^-} + \frac{2}{3} G_{\eta\eta}\tilde{t}_{\eta\eta\to K^+K^-}\Big] \\
&+V_P V_{cs} V_{us}\Big[C\times G_{K^+K^-} t_{K^+K^-\to K^+K^-} \\
&+G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to K^+K^-} - \frac{2}{\sqrt{6}} G_{\pi^0\eta} t_{\pi^0\eta\to K^+K^-}\Big],
\end{aligned}
\tag{14}
$$

where the first term only contains the contribution from isospin $I=0$, and the second term has the contributions of $I=0$ and $I=1$ from the mechanisms of Fig. 1b, c. It is easily done taking $G_{K^0\bar{K}^0}=G_{K^+K^-}$, and rewriting $t_{K^+K^-\to K^+K^-}$ and $t_{K^0\bar{K}^0\to K^+K^-}$ from the mechanisms of Fig. 1b, c as done in Ref. [32],

$$
\begin{aligned}
&G_{K^0\bar{K}^0} t_{K^0\bar{K}^0\to K^+K^-} + C\times G_{K^+K^-} t_{K^+K^-\to K^+K^-} \\
&= G_{K^0\bar{K}^0}\Big[\frac{1+C}{2}\left(t_{K^0\bar{K}^0\to K^+K^-} + t_{K^+K^-\to K^+K^-}\right) \\
&+\frac{1-C}{2}\left(t_{K^0\bar{K}^0\to K^+K^-} - t_{K^+K^-\to K^+K^-}\right)\Big],
\end{aligned}
\tag{15}
$$

where the first two terms are in $I=0$ while the last two terms are in $I=1$.

Thus, the amplitude of Eq. (14) can be rewritten as,

$$
\begin{aligned}
t^{s-\text{wave}}_{\Lambda_c\to pK^+K^-} &= V_P V_{cs} V_{us}\,\{[(1+C) \\
&+\frac{3+C}{2} G_{K^0\bar{K}^0}\left(t_{K^0\bar{K}^0\to K^+K^-} + t_{K^+K^-\to K^+K^-}\right) \\
&+(1+C) G_{\pi^+\pi^-} t_{\pi^+\pi^-\to K^+K^-} \\
&+\frac{1}{2} G_{\pi^0\pi^0}\tilde{t}_{\pi^0\pi^0\to K^+K^-} + \frac{2}{3} G_{\eta\eta}\tilde{t}_{\eta\eta\to K^+K^-}\Big] \\
&+\Big[\frac{1-C}{2} G_{K^0\bar{K}^0}\left(t_{K^0\bar{K}^0\to K^+K^-} - t_{K^+K^-\to K^+K^-}\right) \\
&-\frac{2}{\sqrt{6}} G_{\pi^0\eta} t_{\pi^0\eta\to K^+K^-}\Big]\Big\} \\
&= t^{I=0} + t^{I=1},
\end{aligned}
\tag{16}
$$

where the terms $t^{I=0}$ and $t^{I=1}$ correspond to the contributions from the $I=0$ and $I=1$, respectively.

In Eqs. (13) and (14), we include a factor of 2 from the two way to match the two identical particles of the operators in Eqs. (9) and (10) with the two mesons ($\pi^0\pi^0$ and $\eta\eta$) produced, and a factor $1/2$ in the intermediate loops involving a pair of identical mesons [30,32]. The scattering matrix $t_{i\to j}$ has been calculated within the chiral unitary approach in Refs. [22,28,31,44,45], and we take $\tilde{t}_{\eta\eta\to i}=\sqrt{2}t_{\eta\eta\to i}$, $\tilde{t}_{\pi^0\pi^0\to j}=\sqrt{2}t_{\pi^0\pi^0\to j}$ for the two identical particles [44]. G_l is the loop function for the two mesons propagator in the lth channel, which is given as follows after the integration in dq^0,

$$
\begin{aligned}
G_l &= i\int\frac{d^4q}{(2\pi)^4}\frac{1}{(p-q)^2-m_1^2+i\epsilon}\frac{1}{q^2-m_2^2+i\epsilon} \\
&= \int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\omega_1+\omega_2}{\omega_1\omega_2}\frac{1}{(\sqrt{s}+\omega_1+\omega_2)(\sqrt{s}-\omega_1-\omega_2+i\epsilon)},
\end{aligned}
\tag{17}
$$

where $\sqrt{s}$ is the invariant mass of the meson-meson pair, and the meson energies $\omega_i=\sqrt{(\mathbf{q})^2+m_i^2}$ $(i=1,2)$. The integral on $\mathbf{q}$ in Eq. (17) is performed with a cutoff $|\mathbf{q}_{\max}|=600$ MeV, as used in Refs. [28,31,44]. The transition amplitude t_{ij} is obtained by solving the Bethe–Salpeter equation in coupled channels,

$$
T=[1-VG]^{-1}V,
\tag{18}
$$

where five channels $\pi^+\pi^-$ (1), $\pi^0\pi^0$ (2), K^+K^- (3), $K^0\bar{K}^0$ (4), and $\eta\eta$ (5) are included for $I=0$, and three channels K^+K^- (1), $K^0\bar{K}^0$, (2) and $\pi^0\eta$ (3) are included for $I=1$. The elements of the diagonal matrix G are given by the loop function of Eq. (17), and V is the matrix of the interaction kernel corresponding to the tree level transition amplitudes obtained from phenomenological Lagrangians [22]. The explicit expressions for $I=0$ can be expressed as [44],

$$
\begin{aligned}
&V_{\pi^+\pi^-\to\pi^+\pi^-} = -\frac{1}{2f^2}s, \quad V_{\pi^+\pi^-\to\pi^0\pi^0} \\
&= -\frac{1}{\sqrt{2}f^2}(s-m_\pi^2), \quad V_{\pi^+\pi^-\to K^+K^-} = -\frac{1}{4f^2}s, \\
&\quad V_{\pi^+\pi^-\to K^0\bar{K}^0} = -\frac{1}{4f^2}s, \quad V_{\pi^+\pi^-\to\eta\eta}
\end{aligned}
$$

$$= -\frac{1}{3\sqrt{2}f^2}m_\pi^2, \quad V_{\pi^0\pi^0\to\pi^0\pi^0} = -\frac{1}{2f^2}m_\pi^2,$$
$$V_{\pi^0\pi^0\to K^+K^-} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{\pi^0\pi^0\to K^0\bar{K}^0}$$
$$= -\frac{1}{4\sqrt{2}f^2}s, \quad V_{\pi^0\pi^0\to\eta\eta} = -\frac{1}{6f^2}m_\pi^2,$$
$$V_{K^+K^-\to K^+K^-} = -\frac{1}{2f^2}s, \quad V_{K^+K^-\to K^0\bar{K}^0}$$
$$= -\frac{1}{4f^2}s, \quad V_{K^+K^-\to\eta\eta}$$
$$= -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2),$$
$$V_{K^0\bar{K}^0\to K^0\bar{K}^0} = -\frac{1}{2f^2}s, \quad V_{K^0\bar{K}^0\to\eta\eta}$$
$$= -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2),$$
$$V_{\eta\eta\to\eta\eta} = -\frac{1}{18f^2}(16m_K^2 - 7m_\pi^2), \tag{19}$$

and the ones for $I = 1$ are [28],

$$V_{K^+K^-\to K^+K^-} = -\frac{1}{2f^2}s, \quad V_{K^+K^-\to K^0\bar{K}^0} = -\frac{1}{4f^2}s,$$
$$V_{K^+K^-\to\pi^0\eta} = \frac{-\sqrt{3}}{12f^2}\left(3s - \frac{8}{3}m_K^2 - \frac{1}{3}m_\pi^2 - m_\eta^2\right),$$
$$V_{K^0\bar{K}^0-\to K^0\bar{K}^0} = -\frac{1}{2f^2}s, \quad V_{K^0\bar{K}^0-\to\pi^0\eta}$$
$$= -V_{K^+K^-\to\pi^0\eta}, \quad V_{\pi^0\eta\to\pi^0\eta} = -\frac{m_\pi^2}{3f^2}, \tag{20}$$

where $f = f_\pi = 93$ MeV is the pion decay constant, and m_π, m_K, and m_η are the averaged masses of the pion, kaon, and η mesons, respectively [42].

With the amplitudes of Eqs. (14) and (13), we can write the differential decay width for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ in s-wave,

$$\frac{d\Gamma^{s-\text{wave}}}{dM_{\text{inv}}} = \frac{1}{(2\pi)^3}\frac{p_p\tilde{k}}{4M_{\Lambda_c}^2}\left|t^{s-\text{wave}}_{\Lambda_c\to pK^+K^-,\, p\pi^+\pi^-}\right|^2, \tag{21}$$

where M_{inv} is the invariant mass of the K^+K^- or $\pi^+\pi^-$, p_p is the momentum of the proton in the Λ_c rest frame, and $\tilde{k}$ is the momentum of the K^+ (or π^+) in the rest frame of the K^+K^- (or $\pi^+\pi^-$) system,

$$p_p = \frac{\lambda^{1/2}\left(M_{\Lambda_c}^2, M_p^2, M_{\text{inv}}^2\right)}{2M_{\Lambda_c}},$$
$$\tilde{k} = \frac{\lambda^{1/2}\left(M_{\text{inv}}^2, m_{K^+/\pi^+}^2, m_{K^-/\pi^-}^2\right)}{2M_{\text{inv}}}, \tag{22}$$

with the Källen function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. The masses of the baryons and mesons involved in our calculations are taken from PDG [42].

2.2 Λ_c decays via the intermediate vector mesons ϕ and ρ^0

In this section, we will present the formalism for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ via the intermediate mesons ϕ and ρ^0. The quark level diagrams for the two-body decays of Λ_c into a proton and a vector meson are shown in Fig. 4.

At the quark level, the quark components of the vector mesons are,

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \phi = s\bar{s},$$
$$\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \bar{K}^{*0} = s\bar{d}. \tag{23}$$

The amplitudes can be written as,

$$t_{\Lambda_c\to p\rho^0} = -\frac{1}{\sqrt{2}}V_P' V_{cd}V_{ud}, \qquad t_{\Lambda_c\to p\phi} = V_P' V_{cs}V_{us}, \tag{24}$$
$$t_{\Lambda_c\to p\omega} = \frac{1}{\sqrt{2}}V_P' V_{cd}V_{ud}, \qquad t_{\Lambda_c\to p\bar{K}^{*0}} = V_P' V_{cs}V_{ud}, \tag{25}$$

where V_P' is a normalization factor for the Λ_c decay into proton and a vector meson. The factor of $1/\sqrt{2}$ in the above amplitudes comes from the quark component of the ρ^0 and ω. With those amplitudes, the decay width for the two-body decay of Λ_c into proton and a vector meson in s-wave is,

$$\Gamma_{\Lambda_c\to pV} = \frac{\lambda^{1/2}\left(M_{\Lambda_c}^2, m_V^2, M_p^2\right)}{16\pi M_{\Lambda_c}^3}\left|t_{\Lambda_c\to pV}\right|^2, \tag{26}$$

where V stands for the vector mesons ρ^0, ϕ, ω, and $\bar{K}^{*0}$.

The K^+K^- and $\pi^+\pi^-$ invariant mass distributions respectively for the ϕ and ρ^0 mesons can be obtained by converting the total rate for vector production into a mass distribution as Refs. [30,46],

$$\frac{d\Gamma_{\Lambda_c\to p\rho^0,\rho^0\to\pi^+\pi^-}}{dM_{\text{inv}}} = \frac{2m_\rho^2}{\pi}\frac{\tilde{\Gamma}_\rho\tilde{\Gamma}_{\Lambda_c\to p\rho^0}}{(M_{\text{inv}}^2 - m_\rho^2)^2 + m_\rho^2\tilde{\Gamma}_\rho^2}, \tag{27}$$
$$\frac{d\Gamma_{\Lambda_c\to p\phi,\phi\to K^+K^-}}{dM_{\text{inv}}} = \frac{m_\phi^2}{\pi}\frac{\tilde{\Gamma}_\phi\tilde{\Gamma}_{\Lambda_c\to p\phi}}{(M_{\text{inv}}^2 - m_\phi^2)^2 + m_\phi^2\tilde{\Gamma}_\phi^2}, \tag{28}$$

where we have considered that the K^+K^- decay accounts for 1/2 of the $K\bar{K}$ decay width of the ϕ meson. Since $\rho^0 \to \pi^+\pi^-$ and $\phi \to K^+K^-$ are in p-wave, we take

$$\tilde{\Gamma}_\rho = \Gamma_{\rho^0}\left(\frac{\sqrt{M_{\text{inv}}^2 - 4m_\pi^2}}{\sqrt{m_\rho^2 - 4m_\pi^2}}\right)^3,$$
$$\tilde{\Gamma}_\phi = \Gamma_\phi\left(\frac{\sqrt{M_{\text{inv}}^2 - 4m_K^2}}{\sqrt{m_\phi^2 - 4m_K^2}}\right)^3, \tag{29}$$

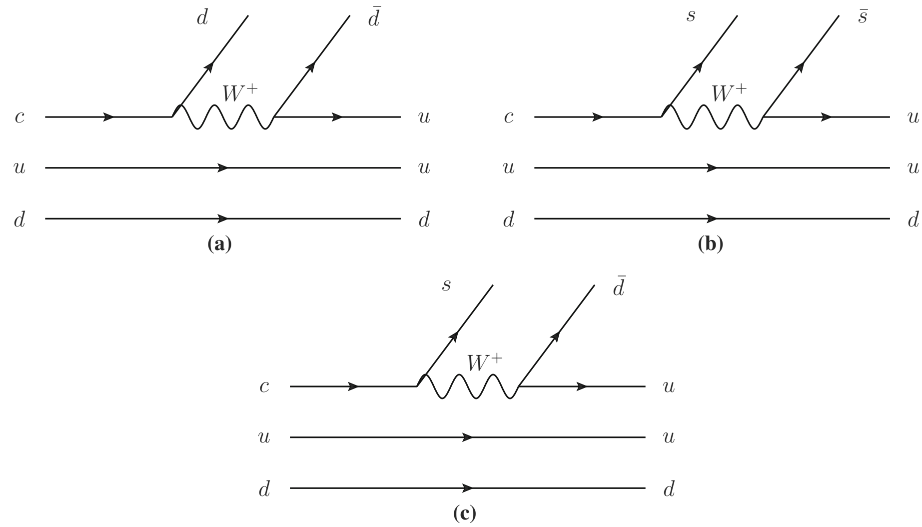

Fig. 4 The quark level diagrams for the two-body decays of Λ_c, a $\Lambda_c \to p\rho^0$, and $p\omega$, b $\Lambda_c \to p\phi$, and c $\Lambda_c \to p\bar{K}^{*0}$

and

$$\tilde{\Gamma}_{\Lambda_c \to pV} = \Gamma_{\Lambda_c \to pV} \frac{\lambda^{1/2}\left(M^2_{\Lambda_c}, M^2_{\rm inv}, M^2_p\right)}{\lambda^{1/2}\left(M^2_{\Lambda_c}, m^2_V, M^2_p\right)} \frac{m_V}{M_{\rm inv}}. \tag{30}$$

For the processes $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$, the contributions from the vector mesons ϕ and ρ^0, which respectively decay into K^+K^- and $\pi^+\pi^-$ in p-wave, should be added to Eq. (21) incoherently.

3 Results and discussion

With Eqs. (24-26), the ratios of the branching fractions of the decays $\Lambda_c \to p\bar{K}^{*0}$, $\Lambda_c \to p\omega$, $\Lambda_c \to p\rho^0$ with respect to the decay $\Lambda_c \to p\phi$ can be obtained with Eq. (26),

$$\begin{aligned} R_1^{\rm th} &= \frac{\mathcal{B}(\Lambda_c \to p\bar{K}^{*0})}{\mathcal{B}(\Lambda_c \to p\phi)} = \frac{\Gamma_{\Lambda_c \to p\bar{K}^{*0}}}{\Gamma_{\Lambda_c \to p\phi}} \\ &= \frac{\lambda^{1/2}\left(M^2_{\Lambda_c}, m^2_{\bar{K}^{*0}}, M^2_p\right)\left|t_{\Lambda_c \to p\bar{K}^{*0}}\right|^2}{\lambda^{1/2}\left(M^2_{\Lambda_c}, m^2_\phi, M^2_p\right)\left|t_{\Lambda_c \to p\phi}\right|^2} \\ &= \frac{\lambda^{1/2}\left(M^2_{\Lambda_c}, m^2_{K^{*0}}, M^2_p\right) V^2_{ud}}{\lambda^{1/2}\left(M^2_{\Lambda_c}, m^2_\phi, M^2_p\right) V^2_{us}} = 21.6, \end{aligned} \tag{31}$$

and analogously,

$$\begin{aligned} R_2^{\rm th} &= \frac{\mathcal{B}(\Lambda_c \to p\omega)}{\mathcal{B}(\Lambda_c \to p\phi)} = 0.636, \\ R_3^{\rm th} &= \frac{\mathcal{B}(\Lambda_c \to p\rho^0)}{\mathcal{B}(\Lambda_c \to p\phi)} = 0.640, \end{aligned} \tag{32}$$

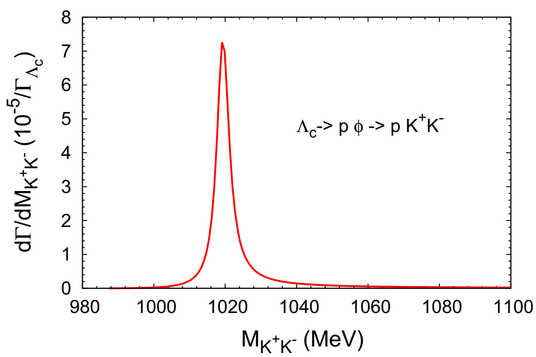

Fig. 5 The K^+K^- invariant mass distribution of the decay $\Lambda_c \to p\phi \to pK^+K^-$ with $(V'_P)^2/\Gamma_{\Lambda_c} = 4.5 \times 10^3$ MeV

and one can find that $R_1^{\rm th}$ and $R_2^{\rm th}$ are consistent with the experimental results [42],

$$\begin{aligned} R_1^{\rm exp} &= \frac{\mathcal{B}(\Lambda_c \to p\bar{K}^{*0})}{\mathcal{B}(\Lambda_c \to p\phi)} \\ &= \frac{(1.94 \pm 0.27)\%}{(1.06 \pm 0.14) \times 10^{-3}} = 18.3 \pm 3.5, \end{aligned} \tag{33}$$

$$\begin{aligned} R_2^{\rm exp} &= \frac{\mathcal{B}(\Lambda_c \to p\omega)}{\mathcal{B}(\Lambda_c \to p\phi)} \\ &= \frac{(9 \pm 4) \times 10^{-4}}{(1.06 \pm 0.14) \times 10^{-3}} = 0.85 \pm 0.39, \end{aligned} \tag{34}$$

which implies that it is reasonable to take the same value of V'_P for the mechanisms of Fig. 4. By fitting to the branching fractions of the decays $\Lambda_c \to p\bar{K}^{*0}$, $\Lambda_c \to p\phi$, and $\Lambda_c \to p\omega$, we can obtain the $(V'_P)^2/\Gamma_{\Lambda_c} = (4.5 \pm 0.4) \times 10^3$ MeV. With this value, the branching fraction of the decay $\Lambda_c \to p\rho^0$ is estimated to be $\mathcal{B}(\Lambda_c \to p\rho^0) = (6.3 \pm 0.6) \times 10^{-4}$, and the K^+K^- and $\pi^+\pi^-$ invariant mass distribution of the decays $\Lambda_c \to p\phi \to pK^+K^-$ and $\Lambda_c \to p\rho \to p\pi^+\pi^-$ are easily calculated as shown in Figs. 5 and 6, respectively.

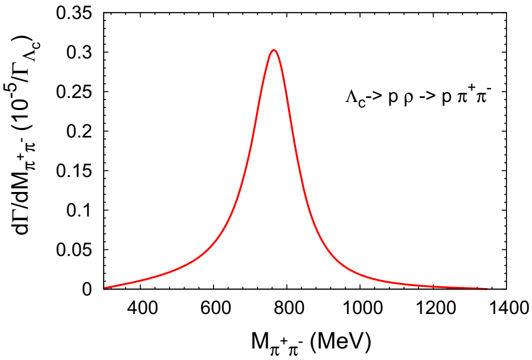

Fig. 6 The $\pi^+\pi^-$ invariant mass distribution of the decay $\Lambda_c \to p\rho \to p\pi^+\pi^-$ with $(V_P')^2/\Gamma_{\Lambda_c} = 4.5 \times 10^3$ MeV

In addition to the factor V_P, we have also the free parameter C, the relative weight of the external emission mechanism with respect to the internal emission mechanisms. The value of C should be around 3 because we take the number of the colors $N_c = 3$, and the relative sign of C is not fixed. We present the K^+K^- and $\pi^+\pi^-$ invariant mass distributions with different values of $C = 3, 2, -2, -3$ in Figs. 7 and 8, respectively. In the K^+K^- invariant mass distribution, one can see that the contributions from the isospin $I = 1$ are much smaller than the ones of the isospin $I = 0$ for the positive values of C, while both contributions from the isospin $I = 0$ and $I = 1$ are comparable for the negative values of C. This is because the coefficients of the terms $t^{I=0}$ and $t^{I=1}$ have the opposite sign before the C, and the contributions from the $\pi^0\eta$ and $(K\bar{K})_{I=1}$ [see Eq. (16)] have the negative interference for positive values of C. In both cases, one can find an enhancement structure close to the threshold, which is stronger for the positive values of C and weaker for the negative values of C. For the $\pi^+\pi^-$ invariant mass distribution of Fig. 8, we can see a clear bump structure around 500 MeV, and a sharp peak around 980 MeV, which correspond to the $f_0(500)$ and $f_0(980)$, respectively. Both the signals are clearer for the positive value of C, and weaker for the negative value of C.

It should be stressed that although the BESIII Collaboration has reported the K^+K^- and $\pi^+\pi^-$ invariant mass distributions, we can not fit our model to to BESIII data which contain the background in the sideband region [18]. In addition, we must bear in mind that the chiral unitary approach only makes reliable predictions up to 1100-1200 MeV. With the value of $(V_P')^2/\Gamma_{\Lambda_c} = 4.5 \times 10^3$ MeV obtained above, we present the K^+K^- and $\pi^+\pi^-$ invariant mass distributions by summing the contributions from the decays in s-

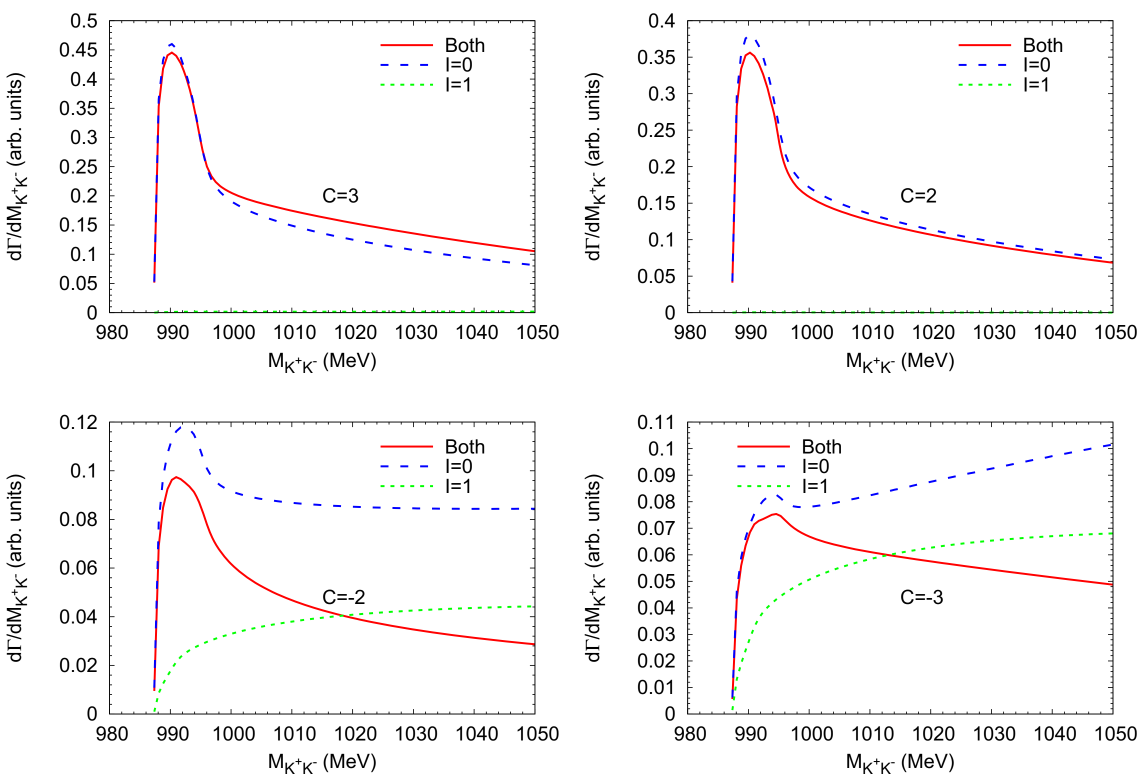

Fig. 7 The K^+K^- invariant mass distribution of the decay $\Lambda_c \to pK^+K^-$ in s-wave with different values of $C = 3, 2, -2, -3$ and an arbitrary normalization factor V_P. The curves labeled as 'both', '$I = 0$', and '$I = 1$' correspond to the contributions from the term of $t^{s\text{-wave}}_{\Lambda_c \to pK^+K^-}$, $t^{I=0}$, and $t^{I=1}$, respectively

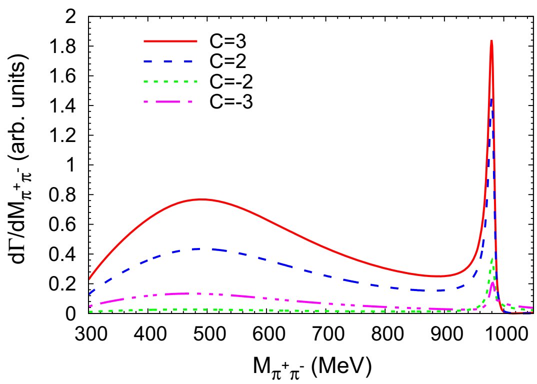

Fig. 8 The $\pi^+\pi^-$ invariant mass distribution of the decay $\Lambda_c \to p\pi^+\pi^-$ in s-wave with different values of $C = 3, 2, -2, -3$ and an arbitrary normalization factor V_P

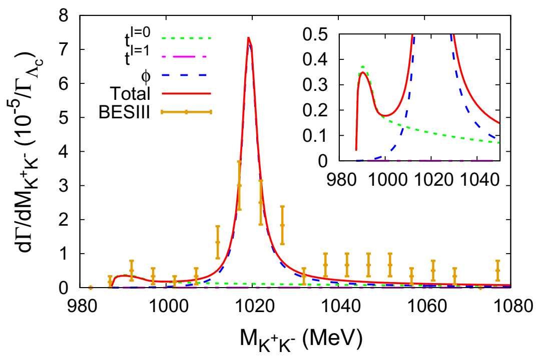

Fig. 9 The K^+K invariant mass distribution of the $\Lambda_c \to pK^+K^-$ decay compared with the experimental data from Ref. [18]. The green dotted curve stands for the contribution from the meson-meson interaction in s-wave, the blue dashed curve corresponds to the results for the intermediate vector ϕ, and the red solid line shows the total contributions

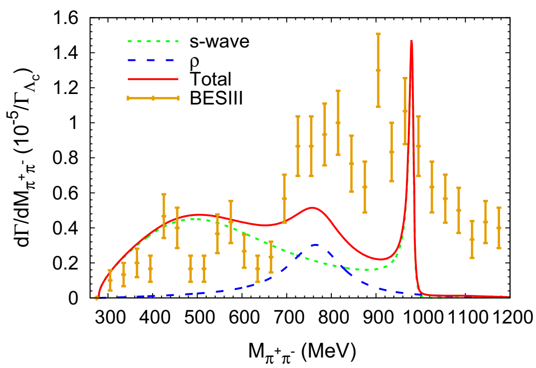

Fig. 10 The $\pi^+\pi^-$ invariant mass distributions of the $\Lambda_c \to p\pi^+\pi^-$ decay compared with the experimental data from Ref. [18]. The green dotted curve stands for the contribution from the meson–meson interaction in s-wave, the blue dashed curve corresponds to the results for the intermediate vector ρ^0, and the red solid line shows the total contributions

wave and the intermediate vector mesons incoherently, as shown in Figs. 9 and 10, respectively. For comparison, the BESIII data [18] have been adjusted to the strength of our theoretical calculations. We take the parameter $C = 2$ and $(V_P)^2/\Gamma_{\Lambda_c} = 0.2$ MeV^{-1}, in order to give rise to the sizeable signals of the $f_0(500)$ and $f_0(980)$ [1]. Both the parameters can be obtained by fitting to the experimental data, when more precise measurement of the processes is available in future. For the K^+K^- invariant mass distribution, our model produces an enhancement structure close to the threshold mainly due to the resonance $f_0(980)$, and a clear peak of the ϕ, which are in good agreement with the BESIII data. It is worth mentioning that, in the K^+K^- invariant mass distribution of the decay $\chi_{cJ} \to p\bar{p}K^+K^-$ measured by the BESIII Collaboration [47], one can find an enhancement structure close to the threshold, which can be associated to the resonance $f_0(980)$ and $a_0(980)$. A similar structure can also be found in the decay $D_s^+ \to K^+K^-\pi^+$ measured by the BABAR Collaboration [48].

For the $\pi^+\pi^-$ invariant mass distribution of the decay $\Lambda_c \to p\pi^+\pi^-$ as shown in Fig. 10, one can see a clear peak around 770 MeV, corresponding to the vector meson ρ^0, and a broad peak around 500 MeV, which can be associated to the scalar meson $f_0(500)$, dynamically generated from the meson-meson interactions in s-wave. In addition, there is a narrow sharp peak around 980 MeV for the scalar state $f_0(980)$. We can see that the broad peak for $f_0(500)$, the peak for ρ^0, and a narrow sharp one for $f_0(980)$ [2] of our results are compatible with the BESIII measurement [18].

From Figs. 9 and 10, one can find that the results with $C = 2$ are in reasonable agreement with the BESIII measurements [18], which implies that the W external emission mechanism is more important than the W internal emission mechanism. According to the topological classification of the weak decays in Refs. [49,50], the strength of W external emission is larger than the one of W internal emission. It should be stressed that our results strongly depend on the sign of C, and the present measurements of the BESIII Collaboration favor $C = 2$ and a much smaller contribution from the $a_0(980)$. Indeed, if there is a sizeable contribution from the $a_0(980)$ in $\Lambda_c \to pK^+K^-$, it implies that we can observe the process $\Lambda_c \to p\pi^0\eta$, and the signal of the $a_0(980)$ in the $\pi^0\eta$ mass distribution experimentally, however there are no any report about this process [42].

[1] It should be pointed out that the dimensions of V_P and V'_P are '1' and 'MeV' in our formalism, respectively, thus one can not obtain the relative weight of the three-body decay and two-body decay of the Λ_c by comparing the V_P with V'_P.

[2] This peak would be a bit broader with the peak strength reduced if it is folded with the experimental resolution.

4 Conclusions

In this work, we have studied the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$, by taking into account contributions of the intermediate vector mesons, and the s-wave meson–meson interactions within the chiral unitary approach, where the $f_0(500)$, $f_0(980)$, and $a_0(980)$ resonances are dynamically generated.

The K^+K^- and $\pi^+\pi^-$ invariant mass distributions for these two decays are calculated. In the K^+K^- invariant mass distribution, one can find a narrow peak for the ϕ, and an enhancement structure close to the K^+K^- threshold, which should be the reflection of the $f_0(980)$ and $a_0(980)$ resonances. For the $\Lambda_c \to p\pi^+\pi^-$ mass distribution, in addition to the broad peak of the ρ^0, one can find a bump structure around 500 MeV for the $f_0(500)$, and a narrow sharp peak around 980 MeV for the $f_0(980)$, in agreement with the BESIII measurement.

According to our calculations, the present measurements of the BESIII Collaboration favor a much smaller contribution from the $a_0(980)$. As we discussed, if there is a sizeable contribution from the $a_0(980)$ in $\Lambda_c \to pK^+K^-$, it implies that we can observe the process $\Lambda_c \to p\pi^0\eta$, and the signal of the $a_0(980)$ in the $\pi^0\eta$ mass distribution experimentally, however there are no any report about this process [42].

We encourage our experimental colleagues to measure these two decays, which can be used to test the molecular nature of the scalar resonances $f_0(500)$, $f_0(980)$, and $a_0(980)$.

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