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Decay properties of *P*-wave bottom baryons within light-cone sum rules

Hui-Min Yang¹, Hua-XingChen^{1,2,a}, Er-Liang Cui³, Atsushi Hosaka^{4,5}, Qiang Mao⁶

¹ School of Physics, Beihang University, Beijing 100191, China

² School of Physics, Southeast University, Nanjing 210094, China

³ College of Science, Northwest A&F University, Yangling 712100, China

⁴ Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan

⁵ Advanced Science Research Center, Japan Atomic Energy Agency (JAEA), Tokai 319-1195, Japan

⁶ Department of Electrical and Electronic Engineering, Suzhou University, Suzhou 234000, China

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Abstract We use the method of light-cone sum rules to study decay properties of *P*-wave bottom baryons belonging to the *SU*(3) flavor **6**_{*F*} representation. In Cui et al. (Phys Rev D 99:094021, 2019) we have studied their mass spectrum and pionic decays, and found that the Σ_b (6097) and Ξ_b (6227) can be well interpreted as *P*-wave bottom baryons of $J^P = 3/2^-$. In this paper we further study their decays into ground-state bottom baryons and vector mesons. We propose to search for a new state Ξ_b (5/2⁻), that is the $J^P = 5/2^-$ partner state of the Ξ_b (6227), in the Ξ_b (5/2⁻) $\rightarrow \Xi_b^* \rho \rightarrow \Xi_b^* \pi \pi$ decay process. Its mass is 12 ± 5 MeV larger than that of the Ξ_b (6227).

1 Introduction

In the past years important progress has been made in the field of heavy baryons, and many heavy baryons were observed in various experiments [2-9]. These heavy baryons are interesting in a theoretical point of view [10-12]: the light degrees of freedom (light quarks and gluons) circle around the nearly static heavy quark, so that the whole system behaves as the QCD analogue of the hydrogen bounded by electromagnetic interaction. To understand them, various phenomenological models have been applied, such as the relativized potential quark model [13,14], the relativistic quark model [15], the constituent quark model [16], the chiral quark model [17], the heavy hadron chiral perturbation theory [18], the hyperfine interaction [19,20], the Feynman–Hellmann theorem [21], the combined expansion in $1/m_0$ and $1/N_c$ [22], the pion induced reactions [23], the variational approach [24], the relativistic flux tube model [25], the Faddeev approach [26],

the Regge trajectory [27], the extended local hidden gauge approach [28], the unitarized dynamical model [29], the unitarized chiral perturbation theory [30], and QCD sum rules [31–47], etc. We refer to reviews [48–52] for their recent progress.

In Refs. [58,59] we have systematically applied the method of QCD sum rules [53,54] to study *P*-wave heavy baryons within the heavy quark effective theory (HQET) [55–57], where we systematically constructed all the *P*-wave heavy baryon interpolating fields, and applied them to study the mass spectrum of *P*-wave heavy baryons. Later in Ref. [60] we further studied their decay properties using light-cone sum rules, including:

- S-wave decays of flavor $\mathbf{\bar{3}}_F$ P-wave heavy baryons into ground-state heavy baryons and pseudoscalar mesons;
- S-wave decays of flavor $\mathbf{6}_F$ P-wave heavy baryons into ground-state heavy baryons and pseudoscalar mesons;
- S-wave decays of flavor $\mathbf{\bar{3}}_F$ P-wave heavy baryons into ground-state heavy baryons and vector mesons.

Very quickly, one notices that in order to make a complete study of *P*-wave heavy baryons, we still need to study:

- S-wave decays of flavor $\mathbf{6}_F$ P-wave heavy baryons into ground-state heavy baryons and vector mesons.

Besides them, we also need to systematically study *D*-wave and radiative decay properties of *P*-wave heavy baryons.

In the present study we will study *S*-wave decays of flavor $\mathbf{6}_F$ *P*-wave heavy baryons into ground-state heavy baryons and vector mesons. We will further use the obtained results to investigate the Σ_b (6097) and Ξ_b (6227) recently observed by

^ae-mail: hxchen@buaa.edu.cn

LHCb [7,8]. Our previous sum rule study in Ref. [1] suggests that they can be well interpreted as *P*-wave bottom baryons of $J^P = 3/2^-$. This conclusion is supported by Refs. [61–67], and we refer to Refs. [68–78] for more relevant discussions. The result of Ref. [1] also suggests that they belong to the bottom baryon doublet [**6**_{*F*}, 2, 1, λ], whose definition will be given below. This doublet contains six bottom baryons, $\Sigma_b(\frac{3^-}{2}/\frac{5^-}{2})$, $\Xi'_b(\frac{3^-}{2}/\frac{5^-}{2})$, and $\Omega_b(\frac{3^-}{2}/\frac{5^-}{2})$. We predicted the mass and decay width of the $\Omega_b(3/2^-)$ state to be

$$M_{\Omega_b(3/2^-)} = 6.46 \pm 0.12 \text{ GeV},$$

$$\Gamma_{\Omega_b(3/2^-)} = 58^{+65}_{-33} \text{ MeV},$$

and masses of the three $J^P = 5/2^-$ states to be

$$M_{\Sigma_b(5/2^-)} = 6.11 \pm 0.12 \text{ GeV},$$

$$M_{\Sigma_b(5/2^-)} - M_{\Sigma_b(3/2^-)} = 13 \pm 5 \text{ MeV},$$

$$M_{\Xi'_b(5/2^-)} = 6.29 \pm 0.11 \text{ GeV},$$

$$M_{\Xi'_b(5/2^-)} - M_{\Xi'_b(3/2^-)} = 12 \pm 5 \text{ MeV},$$

$$M_{\Omega_b(5/2^-)} = 6.47 \pm 0.12 \text{ GeV},$$

$$M_{\Omega_b(5/2^-)} - M_{\Omega_b(3/2^-)} = 11 \pm 5 \text{ MeV}.$$

The three $J^P = 5/2^-$ states are probably quite narrow, because their *S*-wave decays into ground-state bottom baryons and pseudoscalar mesons can not happen, and widths of the following *D*-wave decays are extracted to be zero in Ref. [1]:

$$\begin{split} & (w') \ \Gamma_{\Sigma_b^-(5/2^-) \to \Lambda_b^0 \pi^-} = 0, \\ & (x') \ \Gamma_{\Xi_b'^-(5/2^-) \to \Xi_b^0 \pi^-} = 0, \\ & (y') \ \Gamma_{\Xi_b'^-(5/2^-) \to \Lambda_b^0 K^-} = 0, \\ & (z') \ \Gamma_{\Omega_b^-(5/2^-) \to \Xi_b^0 K^-} = 0. \end{split}$$

To further study their decay properties, in this paper we will investigate their *S*-wave decays into ground-state bottom baryons together with vector mesons ρ and K^* .

This paper is organized as follows. In Sect. 2 we study *S*-wave decays of flavor $\mathbf{6}_F$ *P*-wave bottom baryons into ground-state bottom baryons and vector mesons, separately in several subsections for the four bottom baryon multiplets, $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$, and $[\mathbf{6}_F, 2, 1, \lambda]$. A short summary is given in Sect. 3. Some relevant parameters and formulae are given in Appendix A and Appendix B.

2 Decay properties of *P*-wave bottom baryons

At the beginning let us briefly introduce our notations. A *P*-wave bottom baryon (bqq) consists of one bottom quark (b) and two light quarks (qq). Its orbital excitation can be either between the two light quarks $(l_{\rho} = 1)$ or between the bottom quark and the two-light-quark system $(l_{\lambda} = 1)$, so there are

 ρ -type bottom baryons ($l_{\rho} = 1$ and $l_{\lambda} = 0$) and λ -type ones ($l_{\rho} = 0$ and $l_{\lambda} = 1$). Altogether its internal symmetries are as follows:

- The color structure of the two light quarks is antisymmetric $(\bar{\mathbf{3}}_C)$.
- The SU(3) flavor structure of the two light quarks is either antisymmetric $(\bar{\mathbf{3}}_F)$ or symmetric $(\mathbf{6}_F)$.
- The spin structure of the two light quarks is either antisymmetric ($s_l \equiv s_{qq} = 0$) or symmetric ($s_l = 1$).
- The orbital structure of the two light quarks is either antisymmetric ($l_{\rho} = 1$) or symmetric ($l_{\rho} = 0$).
- Due to the Pauli principle, the total symmetry of the two light quarks is antisymmetric.

According to the above symmetries, one can categorize the *P*-wave bottom baryons into eight baryon multiplets, as shown in Fig. 1. We denote these multiplets as $[F(\text{flavor}), j_l, s_l, \rho/\lambda]$, with j_l the total angular momentum of the light components $(j_l = l_\lambda \otimes l_\rho \otimes s_l)$. Every multiplet contains one or two bottom baryons, whose total angular momenta are $j = j_l \otimes s_b = |j_l \pm 1/2|$, with s_b the spin of the bottom quark. Especially, the heavy quark effective theory tells that the bottom baryons inside the same doublet with $j = j_l - 1/2$ and $j = j_l + 1/2$ have similar masses.

In this section we investigate *S*-wave decays of flavor $\mathbf{6}_F$ *P*-wave bottom baryons into ground-state bottom baryons and vector mesons. To do this we use the method of lightcone sum rules within HQET, and investigate the following decay channels (the coefficients at right hand sides are isospin factors):

$$(a1) \Gamma \Big[\Sigma_b [1/2^-] \to \Lambda_b + \rho \to \Lambda_b + \pi + \pi \Big]$$

= $\Gamma \Big[\Sigma_b^- [1/2^-] \to \Lambda_b^0 + \pi^0 + \pi^- \Big],$ (1)
$$(a2) \Gamma \Big[\Sigma_b [1/2^-] \to \Sigma_b + \rho \to \Sigma_b + \pi + \pi \Big]$$

= $2 \times \Gamma \Big[\Sigma_b^- [1/2^-] \to \Sigma_b^0 + \pi^0 + \pi^- \Big],$ (2)

$$(a3) \Gamma \Big[\Sigma_b [1/2^-] \to \Sigma_b^* + \rho \to \Sigma_b^* + \pi + \pi \Big]$$
$$= 2 \times \Gamma \Big[\Sigma_c^- [1/2^-] \to \Sigma_c^{*0} + \pi^0 + \pi^- \Big]. \tag{3}$$

$$= 2 \times I \left[\mathcal{Z}_{b} \left[1/2^{-} \right] \rightarrow \mathcal{Z}_{b} + \pi + \pi \right], \tag{3}$$
$$(b1) \left[\mathcal{Z}_{b} \left[1/2^{-} \right] \rightarrow \mathcal{Z}_{b} + \rho \rightarrow \mathcal{Z}_{b} + \pi + \pi \right]$$

$$= \frac{3}{2} \times \Gamma \Big[\Xi_b^{\prime-} [1/2^-] \to \Xi_b^0 + \pi^0 + \pi^- \Big], \tag{4}$$

$$= \frac{3}{2} \times \Gamma \Big[\Xi_{b}^{\prime}[1/2^{-}] \to \Lambda_{b}^{0} + K^{0} + \pi^{-} \Big],$$
(5)

$$(b3) \Gamma \Big[\Xi'_{b} [1/2^{-}] \to \Xi'_{b} + \rho \to \Xi'_{b} + \pi + \pi \Big] \\= \frac{3}{2} \times \Gamma \Big[\Xi'_{b} [1/2^{-}] \to \Xi'^{0}_{b} + \pi^{0} + \pi^{-} \Big],$$
(6)

$$\mathbf{\bar{3}}_{\mathbf{C}}(\mathbf{A}) \begin{pmatrix} l_{p}=1\ (\mathbf{A}) \\ l_{\lambda}=0 \end{pmatrix} (\mathbf{A}) \longrightarrow \mathbf{6}_{\mathbf{F}}(\mathbf{S}) \longrightarrow j_{l}=1: \Sigma_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{6}_{\mathbf{F}},1,0,\rho] \\ \mathbf{J}_{l}=0: & \Lambda_{b0}\left(\frac{1}{2}\right) & \Xi_{b0}\left(\frac{1}{2}\right) & [\mathbf{3}_{\mathbf{F}},0,1,\rho] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{3}_{\mathbf{F}},1,1,\rho] \\ \mathbf{J}_{l}=2: & \Lambda_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & \Xi_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{3}_{\mathbf{F}},2,1,\rho] \\ \mathbf{J}_{l}=0: & \Sigma_{b0}\left(\frac{1}{2}\right) & \Xi_{b0}\left(\frac{1}{2}\right) & \Omega_{b0}\left(\frac{1}{2}\right) & [\mathbf{6}_{\mathbf{F}},0,1,\lambda] \\ \mathbf{J}_{l}=0: & \Sigma_{b0}\left(\frac{1}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{6}_{\mathbf{F}},0,1,\lambda] \\ \mathbf{J}_{l}=0: & \Sigma_{b0}\left(\frac{1}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{6}_{\mathbf{F}},0,1,\lambda] \\ \mathbf{J}_{l}=0: & \Sigma_{b0}\left(\frac{1}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{6}_{\mathbf{F}},0,1,\lambda] \\ \mathbf{J}_{l}=0: & \Sigma_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Sigma_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & \Xi_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & \Omega_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Omega_{b2}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Sigma_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Sigma_{b1}\left(\frac{3}{2},\frac{5}{2}\right) & [\mathbf{6}_{\mathbf{F}},2,1,\lambda] \\ \mathbf{J}_{l}=0: & \Lambda_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Xi_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Sigma_{b1}\left(\frac{1}{2},\frac{3}{2}\right) & \Sigma_{$$

Fig. 1 Categorization of *P*-wave bottom baryons. Taken from Ref. [1]

$$(b4) \Gamma \left[\Xi_{b}^{\prime}[1/2^{-}] \rightarrow \Sigma_{b} + K^{*} \rightarrow \Sigma_{b} + K + \pi \right]$$
$$= \frac{9}{2} \times \Gamma \left[\Xi_{b}^{\prime-}[1/2^{-}] \rightarrow \Sigma_{b}^{0} + K^{0} + \pi^{-} \right], \tag{7}$$
$$(b5) \Gamma \left[\Xi_{b}^{\prime}[1/2^{-}] \rightarrow \Xi_{b}^{*} + \rho \rightarrow \Xi_{b}^{*} + \pi + \pi \right]$$

$$= \frac{3}{2} \times \Gamma \Big[\Xi_b^{\prime -} [1/2^-] \to \Xi_b^{*0} + \pi^0 + \pi^- \Big], \tag{8}$$

$$(b6) \Gamma \left[\Xi_{b}^{\prime} [1/2^{-}] \rightarrow \Sigma_{b}^{*} + K^{*} \rightarrow \Sigma_{b}^{*} + K + \pi \right]$$

$$= \frac{9}{2} \times \Gamma \left[\Xi_{b}^{\prime-} [1/2^{-}] \rightarrow \Sigma_{b}^{*0} + K^{0} + \pi^{-} \right],$$

$$(9)$$

$$(c1) \Gamma \Big[\Omega_b [1/2^-] \to \Xi_b + K^* \to \Xi_b + K + \pi \Big]$$

= $3 \times \Gamma \Big[\Omega_b^- [1/2^-] \to \Xi_b^0 + K^0 + \pi^- \Big],$ (10)

$$(c2) \Gamma \Big[\Omega_b [1/2^-] \to \Xi'_b + K^* \to \Xi'_b + K + \pi \Big] = 3 \times \Gamma \Big[\Omega_b^- [1/2^-] \to \Xi'^0_b + K^0 + \pi^- \Big],$$
(11)

$$(c3) \Gamma \Big[\Omega_b [1/2^-] \to \Xi_b^* + K^* \to \Xi_b^* + K + \pi \Big] = 3 \times \Gamma \Big[\Omega_b^- [1/2^-] \to \Xi_b^{*0} + K^0 + \pi^- \Big],$$
(12)

$$(d1) \Gamma \Big[\Sigma_b[3/2^-] \to \Lambda_b + \rho \to \Lambda_b + \pi + \pi \Big]$$

$$= \Gamma \left[\Sigma_b^{-} [3/2^-] \to \Lambda_b^0 + \pi^0 + \pi^- \right], \tag{13}$$

$$(d2) \Gamma \left[\Sigma_b [3/2^-] \to \Sigma_b + a \to \Sigma_b + \pi + \pi \right]$$

$$(a2) I \left[\Sigma_b[3/2^-] \to \Sigma_b + \rho \to \Sigma_b + \pi + \pi \right]$$
$$= 2 \times \Gamma \left[\Sigma_b^-[3/2^-] \to \Sigma_b^0 + \pi^0 + \pi^- \right], \tag{14}$$

$$(d3) \Gamma \Big[\Sigma_b[3/2^-] \to \Sigma_b^* + \rho \to \Sigma_b^* + \pi + \pi \Big]$$
$$= 2 \times \Gamma \Big[\Sigma_c^{-}[3/2^-] \to \Sigma_c^{*0} + \pi^0 + \pi^- \Big]. \tag{1}$$

$$= 2 \times \Gamma \left[\Sigma_b^{-} [3/2^-] \to \Sigma_b^{*0} + \pi^0 + \pi^- \right], \qquad (15)$$

$$(e1) \Gamma \left[\Xi_b' [3/2^-] \to \Xi_b + \rho \to \Xi_b + \pi + \pi \right]$$

$$= \frac{5}{2} \times \Gamma \left[\Xi_b^{\prime -} [3/2^-] \to \Xi_b^0 + \pi^0 + \pi^- \right],$$
(16)
(e2) $\Gamma \left[\Xi_b^{\prime} [3/2^-] \to \Lambda_b + K^* \to \Lambda_b + K + \pi \right]$

$$= \frac{3}{2} \times \Gamma \Big[\Xi_b^{\prime-} [3/2^-] \to \Lambda_b^0 + K^0 + \pi^- \Big],$$
(17)
(e3) $\Gamma \Big[\Xi_b^{\prime} [3/2^-] \to \Xi_b^{\prime} + \rho \to \Xi_b^{\prime} + \pi + \pi \Big]$

$$= \frac{3}{2} \times \Gamma \Big[\Xi_b^{\prime -} [3/2^-] \to \Xi_b^{\prime 0} + \pi^0 + \pi^- \Big],$$
(18)

$$(e4) \Gamma \left[\Xi'_{b}[3/2^{-}] \to \Sigma'_{b} + K^{*} \to \Sigma'_{b} + K + \pi \right]$$
$$= \frac{9}{2} \times \Gamma \left[\Xi'_{b}[3/2^{-}] \to \Sigma'^{0}_{b} + K^{0} + \pi^{-} \right], \qquad (19)$$

$$(e5) \Gamma \Big[\Xi_b'[3/2^-] \to \Xi_b^* + \rho \to \Xi_b^* + \pi + \pi \Big] \\= \frac{3}{2} \times \Gamma \Big[\Xi_b'^-[3/2^-] \to \Xi_b^{*0} + \pi^0 + \pi^- \Big],$$
(20)

$$(e6) \Gamma \left[\Xi_{b}'[3/2^{-}] \to \Sigma_{b}^{*} + K^{*} \to \Sigma_{b}^{*} + K + \pi \right] \\ = \frac{9}{2} \times \Gamma \left[\Xi_{b}'^{-}[3/2^{-}] \to \Sigma_{b}^{*0} + K^{0} + \pi^{-} \right],$$
(21)

$$(f1) \Gamma \Big[\Omega_b[3/2^-] \to \Xi_b + K^* \to \Xi_b + K + \pi \Big]$$

$$2 \mapsto \Gamma \Big[\Omega_b[3/2^-] \to \Xi_b + K^* \to \Xi_b + K + \pi \Big]$$

$$(22)$$

$$= 3 \times \Gamma \left[\Omega_{b} \left[3/2^{-} \right] \rightarrow \Xi_{b}^{+} + K^{*} + \pi^{-} \right], \qquad (22)$$

$$(f2) \Gamma \left[\Omega_{b} \left[3/2^{-} \right] \rightarrow \Xi_{b}^{+} + K^{*} \rightarrow \Xi_{b}^{+} + K + \pi^{-} \right]$$

$$= 3 \times \Gamma \left[\Omega_b^{-}[3/2^-] \to \Xi_b^{\prime 0} + K^0 + \pi^- \right], \qquad (23)$$
$$(f3) \Gamma \left[\Omega_b[3/2^-] \to \Xi_b^* + K^* \to \Xi_b^* + K + \pi \right]$$

$$= 3 \times \Gamma \Big[\Omega_b^-[3/2^-] \to \Xi_b^{*0} + K^0 + \pi^- \Big],$$
(24)

$$(g1) \Gamma \Big[\Sigma_b [5/2^-] \to \Sigma_b^* + \rho \to \Sigma_b^* + \pi \pi \Big]$$
$$= 2 \times \Gamma \Big[\Sigma_b^- [5/2^-] \to \Sigma_b^{*0} + \pi^0 + \pi^- \Big], \qquad (25)$$

$$(h1) \Gamma \Big[\Xi_b'[5/2^-] \to \Sigma_b^* + K^* \to \Sigma_b^* + K + \pi \Big] \\= \frac{9}{2} \times \Gamma \Big[\Xi_b'^-[5/2^-] \to \Sigma_b^{*0} + K^0 + \pi^- \Big],$$
(26)

$$(h2) \Gamma \Big[\Xi'_{b} [5/2^{-}] \to \Xi^{*}_{b} + \rho \to \Xi^{*}_{b} + \pi + \pi \Big] \\= \frac{3}{2} \times \Gamma \Big[\Xi^{\prime -}_{b} [5/2^{-}] \to \Xi^{*0}_{b} + \pi^{0} + \pi^{-} \Big],$$
(27)

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$$(i1) \Gamma \Big[\Omega_b [5/2^-] \to \Xi_b^* + K^* \to \Xi_b^* + K + \pi \Big] = 3 \times \Gamma \Big[\Omega_b^- [5/2^-] \to \Xi_b^{*0} + K^0 + \pi^- \Big].$$
(28)

We can calculate their decay widths through the following Lagrangians

$$\begin{aligned} \mathscr{L}_{X_{b}(1/2^{-}) \to Y_{b}(1/2^{+})V} &= g\bar{X}_{b}(1/2^{-})\gamma_{\mu}\gamma_{5}Y_{b}(1/2^{+})V^{\mu}, \\ \mathscr{L}_{X_{b}(1/2^{-}) \to Y_{b}(3/2^{+})V} &= g\bar{X}_{b}(1/2^{-})Y_{b}^{\mu}(3/2^{+})V_{\mu}, \\ \mathscr{L}_{X_{b}(3/2^{-}) \to Y_{b}(1/2^{+})V} &= g\bar{X}_{b}^{\mu}(3/2^{-})Y_{b}(1/2^{+})V_{\mu}, \\ \mathscr{L}_{X_{b}(3/2^{-}) \to Y_{b}(3/2^{+})V} &= g\bar{X}_{b}^{\nu}(3/2^{-})\gamma_{\mu}\gamma_{5}Y_{b\nu}(3/2^{+})V^{\mu}, \\ \mathscr{L}_{X_{b}(5/2^{-}) \to Y_{b}(3/2^{+})V} &= g\bar{X}_{b}^{\mu\nu}(5/2^{-})Y_{b\mu}(3/2^{+})V_{\nu} \\ &\quad + g\bar{X}_{b}^{\nu\mu}(5/2^{-})Y_{b\mu}(3/2^{+})V_{\nu}, \end{aligned}$$

where $X_b^{(\mu\nu)}$, $Y_b^{(\mu)}$, and V^{μ} denotes the *P*-wave bottom baryon, ground-state bottom baryon, and vector meson, respectively.

As an example, we study the *S*-wave decay of the $\Sigma_b^-(1/2^-)$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ into $\Lambda_b^0(1/2^+)$ and $\rho^-(1^-)$ in the next subsection, and investigate the four bottom baryon multiplets, $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$, and $[\mathbf{6}_F, 2, 1, \lambda]$ separately in the following subsections.

2.1
$$\Sigma_b^-(1/2^-)$$
 of [**6**_{*F*}, 1, 0, ρ] decaying into $\Lambda_b^0(1/2^+)$
and $\rho^-(1^-)$

In this subsection we study the *S*-wave decay of the $\Sigma_b^-(1/2^-)$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ into $\Lambda_b^0(1/2^+)$ and $\rho^-(1^-)$. To do this we consider the following three-point correlation function:

$$\begin{aligned} \Pi(\omega, \, \omega') \\ &= \int d^4 x \, e^{-ik \cdot x} \, \langle 0 | J_{1/2, -, \Sigma_b^-, 1, 0, \rho}(0) \bar{J}_{A_b^0}(x) | \rho^-(q) \rangle \\ &= \frac{1 + \psi}{2} G_{\Sigma_b^-[\frac{1}{2}^-] \to A_b^0 \rho^-}(\omega, \, \omega'), \end{aligned}$$
(29)

where $J_{1/2,-,\Sigma_b^-,1,0,\rho}$ and $J_{A_b^0}$ are the interpolating fields coupling to $\Sigma_b^-(1/2^-)$ and Λ_b^0 :

$$J_{1/2,-,\Sigma_{b}^{-},1,0,\rho} = i\epsilon_{abc} \Big([\mathscr{D}_{t}^{\mu}d^{aT}]C\gamma_{5}d^{b} - d^{aT}C\gamma_{5} [\mathscr{D}_{t}^{\mu}d^{b}] \Big) \gamma_{t}^{\mu}\gamma_{5}h_{v}^{c},$$

$$(30)$$

$$J_{A_{b}^{0}} = \epsilon_{abc} [u^{aT}C\gamma_{5}d^{b}]h_{v}^{c},$$

$$(31)$$

We refer to Refs. [58,96], where we systematically constructed all the *S*- and *P*-wave heavy baryon interpolating fields. In the above expressions $h_v(x)$ is the heavy quark field; k' = k + q, with k', k, and q the momenta of the $\Sigma_b^-(1/2^-)$, Λ_b^0 , and ρ^- , respectively; $\omega = v \cdot k$ and $\omega' = v \cdot k'$. Note that the definitions of ω and ω' in the present study are the same as those used in Refs. [1,60], but different from those used in Refs. [58,59]. At the hadronic level, we write $G_{\Sigma_b^-[\frac{1}{2}^-] \to A_b^0 \rho^-}$ as:

$$G_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \to \Lambda_{b}^{0}\rho^{-}}(\omega, \omega') = g_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \to \Lambda_{b}^{0}\rho^{-}} \times \frac{f_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]}f_{\Lambda_{b}^{0}}}{(\bar{\Lambda}_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]} - \omega')(\bar{\Lambda}_{\Lambda_{b}^{0}} - \omega)}.$$
 (32)

At the quark and gluon level, we calculate $G_{\Sigma_b^-[\frac{1}{2}^-] \to \Lambda_b^0 \rho^-}$ using the method of operator product expansion (OPE):

$$\begin{split} G_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \rightarrow A_{b}^{0}\rho^{-}}(\omega, \omega') \\ &= \int_{0}^{\infty} dt \int_{0}^{1} du e^{i(1-u)\omega't} e^{iu\omega t} \times 8 \\ &\times \left(\frac{if_{\rho}^{\perp}m_{\rho}^{2}}{48} \langle \bar{q}q \rangle \psi_{3;\rho}^{||}(u) + \frac{if_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}t^{4}(v \cdot q)^{2}} \phi_{2;\rho}^{||}(u) \right. \\ &- \frac{if_{\rho}^{||}m_{\rho}^{3}}{4\pi^{2}t^{4}(v \cdot q)^{2}} \phi_{3;\rho}^{\perp}(u) + \frac{if_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}t^{4}(v \cdot q)^{2}} \psi_{4;\rho}^{||}(u) \\ &- \frac{if_{\rho}^{||}m_{\rho}}{4\pi^{2}t^{4}} \phi_{2;\rho}^{||}(u) + \frac{if_{\rho}^{||}m_{\rho}}{2\pi^{2}t^{4}} \phi_{3;\rho}^{\perp}(u) \\ &+ \frac{if_{\rho}^{\perp}m_{\rho}^{2}t^{2}}{768} \langle g_{s}\bar{q}\sigma Gq \rangle \psi_{3;\rho}^{||}(u) - \frac{if_{\rho}^{||}m_{\rho}^{3}}{64\pi^{2}t^{2}} \phi_{4;\rho}^{||}(u) \Big) \\ &- \int_{0}^{\infty} dt \int_{0}^{1} du \int \mathscr{D}\underline{\alpha} e^{i\omega' t(\alpha_{2}+u\alpha_{3})} e^{i\omega t(1-\alpha_{2}-u\alpha_{3})} \\ &\times \left(\frac{if_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}t^{2}} \Phi_{4;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}t^{2}} \widetilde{\Phi}_{4;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}^{3}}{16\pi^{2}t^{2}} \Psi_{4;\rho}^{||}(\underline{\alpha}) \right) \\ &+ \frac{if_{\rho}^{||}m_{\rho}^{3}}{16\pi^{2}t^{2}} \widetilde{\Psi}_{4;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}^{3}u}{4\pi^{2}t^{2}} \Phi_{4;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}^{3}u}{8\pi^{2}t^{2}} \Psi_{4;\rho}^{||}(\underline{\alpha}) \Big). \end{split}$$

After Wick rotations and making double Borel transformation with the variables ω and ω' to be T_1 and T_2 , we obtain

$$\begin{split} g_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \to \Lambda_{b}^{0}\rho^{-}} f_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]} f_{\Lambda_{b}^{0}} e^{-\frac{\tilde{\lambda}_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]}}{T_{1}}} e^{-\frac{\tilde{\lambda}_{A_{b}^{0}}}{T_{2}}} \\ &= 8 \times \left(\frac{-f_{\rho}^{\perp} m_{\rho}^{2}}{48} \langle \bar{q}q \rangle T f_{0} \left(\frac{\omega_{c}}{T} \right) \psi_{3;\rho}^{\parallel}(u_{0}) \right. \\ &\left. - \frac{f_{\rho}^{\parallel} m_{\rho}^{3}}{8\pi^{2}} T^{3} f_{2} \left(\frac{\omega_{c}}{T} \right) \int_{0}^{u_{0}} du_{1} \int_{0}^{u_{1}} du_{2} \phi_{2;\rho}^{\parallel}(u_{2}) \right. \\ &\left. + \frac{f_{\rho}^{\parallel} m_{\rho}^{3}}{4\pi^{2}} T^{3} f_{2} \left(\frac{\omega_{c}}{T} \right) \int_{0}^{u_{0}} du_{1} \int_{0}^{u_{1}} du_{2} \phi_{3;\rho}^{\perp}(u_{2}) \right. \\ &\left. - \frac{f_{\rho}^{\parallel} m_{\rho}^{3}}{8\pi^{2}} T^{3} f_{2} \left(\frac{\omega_{c}}{T} \right) \int_{0}^{u_{0}} du_{1} \int_{0}^{u_{1}} du_{2} \psi_{4;\rho}^{\parallel}(u_{2}) \right. \\ &\left. + \frac{f_{\rho}^{\parallel} m_{\rho}}{4\pi^{2}} T^{5} f_{4} \left(\frac{\omega_{c}}{T} \right) \phi_{2;\rho}^{\parallel}(u_{0}) - \frac{f_{\rho}^{\parallel} m_{\rho}}{2\pi^{2}} T^{5} f_{4} \left(\frac{\omega_{c}}{T} \right) \phi_{3;\rho}^{\parallel}(u_{0}) \right. \\ &\left. + \frac{f_{\rho}^{\perp} m_{\rho}^{2}}{768} \langle g_{s} \bar{q} \sigma G q \rangle \frac{1}{T} \psi_{3;\rho}^{\parallel}(u_{0}) - \frac{f_{\rho}^{\parallel} m_{\rho}^{3}}{64} T^{3} f_{2} \left(\frac{\omega_{c}}{T} \right) \phi_{4;\rho}^{\parallel}(u_{0}) \right) \end{split}$$

$$-\left(\frac{f_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\Phi_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}}{8\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\widetilde{\Phi}_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}}{16\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\Psi_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}}{16\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\widetilde{\Psi}_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}u}{4\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\Phi_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}u}{8\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\Psi_{4;\rho}^{||}(\underline{\alpha}) \\ +\frac{f_{\rho}^{||}m_{\rho}^{3}u}{8\pi^{2}}T^{3}f_{2}\left(\frac{\omega_{c}}{T}\right)\int_{0}^{\frac{1}{2}}d\alpha_{2}\int_{\frac{1}{2}-\alpha_{2}}^{1-\alpha_{2}}d\alpha_{3}\frac{1}{\alpha_{3}}\Psi_{4;\rho}^{||}(\underline{\alpha}) \\ (34)$$

Here $u_0 = \frac{T_1}{T_1+T_2}$, $T = \frac{T_1T_2}{T_1+T_2}$, and $f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$. Explicit forms of the light-cone distribution amplitudes contained in the above expression can be found in Refs. [79–86], and we work at the renormalization scale 2 GeV for the parameters involved. More sum rule examples can be found in Appendix B.

In the present study we work at the symmetric point $T_1 = T_2 = 2T$ so that $u_0 = \frac{1}{2}$. We use the following values for the bottom quark mass and various quark and gluon condensates [2,87–95]:

$$m_b = 4.66 \pm 0.03 \text{ GeV},$$

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3,$$

$$\langle \bar{s}s \rangle = (0.8 \pm 0.1) \times \langle \bar{q}q \rangle,$$

$$\langle g_s \bar{q}\sigma Gq \rangle = M_0^2 \times \langle \bar{q}q \rangle,$$

$$\langle g_s \bar{s}\sigma Gs \rangle = M_0^2 \times \langle \bar{s}s \rangle,$$

$$M_0^2 = 0.8 \text{ GeV}^2,$$

$$\langle g_s^2 GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4.$$
(35)

Now the coupling constant $g_{\Sigma_b^-[\frac{1}{2}^-] \to \Lambda_b^0 \rho^-}$ only depends on two free parameters, the threshold value ω_c and the Borel mass *T*. We choose $\omega_c = 1.485$ GeV to be the average of the threshold values of the $\Sigma_b(1/2^-)$ and Λ_b^0 mass sum rules (see Appendix A and Ref. [1] for the parameters of the $\Sigma_b(1/2^-)$ and Λ_b^0), and extract the coupling constant $g_{\Sigma_b^-[\frac{1}{2}^-] \to \Lambda_b^0 \rho^-}$ to be

$${}^{g}{}_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \to \Lambda_{b}^{0}\rho^{-}} = 0.17 {}^{+0.01}_{-0.01} {}^{+0.06}_{-0.05} {}^{+0.06}_{-0.05} {}^{+0.20}_{-0.17} {}^{+0.10}_{-0.11}, \quad (36)$$

where the uncertainties are due to the Borel mass, the parameters of the Λ_b^0 , the parameters of the $\Sigma_b^-(1/2^-)$, the lightcone distribution amplitudes of vector mesons [79–86], and various quark masses and condensates listed in Eq. (35), respectively. Besides these statistical uncertainties, there is another (theoretical) uncertainty, which comes from the scale dependence. In the present study we do not consider this, and simply work at the renormalization scale 2 GeV, since $\sqrt{M_{\Sigma_b^-(1/2^-)}^2 - M_{A_b^0}^2} = 2.4$ GeV. However, it is useful to give a rough estimation on this. Since the largest uncertainty comes from the light-cone distribution amplitudes of vector mesons, we choose the values for the parameters contained in these amplitudes to be at the renormalization scale 1 GeV (see Tables 1 and 2 of Ref. [86]), and redo the above calculations to obtain:

$$g_{\Sigma_b^-[\frac{1}{2}^-] \to \Lambda_b^0 \rho^-}(1 \text{ GeV}) = 0.42.$$
 (37)

Hence, the scale dependence leads to a significant uncertainty, and the total uncertainty of our results can be even larger.

For completeness, we show $g_{\Sigma_b^-[\frac{1}{2}^-] \to A_b^0 \rho^-}$ in Fig. 2a as a function of the Borel mass *T*, and find that it only slightly depends on the Borel mass, where the working region for *T* has been evaluated in the $\Sigma_b(1/2^-)$ mass sum rules [1,58– 60] to be 0.31 GeV < *T* < 0.34 GeV (we also summarize this in Table 5). Note that the definitions of ω and ω' in this paper are the same as those used in Refs. [1,60], but different from those used in Refs. [58,59], so the Borel windows used in this paper are also the same/similar as those used in Refs. [1,60], but just about half of those used in Refs. [58,59].

The two-body decay $\Sigma_b^-(1/2^-)$ $\rightarrow \Lambda_b^0 \rho^-$ is kinematically forbidden, but the three-body decay process $\Sigma_b^-(1/2^-) \rightarrow \Lambda_b^0 \rho^- \rightarrow \Lambda_b^0 \pi^0 \pi^-$ is kinematically allowed, whose decay amplitude is

$$\mathscr{M} (0 \to 3 + 4 \to 3 + 2 + 1) \equiv \mathscr{M} \left(\Sigma_b^-(1/2^-) \to \Lambda_b^0 + \rho^- \to \Lambda_b^0 + \pi^0 + \pi^- \right) = g_{0 \to 3+4} \times g_{4 \to 2+1} \times \bar{u}_0 \gamma_\mu \gamma_5 u_3 \times \left(g_{\mu\nu} - \frac{p_{4,\mu} p_{4,\nu}}{m_4^2} \right) \times \frac{1}{p_4^2 - m_4^2 + im_4 \Gamma_4} \times \left(p_{1,\nu} - p_{2,\nu} \right).$$
(38)

Here 0 denotes the initial state $\Sigma_b^-(1/2^-)$; 4 denotes the intermediate state ρ^- ; 1, 2 and 3 denote the finial states π^- , π^0 and Λ_b^0 , respectively.

This amplitude can be used to further calculate its decay width

$$\begin{split} \Gamma &(0 \to 3 + 4 \to 3 + 2 + 1) \\ &\equiv \Gamma \left(\Sigma_b^-(1/2^-) \to \Lambda_b^0 + \rho^- \to \Lambda_b^0 + \pi^0 + \pi^- \right) \\ &= \frac{1}{(2\pi)^3} \frac{1}{32m_0^3} \times g_{0 \to 3 + 4}^2 \times g_{4 \to 2 + 1}^2 \times \int dm_{12} dm_{23} \\ &\times \frac{1}{2} \operatorname{Tr} \Big[\left(\not p_3 + m_3 \right) \gamma_{\mu'} \gamma_5 \left(\not p_0 + m_0 \right) \gamma_{\mu} \gamma_5 \Big] \end{split}$$

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$$\times \left(g_{\mu\nu} - \frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right) \left(g_{\mu'\nu'} - \frac{p_{4,\mu'}p_{4,\nu'}}{m_4^2}\right) \\ \times \frac{\left(p_{1,\nu} - p_{2,\nu}\right)\left(p_{1,\nu'} - p_{2,\nu'}\right)}{|p_4^2 - m_4^2 + im_4\Gamma_4|^2},$$
(39)

so that the width of the $\Sigma_b^-(1/2^-) \to \Lambda_b^0 \rho^- \to \Lambda_b^0 \pi^0 \pi^-$ decay is evaluated to be

$$I \Sigma_{b}^{-} [\frac{1}{2}^{-}] \rightarrow \Lambda_{b}^{0} \rho^{-} \rightarrow \Lambda_{b}^{0} \pi^{0} \pi^{-}$$

= 8.9 $^{+32.5}_{-8.9}$ keV = 8.9 $^{+1.1}_{-0.0}$ $^{+6.7}_{-5.0}$ $^{+27.9}_{-4.5}$ $^{+13.6}_{-8.9}$ keV. (40)

In the following subsections we apply the same procedures to separately study the four bottom baryon multiplets, $[\mathbf{6}_F, 1, 0, \rho], [\mathbf{6}_F, 0, 1, \lambda], [\mathbf{6}_F, 1, 1, \lambda], \text{ and } [\mathbf{6}_F, 2, 1, \lambda].$

2.2 The bottom baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$

The bottom baryon doublet $[6_F, 1, 0, \rho]$ consists of six members: $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi'_b(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. We use the method of light-cone sum rules within HQET to study their decays into ground-state bottom baryons and vector mesons.

There are altogether twenty-four non-vanishing decay channels, whose coupling constants are extracted to be

$$\begin{aligned} (a1) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.17, \\ (a2) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.35, \\ (a3) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.20, \\ (b1) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.03, \\ (b2) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.03, \\ (b3) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.24, \\ (b4) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.38, \\ (b5) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.38, \\ (b5) g_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.22, \\ (c1) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.22, \\ (c1) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.24, \\ (c2) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\kappa}} &= 0.28, \\ (d1) g_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.29, \\ (d2) g_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.24, \\ (e1) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.24, \\ (e2) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.42, \\ (e3) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.42, \\ (e3) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} &= 0.14, \\ (e3) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}$$

$$\begin{aligned} & (e4) \ g_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Sigma_{b}\left[\frac{1}{2}^{+}\right]K^{*}} = 0.22, \\ & (e5) \ g_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Xi_{b}^{*}\left[\frac{3}{2}^{+}\right]\rho} = 0.16, \\ & (e6) \ g_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Sigma_{b}^{*}\left[\frac{3}{2}^{+}\right]K^{*}} = 0.25, \\ & (f1) \ g_{\Omega_{b}\left[\frac{3}{2}^{-}\right] \to \Xi_{b}\left[\frac{1}{2}^{+}\right]K^{*}} = 0.34, \\ & (f2) \ g_{\Omega_{b}\left[\frac{3}{2}^{-}\right] \to \Xi_{b}^{'}\left[\frac{1}{2}^{+}\right]K^{*}} = 0.28, \\ & (f3) \ g_{\Omega_{b}\left[\frac{3}{2}^{-}\right] \to \Xi_{b}^{*}\left[\frac{3}{2}^{+}\right]K^{*}} = 0.33. \end{aligned}$$

Then we compute the three-body decay widths, which are kinematically allowed:

$$\begin{array}{l} (a1) \ \Gamma_{\Sigma_{b}[\frac{1}{2}^{-}] \rightarrow A_{b}[\frac{1}{2}^{+}] \rho \rightarrow A_{b}[\frac{1}{2}^{+}] \pi \pi} = 8.9 \ \mathrm{keV}, \\ (a2) \ \Gamma_{\Sigma_{b}[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}[\frac{1}{2}^{+}] \rho \rightarrow \Sigma_{b}[\frac{1}{2}^{+}] \pi \pi} = 2.2 \times 10^{-3} \ \mathrm{keV}, \\ (b1) \ \Gamma_{\Xi_{b}'[\frac{1}{2}^{-}] \rightarrow \Xi_{b}[\frac{1}{2}^{+}] \rho \rightarrow \Xi_{b}[\frac{1}{2}^{+}] \pi \pi} = 0.2 \ \mathrm{keV}, \\ (b3) \ \Gamma_{\Xi_{b}'[\frac{1}{2}^{-}] \rightarrow \Xi_{b}'[\frac{1}{2}^{+}] \rho \rightarrow \Xi_{b}'[\frac{1}{2}^{+}] \pi \pi} = 5.1 \times 10^{-3} \ \mathrm{keV}, \\ (d1) \ \Gamma_{\Sigma_{b}[\frac{3}{2}^{-}] \rightarrow A_{b}[\frac{1}{2}^{+}] \rho \rightarrow A_{b}[\frac{1}{2}^{+}] \pi \pi} = 4.1 \ \mathrm{keV}, \\ (d2) \ \Gamma_{\Sigma_{b}[\frac{3}{2}^{-}] \rightarrow \Sigma_{b}[\frac{1}{2}^{+}] \rho \rightarrow \Sigma_{b}[\frac{1}{2}^{+}] \pi \pi} = 2.1 \times 10^{-4} \ \mathrm{keV}, \\ (e1) \ \Gamma_{\Xi_{b}'[\frac{3}{2}^{-}] \rightarrow \Xi_{b}[\frac{1}{2}^{+}] \rho \rightarrow \Xi_{b}[\frac{1}{2}^{+}] \pi \pi} = 0.2 \ \mathrm{keV}, \\ (e3) \ \Gamma_{\Xi_{b}'[\frac{3}{2}^{-}] \rightarrow \Xi_{b}'[\frac{1}{2}^{+}] \rho \rightarrow \Xi_{b}[\frac{1}{2}^{+}] \pi \pi} = 5.8 \times 10^{-4} \ \mathrm{keV}.$$

We summarize these results in Table 1. For completeness, we also show the coupling constants as functions of the Borel mass T in Fig. 2.

2.3 The bottom baryon singlet $[\mathbf{6}_F, 0, 1, \lambda]$

The bottom baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$ consists of three members: $\Sigma_b(\frac{1}{2}), \Xi'_b(\frac{1}{2})$, and $\Omega_b(\frac{1}{2})$. We use the method of light-cone sum rules within HQET to study their decays into ground-state bottom baryons and vector mesons.

There are altogether eight non-vanishing decay channels, whose coupling constants are extracted to be

$$\begin{array}{l} (a2) \ g_{\Sigma_{b}[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} = 2.25, \\ (a3) \ g_{\Sigma_{b}[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{*}[\frac{3}{2}^{+}]_{\rho}} = 7.74, \\ (b3) \ g_{\Sigma_{b}'[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{*}[\frac{1}{2}^{+}]_{\rho}} = 1.58, \\ (b4) \ g_{\Xi_{b}'[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 1.77, \\ (b5) \ g_{\Xi_{b}'[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{*}[\frac{3}{2}^{+}]_{\rho}} = 5.70, \\ (b6) \ g_{\Xi_{b}'[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{*}[\frac{3}{2}^{+}]_{K^{*}}} = 5.77, \\ (c2) \ g_{\Omega_{b}[\frac{1}{2}^{-}] \rightarrow \Xi_{b}'[\frac{1}{2}^{+}]_{K^{*}}} = 2.37, \\ (c3) \ g_{\Omega_{b}[\frac{1}{2}^{-}] \rightarrow \Xi_{b}'[\frac{3}{2}^{+}]_{K^{*}}} = 8.18. \end{array}$$

Then we compute the three-body decay widths, which are kinematically allowed:

(a2)
$$\Gamma_{\Sigma_b[\frac{1}{2}^-] \to \Sigma_b[\frac{1}{2}^+]\rho \to \Sigma_b[\frac{1}{2}^+]\pi\pi} = 5.9 \times 10^{-2} \text{ keV},$$



Fig. 2 The coupling constants **a** $g_{\Sigma_b^-[\frac{1}{2}^-] \to A_b^0 \rho^-}$, **b** $g_{\Sigma_b^-[\frac{1}{2}^-] \to \Sigma_b^0 \rho^-}$, **c** $g_{\Xi_b^{'-}[\frac{1}{2}^-] \to \Xi_b^0 \rho^-}$, **d** $g_{\Xi_b^{'-}[\frac{1}{2}^-] \to \Xi_b^0 \rho^-}$, **e** $g_{\Sigma_b^-[\frac{3}{2}^-] \to A_b^0 \rho^-}$, **f** $g_{\Sigma_b^-[\frac{3}{2}^-] \to \Sigma_b^0 \rho^-}$, **g** $g_{\Xi_b^{'-}[\frac{3}{2}^-] \to \Xi_b^0 \rho^-}$, and **h** $g_{\Xi_b^{'-}[\frac{3}{2}^-] \to \Xi_b^0 \rho^-}$ as functions

of the Borel mass *T*. Here the baryons $\Sigma_b(\frac{1}{2}^{-}/\frac{3}{2}^{-})$ and $\Xi'_b(\frac{1}{2}^{-}/\frac{3}{2}^{-})$ belong to the bottom baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$, and the working regions for *T* have been evaluated in mass sum rules [1,58-60] and summarized in Table 5

Decay channels	Coupling constant g	Partial width	Total width	
(a1) $\Sigma_b(\frac{1}{2}) \to \Lambda_b(\frac{1}{2}) \rho \to \Lambda_b(\frac{1}{2})\pi\pi$	0.17	8.9 keV	8.9 MeV	
(a2) $\Sigma_b(\frac{1}{2}) \to \Sigma_b(\frac{1}{2}) \rho \to \Sigma_b(\frac{1}{2}) \pi \pi$	0.35	$2.2 \times 10^{-3} \text{ keV}$		
(b1) $\Xi_b'(\frac{1}{2}) \to \Xi_b(\frac{1}{2}) \rho \to \Xi_b(\frac{1}{2}) \pi \pi$	0.03	0.2 keV	0.2 keV	
(b3) $\Xi'_{b}(\frac{1}{2}) \to \Xi'_{b}(\frac{1}{2}) \rho \to \Xi'_{b}(\frac{1}{2})\pi\pi$	0.24	$5.1 \times 10^{-3} \text{ keV}$		
(d1) $\Sigma_b(\frac{3}{2}^-) \to \Lambda_b(\frac{1}{2}^+)\rho \to \Lambda_b(\frac{1}{2}^+)\pi\pi$	0.29	4.1 keV	4.1 keV	
(d2) $\Sigma_b(\frac{3}{2}^-) \rightarrow \Sigma_b(\frac{1}{2}^+)\rho \rightarrow \Sigma_b(\frac{1}{2}^+)\pi\pi$	0.19	$2.1 \times 10^{-4} \text{ keV}$		
(e1) $\Xi_b'(\frac{3}{2}) \to \Xi_b(\frac{1}{2}) \rho \to \Xi_b(\frac{1}{2})\pi\pi$	0.06	0.2 keV	0.2 keV	
(e3) $\Xi'_b(\frac{3}{2}^-) \to \Xi'_b(\frac{1}{2}^+) \rho \to \Xi'_b(\frac{1}{2}^+) \pi \pi$	0.14	$5.8 \times 10^{-4} \text{ keV}$		

Table 1 *S*-wave decays of *P*-wave bottom baryons belonging to the doublet $[\mathbf{6}_F, 1, 0, \rho]$ into ground-state bottom baryons and vector mesons

Table 2 S-wave decays of P-wave bottom baryons belonging to the doublet $[6_F, 0, 1, \lambda]$ into ground-state bottom baryons and vector mesons

Decay channels	Coupling constant g	Partial width	Total width	
(a2) $\Sigma_b(\frac{1}{2}) \to \Sigma_b(\frac{1}{2}) \to \Sigma_b(\frac{1}{2}) \pi \pi$	2.24	$5.9 \times 10^{-2} \text{ keV}$	$5.9 \times 10^{-2} \text{ keV}$	
(b3) $\Xi'_b(\frac{1}{2}) \to \Xi'_b(\frac{1}{2}) \rho \to \Xi'_b(\frac{1}{2})\pi\pi$	1.58	0.2 keV	0.2 keV	

(b3)
$$\Gamma_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Xi'_{b}[\frac{1}{2}^{+}] \rho \to \Xi'_{b}[\frac{1}{2}^{+}] \pi \pi} = 0.2 \text{ keV}.$$
 (44)

We summarize these results in Table 2. For completeness, we also show the coupling constants as functions of the Borel mass T in Fig. 3.

2.4 The bottom baryon doublet $[\mathbf{6}_F, 1, 1, \lambda]$

The bottom baryon doublet $[\mathbf{6}_F, 1, 1, \lambda]$ consists of six members: $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi'_b(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. We use the method of light-cone sum rules within HQET to study their decays into ground-state bottom baryons and vector mesons.

There are altogether twenty-four non-vanishing decay channels, whose coupling constants are extracted to be

$$(a1) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]_{\rho}} = 0.83,$$

$$(a2) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} = 2.21,$$

$$(a3) g_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{3}{2}^{+}]_{\rho}} = 1.28,$$

$$(b1) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} = 0.59,$$

$$(b2) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 1.60,$$

$$(b3) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho}} = 1.45,$$

$$(b4) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 0.13,$$

$$(b5) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{3}{2}^{+}]_{\rho}} = 0.84,$$

$$(b6) g_{\Sigma'_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{3}{2}^{+}]_{K^{*}}} = 0.08,$$

$$(c1) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 0.17,$$

$$(c2) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \Sigma'_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 0.17,$$

$$\begin{aligned} (c3) g_{\Omega_{b}[\frac{1}{2}^{-}] \to \mathcal{E}_{b}^{*}[\frac{3}{2}^{+}]K^{*}} &= 0.10, \\ (d1) g_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]\rho} &= 1.62, \\ (d2) g_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]\rho} &= 0.90, \\ (d3) g_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]\rho} &= 1.48, \\ (e1) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}[\frac{1}{2}^{+}]\rho} &= 0.57, \\ (e2) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \Lambda_{b}[\frac{1}{2}^{+}]K^{*}} &= 1.31, \\ (e3) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{1}{2}^{+}]\rho} &= 0.84, \\ (e4) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{1}{2}^{+}]\rho} &= 0.08, \\ (e5) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{3}{2}^{+}]\rho} &= 0.97, \\ (e6) g_{\mathcal{E}_{b}^{'}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{3}{2}^{+}]K^{*}} &= 0.09, \\ (f1) g_{\Omega_{b}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{1}{2}^{+}]K^{*}} &= 0.10, \\ (f3) g_{\Omega_{b}[\frac{3}{2}^{-}] \to \mathcal{E}_{b}^{'}[\frac{3}{2}^{+}]K^{*}} &= 0.12. \end{aligned}$$

Then we compute the three-body decay widths, which are kinematically allowed:

$$\begin{array}{l} (a1) \ \Gamma_{\Sigma_{b}[\frac{1}{2}^{-}] \to A_{b}[\frac{1}{2}^{+}]\rho \to A_{b}[\frac{1}{2}^{+}]\pi\pi} = 207.4 \ \mathrm{keV}, \\ (a2) \ \Gamma_{\Sigma_{b}[\frac{1}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]\rho \to \Sigma_{b}[\frac{1}{2}^{+}]\pi\pi} = 5.7 \times 10^{-2} \ \mathrm{keV}, \\ (b1) \ \Gamma_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Xi_{b}[\frac{1}{2}^{+}]\rho \to \Xi_{b}[\frac{1}{2}^{+}]\pi\pi} = 61.0 \ \mathrm{keV}, \\ (b3) \ \Gamma_{\Xi'_{b}[\frac{1}{2}^{-}] \to \Xi'_{b}[\frac{1}{2}^{+}]\rho \to \Xi'_{b}[\frac{1}{2}^{+}]\pi\pi} = 0.2 \ \mathrm{keV}, \\ (d1) \ \Gamma_{\Sigma_{b}[\frac{3}{2}^{-}] \to A_{b}[\frac{1}{2}^{+}]\rho \to A_{b}[\frac{1}{2}^{+}]\pi\pi} = 261.1 \ \mathrm{keV}, \\ (d2) \ \Gamma_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]\rho \to \Sigma_{b}[\frac{1}{2}^{+}]\pi\pi} = 3.2 \times 10^{-3} \ \mathrm{keV}, \end{array}$$



Fig. 3 The coupling constants **a** $g_{\Sigma_b^-[\frac{1}{2}^-] \to \Sigma_b^0 \rho^-}$ and **b** $g_{\Xi_b^{'-}[\frac{1}{2}^-] \to \Xi_b^{'0} \rho^-}$ as functions of the Borel mass *T*. Here the baryons $\Sigma_b(\frac{1}{2}^-)$ and $\Xi_b^{'}(\frac{1}{2}^-)$ belong to the bottom baryon sin-

glet $[6_F, 0, 1, \lambda]$, and the working regions for *T* have been evaluated in mass sum rules [1,58-60] and summarized in Table 5

Table 3 S-wave decays of P-wave bottom baryons belonging to the doublet $[6_F, 1, 1, \lambda]$ into ground-state bottom baryons and vector mesons

Decay channels	Coupling constant g	Partial width	Total width	
(a1) $\Sigma_b(\frac{1}{2}) \to \Lambda_b(\frac{1}{2}) \to \Lambda_b(\frac{1}{2}) \to \Lambda_b(\frac{1}{2}) \pi \pi$	0.83	207.5 keV	207.4 keV	
(a2) $\Sigma_b(\frac{1}{2}^-) \to \Sigma_b(\frac{1}{2}^+) \rho \to \Sigma_b(\frac{1}{2}^+) \pi \pi$	2.21	$5.7 \times 10^{-2} \text{ keV}$		
(b1) $\Xi_b^{\prime}(\frac{1}{2}) \to \Xi_b(\frac{1}{2}) \rho \to \Xi_b(\frac{1}{2})\pi\pi$	0.59	61.0 keV	61.2 keV	
(b3) $\Xi'_{b}(\frac{1}{2}^{-}) \to \Xi'_{b}(\frac{1}{2}^{+})\rho \to \Xi'_{b}(\frac{1}{2}^{+})\pi\pi$	1.45	0.2 keV		
(d1) $\Sigma_b(\frac{3}{2}^-) \to \Lambda_b(\frac{1}{2}^+)\rho \to \Lambda_b(\frac{1}{2}^+)\pi\pi$	1.62	261.1 keV	261.1 keV	
(d2) $\Sigma_b(\frac{3}{2}^-) \rightarrow \Sigma_b(\frac{1}{2}^+)\rho \rightarrow \Sigma_b(\frac{1}{2}^+)\pi\pi$	0.90	$3.2 \times 10^{-3} \text{ keV}$		
(e1) $\Xi'_{b}(\frac{3}{2}) \to \Xi_{b}(\frac{1}{2}) \rho \to \Xi_{b}(\frac{1}{2})\pi\pi$	0.57	18.8 keV	18.8 keV	
(e3) $\Xi'_b(\frac{3}{2}^-) \to \Xi'_b(\frac{1}{2}^+)\rho \to \Xi'_b(\frac{1}{2}^+)\pi\pi$	0.84	$2.1 \times 10^{-2} \text{ keV}$		

(e1)
$$\Gamma_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Xi_{b}\left[\frac{1}{2}^{+}\right]\rho \to \Xi_{b}\left[\frac{1}{2}^{+}\right]\pi\pi} = 18.8 \text{ keV},$$

(e3) $\Gamma_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Xi'_{b}\left[\frac{1}{2}^{+}\right]\rho \to \Xi'_{b}\left[\frac{1}{2}^{+}\right]\pi\pi} = 2.1 \times 10^{-2} \text{ MeV}.$ (46)

We summarize these results in Table 3. For completeness, we also show the coupling constants as functions of the Borel mass T in Fig. 4.

2.5 The bottom baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$

The bottom baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$ consists of six members: $\Sigma_b(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi'_b(\frac{3}{2}^-/\frac{5}{2}^-)$, and $\Omega_b(\frac{3}{2}^-/\frac{5}{2}^-)$. We use the method of light-cone sum rules within HQET to study their decays into ground-state bottom baryons and vector mesons.

There are altogether twelve non-vanishing decay channels, whose coupling constants are extracted to be

$$(e^{3}) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Xi'_{b}[\frac{1}{2}^{+}]_{\rho}} = 4.23 + 2.42 - 2.13,$$

$$(e^{4}) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 3.17 + 2.37 - 2.20,$$

$$(e^{5}) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{\rho}} = 0.50 + 0.28 - 0.24,$$

$$(e^{6}) g_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Sigma^{*}_{b}[\frac{3}{2}^{+}]_{K^{*}}} = 0.40 + 0.28 - 0.26,$$

$$(f^{2}) g_{\Omega_{b}[\frac{3}{2}^{-}] \to \Xi^{*}_{b}[\frac{1}{2}^{+}]_{K^{*}}} = 4.56 + 3.19 - 0.26,$$

$$(f^{3}) g_{\Omega_{b}[\frac{3}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{K^{*}}} = 0.60 + 0.38 - 0.35,$$

$$(g^{1}) g_{\Sigma_{b}[\frac{5}{2}^{-}] \to \Sigma^{*}_{b}[\frac{3}{2}^{+}]_{\rho}} = 3.78 + 2.01 - 0.65,$$

$$(h^{1}) g_{\Xi'_{b}[\frac{5}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{\rho}} = 4.09 + 2.08 - 0.72,$$

$$(h^{2}) g_{\Xi'_{b}[\frac{5}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{K^{*}}} = 2.14 + 1.33,$$

$$(i^{1}) g_{\Omega_{b}[\frac{5}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{K^{*}}} = 6.36 + 3.75 - 0.72.$$

$$(47)$$

Then we compute the three-body decay widths, which are kinematically allowed:

(d2)
$$\Gamma_{\Sigma_b[\frac{3}{2}^-] \to \Sigma_b[\frac{1}{2}^+] \rho \to \Sigma_b[\frac{1}{2}^+] \pi \pi} = 0.14 \stackrel{+0.19}{_{-0.12}} \text{keV},$$



Fig. 4 The coupling constants **a** $g_{\Sigma_b^-[\frac{1}{2}^-] \to A_b^0 \rho^-}$, **b** $g_{\Sigma_b^-[\frac{1}{2}^-] \to \Sigma_b^0 \rho^-}$, **c** $g_{\Xi_b^{'-}[\frac{1}{2}^-] \to \Xi_b^0 \rho^-}$, **d** $g_{\Xi_b^{'-}[\frac{1}{2}^-] \to \Xi_b^0 \rho^-}$, **e** $g_{\Sigma_b^-[\frac{3}{2}^-] \to A_b^0 \rho^-}$, **f** $g_{\Sigma_b^-[\frac{3}{2}^-] \to \Sigma_b^0 \rho^-}$, **g** $g_{\Xi_b^{'-}[\frac{3}{2}^-] \to \Xi_b^0 \rho^-}$, and **h** $g_{\Xi_b^{'-}[\frac{3}{2}^-] \to \Xi_b^0 \rho^-}$ as functions

of the Borel mass *T*. Here the baryons $\Sigma_b(\frac{1}{2}^{-}/\frac{3}{2}^{-})$ and $\Xi'_b(\frac{1}{2}^{-}/\frac{3}{2}^{-})$ belong to the bottom baryon doublet $[\mathbf{6}_F, 1, 1, \lambda]$, and the working regions for *T* have been evaluated in mass sum rules [1,58-60] and summarized in Table 5

Table 4 S-wave decays of P-wave bottom baryons belonging to the doublet $[6_F, 2, 1, \lambda]$ into ground-state bottom baryons and vector mesons

Decay channels	Coupling constant g	Partial width	Total width
(d2) $\Sigma_b(\frac{3}{2}) \rightarrow \Sigma_b(\frac{1}{2}) \rho \rightarrow \Sigma_b(\frac{1}{2})\pi\pi$	$5.90 \stackrel{+3.56}{_{-3.08}}$	$0.14 \stackrel{+0.19}{_{-0.12}} \mathrm{keV}$	0.14 ^{+0.19} _{-0.12} keV
(e3) $\Xi_b'(\frac{3}{2}^-) \to \Xi_b'(\frac{1}{2}^+)\rho \to \Xi_b'(\frac{1}{2}^+)\pi\pi$	$4.23 {}^{+2.42}_{-2.13}$	0.53 ^{+0.68} _{-0.46} keV	$0.53 \ ^{+0.68}_{-0.46} \ \mathrm{keV}$
(g1) $\Sigma_b(\frac{5}{2}) \to \Sigma_b^*(\frac{3}{2}) \rho \to \Sigma_b^*(\frac{3}{2}) \pi \pi$	$3.78 \substack{+2.01 \\ -1.65}$	$(3^{+4}_{-2}) \times 10^{-6} \text{ keV}$	$(3^{+4}_{-2}) \times 10^{-6} \text{ keV}$
(h2) $\Xi_b'(\frac{5}{2}^-) \to \Xi_b^*(\frac{3}{2}^+)\rho \to \Xi_b^*(\frac{3}{2}^+)\pi\pi$	$4.09 {}^{+2.08}_{-1.72}$	$0.29 \stackrel{+0.32}{_{-0.23}} \mathrm{keV}$	$0.29 \stackrel{+0.32}{_{-0.23}} \mathrm{keV}$



Fig. 5 The coupling constants **a** $g_{\Sigma_b^-[\frac{3}{2}^-] \to \Sigma_b^0 \rho^-}$, **b** $g_{\Sigma_b^-[\frac{5}{2}^-] \to \Sigma_b^{*0} \rho^-}$, **c** $g_{\Xi_b^{'-}[\frac{3}{2}^-] \to \Xi_b^{*0} \rho^-}$, and **d** $g_{\Xi_b^{'-}[\frac{5}{2}^-] \to \Xi_b^{*0} \rho^-}$ as functions of the Borel mass *T*. Here the baryons $\Sigma_b(\frac{3}{2}^-/\frac{5}{2}^-)$ and $\Xi_b^{'}(\frac{3}{2}^-/\frac{5}{2}^-)$ belong to the

$$(e3) \Gamma_{\Xi'_{b}\left[\frac{3}{2}^{-}\right] \to \Xi'_{b}\left[\frac{1}{2}^{+}\right]\rho \to \Xi'_{b}\left[\frac{1}{2}^{+}\right]\pi\pi} = 0.53 \stackrel{+0.68}{_{-0.46}} \text{ keV},$$

$$(g1) \Gamma_{\Sigma_{b}\left[\frac{5}{2}^{-}\right] \to \Sigma^{*}_{b}\left[\frac{3}{2}^{+}\right]\rho \to \Sigma^{*}_{b}\left[\frac{3}{2}^{+}\right]\pi\pi} = \left(3 \stackrel{+4}{_{-2}}\right) \times 10^{-6} \text{ keV},$$

$$(h2) \Gamma_{\Xi'_{b}\left[\frac{5}{2}^{-}\right] \to \Xi^{*}_{b}\left[\frac{3}{2}^{+}\right]\rho \to \Xi^{*}_{b}\left[\frac{3}{2}^{+}\right]\pi\pi} = 0.29 \stackrel{+0.32}{_{-0.23}} \text{ keV}. \quad (48)$$

We summarize these results in Table 4. For completeness, we also show the coupling constants as functions of the Borel mass T in Fig. 5.

bottom baryon doublet [6_F , 2, 1, λ], and the working regions for T have been evaluated in mass sum rules [1,58–60] and summarized in Table 5

3 Summary and discussions

To summarize this paper, we have used the method of lightcone sum rules within heavy quark effective theory to study decay properties of *P*-wave bottom baryons belonging to the flavor **6**_{*F*} representation. We have studied their *S*-wave decays into ground-state bottom baryons and vector mesons. The possible decay channels are given in Eqs. (1–28), and the extracted decay widths are listed in Tables 1, 2, 3, and 4. These results are obtained separately for the four bottom baryon multiplets of flavor **6**_{*F*}: [**6**_{*F*}, 1, 0, ρ], [**6**_{*F*}, 0, 1, λ], [**6**_{*F*}, 1, 1, λ], and [**6**_{*F*}, 2, 1, λ]. In Ref. [1] we have studied the mass spectrum and pionic decay properties of the $\Sigma_b(6097)$ and $\Xi_b(6227)$ [7,8]. Our results suggest that they can be well interpreted as *P*wave bottom baryons of $J^P = 3/2^-$, belonging to the bottom baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$. This doublet contains altogether six bottom baryons, $\Sigma_b(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi'_b(\frac{3}{2}^-/\frac{5}{2}^-)$, and $\Omega_b(\frac{3}{2}^-/\frac{5}{2}^-)$. In the present study we further investigate their *S*-wave decays into ground-state bottom baryons and vector mesons, and extract:

$$(d2) \Gamma_{\Sigma_{b}[\frac{3}{2}^{-}] \to \Sigma_{b}[\frac{1}{2}^{+}]_{\rho \to \Sigma_{b}[\frac{1}{2}^{+}]_{\pi\pi}} = 0.14 \stackrel{+0.19}{_{-0.12}} \text{ keV},
(e3) \Gamma_{\Xi'_{b}[\frac{3}{2}^{-}] \to \Xi'_{b}[\frac{1}{2}^{+}]_{\rho \to \Xi'_{b}[\frac{1}{2}^{+}]_{\pi\pi}} = 0.53 \stackrel{+0.68}{_{-0.46}} \text{ keV},
(g1) \Gamma_{\Sigma_{b}[\frac{5}{2}^{-}] \to \Sigma^{*}_{b}[\frac{3}{2}^{+}]_{\rho \to \Sigma^{*}_{b}[\frac{3}{2}^{+}]_{\pi\pi}} = \left(3 \stackrel{+4}{_{-2}}\right) \times 10^{-6} \text{ keV},
(h2) \Gamma_{\Xi'_{b}[\frac{5}{2}^{-}] \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{\rho \to \Xi^{*}_{b}[\frac{3}{2}^{+}]_{\pi\pi}} = 0.29 \stackrel{+0.32}{_{-0.23}} \text{ keV}.$$

Hence, these three $J^P = 5/2^-$ states are probably quite narrow, because their *S*-wave decays into ground-state bottom baryons and pseudoscalar mesons can not happen, and widths of the following *D*-wave decays are also calculated to be zero in Ref. [1]:

$$(w') \Gamma_{\Sigma_{b}^{-}(5/2^{-}) \to \Lambda_{b}^{0}\pi^{-}} = 0,$$

$$(x') \Gamma_{\Xi_{b}^{'-}(5/2^{-}) \to \Xi_{b}^{0}\pi^{-}} = 0,$$

$$(y') \Gamma_{\Xi_{b}^{'-}(5/2^{-}) \to \Lambda_{b}^{0}K^{-}} = 0,$$

$$(z') \Gamma_{\Omega_{b}^{-}(5/2^{-}) \to \Xi_{b}^{0}K^{-}} = 0.$$
(49)

We suggest the LHCb and Belle/Belle-II experiments to search for these three narrow states. Especially, we propose to search for the $\Xi_b(5/2^-)$, that is the $J^P = 5/2^-$ partner state of the $\Xi_b(6227)$, in the $\Xi_b(5/2^-) \rightarrow \Xi_b^* \rho \rightarrow \Xi_b^* \pi \pi$ decay process. Its mass is 12 ± 5 MeV larger than that of the $\Xi_b(6227)$.

To end this work, we note that in the present study we have studied *S*-wave decays of flavor $\mathbf{6}_F P$ -wave heavy baryons into ground-state heavy baryons and vector mesons, which is actually a complement to Ref. [60], where we studied *S*wave decays of flavor $\mathbf{3}_F P$ -wave heavy baryons into groundstate heavy baryons together with pseudoscalar and vector mesons, and *S*-wave decays of flavor $\mathbf{6}_F P$ -wave heavy baryons into ground-state heavy baryons together with pseudoscalar mesons. To make a complete QCD sum rule studies of *P*-wave heavy baryons within HQET, we still need to systematically study their *D*-wave and radiative decay properties, which is currently under investigation.

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Appendix A: Parameters of *S*/*P*-wave bottom baryons and *S*-wave mesons

We list masses of ground-state bottom baryons used in the present study, taken from PDG [2]:

$$\begin{split} &\Lambda_b(1/2^+) \ : \ m = 5619.60 \ {\rm MeV}, \\ &\Xi_b(1/2^+) \ : \ m = 5793.20 \ {\rm MeV}, \\ &\Sigma_b(1/2^+) \ : \ m = 5813.4 \ {\rm MeV}, \\ &\Sigma_b^*(3/2^+) \ : \ m = 5833.6 \ {\rm MeV}, \\ &\Xi_b^*(1/2^+) \ : \ m = 5935.02 \ {\rm MeV}, \\ &\Xi_b^*(3/2^+) \ : \ m = 5952.6 \ {\rm MeV}, \\ &\Omega_b(1/2^+) \ : \ m = 6046.1 \ {\rm MeV}, \\ &\Omega_b^*(3/2^+) \ : \ m = 6063 \ {\rm MeV}. \end{split}$$

Their QCD sum rule parameters can be found in Refs. [1,60, 96].

We list masses of *P*-wave bottom baryons used in the present study, taken from the LHCb experiments [7,8] as well as our previous QCD sum rule studies [1,58,59]:

$$\begin{split} &M_{\Sigma_b(1/2^-)} = M_{\Sigma_b(3/2^-)} = 6096.9 \text{ MeV}, \\ &M_{\Sigma_b(5/2^-)} - M_{\Sigma_b(3/2^-)} = 13 \text{ MeV}, \\ &M_{\Xi'_b(1/2^-)} = M_{\Xi'_b(3/2^-)} = 6226.9 \text{ MeV}, \\ &M_{\Xi'_b(5/2^-)} - M_{\Xi'_b(3/2^-)} = 12 \text{ MeV}, \\ &M_{\Omega_b(1/2^-)} = M_{\Omega_b(3/2^-)} = 6460 \text{ MeV}, \\ &M_{\Omega_b(5/2^-)} - M_{\Omega_b(3/2^-)} = 11 \text{ MeV}. \end{split}$$
(A.1)

Their QCD sum rule parameters can be found in Table 5.

We list masses and decay widths of the pseudoscalar and vector mesons used in the present study, taken from PDG [2]:

$$\pi(0^{-}) : m = 138.04 \text{ MeV}, \qquad (A.2)$$

$$K(0^{-}) : m = 495.65 \text{ MeV}, \qquad \rho(1^{-}) : m = 775.21 \text{ MeV}, \qquad \Gamma = 148.2 \text{ MeV}, \qquad g_{\rho\pi\pi} = 5.94,$$

Table 5 Parameters of *P*-wave bottom baryons belonging to the bottom baryon multiplets $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$. Detailed discussions can be found in Refs. [1,58,59]

Multiplets	В	$\omega_c \; ({\rm GeV})$	Working region (GeV)	$\overline{\Lambda}$ (GeV)	Baryons (j^P)	Mass (GeV)	Difference (MeV)	$f (\text{GeV}^4)$
$[6_{F}, 1, 0, \rho]$	Σ_b	1.87	0.31 < T < 0.34	1.35 ± 0.09	$\Sigma_b(1/2^-)$	6.10 ± 0.11	3 ± 1	0.087 ± 0.018
					$\Sigma_b(3/2^-)$	6.10 ± 0.10		0.050 ± 0.011
	Ξ_b'	2.02	0.29 < T < 0.36	1.49 ± 0.09	$E_b'(1/2^-)$	6.24 ± 0.11	3 ± 1	0.080 ± 0.016
					$\Xi_{b}^{\prime}(3/2^{-})$	6.24 ± 0.11		0.046 ± 0.009
	Ω_b	2.17	0.33 < T < 0.38	1.67 ± 0.09	$\Omega_b(1/2^-)$	6.42 ± 0.11	3 ± 1	0.155 ± 0.030
					$\Omega_b(3/2^-)$	6.42 ± 0.11		0.090 ± 0.017
$[6_F, 0, 1, \lambda]$	Σ_b	1.75	0.30 < T < 0.33	1.29 ± 0.08	$\Sigma_b(1/2^-)$	6.09 ± 0.10	-	0.085 ± 0.017
	Ξ_b'	1.90	0.30 < T < 0.34	1.44 ± 0.08	$E_b'(1/2^-)$	6.25 ± 0.10	-	0.077 ± 0.016
	Ω_b	2.05	0.29 < T < 0.35	1.59 ± 0.08	$\Omega_b(1/2^-)$	6.40 ± 0.11	-	0.143 ± 0.030
$[6_F, 1, 1, \lambda]$	Σ_b	1.95	0.33 < T < 0.36	1.28 ± 0.12	$\Sigma_b(1/2^-)$	6.10 ± 0.13	7 ± 3	0.078 ± 0.018
					$\Sigma_b(3/2^-)$	6.10 ± 0.13		0.045 ± 0.010
	Ξ_b'	2.10	0.32 < T < 0.39	1.47 ± 0.11	$\Xi_{b}^{\prime}(1/2^{-})$	6.33 ± 0.13	5 ± 3	0.085 ± 0.017
					$\Xi_b'(3/2^-)$	6.33 ± 0.13		0.049 ± 0.010
	Ω_b	2.25	0.31 < T < 0.42	1.63 ± 0.14	$\Omega_b(1/2^-)$	6.50 ± 0.16	4 ± 2	0.171 ± 0.036
					$\Omega_b(3/2^-)$	6.51 ± 0.16		0.099 ± 0.021
$[6_F, 2, 1, \lambda]$	Σ_b	1.84	0.30 < T < 0.34	1.29 ± 0.09	$\Sigma_b(3/2^-)$	6.10 ± 0.12	13 ± 5	0.102 ± 0.022
					$\Sigma_b(5/2^-)$	6.11 ± 0.12		0.045 ± 0.010
	Ξ_b'	1.99	0.30 < T < 0.36	1.45 ± 0.09	$E_b'(3/2^-)$	6.27 ± 0.12	12 ± 5	0.099 ± 0.021
					$E_{b}^{\prime}(5/2^{-})$	6.29 ± 0.11		0.044 ± 0.009
	Ω_b	2.14	0.32 < T < 0.38	1.62 ± 0.09	$\Omega_b(3/2^-)$	6.46 ± 0.12	11 ± 5	0.194 ± 0.038
					$\Omega_b(5/2^-)$	6.47 ± 0.12		0.087 ± 0.017

$$K^*(1^-)$$
: $m = 893.57$ MeV,
 $\Gamma = 49.1$ MeV, $g_{K^*K\pi} = 3.20$, (A.3)

where the two coupling constants $g_{\rho\pi\pi}$ and $g_{K^*K\pi}$ are evaluated using the experimental decay widths of the ρ and K^* [2] through the following Lagrangians

$$\mathscr{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \times \left(\rho_{\mu}^{0}\pi^{+}\partial^{\mu}\pi^{-} - \rho_{\mu}^{0}\pi^{-}\partial^{\mu}\pi^{+}\right) + \cdots,$$

$$\mathscr{L}_{K^{*}K\pi} = g_{K^{*}K\pi}K_{\mu}^{*+} \times \left(K^{-}\partial^{\mu}\pi^{0} - \partial^{\mu}K^{-}\pi^{0}\right) + \cdots.$$

(A.4)

Appendix B: Sum rule equations

In this appendix we show several examples of sum rule equations, which are used to extract S-wave decays of P-wave bottom baryons into ground-state bottom baryons and vector mesons.

The sum rule for $\Xi_b^{\prime-}[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 0, 1, \lambda]$ is

$$\begin{split} G_{\Xi_{b}^{'}[\frac{1}{2}^{-}] \to \Xi_{b}^{*0}\rho^{-}}(\omega,\omega') &= \frac{g_{\Xi_{b}^{'}[\frac{1}{2}^{-}] \to \Xi_{b}^{*0}\rho^{-}}f_{\Xi_{b}^{-}[\frac{1}{2}^{-}]}f_{\Xi_{b}^{*0}}}{(\bar{\Lambda}_{\Xi_{b}^{-}[\frac{1}{2}^{-}]}-\omega')(\bar{\Lambda}_{\Xi_{b}^{*0}}-\omega)} \\ &= \int_{0}^{\infty} dt \int_{0}^{1} du e^{i(1-u)\omega't}e^{iu\omega t} \times 4 \\ &\times \left(\frac{if_{\rho}^{\parallel}m_{\rho}}{6\pi^{2}t^{4}}\phi_{2;\rho}^{\parallel}(u) - \frac{2if_{\rho}^{\parallel}m_{\rho}}{3\pi^{3}t^{4}}\phi_{3;\rho}^{\perp}(u)\right) \end{split}$$

$$\begin{split} &-\frac{if_{\rho}^{\parallel}m_{\rho}^{3}}{12\pi^{2}t^{4}(\upsilon \cdot q)^{2}}\phi_{2;\rho}^{\parallel}(u)+\frac{if_{\rho}^{\parallel}m_{\rho}^{3}}{6\pi^{2}t^{4}(\upsilon \cdot q)^{2}}\phi_{3;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{\rho}^{3}}{12\pi^{2}t^{4}(\upsilon \cdot q)^{2}}\psi_{4;\rho}^{\parallel}(u)+\frac{f_{\rho}^{\perp}m_{s}m_{\rho}^{2}}{12\pi^{2}t^{3}\upsilon \cdot q}\phi_{2;\rho}^{\perp}(u)\\ &-\frac{f_{\rho}^{\perp}m_{s}m_{\rho}^{2}}{12\pi^{2}t^{3}\upsilon \cdot q}\psi_{4;\rho}^{\perp}(u)-\frac{f_{\rho}^{\parallel}m_{\rho}\upsilon \cdot q}{12\pi^{2}t^{3}}\psi_{3;\rho}^{\parallel}(u)\\ &+\frac{if_{\rho}^{\parallel}m_{\rho}^{3}}{96\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(u)-\frac{if_{\rho}^{\perp}m_{s}m_{\rho}^{2}}{24\pi^{2}t^{2}}\psi_{3;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\perp}m_{s}u(\upsilon \cdot q)^{2}}{12\pi^{2}t^{2}}\phi_{2;\rho}^{\perp}(u)+\frac{if_{\rho}^{\parallel}m_{\rho}u(\upsilon \cdot q)^{2}}{24\pi^{2}t^{2}}\psi_{3;\rho}^{\perp}(u)\\ &+\frac{f_{\rho}^{\perp}m_{\rho}^{2}}{36t\upsilon \cdot q}\langle\bar{s}s\rangle\phi_{2;\rho}^{\perp}(u)-\frac{f_{\rho}^{\perp}m_{\rho}^{2}}{36\upsilon \cdot q}\langle\bar{s}s\rangle\psi_{4;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}^{3}}{288(\upsilon \cdot q)^{2}}\langle\bar{s}s\rangle\phi_{2;\rho}^{\parallel}(u)+\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}}{144}\langle\bar{s}s\rangle\phi_{2;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}}{36}\langle\bar{s}s\rangle\phi_{3;\rho}^{\perp}(u)-\frac{if_{\rho}^{\perp}m_{s}^{2}}{72}\langle\bar{s}s\rangle\psi_{3;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}}{36}\langle\bar{s}s\rangle\phi_{3;\rho}^{\perp}(u)-\frac{if_{\rho}^{\perp}m_{s}m_{\rho}}{12\pi^{2}}\langle\bar{s}s\rangle\psi_{3;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}}{36}\langle\bar{s}s\rangle\phi_{3;\rho}^{\perp}(u)-\frac{if_{\rho}^{\perp}m_{s}m_{\rho}}{72}\langle\bar{s}s\rangle\psi_{3;\rho}^{\parallel}(u)\\ &+\frac{f_{\rho}^{\perp}m_{s}^{2}}{36}\langle\bar{s}s\sigma Gs\rangle\phi_{2;\rho}^{\perp}(u)-\frac{f_{\rho}^{\perp}m_{s}m_{\rho}}{576\upsilon \cdot q}\langle g_{s}\bar{s}\sigma Gs\rangle\psi_{4;\rho}^{\perp}(u) \end{split}$$

$$\begin{split} &-\frac{f_{\rho}^{\parallel}m_{s}m_{\rho}tv\cdot q}{288}\langle\bar{s}s\rangle\psi_{3;\rho}^{\perp}(u)\frac{if_{\rho}^{\parallel}m_{s}m_{\rho}^{3}t^{2}}{2304}\langle\bar{s}s\rangle\phi_{4;\rho}^{\parallel}(u)\\ &-\frac{if_{\rho}^{\perp}m_{\rho}^{2}t^{2}}{1152}\langle g_{s}\bar{s}\sigma\,Gs\rangle\psi_{3;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\perp}m_{\rho}^{2}ut^{2}(v\cdot q)^{2}}{576}\langle g_{s}\bar{s}\sigma\,Gs\rangle\phi_{4;\rho}^{\perp}(u)\\ &+\frac{if_{\rho}^{\parallel}m_{\rho}m_{\mu}ut^{2}(v\cdot q)^{2}}{576}\langle\bar{s}s\rangle\phi_{4;\rho}^{\perp}(u)\\ &+\frac{if_{\rho}^{\parallel}m_{\rho}m_{\mu}ut^{2}(v\cdot q)^{2}}{576}\langle\bar{s}s\rangle\phi_{4;\rho}^{\perp}(u)\\ &+\frac{f_{\rho}^{\perp}m_{\rho}^{2}t^{3}v\cdot q}{2304}\langle g_{s}\bar{s}\sigma\,Gs\rangle\phi_{4;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\perp}m_{\rho}^{2}t^{3}v\cdot q}{2304}\langle g_{s}\bar{s}\sigma\,Gs\rangle\phi_{4;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\perp}m_{\rho}^{2}ut^{4}(v\cdot q)^{2}}{9216}\langle g_{s}\bar{s}\sigma\,Gs\rangle\phi_{4;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{\rho}^{2}ut^{4}(v\cdot q)^{2}}{9216}\langle g_{s}\bar{s}\sigma\,Gs\rangle\phi_{4;\rho}^{\perp}(u)\\ &-\frac{if_{\rho}^{\parallel}m_{\rho}^{2}u}{12\pi^{2}t^{2}}\Psi_{4;\rho}^{\parallel}(\underline{\omega})+\frac{if_{\rho}^{\parallel}m_{\rho}^{3}}{24\pi^{2}t^{2}}\tilde{\Psi}_{4;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{if_{\rho}^{\parallel}m_{\rho}(v\cdot q)^{2}}{12\pi^{2}t^{2}}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{if_{\rho}^{\parallel}m_{\rho}(v\cdot q)^{2}}{12\pi^{2}t^{2}}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &+\frac{if_{\rho}^{\parallel}m_{\rho}u(v\cdot q)^{2}}{12\pi^{2}t^{2}}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &+\frac{f_{\rho}^{\parallel}m_{\rho}u(v\cdot q)^{2}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &+\frac{f_{\rho}^{\parallel}m_{\rho}u(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u\alpha_{2}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{12\pi^{2}t}\tilde{\Phi}_{3;\rho}^{\parallel}(\underline{\omega})\\ &-\frac{f_{\rho}^{\parallel}m_{\rho}$$

The sum rule for $\mathcal{Z}_b^{\prime -}[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$\begin{split} G_{\Xi_{b}^{'-}[\frac{1}{2}^{-}] \to \Sigma_{b}^{0}K^{*}}(\omega, \omega') &= \frac{g_{\Xi_{b}^{'-}[\frac{1}{2}^{-}] \to \Sigma_{b}^{0}K^{*}}f_{\Xi_{b}^{'-}[\frac{1}{2}^{-}]}f_{\Sigma_{b}^{0}}}{(\bar{\Lambda}_{\Xi_{b}^{'-}[\frac{1}{2}^{-}]} - \omega')(\bar{\Lambda}_{\Sigma_{b}^{0}} - \omega)} \\ &= \int_{0}^{\infty} dt \int_{0}^{1} du e^{i(1-u)\omega't} e^{iu\omega t} \times 4 \\ &\times \left(\frac{if_{K^{*}}^{\perp}v \cdot q}{2\pi^{2}t^{4}}\phi_{2;K^{*}}^{\perp}(u) - \frac{if_{k}^{\perp}m_{K^{*}}^{2}}{2\pi^{2}t^{4}v \cdot q}\phi_{2;K^{*}}^{\perp}(u) \right. \\ &+ \frac{if_{K^{*}}^{\perp}m_{K^{*}}^{2}}{2\pi^{2}t^{4}v \cdot q}\psi_{4;K^{*}}^{\perp}(u) - \frac{f_{K^{*}}^{\perp}u(v \cdot q)^{2}}{4\pi^{2}t^{3}}\phi_{2;K^{*}}^{\perp}(u) \\ &+ \frac{if_{K^{*}}^{\perp}m_{K^{*}}^{2}v \cdot q}{32\pi^{2}t^{2}}\phi_{4;K^{*}}^{\perp}(u) - \frac{f_{K^{*}}^{\perp}m_{K^{*}}^{2}u(v \cdot q)^{2}}{64\pi^{2}t}\phi_{4;K^{*}}^{\perp}(u) \\ &- \frac{if_{K^{*}}^{\parallel}m_{K^{*}}v \cdot q}{48}\langle \bar{q}q\rangle\psi_{3;K^{*}}^{\perp}(u) \end{split}$$

$$\begin{split} &-\frac{f_{k}^{\dagger}m_{k}^{\ast}m_{k}^{\ast}ut(v\cdot q)^{2}}{96}\langle\bar{q}q\rangle\psi_{3;K^{\ast}}^{\dagger}(u)\\ &-\frac{if_{k}^{\dagger}m_{k}^{\ast}m_{k}^{\ast}t^{2}v\cdot q}{768}\langle g_{s}\bar{q}\sigma Gq\rangle\psi_{3;K^{\ast}}^{\dagger}(u)\\ &-\frac{f_{k}^{\dagger}m_{k}^{\ast}m_{k}^{\ast}u^{3}(v\cdot q)^{2}}{1536}\langle g_{s}\bar{q}\sigma Gq\rangle\psi_{3;K^{\ast}}^{\dagger}(u) \end{pmatrix}\\ &-\int_{0}^{\infty}dt\int_{0}^{1}du\int\mathcal{D}\underline{Q}\underline{e}^{i\omega't(a_{2}+ua_{3})}e^{i\omega(1-a_{2}-ua_{3})}\\ &\times\frac{1}{2}\left(-\frac{if_{K}^{\dagger}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(1)}(\underline{a})+\frac{if_{K^{\ast}}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\right)\\ &-\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(1)}(\underline{a})+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{4\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{8\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{4\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{if_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{4\pi^{2}t^{2}}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\perp(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\ast(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\ast(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\ast(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{2}t}\Phi_{4;K^{\ast}}^{\ast(2)}(\underline{a})\\ &+\frac{f_{K^{\ast}}m_{K^{\ast}}^{2}v\cdot q^{2}}{16\pi^{$$

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$$+ \frac{f_{K^*}^{\perp} m_{K^*}^2 u^2 \alpha_3 (v \cdot q)^2}{8\pi^2 t} \Phi_{4;K^*}^{\perp(3)}(\underline{\alpha}) \\ - \frac{f_{K^*}^{\perp} m_{K^*}^2 u^2 \alpha_3 (v \cdot q)^2}{8\pi^2} \Phi_{4;K^*}^{\perp(4)}(\underline{\alpha}) \\ - \frac{f_{K^*}^{\perp} m_{K^*}^2 u^2 \alpha_3 (v \cdot q)^2}{8\pi^2} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ - \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \Phi_{4;K^*}^{\perp(1)}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \Phi_{4;K^*}^{\perp(2)}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \Phi_{4;K^*}^{\perp(3)}(\underline{\alpha}) \\ - \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \Phi_{4;K^*}^{\perp(4)} \\ - \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{4;K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}_{K^*}^{\perp}(\underline{\alpha}) \\ + \frac{f_{K^*}^{\perp} m_{K^*}^2 u \alpha_3 (v \cdot q)^2}{16\pi^2 t} \widetilde{\Psi}$$

The sum rule for $\Sigma_{b}^{-}[\frac{1}{2}^{-}]$ belonging to $[\mathbf{6}_{F}, 1, 1, \lambda]$ is

$$\begin{split} G_{\Sigma_{b}^{-}[\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{0}\rho^{-}}(\omega,\omega') &= \frac{s_{\Sigma_{b}^{-}\frac{1}{2}^{-}] \rightarrow \Sigma_{b}^{0}\rho^{-}}{(\bar{\Lambda}_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]}f_{\Sigma_{b}^{0}})}{(\bar{\Lambda}_{\Sigma_{b}^{-}[\frac{1}{2}^{-}]}-\omega')(\bar{\Lambda}_{\Sigma_{b}^{0}}-\omega)} \\ &= \int_{0}^{\infty} dt \int_{0}^{1} du e^{i(1-u)\omega't} e^{iu\omega t} \times 8 \\ &\times \left(-\frac{f_{\rho}^{||}m_{\rho}}{2\pi^{2}t^{4}}\phi_{2;\rho}^{||}(u) - \frac{f_{\rho}^{||}m_{\rho}}{2\pi^{2}t^{4}}\phi_{3;\rho}^{\perp}(u) \right. \\ &+ \frac{f_{\rho}^{||}m_{\rho}^{3}}{4\pi^{2}t^{2}(v\cdot q)^{2}}\phi_{2;\rho}^{||}(u) - \frac{f_{\rho}^{||}m_{\rho}}{8\pi^{2}t^{3}}\psi_{3;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{||}m_{\rho}}{4\pi^{2}t^{2}(v\cdot q)^{2}}\psi_{3;\rho}^{\perp}(u) - \frac{f_{\rho}^{||}m_{\rho}^{3}}{32\pi^{2}t^{2}}\phi_{3;\rho}^{||}(u) \\ &- \frac{f_{\rho}^{||}m_{\rho}u(v\cdot q)^{2}}{16\pi^{2}t^{2}}\psi_{3;\rho}^{\perp}(u) - \frac{f_{\rho}^{\perp}m_{\rho}^{2}}{32\pi^{2}t^{2}}\phi_{3;\rho}^{||}(u) \\ &+ \frac{f_{\rho}^{\perp}m_{\rho}^{2}}{4tv\cdot q}\langle\bar{q}q\rangle\phi_{2;\rho}^{\perp}(u) - \frac{f_{\rho}^{\perp}m_{\rho}^{2}}{24tv\cdot q}\langle\bar{q}q\rangle\psi_{3;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\perp}u(v\cdot q)^{2}}{24}\langle\bar{q}q\rangle\phi_{2;\rho}^{\perp}(u) - \frac{f_{\rho}^{\perp}m_{\rho}^{2}}{384v\cdot q}\langle g_{s}\bar{q}\sigma Gq\rangle\psi_{3;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\perp}u^{2}(v\cdot q)^{2}}{384}\langle g_{s}\bar{q}\sigma Gq\rangle\phi_{2;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\perp}m_{\rho}^{2}u^{2}(v\cdot q)^{2}}{384}\langle g_{s}\bar{q}\sigma Gq\rangle\phi_{2;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\perp}m_{\rho}^{2}u^{2}(v\cdot q)^{2}}{6144}\langle g_{s}\bar{q}\sigma Gq\rangle\phi_{2;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\perp}m_{\rho}^{2}u^{4}(v\cdot q)^{2}}{6144}\langle g_{s}\bar{q}\sigma Gq\rangle\phi_{4;\rho}^{\perp}(u) \\ &+ \frac{f_{\rho}^{\parallel}m_{\rho}^{2}}{8\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(\omega) \\ &- \int_{0}^{\infty} dt \int_{0}^{1} du \int \mathscr{D}\underline{\alpha}e^{i\omega't(\alpha_{2}+u\alpha_{3})}e^{i\omegat(1-\alpha_{2}-u\alpha_{3})} \\ &\times \left(\frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{8\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(\omega) - \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{8\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(\omega) \\ &+ \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{4\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(\omega) - \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{8\pi^{2}t^{2}}\phi_{3;\rho}^{\parallel}(\omega) \\ &+ \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{4\pi^{2}t^{2}}\phi_{4;\rho}^{\parallel}(\omega) - \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{8\pi^{2}t^{2}}\phi_{3;\rho}^{\parallel}(\omega) \\ &- \int_{0}^{\infty} dt \int_{0}^{1} du \int \mathscr{D}\underline{\alpha}e^{i\omega't(\alpha_{2}+u\alpha_{3})}e^{i\omegat(1-\alpha_{2}-u\alpha_{3})} \\ &\times \left(\frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{4\pi^{2}t^{2}}}\phi_{4;\rho}^{\parallel}(\omega) - \frac{f_{\rho}^{\parallel}m_{\rho}^{3}}{8\pi^{2}t^{2}}}\phi_{3;\rho}^{\parallel}(\omega) \right) \\ \end{array}$$

$$-\frac{f_{\rho}^{||}m_{\rho}(v\cdot q)^{2}}{8\pi^{2}t^{2}}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) + \frac{3f_{\rho}^{||}m_{\rho}u(v\cdot q)^{2}}{8\pi^{2}t^{2}}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) - \frac{f_{\rho}^{||}m_{\rho}u(v\cdot q)^{2}}{8\pi^{2}t^{2}}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) - \frac{if_{\rho}^{||}m_{\rho}\alpha_{2}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}\alpha_{2}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) - \frac{if_{\rho}^{||}m_{\rho}\alpha_{2}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}u\alpha_{2}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) - \frac{if_{\rho}^{||}m_{\rho}u^{2}\alpha_{3}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}^{||}m_{\rho}u\alpha_{3}(v\cdot q)^{3}}{8\pi^{2}t}\widetilde{\varphi}_{3;\rho}^{||}(\underline{\alpha}) + \frac{if_{\rho}$$

The sum rule for $\Omega_b^-[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ is

$$G_{\Omega_{b}^{-}[\frac{1}{2}^{-}] \to \Xi_{b}^{0} K^{*}}(\omega, \omega') = \frac{g_{\Omega_{b}^{-}[\frac{1}{2}^{-}] \to \Xi_{b}^{0} K^{*}} f_{\Omega_{b}^{-}[\frac{1}{2}^{-}]} f_{\Xi_{b}^{0}}}{(\bar{A}_{\Omega_{b}^{-}[\frac{1}{2}^{-}]} - \omega')(\bar{A}_{\Xi_{b}^{0}} - \omega)} = 0.$$
(B.8)

References

- 1. E.L. Cui, H.M. Yang, H.X. Chen, A. Hosaka, Identifying the $\Xi_b(6227)$ and $\Sigma_b(6097)$ as *P*-wave bottom baryons of $J^P = 3/2^-$. Phys. Rev. D **99**, 094021 (2019)
- M. Tanabashi et al. [Particle Data Group], Review of particle physics. Phys. Rev. D 98, 030001 (2018)
- J. Yelton et al., [Belle Collaboration], Study of excited Z_c states decaying into Z⁰_c and Z⁺_c Baryons. Phys. Rev. D 94, 052011 (2016)
- Y. Kato et al. [Belle Collaboration], Studies of charmed strange baryons in the *AD* final state at Belle. Phys. Rev. D 94, 032002 (2016)
- 5. R. Aaij et al. [LHCb Collaboration], Observation of five new narrow Ω_c^0 states decaying to $\Xi_c^+ K^-$. Phys. Rev. Lett. **118**, 182001 (2017)
- 6. R. Aaij et al. [LHCb Collaboration], Study of the $D^0 p$ amplitude in $\Lambda_b^0 \to D^0 p \pi^-$ decays. JHEP **1705**, 030 (2017)
- 7. R. Aaij et al. [LHCb Collaboration], Observation of a new Ξ_b^- resonance. Phys. Rev. Lett. **121**, 072002 (2018)
- 8. R. Aaij et al. [LHCb Collaboration], Observation of two resonances in the $\Lambda_b^0 \pi^{\pm}$ systems and precise measurement of Σ_b^{\pm} and $\Sigma_b^{*\pm}$ properties. Phys. Rev. Lett. **122**, 012001 (2019)
- 9. R. Aaij et al. [LHCb Collaboration], Observation of a new resonance in the $\Lambda_b^0 \pi^+ \pi^-$ system. arXiv:1907.13598 [hep-ex]
- J.G. Körner, M. Kramer, D. Pirjol, Heavy baryons. Prog. Part. Nucl. Phys. 33, 787 (1994)
- A.V. Manohar, M.B. Wise, Heavy quark physics. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10, 1 (2000)
- E. Klempt, J.M. Richard, Baryon spectroscopy. Rev. Mod. Phys. 82, 1095 (2010)
- S. Capstick, N. Isgur, Baryons in a relativized quark model with chromodynamics. Phys. Rev. D 34, 2809 (1986)
- 14. S. Capstick, N. Isgur, Baryons in a relativized quark modelwith chromodynamics. AIP Conf. Proc. **132**, 267 (1985)

- D. Ebert, R.N. Faustov, V.O. Galkin, Masses of excited heavy baryons in the relativistic quark model. Phys. Lett. B 659, 612 (2008)
- P.G. Ortega, D.R. Entem, F. Fernandez, Quark model description of the Λ_c(2940)⁺ as a molecular D*Nstate and the possible existence of the Λ_b(6248). Phys. Lett. B **718**, 1381 (2013)
- X.H. Zhong, Q. Zhao, Charmed baryon strong decays in a chiral quark model. Phys. Rev. D 77, 074008 (2008)
- H.Y. Cheng, C.K. Chua, Strong decays of charmed baryons in heavy hadron chiral perturbation theory: an update. Phys. Rev. D 92, 074014 (2015)
- L.A. Copley, N. Isgur, G. Karl, Charmed baryons in a quark model with hyperfine interactions. Phys. Rev. D 20, 768 (1979). Erratum: [Phys. Rev. D 23, 817 (1981)]
- 20. M. Karliner, B. Keren-Zur, H.J. Lipkin, J.L. Rosner, The quark model and *b* baryons. Ann. Phys. **324**, 2 (2009)
- R. Roncaglia, D.B. Lichtenberg, E. Predazzi, Predicting the masses of baryons containing one or two heavy quarks. Phys. Rev. D 52, 1722 (1995)
- E.E. Jenkins, Heavy baryon masses in the 1/m(Q) and 1/N(c) expansions. Phys. Rev. D 54, 4515 (1996)
- S.H. Kim, A. Hosaka, H.C. Kim, H. Noumi, K. Shirotori, Pion induced reactions for charmed baryons. PTEP 2014, 103D01 (2014)
- W. Roberts, M. Pervin, Heavy baryons in a quark model. Int. J. Mod. Phys. A 23, 2817 (2008)
- 25. B. Chen, K.W. Wei, A. Zhang, Assignments of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture. Eur. Phys. J. A **51**, 82 (2015)
- H. Garcilazo, J. Vijande, A. Valcarce, Faddeev study of heavy baryon spectroscopy. J. Phys. G 34, 961 (2007)
- X.H. Guo, K.W. Wei, X.H. Wu, Some mass relations for mesons and baryons in Regge phenomenology. Phys. Rev. D 78, 056005 (2008)
- W.H. Liang, C.W. Xiao, E. Oset, Baryon states with open beauty in the extended local hidden gauge approach. Phys. Rev. D 89, 054023 (2014)
- C. Garcia-Recio, J. Nieves, O. Romanets, L.L. Salcedo, L. Tolos, Odd parity bottom-flavored baryon resonances. Phys. Rev. D 87, 034032 (2013)
- 30. J.X. Lu, Y. Zhou, H.X. Chen, J.J. Xie, L.S. Geng, Dynamically generated $J^P = 1/2^{-}(3/2^{-})$ singly charmed and bottom heavy baryons. Phys. Rev. D **92**, 014036 (2015)
- H.X. Chen, Q. Mao, A. Hosaka, X. Liu, S.L. Zhu, D-wave charmed and bottomed baryons from QCD sum rules. Phys. Rev. D 94, 114016 (2016)
- 32. Q. Mao, H.X. Chen, A. Hosaka, X. Liu, S.L. Zhu, *D*-wave heavy baryons of the SU(3) flavor 6_F . Phys. Rev. D **96**, 074021 (2017)
- E. Bagan, P. Ball, V.M. Braun, H.G. Dosch, QCD sum rules in the effective heavy quark theory. Phys. Lett. B 278, 457 (1992)
- M. Neubert, Heavy meson form-factors from QCD sum rules. Phys. Rev. D 45, 2451 (1992)
- D.J. Broadhurst, A.G. Grozin, Operator product expansion in static quark effective field theory: large perturbative correction. Phys. Lett. B 274, 421 (1992)
- T. Huang, C.W. Luo, Light quark dependence of the Isgur–Wise function from QCD sum rules. Phys. Rev. D 50, 5775 (1994)
- Y.B. Dai, C.S. Huang, M.Q. Huang, C. Liu, QCD sum rules for masses of excited heavy mesons. Phys. Lett. B 390, 350 (1997)
- P. Colangelo, F. De Fazio, N. Paver, Universal tau(1/2)(y) Isgur-Wise function at the next-to-leading order in QCD sum rules. Phys. Rev. D 58, 116005 (1998)
- S. Groote, J.G. Körner, O.I. Yakovlev, QCD sum rules for heavy baryons at next-to-leading order in alpha-s. Phys. Rev. D 55, 3016 (1997)

- 40. S.L. Zhu, Strong and electromagnetic decays of p wave heavy baryons Λ_{c1} , Λ_{c1}^* . Phys. Rev. D **61**, 114019 (2000)
- 41. J.P. Lee, C. Liu, H.S. Song, QCD sum rule analysis of excited Lambda(c) mass parameter. Phys. Lett. B **476**, 303 (2000)
- 42. C.S. Huang, Al Zhang, S.L. Zhu, Excited heavy baryon masses in HQET QCD sum rules. Phys. Lett. B **492**, 288 (2000)
- D.W. Wang, M.Q. Huang, Excited heavy baryon masses to order Lambda(QCD)/m(Q) from QCD sum rules. Phys. Rev. D 68, 034019 (2003)
- F.O. Duraes, M. Nielsen, QCD sum rules study of Z_c and Z_b baryons. Phys. Lett. B 658, 40 (2007)
- D. Zhou, E.L. Cui, H.X. Chen, L.S. Geng, X. Liu, S.L. Zhu, Dwave heavy-light mesons from QCD sum rules. Phys. Rev. D 90, 114035 (2014)
- D. Zhou, H.X. Chen, L.S. Geng, X. Liu, S.L. Zhu, F-wave heavylight meson spectroscopy in QCD sum rules and heavy quark effective theory. Phys. Rev. D 92, 114015 (2015)
- H.X. Chen, W. Chen, S.L. Zhu, Possible interpretations of the *P_c*(4312), *P_c*(4440), and *P_c*(4457). Phys. Rev. D 100, 051501 (2019)
- H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, A review of the open charm and open bottom systems. Rep. Prog. Phys. 80, 076201 (2017)
- R.M. Albuquerque, J.M. Dias, K.P. Khemchandani, A. Martinez Torres, F.S. Navarra, M. Nielsen, C.M. Zanetti, QCD sum rules approach to the *X*, *Y* and *Z* states. J. Phys. G 46, 093002 (2019)
- H.Y. Cheng, Charmed baryons circa 2015. Front. Phys. (Beijing) 10, 101406 (2015)
- V. Crede, W. Roberts, Progress towards understanding baryon resonances. Rep. Prog. Phys. 76, 076301 (2013)
- S. Bianco, F.L. Fabbri, D. Benson, I. Bigi, A Cicerone for the physics of charm. Riv. Nuovo Cim. 26N7, 1 (2003)
- M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, QCD and resonance physics. Theoretical foundations. Nucl. Phys. B 147, 385 (1979)
- L.J. Reinders, H. Rubinstein, S. Yazaki, Hadron properties from QCD sum rules. Phys. Rep. 127, 1 (1985)
- B. Grinstein, The static quark effective theory. Nucl. Phys. B 339, 253 (1990)
- E. Eichten, B.R. Hill, An effective field theory for the calculation of matrix elements involving heavy quarks. Phys. Lett. B 234, 511 (1990)
- 57. A.F. Falk, H. Georgi, B. Grinstein, M.B. Wise, Heavy meson formfactors from QCD. Nucl. Phys. B **343**, 1 (1990)
- H.X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu, S.L. Zhu, P-wave charmed baryons from QCD sum rules. Phys. Rev. D 91, 054034 (2015)
- Q. Mao, H.X. Chen, W. Chen, A. Hosaka, X. Liu, S.L. Zhu, QCD sum rule calculation for P-wave bottom baryons. Phys. Rev. D 92, 114007 (2015)
- H.X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu, S.L. Zhu, Decay properties of *P*-wave charmed baryons from light-cone QCD sum rules. Phys. Rev. D 95, 094008 (2017)
- 61. B. Chen, K.W. Wei, X. Liu, A. Zhang, Role of newly discovered $\Xi_b(6227)^-$ for constructing excited bottom baryon family. Phys. Rev. D **98**, 031502 (2018)
- 62. B. Chen, X. Liu, Assigning the newly reported $\Sigma_b(6097)$ as a *P*-wave excited state and predicting its partners. Phys. Rev. D **98**, 074032 (2018)
- 63. P. Yang, J.J. Guo, A. Zhang, Identification of the newly observed $\Sigma_b(6097)^{\pm}$ baryons from their strong decays. Phys. Rev. D **99**, 034018 (2019)
- 64. K.L. Wang, Q.F. Lü, X.H. Zhong, Interpretation of the newly observed $\Sigma_b (6097)^{\pm}$ and $\Xi_b (6227)^{-}$ states as the *P*-wave bottom baryons. Phys. Rev. D **99**, 014011 (2019)
- T.M. Aliev, K. Azizi, Y. Sarac, H. Sundu, Phys. Rev. D 99, 094003 (2019)

- 66. T.M. Aliev, K. Azizi, Y. Sarac, H. Sundu, Structure of the $\Xi_b(6227)^-$ resonance. Phys. Rev. D **98**, 094014 (2018)
- 67. K. Azizi, H. Sundu, Mass and magnetic dipole moment of negative parity heavy baryons with spin 3/2. Eur. Phys. J. Plus **132**, 22 (2017)
- C.K. Chua, Color-allowed bottom baryon to charmed baryon nonleptonic decays. Phys. Rev. D 99, 014023 (2019)
- M. Karliner, J.L. Rosner, Scaling of P-wave excitation energies in heavy-quark systems. Phys. Rev. D 98, 074026 (2018)
- 70. Y. Huang, C.J. Xiao, L.S. Geng, J. He, Strong decays of the $\Xi_b(6227)$ as a $\Sigma_b \bar{K}$ molecule. Phys. Rev. D **99**, 014008 (2019)
- 71. Q.X. Yu, R. Pavao, V.R. Debastiani, E. Oset, Description of the \mathcal{Z}_c and $i \mathcal{Z}_b$ states as molecular states. Eur. Phys. J. C **79**, 167 (2019)
- D. Jia, W.N. Liu, A. Hosaka, Regge behaviors in low-lying singly charmed and bottom baryons. arXiv:1907.04958 [hep-ph]
- 73. W. Liang, Q.F. Lü, X.H. Zhong, Canonical interpretation of the newly observed Λ_b (6146)⁰ and Λ_b (6152)⁰ via strong decay behaviors. Phys. Rev. D **100**, 054013 (2019)
- 74. Z.H. Guo, J.A. Oller, Anatomy of the newly observed hidden-charm pentaquark states: $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$. Phys. Lett. B **793**, 144 (2019)
- M. Padmanath, R.G. Edwards, N. Mathur, M. Peardon, Excitedstate spectroscopy of singly, doubly and triply-charmed baryons from lattice QCD. arXiv:1311.4806 [hep-lat]
- 76. M. Padmanath, N. Mathur, Quantum numbers of recently discovered Ω_c^0 baryons from lattice QCD. Phys. Rev. Lett. **119**, 042001 (2017)
- H.Y. Cheng, C.K. Chua, Strong decays of charmed baryons in heavy hadron chiral perturbation theory. Phys. Rev. D 75, 014006 (2007)
- H. Nagahiro, S. Yasui, A. Hosaka, M. Oka, H. Noumi, Structure of charmed baryons studied by pionic decays. Phys. Rev. D 95, 014023 (2017)
- P. Ball, Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: the nonsinglet case. JHEP **9901**, 010 (1999)
- P. Ball, V.M. Braun, A. Lenz, Higher-twist distribution amplitudes of the K meson in QCD. JHEP 0605, 004 (2006)
- 81. P. Ball, R. Zwicky, $B_{d,s} \rightarrow \rho, \omega, K^*, \phi$ decay form-factors from light-cone sum rules revisited. Phys. Rev. D **71**, 014029 (2005)

- P. Ball, V.M. Braun, Exclusive semileptonic and rare B meson decays in QCD. Phys. Rev. D 58, 094016 (1998)
- P. Ball, V.M. Braun, Y. Koike, K. Tanaka, Higher twist distribution amplitudes of vector mesons in QCD: Formalism and twist-three distributions. Nucl. Phys. B 529, 323 (1998)
- P. Ball, V.M. Braun, Higher twist distribution amplitudes of vector mesons in QCD: twist-4 distributions and meson mass corrections. Nucl. Phys. B 543, 201 (1999)
- P. Ball, G.W. Jones, Twist-3 distribution amplitudes of K* and phi mesons. JHEP 0703, 069 (2007)
- P. Ball, V.M. Braun, A. Lenz, Twist-4 distribution amplitudes of the K* and phi mesons in QCD. JHEP 0708, 090 (2007)
- K.C. Yang, W.Y.P. Hwang, E.M. Henley, L.S. Kisslinger, QCD sum rules and neutron proton mass difference. Phys. Rev. D 47, 3001 (1993)
- 88. W.Y.P. Hwang, K.C. Yang, QCD sum rules: Δ N and $\Sigma_0 \Lambda$ mass splittings. Phys. Rev. D 49, 460 (1994)
- A.A. Ovchinnikov, A.A. Pivovarov, QCD sum rule calculation of the quark gluon condensate. Sov. J. Nucl. Phys. 48, 721 (1988)
- A.A. Ovchinnikov, A.A. Pivovarov, QCD sum rule calculation of the quark gluon condensate. Yad. Fiz. 48, 1135 (1988)
- M. Jamin, Flavor symmetry breaking of the quark condensate and chiral corrections to the Gell–Mann–Oakes–Renner relation. Phys. Lett. B 538, 71 (2002)
- B.L. Ioffe, K.N. Zyablyuk, Gluon condensate in charmonium sum rules with three loop corrections. Eur. Phys. J. C 27, 229 (2003)
- S. Narison, Withdrawn: QCD as a theory of hadrons from partons to confinement. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17, 1 (2002). arXiv:hep-ph/0205006
- V. Gimenez, V. Lubicz, F. Mescia, V. Porretti, J. Reyes, Operator product expansion and quark condensate from lattice QCD in coordinate space. Eur. Phys. J. C 41, 535 (2005)
- P. Colangelo, A. Khodjamirian, At the Frontier of Particle Physics/Handbook of QCD (World Scientific, Singapore, 2001), p. 1495
- X. Liu, H.X. Chen, Y.R. Liu, A. Hosaka, S.L. Zhu, Bottom baryons. Phys. Rev. D 77, 014031 (2008)