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### Precision protection through cosmic string in quantum metrology

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Abstract The dynamics of the quantum Fisher information of the parameters of the initial atomic state and atomic transition frequency is studied, in the framework of open quantum systems, for a static polarizable two-level atom coupled in the multipolar scheme to a bath of fluctuating vacuum electromagnetic fields in cosmic string space-time. Our results show that with the presence of cosmic string, the quantum Fisher information becomes position and atomic polarization dependent. It may be enhanced or depressed as compared to that in flat space-time case. Remarkably, when the atom is extremely close to the cosmic string and the polarization direction of the atom is perpendicular to the direction of the cosmic string, the quantum Fisher information has been totally protected from the fluctuating vacuum electromagnetic fields. So on the one hand, near a cosmic string, precision of estimation can be enhanced by ranging the radial distance between the probe atom and the cosmic string; on the other hand, the cosmic string can be sensed by studying the distribution of parameter induced state-separation.

### 1 Introduction

In estimation theory, the Fisher information which deduced by the Cramér–Rao bound [1,2] describe how well one can estimate a parameter from the probability distribution of the measures. When quantum measurements are performed on quantum mechanical systems, the observed outcomes follow a probability distribution. Optimizing the measurements and estimator, the Fisher information is extended to the so-called quantum Fisher information (QFI) which gives the precision limit of the parameter estimation in quantum metrology [1–5]. With different models of the probe systems and different parameters to be estimated, many applications of quantum metrology have been done such as quantum frequency standards [6], optimal quantum clock [7], measurement of gravity accelerations [8], clock synchronization [9], only to name a few. Since the central task in quantum metrology is to improve the precision of parameter estimation, a straightforward way to enhance the precision is to increase the QFI of the state of probe systems. It was shown that the use of correlated systems such as entangled states can improve the precision of parameter estimation [6,10–23]. However, in reality, interaction between a system and an environment is unavoidable and the quantum decoherence induced by such interactions may decrease the QFI and destroy the quantum entanglement in the probe system exploited to improve the precision, which in turn degrades the estimation precision [24–43]. As a result, how to control the effects of environment on estimation precision becomes an important issue in quantum metrology.

One environment which no system can be isolated from is the vacuum that fluctuates all the time in quantum sense. In free flat space-time, vacuum fluctuations always make the QFI of initial parameter of the state of probe two-level atom decay [44]. However, vacuum fluctuations can be modified. In the presence of the boundary, field modes are reflected by the boundary and the superposition of the incoming and outgoing modes make the QFI position and polarization dependent. The precision of the estimation may even be shielded from the vacuum fluctuations in certain circumstances [44]. Even without the boundary, field modes may be reflected by some curved space-times, and QFI may becomes position dependent in such space-times.

The cosmic string space-time is one of those space-times. The cosmic strings appear as predictions of grand unification theories which are extended, one-dimensional, closed and infinite linear objects leading to topological defects of space-time. They emerge during the symmetry breaking phase transitions in the early universe [45,46] and become the probable sources of gravitational waves [47–49], gamma-ray bursts [50], and high-energy cosmic rays [51]. The idea of cosmic strings draws great attention in the context of the brane-world scenarios of the superstring theory [52–57]. In cosmic string

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space-time, space-time is locally flat and what distinguishes it from a Minkowski spacetime is its nontrivial topology characterized by the deficit angle. Since the cosmic string only modifies the global space-time topology while leaving the local space-time flatness, it is pretty like what a conducting boundary does to a flat space-time. In this regard, QFI may also be protected through cosmic string with proper arrangement of the probe atom. So we plan to study the dynamics of the QFI of the parameters of the initial atomic state and atomic transition frequency, in the framework of open quantum systems, for a static polarizable two-level atom coupled in the multipolar scheme to a bath of fluctuating vacuum electromagnetic fields in cosmic string space-time and to analyze the precision-protection arrangement in cosmic string spacetime. Then we will further discuss the probability of detection of properties of cosmic string through parameter induced state-separation which is also determined by QFI of the probe atom.

### 2 Quantum Fisher information and dynamical evolution of a two-level atom coupled with vacuum fluctuations in cosmic string spacetime

When we estimate an unknown parameter X from a given quantum state P(X), the unknown parameter can be inferred from a set of positive-operator valued measures on the state. After optimizing the measurement and the estimator, a precision limit of the unknown parameter estimation is given by [5]

$$Var(X) \ge \frac{1}{NF_X},$$
 (1)

where N represents the repeated times and  $F_X$  denotes the quantum Fisher information of parameter X. For two-level system, the state of the system can be expressed in the Bloch sphere representation as

$$P = \frac{1}{2}(I + \boldsymbol{\omega} \cdot \boldsymbol{\sigma}), \tag{2}$$

where  $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$  is the Bloch vector and  $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$  denotes the Pauli matrices. As a result,  $F_X$  can be expressed in a simple form [42]

$$F_X = \begin{cases} |\partial_X \boldsymbol{\omega}|^2 + \frac{(\boldsymbol{\omega} \cdot \partial_X \boldsymbol{\omega})^2}{1 - |\boldsymbol{\omega}|^2}, & |\boldsymbol{\omega}| < 1, \\ |\partial_X \boldsymbol{\omega}|^2, & |\boldsymbol{\omega}| = 1. \end{cases}$$
(3)

For an arbitrary two-level system with initial state

$$|\psi\rangle = \cos\frac{\theta}{2}|+\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\rangle, \tag{4}$$

where  $\theta$  and  $\phi$  correspond to the weight parameter and phase parameter,  $|+\rangle$ ,  $|-\rangle$  denote the excited state and ground state of the probe system respectively. The Bloch vector of the state can be represented as  $\boldsymbol{\omega} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ , and the quantum Fisher information of  $\theta$  and  $\phi$  can be easily calculated as  $F_{\theta} = 1$  and  $F_{\phi} = \sin^2 \theta$ . However, the environment always cause the decoherence of the state of probe system, which affect the future evolution of OFI as well as the precision of estimation unavoidably. An environment no system can be isolated from is the vacuum that fluctuates all time. If we use a two-level atom as the probe system, the electromagnetic vacuum fluctuations becomes the unavoidable environment. If we put the atom in free flat space-time, the QFI for initial parameters is found to be decay with time  $F_{\phi} = \sin^2 \theta \ e^{-\gamma_0 \tau}$ ,  $F_{\theta} = e^{-\gamma_0 \tau}$  [44]. Here the decay rate is determined by the spontaneous emission rate  $\gamma_0$  in Minkowski vacuum. However, the vacuum fluctuations can be modified. If we put the atom near a boundary, the QFI becomes position and atomic polarization dependent. The QFI as well as the precision of the estimation may be enhanced in compared to those in unbounded case [44]. Just like what happens in the boundary case, the vacuum fluctuations can also be modified by the curved space-time itself even without boundary. Where and how to put the atom in the curved space-time to get better precision becomes an important issue in curved space-time quantum metrology. Since the cosmic string only modifies the global space-time topology while leaving the local space-time flatness intact, which is pretty like what a conducting boundary does to a flat space, we firstly consider the quantum metrology in cosmic string space-time.

For this purpose, let us study a static polarizable two-level atom interacting with fluctuating electromagnetic fields in vacuum in cosmic string space-time. We suppose that a static, straight cosmic string lies along the z-direction. So the metric in the cylindrical coordinate system  $(t, \rho, \theta, z)$  is given by

$$ds^{2} = dt^{2} - d\rho^{2} - \rho^{2}d\theta^{2} - dz^{2},$$
(5)

where  $0 \le \theta < \frac{2\pi}{\nu}$ ,  $\nu = (1 - 4G\mu)^{-1}$  with *G* and  $\mu$  being the Newton's constant and the mass per unit length of the string respectively. In this space-time background, the vector potential of electromagnetic field can be solved as [58,59]

$$A_{\rho}(t, \vec{x}) = \frac{1}{\sqrt{2}} \int d\mu_{j} \\ [(c_{+j}f_{+j} + c_{-j}f_{-j}) + (c_{+j}^{\dagger}f_{+j}^{*} + c_{-j}^{\dagger}f_{-j}^{*})], \\ A_{\theta}(t, \vec{x}) = -\frac{i\rho}{\sqrt{2}} \int d\mu_{j} \\ [(c_{+j}f_{+j} - c_{-j}f_{-j}) - (c_{+j}^{\dagger}f_{+j}^{*} - c_{-j}^{\dagger}f_{-j}^{*})], \\ A_{z,t}(t, \vec{x}) = \int d\mu_{j} [c_{3j,0j}f_{0j} + c_{3j,0j}^{\dagger}f_{0j}^{*}]$$
(6)

with

$$f_{\xi j}(\vec{x}) = \frac{1}{2\pi} \sqrt{\frac{\nu}{2\omega}} J_{|\nu m + \xi|}(k_{\perp}\rho) e^{i(\nu m\theta + k_3 z)},$$
(7)

in which

$$\int d\mu_j = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dk_3 \int_0^{\infty} dk_\perp k_\perp, \tag{8}$$

 $c_{\xi j}(t) = c_{\xi j}(0)e^{-i\omega t}, c_{\xi j}^{\dagger} = c_{\xi j}^{\dagger}(0)e^{i\omega t}$  are respectively the annihilation and creation operators for a photon with quantum numbers  $j = (k_3, k_{\perp}, m)$  at time *t* with commutation relations

$$[c_{\xi j}(t), c^{\dagger}_{\xi j'}(t)] = \delta_{j,j'} \quad for \quad \xi = \pm 1, 3, \tag{9}$$

$$[c_{0j}(t), c_{0j'}^{\dagger}(t)] = -\delta_{j,j'} \quad for \quad \xi = 0.$$
<sup>(10)</sup>

Here  $\omega = \sqrt{k_3^2 + k_{\perp}^2}$ , and the symbol *J* denotes BesselJ function.

The total Hamiltonian of the coupled system can be written as  $H = H_s + H_f + H'$ , where  $H_s = \frac{1}{2}\omega_0\sigma_3$  is the Hamiltonian of the atom with  $\omega_0$  denoting the transition frequency.  $H_f$  denotes the Hamiltonian of the free electromagnetic field and its explicit expression is not required here. The Hamiltonian that describes the interaction between the atom and the electromagnetic field can be written in the multipolar coupling scheme as

$$H'(\tau) = -e\mathbf{r} \cdot \mathbf{E}(x(\tau)), \tag{11}$$

where *e* is the electron electric charge,  $e \mathbf{r}$  is the atomic electric dipole moment, and  $\mathbf{E}(x)$  denotes the electric field strength.

We let  $\rho_{tot} = \rho(0) \otimes |0\rangle \langle 0|$  denoting the initial total density matrix of the system. Here  $\rho(0)$  is the initial reduced density matrix of the atom which corresponds to the atomic state in Eqs. (4), and  $|0\rangle$  is the vacuum state of the field. The evolution of the total density matrix  $\rho_{tot}$  in the proper time  $\tau$  reads [60,61]

$$\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -i[H, \rho_{tot}(\tau)].$$
(12)

We assume that the interaction between the atom and field is weak. So, the evolution of the reduced density matrix  $\rho(\tau)$ can be written in the Kossakowski–Lindblad form [62–64]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i \left[ H_{\text{eff}}, \, \rho(\tau) \right] + \mathcal{L}[\rho(\tau)] \,, \tag{13}$$

where

$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{i,j=1}^{3} a_{ij} \left[ 2 \sigma_j \rho \sigma_i - \sigma_i \sigma_j \rho - \rho \sigma_i \sigma_j \right].$$
(14)

The coefficients of the Kossakowski matrix  $a_{ij}$  can be expressed as

$$a_{ij} = A\delta_{ij} - iB\epsilon_{ijk}\delta_{k3} - A\delta_{i3}\delta_{j3}, \tag{15}$$

with

$$A = \frac{1}{4} [\mathcal{G}(\omega_0) + \mathcal{G}(-\omega_0)], \quad B = \frac{1}{4} [\mathcal{G}(\omega_0) - \mathcal{G}(-\omega_0)].$$
(16)

We define a two-point correlation function  $G^+(x-x')$  which is related to the two-point functions of the electromagnetic fields  $\langle 0|E_i(x)E_j(x')|0\rangle$  as

$$G^{+}(x-x') = e^{2} \sum_{i,j=1}^{3} \langle \exists r_{i} | - \rangle \langle -|r_{j} \rangle \langle 0|E_{i}(x)E_{j}(x')|0\rangle.$$
(17)

Then its Fourier and Hilbert transforms  $\mathcal{G}(\lambda)$  and  $\mathcal{K}(\lambda)$  follows

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} d\Delta\tau \ e^{i\lambda\Delta\tau} \ G^+(\Delta\tau) ,$$
  
$$\mathcal{K}(\lambda) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega \ \frac{\mathcal{G}(\omega)}{\omega - \lambda}.$$
(18)

By absorbing the Lamb shift term, the effective Hamiltonian  $H_{\text{eff}}$  can be written as

$$H_{\rm eff} = \frac{1}{2}\Omega\sigma_3 = \frac{1}{2}\{\omega_0 + \frac{i}{2}[\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)]\}\sigma_3, \quad (19)$$

where  $\Omega$  is the effective level spacing of the atom. By applying Eq. (2) to Eq. (13), the Bloch vector with proper time  $\tau$  can be solved as:

$$\omega_{1}(\tau) = \sin\theta \cos(\Omega\tau + \phi) e^{-2A\tau},$$
  

$$\omega_{2}(\tau) = \sin\theta \sin(\Omega\tau + \phi) e^{-2A\tau},$$
  

$$\omega_{3}(\tau) = \cos\theta e^{-4A\tau} - \frac{B}{A}(1 - e^{-4A\tau}).$$
(20)

# **3** Influence of vacuum fluctuations on initial parameter estimation

Let us now examine how the vacuum fluctuations affect the quantum Fisher information as well as the precision of the initial parameter estimation. The electric two-point function can be written as

$$\langle 0|E_i(x)E_j(x')|0\rangle = \partial_0 \partial'_0 \langle 0|A_i(x)A_j(x')|0\rangle + \partial_i \partial'_j \langle 0|A_0(x)A_0(x')|0\rangle.$$
(21)

Applying the vector potential in Eqs. (6) and the trajectory of the static atom

$$t(\tau) = \tau, \quad \rho(\tau) = \rho, \quad \theta(\tau) = \theta, \quad z(\tau) = z,$$
 (22)

the electric-field two-point functions can be calculated as

$$\langle 0|E_{\rho}(x)E_{\rho}(x')|0\rangle = \frac{\nu}{8\pi^2} \int d\mu_j e^{i\omega(\tau-\tau')} \\ \times \left[\frac{\omega}{2}(J_{|\nu m+1|}^2(k_{\perp}\rho) + J_{|\nu m-1|}^2(k_{\perp}\rho)) - \frac{1}{\omega}\left(\frac{dJ_{|\nu m|}(k_{\perp}\rho)}{d\rho}\right)^2\right],$$
(23)

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$$\langle 0|E_{\theta}(x)E_{\theta}(x')|0\rangle = \frac{\nu\rho^{2}}{8\pi^{2}} \int d\mu_{j}e^{i\omega(\tau-\tau')} \\ \times \left[\frac{\omega}{2}(J^{2}_{|\nu m+1|}(k_{\perp}\rho) + J^{2}_{|\nu m-1|}(k_{\perp}\rho)) - \frac{1}{\omega}\frac{\nu^{2}m^{2}}{\rho^{2}}J^{2}_{|\nu m|}(k_{\perp}\rho)\right],$$
(24)

$$\langle 0|E_z(x)E_z(x')|0\rangle = \frac{\nu}{8\pi^2} \int d\mu_j \; \frac{k_\perp^2}{\omega} J_{|\nu m|}^2(k_\perp \rho) e^{i\omega(\tau-\tau')}.$$
(25)

So the Fourier transform of the correlation functions is given by:

$$\mathcal{G}(\lambda) = \sum_{i} \frac{e^2 |\langle -|r_i| + \rangle|^2 \lambda^3}{3\pi} f_i(\lambda, \rho, \nu) \theta(\lambda), \qquad (26)$$

with

$$f_{\rho}(\lambda, \rho, \nu) = \frac{3\nu}{4} \sum_{m} \\ \times \int_{0}^{1} dt \frac{t}{\sqrt{1 - t^{2}}} [(2 - t^{2})J_{|\nu m + 1|}^{2}(\lambda\rho t) \\ + t^{2}J_{|\nu m| + 1}(\lambda\rho t)J_{|\nu m| - 1}(\lambda\rho t)],$$
(27)

$$f_{\theta}(\lambda, \rho, \nu) = \frac{3\nu}{4} \sum_{m} \\ \times \int_{0}^{1} dt \frac{t}{\sqrt{1 - t^{2}}} [(2 - t^{2}) J_{|\nu m + 1|}^{2} (\lambda \rho t) \\ -t^{2} J_{|\nu m| + 1} (\lambda \rho t) J_{|\nu m| - 1} (\lambda \rho t)],$$
(28)

$$f_z(\lambda, \rho, \nu) = \frac{3\nu}{2} \sum_m \int_0^1 dt \frac{t^3}{\sqrt{1 - t^2}} J^2_{|\nu m|}(\lambda \rho t).$$
(29)

Thus the coefficients of the Kossakowski matrix  $a_{ij}$  and the effective level spacing of the atom now become

$$A = B = \frac{\gamma_0}{4} \left( \sum_i \alpha_i f_i(\omega_0, \rho, \nu) \right),$$
(30)  

$$\Omega = \omega_0 + \frac{\gamma_0}{2\pi\omega_0^3} P \int_0^\infty d\omega \, \omega^3 \times \left( 1 - \sum_i \alpha_i f_i(\omega_0, \rho, \nu) \right) \left( \frac{1}{\omega + \omega_0} - \frac{1}{\omega - \omega_0} \right),$$
(31)

where  $\gamma_0 = e^2 |\langle -|\mathbf{r}| + \rangle|^2 \omega_0^3 / 3\pi$ ,  $\alpha_i = |\langle -|r_i| + \rangle|^2 / |\langle -|\mathbf{r}| + \rangle|^2$ . Physically,  $\alpha_i$  represents the relative polarizability and they satisfy  $\sum_i \alpha_i = 1$ . We let  $f(\omega_0, \rho, \nu) \equiv \sum_i \alpha_i f_i(\omega_0, \rho, \nu)$ . As a result, the quantum Fisher information of the initial weight and phase parameters can be expressed as follows

$$F_{\phi} = \sin^2 \theta \ e^{-\gamma_0 f(\omega_0, \rho, \nu)\tau} \tag{32}$$

and

$$F_{\theta} = e^{-\gamma_0 f(\omega_0, \rho, \nu)\tau}.$$
(33)

From the results, we find that the decay rates are multiplied by a factor  $f(\omega_0, \rho, \nu)$  in compare to those in flat space-time case.

3.1 The case for 
$$v = 1$$
.

When v = 1, the deficit angle disappears and the cosmic string space-time reduces to the flat space-time without cosmic strings. Following the properties of the BesselJ function

$$\sum_{m} J_{|m|}^{2}(x) = 1, \qquad \sum_{m} J_{|m|+1}(x) J_{|m|-1}(x) = 0, \qquad (34)$$

 $f_i(\omega_0, \rho, \nu) = 1$  ( $i = \rho, \theta, z$ ). So the dacay rate of QFI becomes  $\gamma_0$  which is just the decay rate of QFI in a free Minkowski space-time. As a result, for  $\nu = 1$ , the results in Minkowski space-time are recovered as expected.

### 3.2 The case for $\nu > 1$ .

We first consider the asymptotic behavior when  $\omega \rho \ll 1$ , one has

$$f_{\rho}(\omega_{0},\rho,\nu) \approx f_{\theta}(\omega_{0},\rho,\nu) \approx \frac{3\nu^{2}(\nu+1)}{\Gamma[2\nu+2]}(\omega_{0}\rho)^{2(\nu-1)},$$
  
$$f_{z}(\omega_{0},\rho,\nu) \approx \nu.$$
 (35)

So atoms with different polarization behave differently. When the polarization is along the *z*-axis, i.e.,  $\alpha_{\rho} = \alpha_{\theta} = 0$ , the decay rate becomes  $\nu$  times of that in the Minkowski vacuum case, which makes the QFI decay even faster than that in flat space-time. However, when the polarization is perpendicular to the *z*-axis, i.e.,  $\alpha_z = 0$ , the decay rate approaches zero, which means that the QFI is protected from electromagnetic vacuum fluctuations when the atom is extremely close to the cosmic string and the polarization direction of the atom is perpendicular to the direction of the cosmic string.

For a generic atom-string distance, the QFI may be decreased, or enhanced as compared to that in flat spacetime case. Since the expressions of QFI is really complicated, we now give some results of numerical in this case. To show the properties of relations among decay rate, atom-string distance and atomic polarization graphically, in Fig. 1, we plot  $f(\omega_0, \rho, 1.5)$  as a function of  $\rho$  for  $\alpha = (1, 0, 0), (0, 1, 0), (0, 0, 1)$  which correspond to the radial polarization, tangential polarization, polarization parallel to the string cases respectively.

The oscillatory behavior displayed in Fig. 1 is similar to that in boundary flat space case [44], the QFI can either be decreased, or enhanced as compared to that in flat space-time case with proper atom-string distance and proper polarization. Apparently, the QFI is protected from electromagnetic



Fig. 1 f as a function of  $\rho$  for  $\alpha = (1, 0, 0), (0, 1, 0), (0, 0, 1)$  respectively. Here  $\rho$  is in the unit of  $1/\omega_0$ 

vacuum fluctuations when the atom is extremely close to the cosmic string and the polarization direction of the atom is perpendicular to the direction of the cosmic string. Similar conclusion is obtained in boundary flat space case [44]. When the atom is extremely close to the boundary and the polarization direction of the atom is perpendicular to the direction of the boundary, the QFI is protected from electromagnetic vacuum fluctuations. However, we should note that the behaviors of the QFI for probe atoms in the same position with different polarization directions perpendicular to the direction of the boundary are the same in bounded flat space-time, while the behaviors of the QFI for probe atoms in the same position with different polarization directions perpendicular to the direction of the cosmic string are different in cosmic string space-time. Furthermore, the oscillatory behavior of QFI in cosmic string space-time is also determined by the property factor  $\nu$  of cosmic string, which indicates different cosmic strings lead to different behaviors of QFI.

## 4 Sensing cosmic string through oscillatory behavior of QFI

From the point of statistical distance between states, quantum Fisher information [1–5] which is the metric in parameter space is a proper candidate to describe the discrimination of states corresponding to small increment of the parameter:  $\{d_{Bures}[P(X), P(X + dX)]\}^2 = \frac{1}{4}F_X dX^2$  [5]. Here  $d_{Bures}$  denotes the Bures distance which can be estimated through probability distributions of proper measurements on the states. As a result,  $F_{\phi}$  can be calculated through the Bures distance between states  $P(\phi)$  and  $P(\phi + d\phi)$ , which makes  $F_{\phi}$  itself observable from probability distributions of the measurements.

As the oscillation appears when  $\nu > 1$ , we can use  $F_{\phi}$  to examine the existence of cosmic string. Since the oscillatory behavior of decay rate of  $F_{\phi}$  depends on  $\nu$ , the magnitude of  $\nu$  can be deduced from the distribution of  $F_{\phi}$ . Since  $F_{\phi}$  is independent on z and  $\theta$ , one can easily find the radial direction of location of cosmic string through the increments of  $F_{\phi}$  corresponding to the increments of coordinates. Then along the direction of  $\rho$ , the cosmic string is located at where  $F_{\phi}$  of a probe atom with radial polarization reaches to its maximum value.

# 5 Effects of vacuum fluctuations on atomic frequency estimation

As is shown, the presence of cosmic string may protect the estimation precision of the initial parameters from the influence of the environment in proper circumstances. We may also wonder what happens if the parameter to be estimated is the property of the probe atom. Now we consider the parameter to be estimated is the atomic frequency  $\omega_0$ . By using Eqs. (3) and (20), the quantum Fisher information of  $\omega_0$  can be calculated after dropping the terms which are of higher order in terms of the electric dipole moment as

$$F_{\omega_0} \approx \sin^2 \theta \, e^{-\gamma_0 f(\omega_0, \rho, \nu)\tau} \tau^2 \tag{36}$$

So the maximum quantum Fisher information is obtained for  $\theta = \frac{\pi}{2}$  with the form

$$F_{\omega_0} \approx e^{-\gamma_0 f(\omega_0,\rho,\nu)\tau} \tau^2.$$
(37)

Applying the equation  $\frac{\partial F_{\omega_0}}{\partial \tau} = 0$ , we find that the maximum of  $F_{\omega_0} = \frac{4}{\gamma_0^2 f(\omega_0, \rho, \nu)^2} e^{-2}$  can be reached at  $\tau = \frac{2}{\gamma_0 f(\omega_0, \rho, \nu)}$ . Due to the oscillatory behavior of  $f(\omega_0, \rho, \nu)$  as is shown in Fig. 1, both the optimal measurement time and the maximal quantum Fisher information may either be enhanced or depressed in compare to those in flat space-time. For an arbitrary polarization, in regions where  $f(\omega_0, \rho, \nu) > 0$ , both the optimal measurement time and the maximal quantum Fisher information are enhanced as compared to those in flat spacetime case, while in regions where  $f(\omega_0, \rho, \nu) < 0$ , they are depressed.

#### 6 Conclusion

In summary, the dynamics of QFI for a static polarizable two-level atom coupled in the multipolar scheme to a bath of fluctuating vacuum electromagnetic fields is studied in cosmic string space-time. When we estimate the parameters of initial atomic state, we find that with the presence of the cosmic string, QFI becomes position and atomic polarization dependent, which makes the precision of estimation decreased, enhanced or remain unchanged depending on the atom-string distance and atomic polarization. When the atom is extremely close to the cosmic string and the polarization direction of the atom is perpendicular to the direction of cosmic string, the QFI may even be shielded from the influence of the vacuum fluctuations as if it were a closed system. The oscillatory behavior of decay rate of QFI is also in relation with the property  $\nu$  of the cosmic string, which makes it possible to sense the cosmic string through QFI-determined Bures distance.

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