Eur. Phys. J. C (2020) 80:1046
https://doi.org/10.1140/epjc/s10052-020-08631-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Revisiting the production of J/ψ pairs at the LHC

A.A. Prokhorov[1,2], **A.V. Lipatov**[2,3,a], **M.A. Malyshev**[3], **S.P. Baranov**[4]

[1] Faculty of Physics, Lomonosov Moscow State University, 119991 Moscow, Russia
[2] Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia
[3] Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991 Moscow, Russia
[4] P.N. Lebedev Institute of Physics, Moscow 119991, Russia

Received: 1 September 2020 / Accepted: 2 November 2020 / Published online: 11 November 2020

Abstract We consider the prompt double J/ψ production in pp collisions at the LHC in the framework of k_T-factorization QCD approach. Using the fragmentation mechanism, we evaluate the color octet contributions to the production cross sections taking into account the combinatorial effects of multiple gluon radiation in the initial state driven by the Ciafaloni–Catani–Fiorani–Marchesini evolution equation. We demonstrate the importance of these contributions in a certain kinematical region covered by the CMS and ATLAS measurements. On the other hand, the experimental data taken by the LHCb Collaboration at forward rapidities and moderate transverse momenta can be described well by $\mathcal{O}(\alpha_s^4)$ color singlet terms and contributions from the double parton scattering mechanism. The extracted value of the effective cross section is compatible with many other estimations based on different final states.

1 Introduction

Prompt production of J/ψ meson pairs at high energies is a very intriguing subject of studies [1–4]. It provides a unique laboratory to investigate the quarkonia production mechanisms predicted by the non-relativistic QCD (NRQCD) factorization [5–7], which is a rigorous framework for the description of heavy quarkonia production or decays. The NRQCD implies a separation of perturbatively calculated short distance cross sections for the production of a heavy quark pair in an intermediate Fock state ${}^{2S+1}L_J^{(a)}$ with spin S, orbital angular momentum L, total angular momentum J and color representation a from its subsequent non-perturbative transition into a physical quarkonium via soft gluon radiation. The latter is described by the long-distance non-perturbative matrix elements (LDMEs), which obey certain hierarchy in powers of the relative heavy quark velocity v [5–7]. At the next-to-leading order (NLO), NRQCD can explain the LHC data on the prompt J/ψ, ψ' and χ_c transverse momentum distributions (see, for example, [8–15]). However, it has a long-standing challenge in the J/ψ and ψ' polarization and provides inadequate description [16–18] of the η_c production data[1] (see also discussions [23–25]). Studying the J/ψ meson pair production can shed light on the puzzling aspects above since $c\bar{c}$ bound state formation takes place here twice.

In the last few years, significant progress has been made in the NRQCD evaluations of prompt double J/ψ production. The complete leading-order (LO) calculations, including both the color singlet (CS) and color octet (CO) terms, were done [26]. The relativistic corrections to the J/ψ pair production are carried out [27]. The NLO contributions to the CS mechanism are known [28] and partial tree-level NLO* contributions to the both CS and CO terms were calculated [29]. The latter were found to be essential for both low and large transverse momenta, as compared to the LO results.[2] However, being comparable with the LHCb measurements [3,4], all these evaluations have sizeble discrepancies with the latest CMS [1] and ATLAS [2] data, especially at large transverse momentum $p_T(J/\psi, J/\psi)$, invariant mass $m(J/\psi, J/\psi)$ and rapidity separation $\Delta y(J/\psi, J/\psi)$ of the J/ψ pairs. For example, the CMS data are underestimated by the NRQCD predictions with a factor of about 10 [26,28]. The difference between the theoretical calculations and more recent ATLAS data at large $p_T(J/\psi, J/\psi)$ or $m(J/\psi, J/\psi)$ is typically smaller but still essential. It was argued [26] that new processes or mechanisms are needed to better describe the LHC data.

At large invariant mass $m(J/\psi, J/\psi)$ the processes with large angular separation between the J/ψ mesons could play

[a] e-mail: lipatov@theory.sinp.msu.ru (corresponding author)

[1] One possible solution, which, however, implies certain modification of the NRQCD rules, has been proposed [19] (see also [20–22]).

[2] At the moment, full NLO NRQCD predictions for double J/ψ production are not available yet.

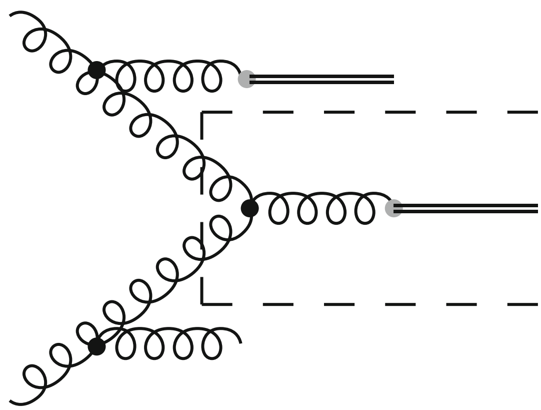

Fig. 1 Contribution to the J/ψ pair production from the fragmentation of gluon cascade. The dashed line encloses the hard subprocess $g^*g^* \to g^*$, the rest of the diagram describes the initial state radiation cascade

a role. One of such processes are the gluon or quark fragmentation shown in Fig. 1. The gluon fragmentation into ${}^3S_1^{(8)}$ intermediate state scales as $1/p_T^4$ and govern the single J/ψ production at high transverse momenta (see, for example, [8–12] and references therein). In the case of J/ψ pair production, such terms were found to be negligible since they are suppressed by powers of QCD coupling α_s [26]. However, at large $p_T(J/\psi, J/\psi)$ or $m(J/\psi, J/\psi)$ one can expect a sizeble combinatorial contribution to the fragmentation yield from the multiple gluon radiation originating during the QCD evolution of the initial gluon cascade. The latter determines the perturbative QCD corrections to the production cross sections at high energies, which can be effectively taken into account using the Ciafaloni–Catani–Fiorani–Marchesini (CCFM) evolution equation [30–33]. Main goal of our study is to clarify this point and investigate the role of combinatorial cascade gluon fragmentation contributions to the double J/ψ production in different kinematical regimes at the LHC.

Our other goal is connected with the investigation of additional production mechanism, double parton scattering (DPS), which is widely discussed in the literature at present (see, for example, [34–41] and references therein). Apart from the single parton scattering (SPS), where J/ψ meson pair is produced in a single gluon–gluon collision, DPS events originate from two independent parton interactions. Studying the DPS mechanism is of great importance since it can help in understanding various backgrounds in searches for new physics at the collider experiments. Despite the relative low total production rate, the DPS mechanism is expected to be important for double J/ψ production at forward rapidities [42–44]. Therefore, the latter can be used to determine the DPS key parameter, the effective cross section σ_{eff}, which is related to the transverse overlap function between partons in the proton and supposed to be universal for all processes with different kinematics and energy scales. Most of the measured values of σ_{eff} lie between 12 and 20 mb (see, for example, [45,46]). However, somewhat lower value $\sigma_{\text{eff}} = 8.8-12.5$ mb was extracted from the latest LHCb data on J/ψ pair production within the NRQCD [4]. Moreover, the values $\sigma_{\text{eff}} = 8.2 \pm 2.2$ mb [47], $\sigma_{\text{eff}} = 6.3 \pm 1.9$ mb [2], $\sigma_{\text{eff}} = 4.8 \pm 2.5$ mb [48] and even $\sigma_{\text{eff}} = 2.2 \pm 1.1$ mb [49], $\sigma_{\text{eff}} = 2.2 - 6.6$ mb [50] were obtained from recent Tevatron and LHC experiments. Below we will try to extract the effective cross section σ_{eff} from combined analysis of the LHCb data [3,4] on double J/ψ production taken at $\sqrt{s} = 7$ and 13 TeV.

To calculate the physical cross sections we use the k_T-factorization approach [51–54]. We see certain technical advantages in the fact that, even with the LO amplitudes for hard subprocesses, one can include a large piece of higher-order pQCD corrections (NLO + NNLO + ...) taking them into account in the form of CCFM-evolved Transverse Momentum Dependent (TMD) gluon densities in a proton[3]. In this way we preserve consistency with our previous studies [19–22] and automatically incorporate the wanted effects of initial state gluon radiation. To reconstruct the CCFM evolution ladder, that is the key point of our consideration, we employ the TMD parton shower routine implemented into the Monte-Carlo event generator CASCADE [56]. The k_T-factorization approach can be considered as a convenient alternative to explicit high-order calculations in the collinear DGLAP-based scheme. The situation in J/ψ pair production is specific since calculating even the LO hard scattering amplitudes is already complicated enough, so that extending to higher orders seems to be a rather cumbersome task. Thus, the k_T-factorization remains the only way open to potentially important higher-order effects. To evaluate the DPS contributions to the double J/ψ production we will use the results of our previous analysis [20].

The outline of the paper is the following. In Sect. 2 we briefly describe the basic steps of our calculations. In Sect. 3 we present the numerical results and discussion. Our conclusions are summarised in Sect. 4.

2 The model

The neccessary starting point of our consideration is related with CS contribution to the double J/ψ production, that refers to $\mathcal{O}(\alpha_s^4)$ gluon–gluon fusion subprocess

$$g^*(k_1) + g^*(k_2) \to c\bar{c}\left[{}^3S_1^{(1)}\right](p_1) + c\bar{c}\left[{}^3S_1^{(1)}\right](p_2), \quad (1)$$

where the four-momenta of all particles are indicated in the parentheses. Some typical Feynman diagrams are depicted in Fig. 2. It is important that both initial gluons are off mass

[3] The description of this approach can be found, for example, in review [55].

shell. That means that they have non-zero transverse four-momenta $k_{1T}^2 = -\mathbf{k}_{1T}^2 \neq 0$ and $k_{2T}^2 = -\mathbf{k}_{2T}^2 \neq 0$ and an admixture of longitudinal component in the polarization four-vectors (see [51–54] for more information). The corresponding off-shell (k_T-dependent) production amplitude contains widely used projection operators for spin and color [57–60] which guarantee the proper quantum numbers of final state charmonia. Below we apply the gauge invariant expression obtained earlier [61]. The derivation steps are explained in detail there. The respective cross section can be written as

$$\sigma(pp \to J/\psi J/\psi + X) = \int \frac{1}{16\pi(x_1x_2s)^2}|\bar{\mathcal{A}}(g^*g^* \to J/\psi J/\psi)|^2 \times f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 d\mathbf{p}_{1T}^2 dy_1 dy_2 \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}\frac{d\psi_1}{2\pi}, \quad (2)$$

where ψ_1 is the azimuthal angle of outgoing J/ψ meson, ϕ_1 and ϕ_2 are the azimuthal angles of initial gluons having the longitudinal momentum fractions x_1 and x_2, y_1 and y_2 are the center of mass rapidities of produced particles and $f_g(x, \mathbf{k}_T^2, \mu^2)$ is the TMD gluon density in a proton taken at the scale μ^2.

In addition to the CS terms above, we have considered some of CO contributions using the fragmentation approach. At high transverse momenta, $p_T \gg m_\psi$, large logaritmic corrections proportional to $\alpha_s^n \ln^n p_T/m_\psi$ occur and, therefore, description in terms of fragmentation functions (FFs), evolving with the energy scale μ^2, appears to be appropriate. In general, the FF $D_a^{\mathcal{H}}(z, \mu^2)$ describing the transition of parton a into the charmonium state $\mathcal{H}$ can be expressed as follows (see, for example, [62] and references therein):

$$D_a^{\mathcal{H}}(z, \mu^2) = \sum_n d_a^n(z, \mu^2)\langle\mathcal{O}^{\mathcal{H}}[n]\rangle, \quad (3)$$

where n labels the intermediate (CS or CO) state of charmed quark pair produced in the hard parton interaction and $\langle\mathcal{O}^{\mathcal{H}}[n]\rangle$ are the corresponding LDMEs. In the leading logarithmic approximation, $g^* \to c\bar{c}[^3S_1^{(8)}]$ transition is the only one giving a sizeble contribution to S-wave charmonia production at $p_T \gg m_\psi$ [62], so that the cross section of inclusive single J/ψ production in pp collisions could be approximately calculated as

$$\frac{d\sigma(pp \to J/\psi + X)}{dp_T} = \int dz \frac{d\sigma(pp \to g^*)}{dp_T^{(g^*)}} d_g^{[^3S_1^{(8)}]}(z, \mu^2)\langle\mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle, \quad (4)$$

where $p = zp^{(g^*)}$ and $p^{(g^*)}$ are the outgoing J/ψ meson and intermediate gluon momenta. One can easily obtain

$$\sigma(pp \to g^*) = \int \frac{\pi}{x_1x_2s\lambda^{1/2}(m_\psi^2, k_1^2, k_2^2)}|\bar{\mathcal{A}}(g^*g^* \to g^*)|^2 \times f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}, \quad (5)$$

where $p^{(g^*)} = k_1 + k_2$ and $\lambda(m_\psi^2, k_1^2, k_2^2)$ is the known kinematical function [63]. Evaluation of the off-shell production amplitude $|\bar{\mathcal{A}}(g^*g^* \to g^*)|^2 = (3/2)\pi\alpha_s(\mu^2)|\mathbf{p}_T^{(g^*)}|^2$ is an extremely straightforward and, in our opinion, needs no explanation. We only note that, according to the k_T-factorization prescription [51–54], the summation over the polarizations of initial off-shell gluons is carried out with $\sum \epsilon^\mu \epsilon^{*\nu} = \mathbf{k}_T^\mu \mathbf{k}_T^\nu/\mathbf{k}_T^2$. In the collinear limit $\mathbf{k}_T \to 0$ this expression converges to the ordinary one after averaging on the azimuthal angle.

The key point of our consideration is that the gluon, produced in the hard scattering and fragmented into the J/ψ meson according to main formula (4), is accompanied by a number of gluons radiated during the non-collinear QCD evolution, which also give rise to final J/ψ mesons. Thus, taking into account all their possible combinations into the meson pairs, one can calculate corresponding gluon fragmentation contribution to the double J/ψ production up to all orders in the pQCD expansion. At high energies, the QCD evolution of gluon cascade can be described by the CCFM equation [30–33], which smoothly interpolates between the small-x BFKL gluon dynamics and high-x DGLAP one, and, therefore, provides us with the suitable tool for our phenomenological study. To reconstruct the CCFM evolution ladder, we generate a Les Houches Event file [64] in the numerical calculations according to (4) and (5) and then process the file with a TMD shower tool implemented into the Monte-Carlo event generator CASCADE [56]. This approach gives us the possibility to take into account the contributions from initial state gluon emissions in a consistent way (see also [65]).

Of course, the same scenario can be applied to fragmentation of charmed quark pairs into J/ψ mesons. So, one can first simulate the perturbative production of $c\bar{c}$ pair in the off-shell gluon–gluon fusion and then reconstruct the CCFM evolution ladder using the CASCADE tool. After that, one can easily produce J/ψ pairs by taking into account all possible combinations of mesons originating from the charmed quarks and/or cascade gluon fragmentation. Unlike the conventional (collinear) QCD factorization, where only fragmentation of both charmed quarks into J/ψ mesons gives contribution, the model above can lead to increase in the double J/ψ production cross section due to additional combinatorial contributions from gluons and quarks.

The charm and gluon FFs at the any scale μ^2, $D_c^{J/\psi}(z, \mu^2)$ and $D_g^{J/\psi}(z, \mu^2)$, can be obtained by solving the LO DGLAP

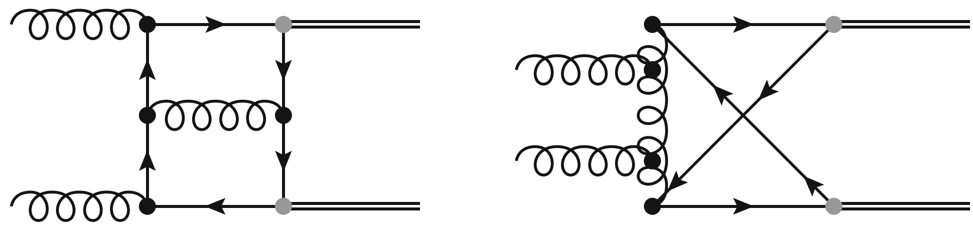

Fig. 2 Examples of the Feynman diagrams, contributing to the J/ψ pair production via CS mechanism

evolution equations:

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} D_c \\ D_g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_c \\ D_g \end{pmatrix}, \quad (6)$$

where P_{ab} are the standard LO DGLAP splitting functions. The initial conditions for these FFs are calculated with [56]

$$d_g^{[^3S_1^{(8)}]}(z,\mu_0^2) = \frac{\alpha_s(\mu_0^2)}{24m_c^3}\pi\delta(1-z), \quad (7)$$

$$d_c^{[^3S_1^{(1)}]}(z,\mu_0^2) = \frac{\alpha_s^2(\mu_0^2)}{m_c^3}\frac{16z(1-z)^2}{243(2-z)^6} \left(5z^4 - 32z^3 + 72z^2 - 32z + 16\right), \quad (8)$$

where starting scale $\mu_0^2 = m_\psi^2$. As it was noted above, we keep only the leading contributions to corresponding FFs (see, for example, [62] and references therein). According to the non-relativistic QCD approximation, we set the charmed quark mass to $m_c = m_\psi/2$ and then solve the DGLAP equations (6) numerically. The obtained charm and gluon FFs, $D_c^{J/\psi}(z,\mu^2)$ and $D_g^{J/\psi}(z,\mu^2)$, are shown in Fig. 3 as functions of z for several values of scale μ^2. Using these FFs, we reproduce well the results of calculations performed with the Monte Carlo event generator PEGASUS [66] (see Fig. 4).

Finally, we turn to the DPS contribution to the double J/ψ production. We apply a commonly used factorization formula (for details see reviews [34–41] and references therein):

$$\sigma_{\rm DPS}(pp \to J/\psi\, J/\psi + X) = \frac{1}{2}\frac{\sigma^2(pp \to J/\psi + X)}{\sigma_{\rm eff}}, \quad (9)$$

where factor 1/2 accounts for two identical particles in the final state. The effective cross section $\sigma_{\rm eff}$ can be considered as a normalization constant which incorporates all "DPS unknowns" in to a single phenomenological parameter. Derivation of the factorization formula (9) relies on the two approximations: first, the double parton distribution function can be decomposed into longitudinal and transverse components and, second, the longitudinal component reduces to the diagonal product of two independent single parton densities. The latter is generally acceptable for such collider experiments where small-x values are probed. The typical values of the variable x in the considered process are of order $x \sim (2m_\psi^2 + p_T^2)^{1/2}/\sqrt{s} \sim 10^{-3}$, that approximately corresponds to the kinematical region of CMS [1], ATLAS [2] and even LHCb [3,4] measurements (due to relatively small invariant mass of produced J/ψ pair, see discussion below). Therefore, one can safely omit the kinematical constraint [67,68] often applied at the edge of phase space[4]. Detailed description of evaluation of inclusive cross section $\sigma(pp \to J/\psi + X)$ in the k_T-factorization approach supplemented with the NRQCD formalism can be found [20].

In the numerical calculations below we will use TMD gluon densities in a proton obtained from the numerical solution of CCFM evolution equation, namely, A0 [70] and more recent JH'2013 set 2 [71] gluons. Their input parameters have been fitted to the proton structure function $F_2(x, Q^2)$. At present, both these gluon distributions are widely used in the phenomenological applications[5] (see, for example, [20–22]). The renormalization and factorization scales, μ_R and μ_F, were set to $\mu_R^2 = \mu_F^2 = \hat{s} + \mathbf{Q}_T^2$, where $\hat{s} = (k_1 + k_2)^2$ and $\mathbf{Q}_T^2$ is the transverse momentum of initial off-shell gluon pair. This choice is dictated mainly by the CCFM evolution algorithm (see [70,71] for more information). As it is often done, the fragmentation scale $\mu_{\rm fr}$ is choosen to be equal to $\mu_{\rm fr} = m_T$, the transverse mass of fragmenting parton. We use the one-loop formula for the QCD coupling α_s with $n_f = 4$ active quark flavors at $\Lambda_{\rm QCD}^{(4)} = 250$ MeV for A0 gluon and two-loop expression for α_s with $n_f = 4$ and $\Lambda_{\rm QCD}^{(4)} = 200$ MeV for JH'2013 set 2 distribution. Following [73], we set the J/ψ meson mass $m_\psi = 3.097$ GeV. We take corresponding CS LDME from the known $J/\psi \to \mu^+\mu^-$ decay width: $\langle\mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle = 1.16$ GeV^3 [8–12].

3 Numerical results and discussion

We are now in a position to present the results of our simulations. First we discuss the role of cascade gluon fragmentation in different kinematical regimes, which correspond to the CMS, ATLAS and LHCb experiments.

In Fig. 5 we show the differential cross sections of double J/ψ production calculated as a functions of J/ψ pair invariant mass $m(J/\psi, J/\psi)$ and difference in rapidity between the J/ψ mesons $|\Delta y(J/\psi, J/\psi)|$ at $\sqrt{s} = 13$ TeV. We have required $p_T(J/\psi) > 10$ GeV for both produced mesons,

[4] Phenomenological consequences of the kinematical constraint [67,68] at the large x were investigated [69].

[5] A comprehensive collection of the TMD gluon densities can be found in the TMDLIB package [72], which is a C++ library providing a framework and an interface to the different parametrizations.

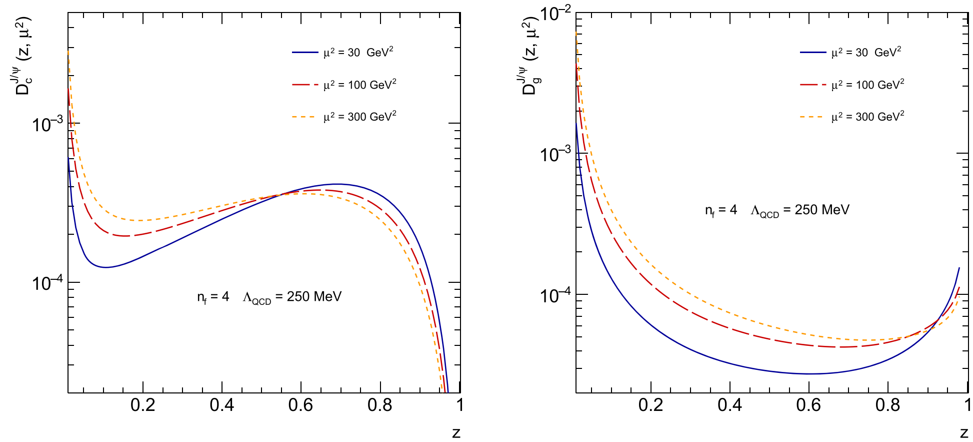

Fig. 3 The charm (left panel) and gluon (right panel) FFs, $D_c^{J/\psi}(z, \mu^2)$ and $D_g^{J/\psi}(z, \mu^2)$, calculated as functions of z for several values of scale μ^2. We have applied one-loop QCD coupling with $n_f = 4$, $\Lambda_{\rm QCD} = 250$ MeV and took $\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle = 1.16$ GeV^3, $\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle = 2.5 \cdot 10^{-3}$ GeV^3

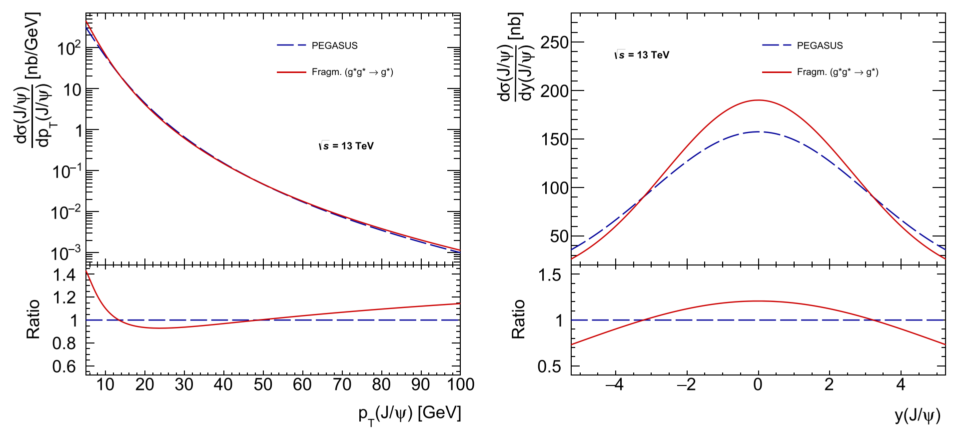

Fig. 4 Transverse momentum and rapidity distributions of inclusive J/ψ production at $\sqrt{s} = 13$ TeV, calculated using the fragmentation approach and Monte-Carlo event generator PEGASUS [63]. The contributions from the $^3S_1^{(8)}$ transition are only taken into account with $\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle = 2.5 \cdot 10^{-3}$ GeV^3 [20]. The A0 gluon distribution in proton is applied

that ensures the validity of the fragmentation approach used. Moreover, this restriction close to the CMS or ATLAS conditions. To illustrate the effect of using different TMD gluon densities we present the results obtained with the A0 and JH'2013 set 2 distributions. One can see that an accurate account of combinatorial contributions, originated from the cascade gluon fragmentation into the J/ψ mesons (labeled as "fragm. comb."), significantly (up to an order of magnitude) increase the cross section compared to the single gluon fragmentation, governed by the LO gluon–gluon fusion subprocess[6] (labeled as "fragm. coll."). For the latter, we reproduce the results [28]. To highlight the importance of the combinatorial gluon fragmentation, we show the results obtained using the simplified selection of J/ψ pair in each event, where one of the J/ψ mesons is originated from the gluon produced in the hard scattering subprocess and another one is produced from the leading cascade gluon (labeled as "fragm. lead."). This selection criterion almost corresponds to the collinear limit, as it is clearly demonstrated in Fig. 5. Next, we find that the cascade gluon fragmentation plays a dominant role at large invariant masses $m(J/\psi, J/\psi) \geq 25$ GeV for A0 gluon density (or at $m(J/\psi, J/\psi) \geq 35$ GeV for JH'2013 set 2 one) and $|\Delta y(J/\psi, J/\psi)| \geq 1$, where it greatly overestimates the CS contributions. Note that the quantitative difference between the A0 and JH'2013 set 2 predictions observed in Figs. 5 and 6 is connected mainly with the different LDMEs used in the calculations[7]. Taking into account these combinatorial contributions results in the drastical rise of the double J/ψ production cross sections at large $m(J/\psi, J/\psi)$, where the strong discrepancy between the NRQCD estimations (including both the CS and CO terms) and experimental data, taken by the CMS and ATLAS Collaborations, is observed. Contrary, the combinatorial fragmentation effects should be significantly less pronounced at forward rapidities, which are covered by the LHCb measurements. To demonstrate it, we have repeated the calculations under the requirements $4.5 < p_T(J/\psi) < 10$ GeV and $2 < y(J/\psi) < 4.5$. The upper limit of $p_T(J/\psi)$ is set to be the same as in LHCb analyses [3,4] while lower limit corresponds to the region, where the fragmentation approach is valid. Our results for distributions in $m(J/\psi, J/\psi)$ and $|\Delta y(J/\psi, J/\psi)|$ are shown in Fig. 6. One can see that the

[6] Here we have applied the MMHT'2014 (LO) parton density set [74].

[7] We took the fitted LDMEs from [20].

cascade gluon fragmentation gives only small contribution to the forward J/ψ pair production and, in principle, can be safely neglected. It can be easily understood since at large rapidities (or, equivalently, at large momentum fraction x of one of the interacting gluons) the gluon emissions in the initial state are insufficient.

Concerning the contributions from charm fragmentation, their role (compared to the LO predictions of conventional pQCD) is also a bit enhanced due to the multiple gluon emissions in the initial state. We find that these processes amount several percent of the J/ψ pair production cross section (see Figs. 5 and 6) and, of course, can be considered as additional non-leading terms.[8]

Thus, we have shown that taking into account the combinatorial contributions from the cascade gluon fragmentation could fill the gap between the NRQCD predictions and experimental data. However, to perform the quantitative comparison with the available CMS [1] and ATLAS [2] measurements one has to include into the analysis a number of other possible fragmentation channels playing role at low and moderate transverse momenta. Moreover, additional feeddown contributions to the double J/ψ production from the χ_c and ψ' decays should be taken into account. An accurate theoretical description requires a rather long-time numerical calculations. So, here we only claim the possible importance of the combinatorial fragmentation terms above and left their further cumbersome analysis for a forthcoming dedicated study.

Now we turn to available LHCb data collected at $\sqrt{s} = 7$ and 13 TeV [3,4]. These data refer to $p_T(J/\psi) < 10$ GeV, $m(J/\psi, J/\psi) < 15$ GeV and forward rapidity region, $2 < y(J/\psi) < 4.5$. Since the combinatorial contributions from gluon and/or charmed quark fragmentation are almost negligible there, only the CS terms and DPS production mechanism play the role. The latter give us the possibility to easily extract the key parameter of DPS mechanism, the effective cross section $\sigma_{\rm eff}$, from the LHCb measurements. The feeddown contributions from radiative χ_c and ψ' decays to the SPS cross section, which is governed by the subprocess (1), are also unimportant at small transverse momenta and invariant mass $m(J/\psi, J/\psi)$, see discussions [44,75]. Thus, we neglect below all these terms for simplicity. To evaluate the DPS contribution to the J/ψ pair production we use the results of our previous studies and strictly follow the approach [20] for the inclusive cross section $\sigma(pp \to J/\psi + X)$, entering to the DPS factorization formula (9). So, the determination of $\sigma_{\rm eff}$ can be performed in a self-consistent way. Note that only the SPS terms are affected by the choice of TMD gluon density in a proton. The DPS predictions are stable, because when we switch to a different set of TMD gluons we have to accordingly change the LDME values, in order to keep the calculation consistent with the single inclusive J/ψ production data. It means that the changes in TMD are compensated by the changes in LDMEs, making the net result the same.

The following kinematical variables have been investigated in the LHCb analyses [3,4]: transverse momentum $p_T(J/\psi, J/\psi)$, rapidity $y(J/\psi, J/\psi)$ and invariant mass of the J/ψ pair, transverse momentum and rapidity of J/ψ mesons, differences in the azimuthal angle $|\Delta\phi(J/\psi, J/\psi)|$ and rapidity $|\Delta y(J/\psi, J/\psi)|$ between the produced mesons and transverse momentum asymmetry $\mathcal{A}_T$, defined as

$$\mathcal{A}_T = \left| \frac{p_T(J/\psi_1) - p_T(J/\psi_2)}{p_T(J/\psi_1) + p_T(J/\psi_2)} \right|. \tag{10}$$

The measurements have been performed for $p_T(J/\psi, J/\psi) > 1$ GeV, $p_T(J/\psi, J/\psi) > 3$ GeV and in the whole $p_T(J/\psi, J/\psi)$ range. We consider $\sigma_{\rm eff}$ as an independent parameter and perform a simultaneous fit to the LHCb data. The fitting procedure was separately done for each of the measured kinematical distributions employing the fitting algorithm as implemented in the commonly used GNUPLOT package [76].

Not all of the existing data sets are equally informative for the $\sigma_{\rm eff}$ extraction. Using the data where the DPS contribution is smaller than the uncertainty of the "main" contribution would only increase the total error. So, our fit is based on the following distributions (all measured at $\sqrt{s} = 7$ TeV and 13 TeV): single J/ψ transverse momentum $p_T(J/\psi)$; single J/ψ rapidity $y(J/\psi)$; invariant mass $m(J/\psi, J/\psi)$; transverse momentum of J/ψ pair; rapidity of J/ψ pair; transverse momentum asymmetry $\mathcal{A}_T$; rapidity separation between the two J/ψ mesons $|\Delta y(J/\psi, J/\psi)|$. For all observables except $|\Delta y(J/\psi, J/\psi)|$ we used the data without cuts on $p_T(J/\psi, J/\psi)$ and with $p_T(J/\psi, J/\psi) > 1$ GeV, while for the rapidity separation $|\Delta y(J/\psi, J/\psi)|$ we used the sets without cuts on $p_T(J/\psi, J/\psi)$, with $p_T(J/\psi, J/\psi) > 1$ GeV, and with $p_T(J/\psi, J/\psi) > 3$ GeV.

The obtained mean-square averages of the fitted values are $\sigma_{\rm eff} = 17.5 \pm 4.1$ mb (for A0 gluon) and $\sigma_{\rm eff} = 13.8 \pm 0.9$ mb (for JH'2013 set 2 gluon), where corresponding uncertainties are estimated in the conventional way using Student's t-distribution at the confidence level $P = 95\%$. These values coincide to each other within the uncertainties. Here we achieve a remarkable agreement with the majority of other $\sigma_{\rm eff}$ estimations based on different final states, such as, for example, $W + 2$ jets [77,78], $2\gamma + 2$ jets [79], $\gamma + 3$ jets [80], 4 jets [34,45], $J/\psi + D^+$, $J/\psi + D^0$, $J/\psi + \Lambda_c^+$ [81], $\Upsilon(1S) + D^0$ [46]. Thus, our results support the expectation about the universality of this parameter for a wide range of processes with essentially different kinematics, energies and hard scales. The both obtained values of $\sigma_{\rm eff}$ significantly

[8] To generate $c\bar{c}$ events in the off-shell gluon–gluon fusion the Monte-Carlo event generator PEGASUS [66] has been used.

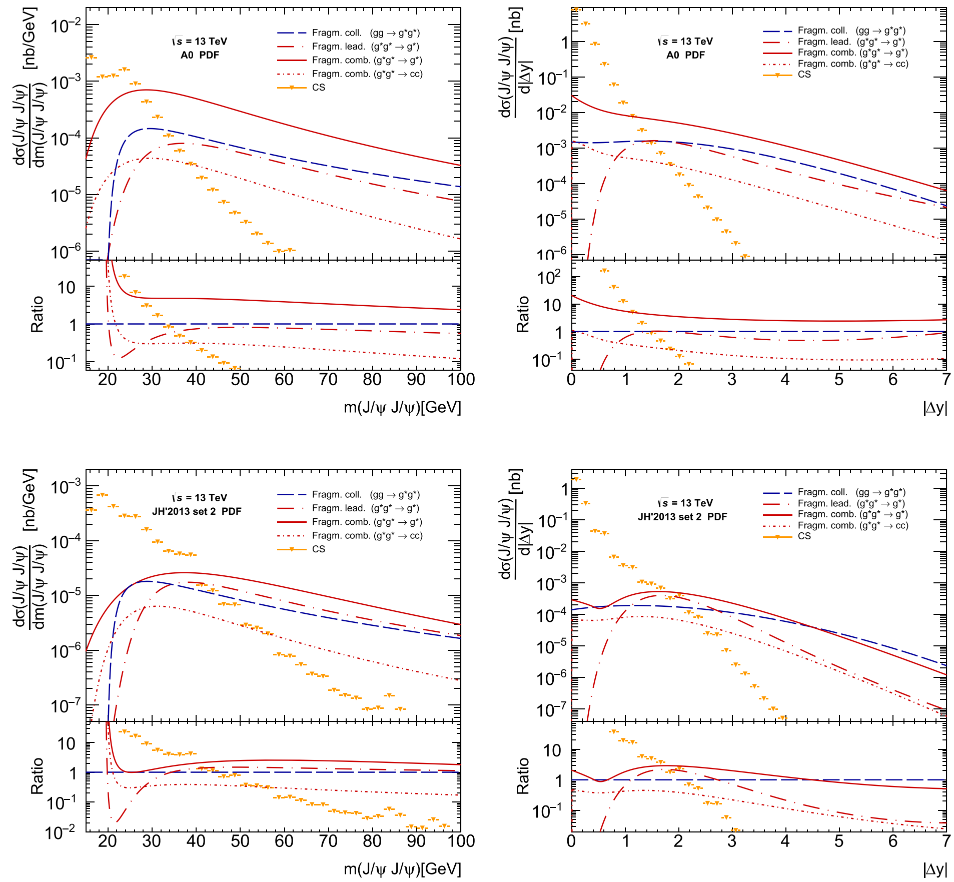

Fig. 5 Different contributions to the double J/ψ production calculated as functions of invariant mass $m(J/\psi, J/\psi)$ and rapidity separation $|\Delta y(J/\psi, J/\psi)|$ at $\sqrt{s} = 13$ TeV. The kinematical cut $p_T(J/\psi) > 10$ GeV is applied for both J/ψ mesons. The A0 (upper panels) and JH'2013 set 2 (lower panels) gluon distributions in proton are used

exceed previous estimations based on the same final state, $J/\psi + J/\psi$, which are typically of about $2-5$ mb [48–50]. Of course, the results [48–50] also contradict to the most of the measured $\sigma_{\rm eff}$ values [77–81]. The disparity between the results can be attributed to using different schemes for parton densities (collinear in [2,46,47] versus k_T-factorization in our case) or to a different choice of the renormalization scale μ_R (regarding the k_T-based part of the analysis presented in [4]). The missing feed-down contributions may also play some role.

A comparison of our predictions with the LHCb experimental data is displayed in Figs. 7, 8, 9 and10. The theoretical uncertainty bands are shown for A0 gluon density and include both scale uncertainties and uncertainties coming from the $\sigma_{\rm eff}$ fitting procedure. First of them have been estimated in a usual way, by varying the μ_R scale around its default value by a factor of 2. This was accompanied with using the A0+ and A0− gluon densities instead of default A0 distribution, in accordance with [70]. As one can see, we achieved a reasonably good agreement between the results of our calculations with both considered TMD gluons and LHCb measurements performed for $\sqrt{s} = 7$ and 13 TeV. There is only exception in the threshold region, $m(J/\psi, J/\psi) \leq 9$ GeV, where our predictions systematically overshoot the data. However, at such low $m(J/\psi, J/\psi)$ an accurate treatment of multiple soft gluon emissions, relativistic corrections and other nonperturbative effects becomes necessary to produce the theoretical estimations. All these issues are out from our present consideration. Next, we find that neither the SPS terms, nor the DPS contributions alone are able to describe the LHCb data, but only their sum. In particular, the DPS contributions are essential to reproduce the measured $|\Delta y(J/\psi, J/\psi)|$ distributions at $|\Delta y(J/\psi, J/\psi)| \geq 1$ or 1.5, that confirms the previous expectations [42–44]. They are important to describe also the normalization of J/ψ rapidity distributions and shape of transverse momentum asymmetry $\mathcal{A}_T$ at $\mathcal{A}_T \leq 0.4$, see Figs. 8, 9 and 10.

The presented results, being considered altogether with the ones for inclusive single production of charmonia states [20], can give a significant impact on the understanding of charmonia production within the NRQCD framework and, in particular, on the further understanding of DPS mechanism. The most interesting outcome of our study is that the extremely low value of DPS effective cross section, $\sigma_{\rm eff} \sim 2-5$ mb, obtained in earlier analyses of double J/ψ production at the LHC, is not confirmed.

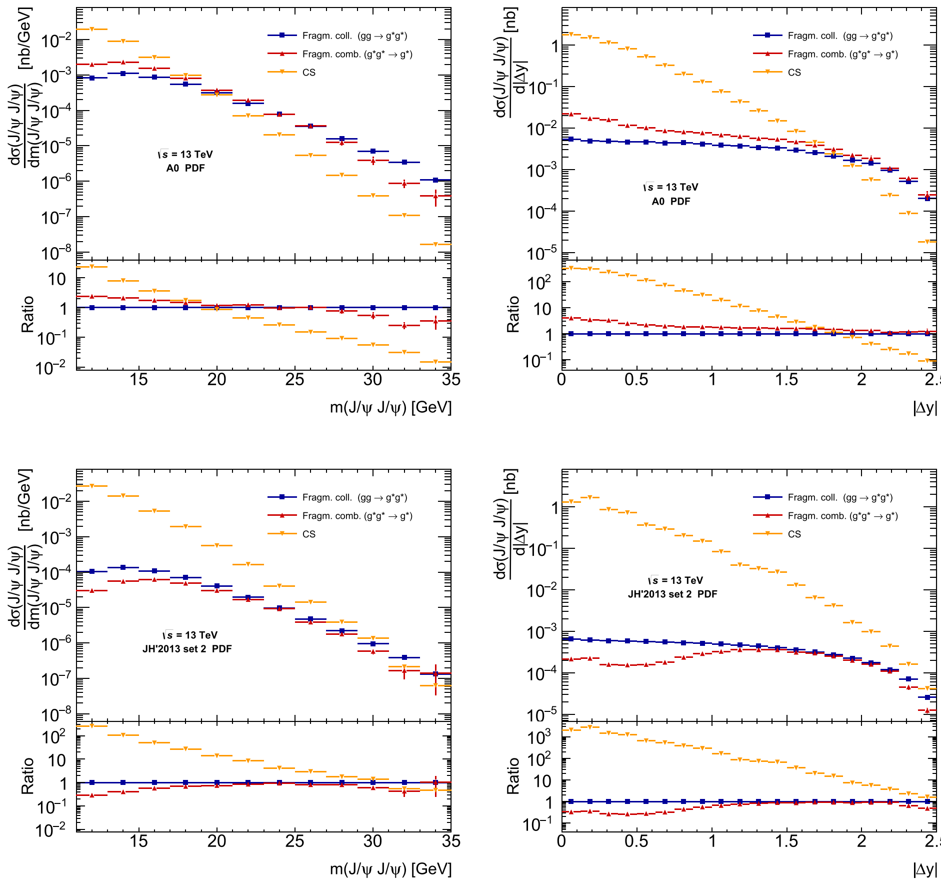

Fig. 6 Different contributions to the double J/ψ production calculated as functions of invariant mass $m(J/\psi, J/\psi)$ and rapidity separation $|\Delta y(J/\psi, J/\psi)|$ at $\sqrt{s} = 13$ TeV. The kinematical cuts $4.5 < p_T(J/\psi) < 10$ GeV and $2 < y(J/\psi) < 4.5$ are applied for both J/ψ mesons. The A0 (upper panels) and JH'2013 set 2 (lower panels) gluon distributions in proton are used

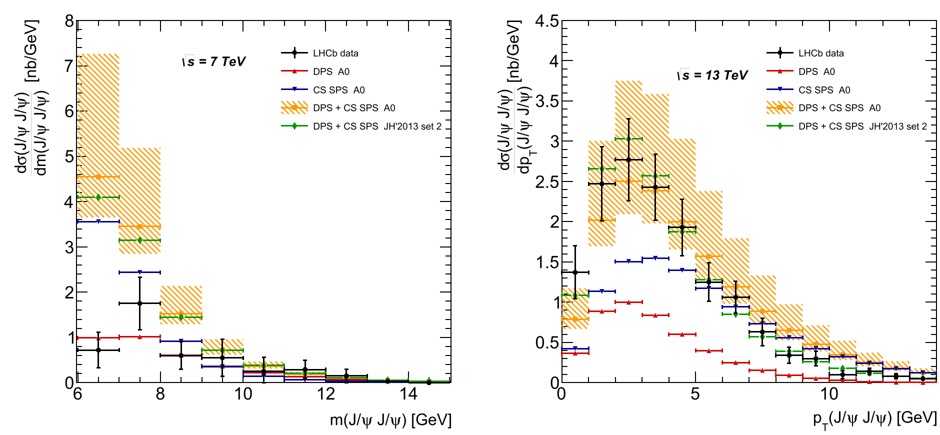

Fig. 7 Differential cross sections of double J/ψ production as functions of invariant mass $m(J/\psi, J/\psi)$ and transverse momentum $p_T(J/\psi, J/\psi)$ calculated at $\sqrt{s} = 7$ TeV (left panel) and $\sqrt{s} = 13$ TeV (right panel). The kinematical cuts applied are described in the text

4 Conclusion

We have considered the prompt production of J/ψ meson pairs in pp collisions at the LHC using the k_T-factorization approach of QCD. We employ the fragmentation mechanism to evaluate the color octet contributions to the production cross sections and take into account the combinatorial effects of multiple gluon radiation in the initial state using the CCFM evolution equation. The latter could be essential in the kinematical region covered by the CMS and ATLAS measurements. On the other hand, we have demonstrated that the experimental data taken by the LHCb Collaboration at forward rapidities can be described well by the color singlet terms and contributions from the double parton scattering mechanism. We determine the DPS effective cross section $\sigma_{\rm eff} = 17.5 \pm 4.1$ mb (for A0 gluon) and $\sigma_{\rm eff} = 13.8 \pm 0.9$ mb (for JH'2013 set 2 gluon) from the combined analysis of recent LHCb data collected at $\sqrt{s} = 7$ and 13 TeV. The

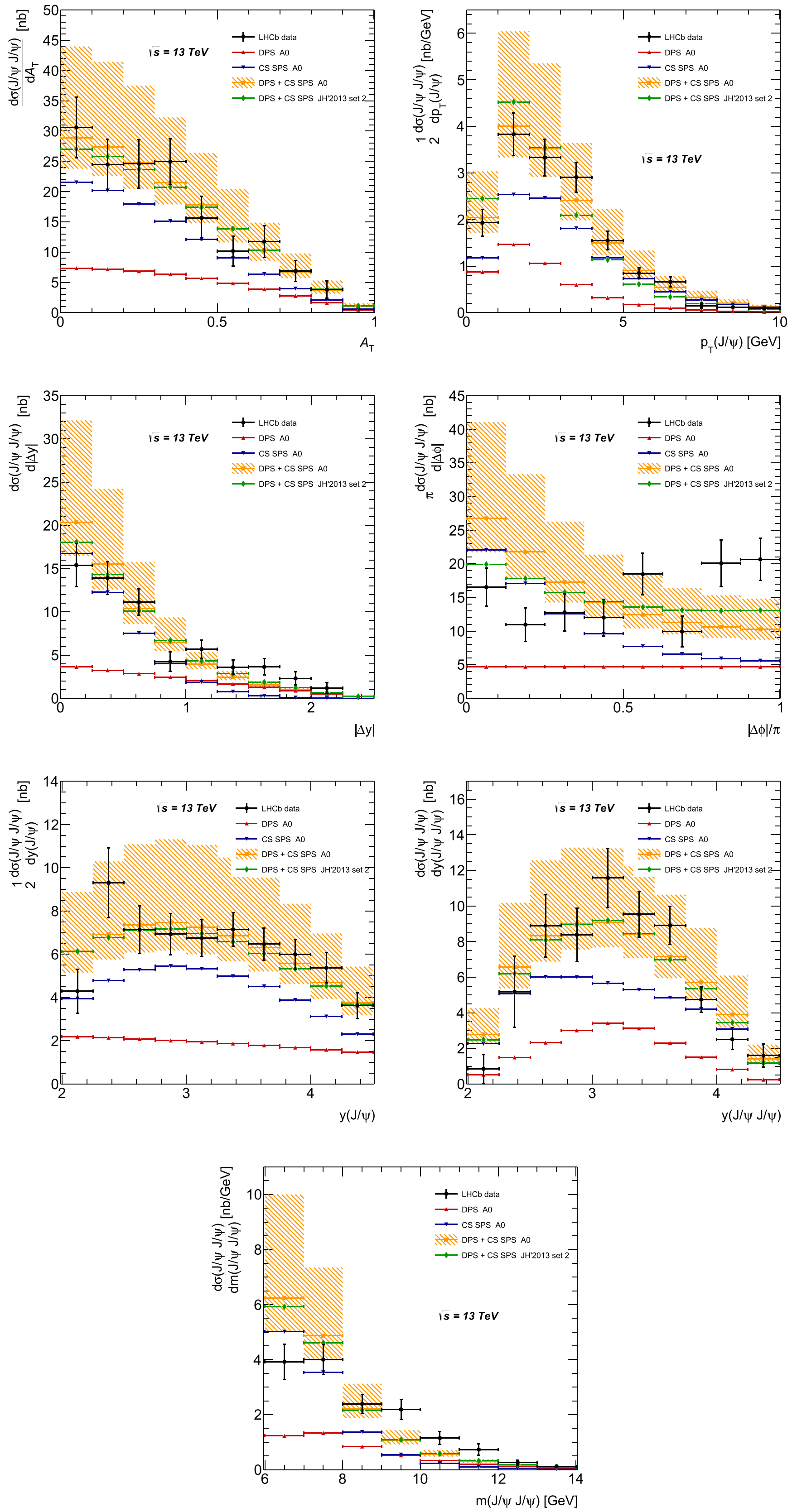

Fig. 8 Prompt double J/ψ production cross sections as functions of different kinematical variables calculated at $\sqrt{s} = 13$ TeV. The kinematical cuts applied are described in the text

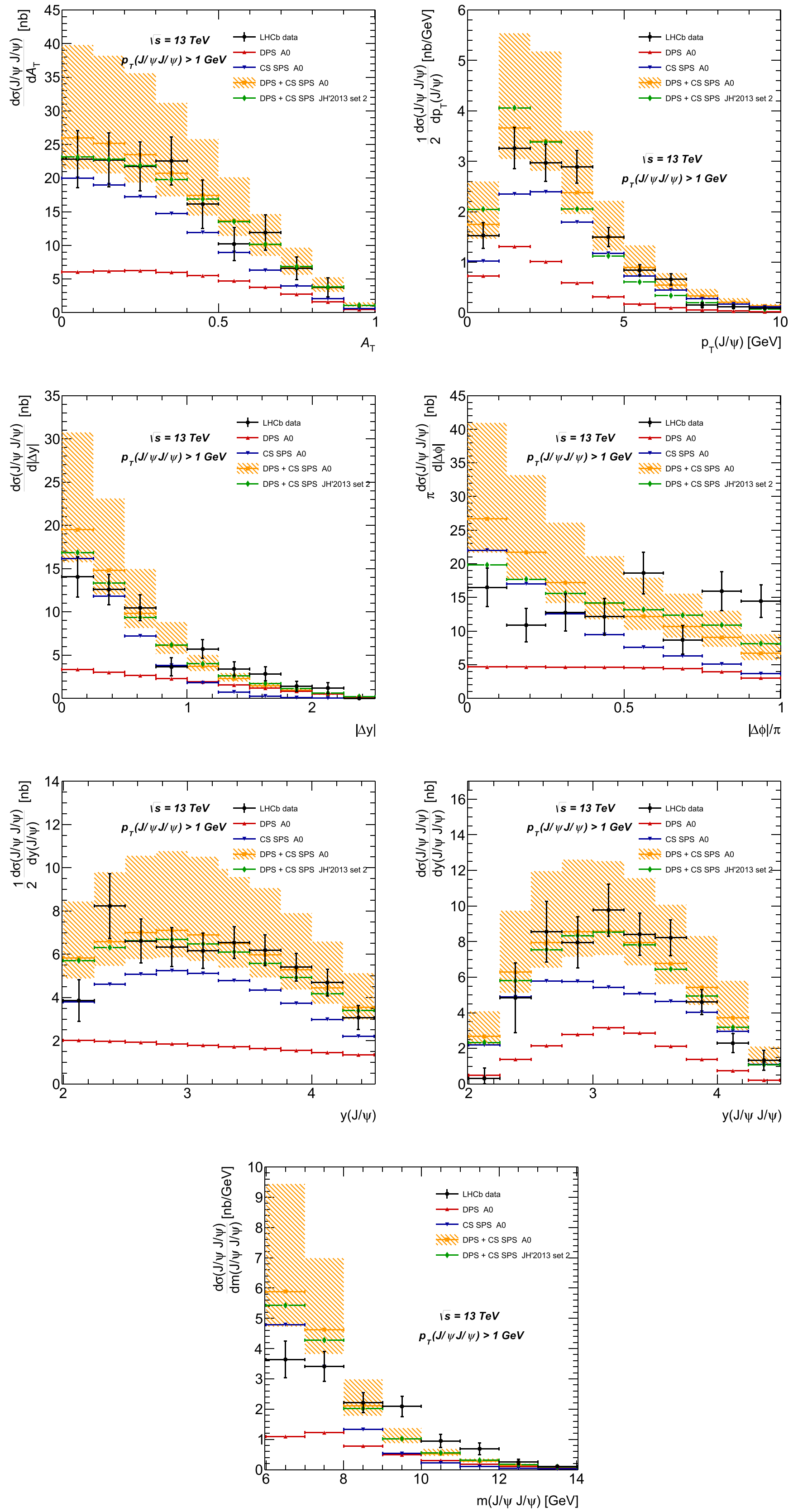

Fig. 9 Prompt double J/ψ production cross sections as functions of different kinematical variables calculated at $p_T(J/\psi, J/\psi) > 1$ GeV and $\sqrt{s} = 13$ TeV. Other kinematical cuts applied are described in the text

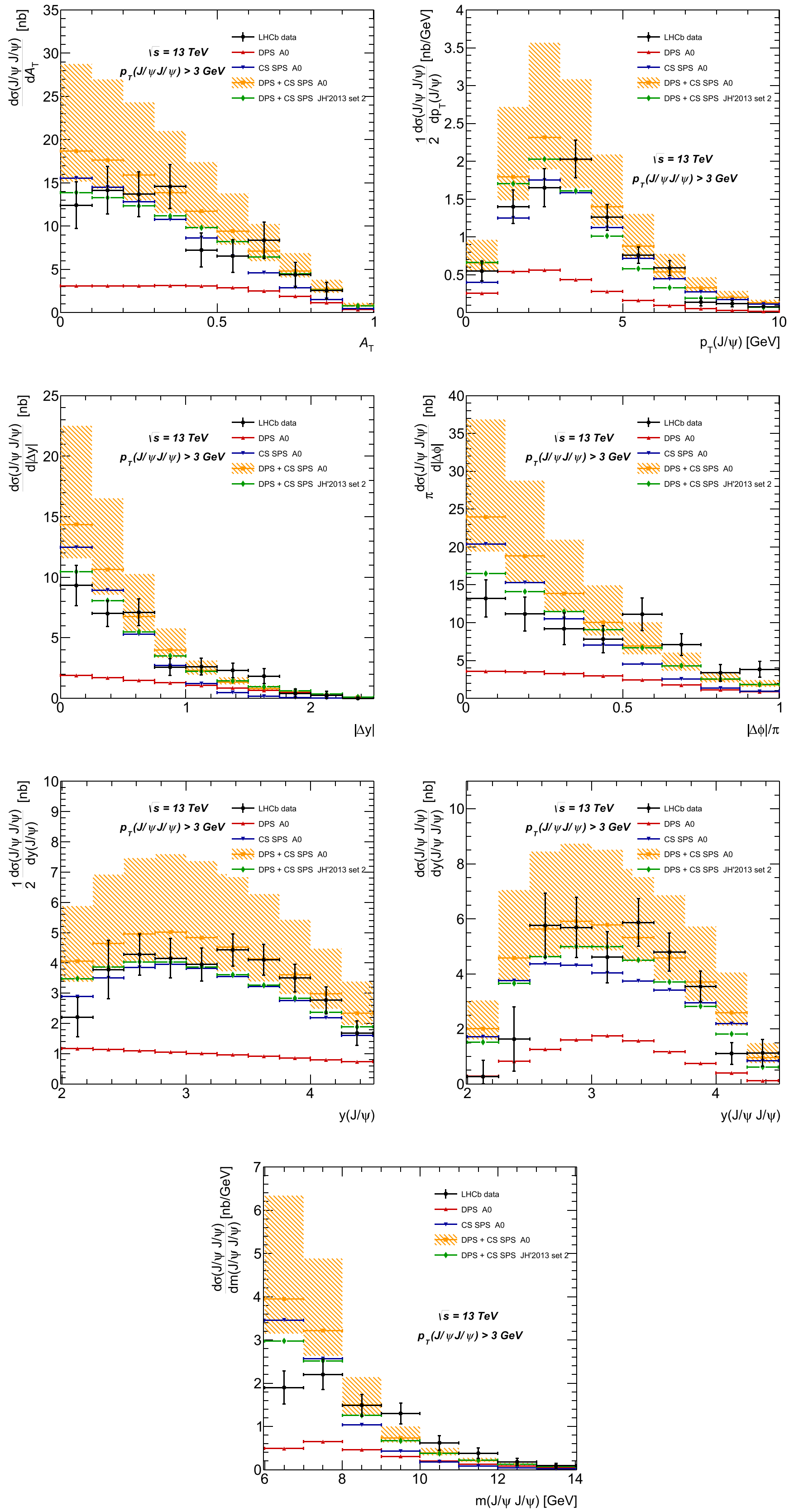

Fig. 10 Prompt double J/ψ production cross sections as functions of different kinematical variables calculated at $p_T(J/\psi, J/\psi) > 3$ GeV and $\sqrt{s} = 13$ TeV. Other kinematical cuts applied are described in the text

extracted values are compatible with many other estimations based on essentially different final states. The extremely low $\sigma_{\mathrm{eff}} \sim 2 - 5$ mb, obtained earlier from the double J/ψ production data, is not confirmed.

Acknowledgements The authors thank G.I. Lykasov for very useful discussions on the topic. M.A.M. and A.A.P. were supported by grants of the foundation for the advancement of theoretical physics and mathematics "Basis" 20-1-3-11-1 and 18-2-6-129-1, respectively. A.A.P. is also supported by the RFBR grant 20-32-90105.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the data is included in our article.]

Funded by SCOAP^3.

References

1. CMS Collaboration, JHEP **09**, 094 (2014)
2. ATLAS Collaboration, Eur. Phys. J. C **77**, 76 (2017)
3. LHCb Collaboration, Phys. Lett. B **707**, 52 (2012)
4. LHCb Collaboration, JHEP **06**, 047 (2017)
5. G. Bodwin, E. Braaten, G. Lepage, Phys. Rev. D **51**, 1125 (1995)
6. P. Cho, A.K. Leibovich, Phys. Rev. D **53**, 150 (1996)
7. P. Cho, A.K. Leibovich, Phys. Rev. D **53**, 6203 (1996)
8. B. Gong, X.Q. Li, J.-X. Wang, Phys. Lett. B **673**, 197 (2009)
9. Y.-Q. Ma, K. Wang, K.-T. Chao, Phys. Rev. Lett. **106**, 042002 (2011)
10. M. Butenschön, B.A. Kniehl, Phys. Rev. Lett. **108**, 172002 (2012)
11. K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, Y.-J. Zhang, Phys. Rev. Lett. **108**, 242004 (2012)
12. B. Gong, L.-P. Wan, J.-X. Wang, H.-F. Zhang, Phys. Rev. Lett. **110**, 042002 (2013)
13. Y.-Q. Ma, K. Wang, K.-T. Chao, H.-F. Zhang, Phys. Rev. D **83**, 111503 (2011)
14. A.K. Likhoded, A.V. Luchinsky, S.V. Poslavsky, Phys. Rev. D **90**, 074021 (2014)
15. H.-F. Zhang, L. Yu, S.-X. Zhang, L. Jia, Phys. Rev. D **93**, 054033 (2016)
16. H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, K.-T. Chao, Phys. Rev. Lett. **114**, 092005 (2015)
17. H.-F. Zhang, Z. Sun, W.-L. Sang, R. Li, Phys. Rev. Lett. **114**, 092006 (2015)
18. M. Butenschön, Z.G. He, B.A. Kniehl, Phys. Rev. Lett. **114**, 092004 (2015)
19. S.P. Baranov, Phys. Rev. D **93**, 054037 (2016)
20. S.P. Baranov, A.V. Lipatov, Phys. Rev. D **100**, 114021 (2019)
21. N.A. Abdulov, A.V. Lipatov, Eur. Phys. J. C **79**, 830 (2019)
22. N.A. Abdulov, A.V. Lipatov, Eur. Phys. J. C **80**, 5 (2020)
23. J.-P. Lansberg, H.-S. Shao, H.-F. Zhang, Phys. Lett. B **786**, 342 (2018)
24. Y. Feng, J. He, J.-P. Lansberg, H.-S. Shao, A. Usachov, H.-F. Zhang, Nucl. Phys. B **945**, 114662 (2019)
25. J.-P. Lansberg, arXiv:1903.09185 [hep-ph]
26. Z.-G. He, B.A. Kniehl, Phys. Rev. Lett. **115**, 022002 (2015)
27. Y.-J. Li, G.-Z. Xu, K.-Y. Liu, Y.-J. Zhang, JHEP **07**, 051 (2013)
28. L.-P. Sun, H. Han, K.-T. Chao, Phys. Rev. D **94**, 074033 (2016)
29. J.P. Lansberg, H.S. Shao, Phys. Rev. Lett. **111**, 122001 (2013)
30. M. Ciafaloni, Nucl. Phys. B **296**, 49 (1988)
31. S. Catani, F. Fiorani, G. Marchesini, Phys. Lett. B **234**, 339 (1990)
32. S. Catani, F. Fiorani, G. Marchesini, Nucl. Phys. B **336**, 18 (1990)
33. G. Marchesini, Nucl. Phys. B **445**, 49 (1995)
34. B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, Phys. Rev. D **83**, 071501 (2011)
35. B. Blok, Yu. Dokshitser, L. Frankfurt, M. Strikman, Eur. Phys. J. C **72**, 1963 (2012)
36. B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, Eur. Phys. J. C **74**, 2926 (2014)
37. M. Diehl, D. Ostermeier, A. Schäfer, JHEP **12**, 89 (2012)
38. P. Bartalini et al., arXiv:1111.0469 [hep-ph]
39. H. Abramowicz et al., arXiv:1306.5413 [hep-ph]
40. S. Bansal et al., arXiv:1410.6664 [hep-ph]
41. R. Astalos et al., arXiv:1506.05829 [hep-ph]
42. S.P. Baranov, A.M. Snigirev, N.P. Zotov, Phys. Lett. B **705**, 116 (2011)
43. C.H. Kom, A. Kulesza, W.J. Stirling, Phys. Rev. Lett. **107**, 082002 (2011)
44. S.P. Baranov, A.M. Snigirev, N.P. Zotov, A. Szczurek, W. Schäfer, Phys. Rev. D **87**, 034035 (2013)
45. ATLAS Collaboration, JHEP **11**, 110 (2016)
46. LHCb Collaboration, JHEP **07**, 052 (2016))
47. J.-P. Lansberg, H.-S. Shao, Phys. Lett. B **751**, 479 (2015)
48. D0 Collaboration, Phys. Rev. D **90**, 111101R (2014)
49. D0 Collaboration, Phys. Rev. Lett. **116**, 082002 (2016)
50. CMS Collaboration, JHEP **05**, 013 (2017)
51. S. Catani, M. Ciafaloni, F. Hautmann, Nucl. Phys. B **366**, 135 (1991)
52. J.C. Collins, R.K. Ellis, Nucl. Phys. B **360**, 3 (1991)
53. L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rep. **100**, 1 (1983)
54. E.M. Levin, M.G. Ryskin, YuM Shabelsky, A.G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991)
55. R. Angeles-Martinez et al., Acta Phys. Polon. B **46**, 2501 (2015)
56. H. Jung, S.P. Baranov, M. Deak, A. Grebenyuk, F. Hautmann, M. Hentschinski, A. Knutsson, M. Kramer, K. Kutak, A.V. Lipatov, N.P. Zotov, Eur. Phys. J. C **70**, 1237 (2010)
57. C.-H. Chang, Nucl. Phys. B **172**, 425 (1980)
58. E.L. Berger, D.L. Jones, Phys. Rev. D **23**, 1521 (1981)
59. R. Baier, R. Rückl, Phys. Lett. B **102**, 364 (1981)
60. S.S. Gershtein, A.K. Likhoded, S.R. Slabospitsky, Sov. J. Nucl. Phys. **34**, 128 (1981)
61. S.P. Baranov, Phys. Rev. D **84**, 054012 (2011)
62. Y.-Q. Ma, J.-W. Qiu, H. Zhang, Phys. Rev. D **89**, 094029 (2014)
63. E. Bycling, K. Kajantie, *Particle Kinematics* (Wiley, Hoboken, 1973)
64. J. Alwall, A. Ballestrero, P. Bartalini, S. Belov, E. Boos, A. Buckley, J.M. Butterworth, L. Dudko, S. Frixione, L. Garren, S. Gieseke, A. Gusev, I. Hinchliffe, J. Huston, B. Kersevan, F. Krauss, N. Lavesson, L. Lönnblad, E. Maina, F. Maltoni, M.L. Mangano, F. Moortgat, S. Mrenna, C.G. Papadopoulos, R. Pittau, P. Richardson, M.H. Seymour, A. Sherstnev, T. Sjöstrand, P. Skands, S.R. Slabospitsky, Z. Wcas, B.R. Webber, M. Worek, D. Zeppenfeld, Comput. Phys. Commun. **176**, 300 (2007)
65. A.V. Lipatov, M.A. Malyshev, H. Jung, Phys. Rev. D **100**, 034028 (2019)

66. A.V. Lipatov, S.P. Baranov, M.A. Malyshev, Eur. Phys. J. C **80**, 330 (2020), https://theory.sinp.msu.ru/doku.php/pegasus/news
67. V.L. Korotkikh, A.M. Snigirev, Phys. Lett. B **594**, 171 (2004)
68. A.M. Snigirev, Phys. Rev. D **83**, 034028 (2011)
69. S.P. Baranov, A.V. Lipatov, M.A. Malyshev, A.M. Snigirev, N.P. Zotov, Phys. Rev. D **93**, 094013 (2016)
70. H. Jung, arXiv:hep-ph/0411287
71. F. Hautmann, H. Jung, Nucl. Phys. B **883**, 1 (2014)
72. F. Hautmann, H. Jung, M. Krämer, P.J. Mulders, E.R. Nocera, Eur. Phys. J. C **74**, 3220 (2014), http://tmdlib.hepforge.org
73. PDG Collaboration, Phys. Rev. D **98**, 030001 (2018)
74. L.A. Harland-Lang, A.D. Martin, P. Motylinski, R.S. Thorne, Eur. Phys. J. C **75**, 204 (2015)
75. S.P. Baranov, A.H. Rezaeian, Phys. Rev. D **93**, 114011 (2016)
76. www.gnuplot.info
77. CMS Collaboration, JHEP **03**, 32 (2014)
78. ATLAS Collaboration, New J. Phys. **15**, 033038 (2013)
79. D0 Collaboration, Phys. Rev. D **93**, 052008 (2016)
80. D0 Collaboration, Phys. Rev. D **89**, 072006 (2014)
81. LHCb Collaboration, JHEP **06**, 141 (2012)