



Superheavy dark matter in $R + R^2$ cosmology with conformal anomaly

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Abstract Cosmological evolution and particle creation in R^2 -modified gravity are considered for the case of the dominant decay of the scalaron into a pair of gauge bosons due to conformal anomaly. It is shown that in the process of thermalization superheavy dark matter with the coupling strength typical for the GUT SUSY can be created. Such dark matter would have the proper cosmological density if the particle mass is close to 10^{12} GeV.

1 Introduction

The most popular and natural hypothesis that dark matter consists of the lightest supersymmetric particles (LSP) somewhat lost its popularity since no manifestation of supersymmetry (SUSY) was observed at LHC [1]. The LHC data significantly restricted parameter space open for SUSY. Though strictly speaking low energy SUSY, around 1 TeV, is not excluded and no direct limits from below on the LSP mass were presented, see [2], still a study of higher energy SUSY and heavier LSPs can be of interest.

Different mechanisms of LSP production in cosmology are summarized in Ref. [3]. If they behave as the usual WIMPs, then their frozen number density is governed by the Zeldovich equation [4], see also [5,6]. This equation was first derived by Zeldovich [4] and later rederived in detail in several textbook, see e.g. [7,8]. According to them the number density of thermal relics can be estimated as

$$\frac{n_X}{n_\gamma} \approx \frac{1}{m_P m_X \sigma_{ann} v}, \quad (1.1)$$

where m_X is the mass of would-be dark matter particles X and n_X and n_γ are the contemporary number densities of X -particles and CMB photons respectively, $m_P = 1.2 \cdot 10^{19}$ GeV is the Planck mass, and σv is the product of the annihilation cross-section of $X\bar{X}$ by their center of mass velocity. This result is valid for a simple order of magnitude estimates with some numerical and logarithmic factor of order 10 neglected.

For S-wave annihilation

$$\sigma_{ann} v = \alpha^2 / m_X^2 \quad (1.2)$$

In what follows we assume that $\alpha \approx 0.01$, which is typical for SUSY. If the coupling is different and/or the annihilation is enhanced or suppressed, the result would be evidently changed. Anyhow the presented expressions are the conventional ones for the estimates of usual WIMPs number and energy densities.

Though there exist other mechanisms of LSP production/annihilation, which may be realized in cosmology, nevertheless a study of alternative cosmological models for LSPs as viable dark matter candidate can be of interest.

Our results obtained in Ref. [9] as well as in the present paper do not demand full supersymmetry and are valid for any massive stable particle with the coupling strength typical to that in supersymmetry. So in what follows we will not use the abbreviation LSP for these particles but instead call them X -particles.

In our recent work [9], we have shown that in modified $R + R^2$ cosmology the relative density of LSP can be considerably smaller than that predicted in the standard scenario. This opens the window for the lightest supersymmetric particle with the mass about 1000 TeV to be a viable dark matter candidate. The frozen number density of massive relics is calculated in terms of the present day density of photons of the cosmic microwave background radiation, see e.g. Ref. [10]. The relative decrease of the LSP density in R^2 -cosmology

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is related to an efficient particle production by the oscillating curvature scalar after freezing of the LSP production and annihilation, as it is shown in our paper [11]. Consequently, the number density of CMB photons rises and the ratio of the frozen number density of LSP to the number density of photons drops down.

Production of dark matter particles by the scalaron decay in a different aspect was considered also in Ref. [12].

According to our work [11], the cosmological evolution in $R + R^2$ theory is considerably different from the standard one, based on the classical General Relativity (GR). In modified gravity the cosmological evolution can be categorized into the following four epochs. At first, there was the exponential expansion (Starobinsky inflation [13]), when the curvature scalar $R(t)$ (called scalaron) was very large and was slowly decreasing down to zero. The next epoch began when $R(t)$ dropped down to zero and started to oscillate, periodically changing sign. The oscillations of R led to particle production and this epoch can be called Big Bang. Next, there was the transition period from the scalaron domination to the relativistic matter domination. Finally, after scalaron had decayed completely, we arrived to the standard cosmology which is governed by General Relativity.

The frozen density of massive species strongly depends upon the probability of particle production by $R(t)$. In our previous papers [9, 11], we considered the decays into minimally coupled massless scalar particles and into massive fermions or conformally coupled scalars. However, as it is argued in Ref. [14], the production of massless gauge bosons due to conformal anomaly may be significant. We avoided this problem by assuming a version of supersymmetric model, where conformal anomaly is absent. Here we clear out this restriction and consider freezing of massive species in the theory where the particle production by oscillating curvature predominantly proceeds through anomalous coupling to gauge bosons.

The paper is organized as follows. In Sect. 2 we summarize our results on cosmological evolution in R^2 -gravity and present the known theoretical estimates of the variation of the coupling constants with changing momentum transfer. In Sect. 3 an estimate of the cosmological number density of X -particles created by direct decay of the scalaron is presented. It is shown that X -particles energy density would have the proper for DM value if their mass is rather small, $M_X \sim 10^7$ GeV. However, as shown in Sect. 4, in this case the X -particle production by thermal processes in plasma, in turn, would be unacceptably strong. To avoid this crunch we assume that X -particles are Majorana fermions because in this case their direct production by the scalaron is forbidden. According to the calculations in Sect. 4 the cosmological density of X -particles would be equal to the observed density of DM, $\rho_{DM} \approx 1 \text{ keV/cm}^3$, if $M_X \sim 10^{10}$ GeV. In Sect. 5 possible manifestations of X -particles in cosmic rays are considered.

In conclusion, the results are discussed and compared to the other cases studied earlier.

2 Cosmological evolution in R^2 gravity

This section contains a condensed summary of the main results of our works [9, 11]. The action of the theory has the form:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6M_R^2} \right) + S_m, \quad (2.1)$$

where $m_{Pl} = 1.2 \cdot 10^{19}$ GeV is the Planck mass, S_m is a matter action. Here g is the determinant of the metric tensor $g_{\mu\nu}$ taken with the signature convention $(+, -, -, -)$. The Riemann tensor describing the curvature of space-time is determined according to $R^\alpha_{\mu\beta\nu} = \partial_\beta \Gamma^\alpha_{\mu\nu} + \dots$, $R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}$, and $R = g^{\mu\nu} R_{\mu\nu}$. We use here the natural system of units $\hbar = c = k_B = 1$. As we see in what follows, M_R is the mass of the scalaron field. The spectrum of the temperature fluctuations of the cosmic microwave background radiation (CMB) demands [12, 15]:

$$M_R = 3 \cdot 10^{13} \text{ GeV}. \quad (2.2)$$

We consider homogeneous and isotropic matter distribution with the linear equation of state:

$$P = w\rho, \quad (2.3)$$

where w is usually a constant parameter. For non-relativistic matter $w = 0$, for relativistic matter $w = 1/3$, and for the vacuum-like state $w = -1$.

Equation of motion for the curvature which follows from action (2.1) has the form:

$$\ddot{R} - \frac{\Delta R}{a^2} + (3H + \Gamma/2)\dot{R} + M_R^2 R = -\frac{8\pi M_R^2}{m_{Pl}^2} (1 - 3w)\rho, \quad (2.4)$$

where $H = \dot{a}/a$ is the Hubble parameter and Γ is the total scalaron decay rate, which is determined by the dominant decay channel. See discussion and the list of references in our works [9, 11]. Note that our definition of Γ in the present paper differs by factor 2 from that used in our earlier works.

The appearance of the damping term, $\Gamma \dot{R}$, in this equation is a result of the back-reaction of particle production by oscillating curvature on the curvature field. This equation has been derived in one-loop approximation in several papers [16–18]. The resulting impact of particle production on the evolution of R is described by non-local in time equation, which for harmonic oscillations of the source is reduced to simple liquid friction term, as given above in Eq. (2.4). It is noteworthy that the quantum average of the energy-momentum tensor over

vacuum or in external (gravitational, as in our case) field does not have the same value of w as for real fields, for example, vacuum expectation value of massless fields has $w = -1$ instead of $w = 1/3$.

We assume that the scalaron field is homogeneous $R = R(t)$, neglecting small perturbations generated in the course of inflation. After inflation is over, the scalaron field starts to oscillate as

$$R(t) = \frac{4M_R \cos(M_R t + \theta)}{t}, \quad (2.5)$$

where θ is a constant phase determined by initial conditions. This and the subsequent equations are valid in the limit $M_R t \gg 1$, but $\Gamma t \lesssim 1$.

The Hubble parameter is similar to that at the matter dominated stage (MD), but with fast oscillations around the MD value:

$$H(t) = \frac{2}{3t} [1 + \sin(M_R t + \theta)]. \quad (2.6)$$

The cosmological energy density of matter at this period depends upon the decay width of the scalaron, which in turn depends upon the dominant decay channel.

If there exists scalar particle minimally coupled to gravity, the decay width of scalaron into massless scalars would be:

$$\Gamma_S = \frac{M_R^3}{24m_{Pl}^2}. \quad (2.7)$$

In this case, the energy density of predominantly relativistic matter is equal to:

$$\rho_S(t) = \frac{M_R^3}{120\pi t} \approx 2.7 \cdot 10^{-3} \frac{M_R^3}{t}. \quad (2.8)$$

If there are several species of massless scalars, the expressions (2.7) and (2.8) should be multiplied by g_S , where g_S denotes the number of species. For massive scalar with the mass m_s the width of two-body decay would be somewhat suppressed due to the phase space factor proportional to $\sqrt{1 - 4m_s^2/M_R^2}$.

If scalaron predominantly decays into fermions or conformally coupled scalars the decay width vanishes in the limit of massless final state particles and is equal to [12]:

$$\Gamma_f = \frac{M_R m_f^2}{24m_{Pl}^2}, \quad (2.9)$$

where m_f is the mass of fermion or conformally coupled scalar. The width is dominated by the heaviest final particle. The corresponding matter density is:

$$\rho_f(t) = \frac{M_R m_f^2}{120\pi t}. \quad (2.10)$$

Now let us turn to the scalaron decay induced by the conformal anomaly. Production of massless gauge bosons by conformally flat gravitational field was first studied in

Refs. [19, 20] and applied to the problem of heating in R^2 -inflation in Ref. [14]. The scalaron decay width for this channel is equal to:

$$\Gamma_{an} = \frac{\beta_1^2 \alpha^2 N}{96\pi^2} \frac{M_R^3}{m_{Pl}^2}, \quad (2.11)$$

where β_1 is the first coefficient of the beta-function, N is the rank of the gauge group, and α is the gauge coupling constant. We take $\beta_1^2 = 49$, $N = 8$. The coupling constant α at very high energies depends upon the theory and is, strictly speaking, unknown. The evolution of α in the minimal standard model (MSM) is presented in Fig. 1, left panel, and the same in the minimal standard supersymmetric model (MSSM) with supersymmetry at TeV scale is presented in the right panel. We can conclude that at the scalaron mass scale, $Q = 3 \cdot 10^{13}$ GeV, $\alpha_3 \approx 0.025$ in MSM, while in MSSM it is $\alpha_3 \approx 0.04$. At $Q = 10^{10}$ GeV they are $\alpha_3 \approx 0.033$ for MSM and $\alpha_3 \approx 0.05$ for MSSM.

The values of the running coupling constants are known to depend upon the particle spectrum. In the case of MSM we assumed that there exists only already known set of particles, while in MSSM there is some freedom depending on the explicit form of the SUSY model. However, the variation of the couplings related to this uncertainty does lead to a strong variation of our order of magnitude estimate of the allowed value of the mass of dark matter particles.

Since, according to our results presented below, supersymmetry may possibly be realized at energies about 10^{12} – 10^{13} GeV, the running of couplings according to MSM without inclusion of SUSY particles is probably correct below the SUSY scale. Recall that for particles produced at the scalaron decay $Q = 3 \cdot 10^{13}$ GeV, while at the universe heating temperature after the complete decay of the scalaron it is near 10^{10} GeV.

So numerically the decay width is:

$$\Gamma_{an} = 2.6 \cdot 10^{-4} \left(\frac{\alpha}{0.025} \right)^2 \frac{M_R^3}{m_{Pl}^2}. \quad (2.12)$$

Correspondingly the energy density of matter created by the decay into this channel would be:

$$\rho_{an} = \frac{\beta_1^2 \alpha^2 N}{4\pi^2} \frac{M_R^3}{120\pi t} \approx 1.65 \cdot 10^{-5} \left(\frac{\alpha}{0.025} \right)^2 \frac{M_R^3}{t}. \quad (2.13)$$

It is instructive to compare the rate of the energy transferred to matter produced in three different cases of the scalaron decay into minimally coupled scalars, fermions, and gauge bosons due to conformal anomaly with the energy density of the scalaron. To this end we need to define the energy density of the oscillating scalar curvature. The first term of action (2.1) in the Jordan frame in the high frequency limit can be rewritten in terms of the cosmological scale factor $a(t)$ in the way analogous to the derivation of the

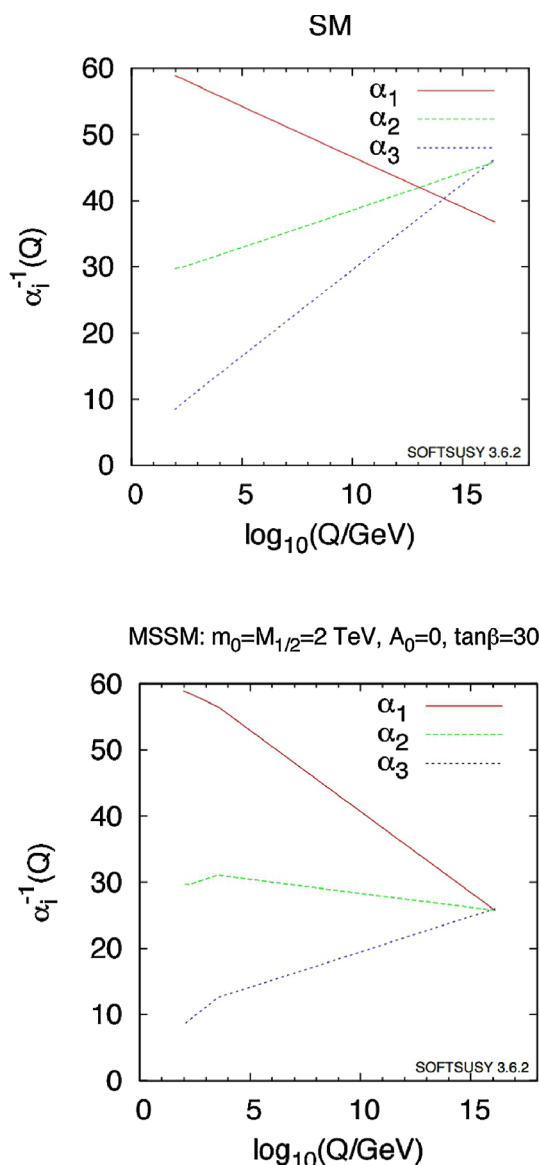


Fig. 1 Evolution of the coupling constants of $U(1)$, $SU(2)$ and $SU(3)$ (color) groups as function of the momentum transfer [21]

Friedmann equations performed in Ref. [22]. For high frequency oscillations and large value of $M_R t$ we have found the solutions [11]

$$H = \frac{2}{3t} [1 + \sin(M_R t + \theta)], \quad R = -\frac{4M_R \cos(M_R t + \theta)}{t}. \quad (2.14)$$

Curvature scalar is related to the Hubble parameter according to:

$$R = -6\dot{H} - 12H^2 \rightarrow -6\dot{H}. \quad (2.15)$$

The last relation is valid in high frequency limit and for the oscillating parts of H and R which presumably give dominant contribution to the energy density.

Keeping this in mind we can rewrite action (2.1) as:

$$\begin{aligned} S(a) &= \frac{6m_{Pl}^2}{16\pi M_R^2} \int d^4x a^3 \left[M_R^2 (\dot{H} + 2H^2) + \dot{H}^2 \right] \\ &= \frac{3m_{Pl}^2}{4\pi M_R^2} \int d^4x a^3 \left[-\frac{M_R^2 H^2}{2} + \frac{\dot{H}^2}{2} \right]. \end{aligned} \quad (2.16)$$

The last equality is obtained through integration by parts.

Varying over the scalar field H we obtain the equation of motion with the left hand side:

$$\ddot{H} + M_R^2 H = r.h.s., \quad (2.17)$$

which is exactly the same as the r.h.s. of Eq. (2.4). This equation has the oscillating solution multiplied by a slow function of time, such as the presented above solution $H \sim \sin(M_R t + \theta)/t$.

Now we need to introduce a canonically normalized scalar field Φ linearly connected with H for which the kinetic term in the Lagrangian is equal to $(\partial\Phi)^2/2$:

$$\Phi = \sqrt{\frac{3}{4\pi}} \frac{m_{Pl}}{M_R} H. \quad (2.18)$$

According to the standard prescription the energy density of the scalar field Φ is

$$\rho_\Phi = \frac{\dot{\Phi}^2 + M_R^2 \Phi^2}{2}. \quad (2.19)$$

Since, according to Eq. (2.15), in high frequency limit $R = -6\dot{H} \sim M_R \cos(M_R t)$, Eq. (2.19) can be identically rewritten in terms of R as

$$\rho_R = \frac{m_{Pl}^2 (\dot{R}^2 + M_R^2 R^2)}{96\pi M_R^4} = \frac{m_{Pl}^2}{6\pi t^2}, \quad (2.20)$$

where expression (2.5) has been used. This result coincides with the expression for the total cosmological energy density in spatially flat, matter dominated universe. This agreement confirms the validity of our approach.

The presented equations are valid if the energy density of matter remains smaller than the energy density of the scalaron until it decays. Comparing Eqs. (2.8), (2.10), and (2.13) with (2.20) we find that in all the cases $t_{cr} \Gamma = 5/6$, where t_{cr} is the time when the matter energy density, formally taken, is equal to the scalaron energy density. So the used above equations are not unreasonable. The scalaron completely decays at $t = 1/\Gamma$ (up to log-correction) and the cosmology turns into the usual Friedmann one governed by the equations of General Relativity (GR). Before that moment the universe expansion was dominated by the scalaron.

If the primeval plasma is thermalized, the following relation between the cosmological time and the temperature is

valid:

$$\rho_{an} = 2.6 \cdot 10^{-2} \alpha_R^2 \frac{M_R^3}{t} = \frac{\pi^2 g_*}{30} T^4, \quad (2.21)$$

where subindex R at α_R means that the coupling is taken at the energies equal to the scalaron mass, since the energy influx to the plasma is supplied by the scalaron decay, and $g_* \approx 100$ is the number of relativistic species. Consequently,

$$t T^4 = \frac{0.78}{\pi^2 g_*} \alpha_R^2 M_R^3 \equiv C \equiv C_0 M_R^3 \quad (2.22)$$

with $C_0 = 5 \cdot 10^{-7} (\alpha_R/0.025)^2$.

Thermal equilibrium is established if the reaction rate is larger than the Hubble expansion rate $H = 2/(3t)$. The reaction rate is determined by the cross-section of two-body reactions between relativistic particles. The typical value of this cross-section at high energies, $E \gg m$, is [23]:

$$\sigma_{rel} = \frac{4\pi\beta\alpha^2}{s} \left(\ln \frac{s}{4m^2} + 1 \right), \quad (2.23)$$

where $\beta \sim 10$ is the number of the open reaction channels and $s = (p_1 + p_2)^2 = 4E^2$ is the total energy of the scattering particles in their center-of-mass frame, where E is the energy of an individual particle.

Hence the reaction rate is

$$\Gamma_{rel} \equiv \frac{\dot{n}}{n} = \langle \sigma_{rel} v n_{rel} \rangle, \quad (2.24)$$

where angular brackets mean averaging over thermal bath with temperature T , $n_{rel} \approx 0.1 g_* T^3$ (we do not distinguish between bosons and fermions in the expression), $v = 1$ is the particle velocity in the center-of-mass system.

We perform thermal averaging naively taking $E = T$ in all expressions so $s \rightarrow 4T^2$, instead of m^2 we substitute the particle thermal mass in plasma, i.e. $m^2 \rightarrow 4\pi\alpha T^2/3$ [24–26]. Correspondingly we arrive at the following thermal equilibrium condition:

$$\frac{3}{2} t \Gamma_{rel} = 0.15 \pi \alpha^2 \beta g_* \left(\ln \frac{3}{4\pi\alpha} + 1 \right) T t > 1. \quad (2.25)$$

Using Eq. (2.22), we find that equilibrium is established at the temperatures below

$$T_{eq} = \left(0.15 \pi \alpha^2 \beta g_* C_0 \right)^{1/3} M_R = 9.2 \cdot 10^{-3} M_R. \quad (2.26)$$

Here we took $\alpha_R = 0.025$ and $\alpha = 0.033$.

The time corresponding to this temperature is equal to

$$t_{eq} = C/T_{eq}^4 \approx 70 M_R^{-1}, \quad (2.27)$$

where C is defined in Eq. (2.22). Hence $M_R t_{eq} \gg 1$, which is sufficiently long time for efficient particle production.

Another essential temperature for our consideration, is the temperature of the universe heating, when scalaron essentially decayed and the expansion regime turned to the con-

ventional GR one. This temperature is determined by the scalaron energy density at the moment $t = 1/\Gamma_{an}$:

$$\rho_R = \frac{m_{Pl}^2 \Gamma_{an}^2}{6\pi} = \frac{\pi^2 g_*}{30} T_h^4, \quad (2.28)$$

so

$$T_h = 3.2 \cdot 10^{-3} \sqrt{M_R/m_{Pl}} M_R = 5.1 \cdot 10^{-6} M_R. \quad (2.29)$$

3 X-particle production through the scalaron decay

There are two possible channels to produce massive stable X-particles: first, directly through the scalaron decay into a pair of $X\bar{X}$ and another by inverse annihilation of relativistic particles in plasma.

Firstly, let us consider the scalaron decay. The probability of the scalaron decay into a pair of fermions is determined by decay width (2.9) with the substitution M_X instead of m_f :

$$\Gamma_X = \frac{M_R M_X^2}{24 m_{Pl}^2}. \quad (3.1)$$

The branching ratio of this decay is equal to:

$$BR(R \rightarrow X\bar{X}) = \frac{\Gamma_X}{\Gamma_{an}} \approx 1.6 \cdot 10^2 \left(\frac{M_X}{M_R} \right)^2. \quad (3.2)$$

The number density of X-particles created by the scalaron decay only, but not by inverse annihilation of relativistic particles in plasma, is governed by the equation:

$$\dot{n}_X + 3Hn_X = \Gamma_X n_R, \quad (3.3)$$

where Γ_X is given by Eq. (3.1), $n_R = \rho_R/M_R$, and ρ_R is defined in Eq. (2.20). So Eq. (3.3) turns into

$$\dot{n}_X + 3Hn_X = \frac{1}{24} \frac{M_X^2}{6\pi t^2}. \quad (3.4)$$

It is solved as

$$n_X = \frac{1}{144\pi} \frac{M_X^2}{t}. \quad (3.5)$$

The equations presented above are valid if the inverse decay of the scalaron can be neglected. This approximation is true if the produced particles are quickly thermalized down to the temperatures much smaller than the scalaron mass.

We are interested in the ratio of n_X to the number density of relativistic species at the moment of the complete scalaron decay when the temperature dropped down to T_h (2.29) and after which the universe came to the conventional Friedmann cosmology and the ratio n_X/n_{rel} remained constant to the present time. This ratio is equal to:

$$F \equiv \frac{n_X}{n_{rel}}|_{T=T_h} = \left[\frac{0.04 M_X^2}{6\pi t_h} \right] \times \left[\frac{\pi^2 g_* T_h^4}{90 T_h} \right]^{-1}$$

$$= 2.3 \cdot 10^{-3} \left(\frac{0.025}{\alpha_R} \right)^2 \left(\frac{M_X}{M_R} \right)^2. \quad (3.6)$$

Consequently, the energy density of X -particles in the present day universe would be:

$$\rho_X^{(0)} = 412/\text{cm}^{-3} M_X F = \rho_{DM} \approx 1\text{keV}/\text{cm}^3. \quad (3.7)$$

The last approximate equality in the r.h.s. is the condition that the energy density of X -particles is equal to the observed energy density of dark matter.

From this condition it follows that $M_X \approx 10^7$ GeV. For larger masses $\rho_X^{(0)}$ would be unacceptably larger than ρ_{DM} . On the other hand, for such a small, or smaller M_X , the probability of X -particle production through the inverse annihilation would be too strong and would again lead to very large energy density of X -particles, see the following section.

A possible way out of this “catch-22” is to find a mechanism to suppress the scalaron decay into a pair of X -particles. And it does exist. If X -particles are Majorana fermions, then in this case particles and antiparticles are identical and so they must be in antisymmetric state. Thus the decay of a scalar field, scalaron, into a pair of identical fermions is forbidden since the scalaron can produce a pair of identical particles in symmetric state only.

4 Production of X -particles in thermal plasma

Here we turn to the X -production through the inverse annihilation of relativistic particles in the thermal plasma. The number density n_X is governed by the Zeldovich equation:

$$\dot{n}_X + 3Hn_X = \langle \sigma_{ann} v \rangle (n_{eq}^2 - n_X^2), \quad (4.1)$$

where $\langle \sigma_{ann} v \rangle$ is the thermally averaged annihilation cross-section of X -particles and n_{eq} is their equilibrium number density.

This equation was originally derived by Zeldovich in 1965 [4], and in 1977 it was applied to freezing of massive stable neutrinos in the papers [5,6]. After that it was unjustly named as Lee-Weinberg equation.

The thermally averaged annihilation cross-section of non-relativistic X -particles, which enters Eq. (4.1), for our case can be taken as

$$\langle \sigma_{ann} v \rangle = \frac{\pi \alpha^2 \beta_{ann}}{M_X^2} \frac{T}{M_X}, \quad (4.2)$$

where the last factor came from thermal averaging of the velocity squared of X -particles, equal to $\langle v^2 \rangle = T/M_X$, which appears because the annihilation of Majorana fermions proceeds in P-wave. We take the coupling constant at the energy scale around M_X equal to $\alpha = 0.033$ and the number of the annihilation channels $\beta_{ann} = 10$. This expression is only an order of magnitude. The exact form depends upon

particle spins, the form of the interaction, and may contain the statistical factor $1/n!$, if there participate n identical particles. In what follows we neglect these subtleties.

The equilibrium distribution of non-relativistic X -particles has the form:

$$\begin{aligned} n_{eq} &= g_s \left(\frac{M_X T}{2\pi} \right)^{3/2} \exp\left(-\frac{M_X}{T}\right) \\ &= g_s M_X^3 (2\pi y)^{-3/2} \exp(-y), \end{aligned} \quad (4.3)$$

where $y = M_X/T$ and g_s is the number of spin states of X -particles. The non-relativistic approximation is justified if $M_X > T_{eq} \approx M_R/100 = 3 \cdot 10^{11}$ GeV, see Eq. (2.26).

Equation (4.1) will be solved with the initial condition $n_X(t_{in}) = 0$. This condition is essentially different from the solution of this equation in the canonical case, when it is assumed that initially $n_X = n_{eq}$ and in the course of the evolution n_X becomes much larger than n_{eq} , reaching the so called frozen density. As we see in what follows, for certain values of the parameters a similar situation can be realized, when n_X approaches the equilibrium value and freezes at much larger value. The other limit when n_X always remains smaller than n_{eq} is also possible.

For better insight into the problem we first make simple analytic estimates of the solution when $n_X \ll n_{eq}$ and after that solve exact Eq. (4.1) numerically.

In the limit $n_X \ll n_{eq}$ Eq. (4.1) is trivially integrated:

$$\begin{aligned} n_{X0}(y) &= \frac{4\pi \alpha^2 \beta_{ann} g_s^2}{(2\pi)^3} \frac{C_0 M_R^3}{y^8} \int_{y_{in}}^y dy_1 y_1^7 e^{-2y_1} \\ &= 5 \cdot 10^{-7} \frac{\alpha^2 \beta_{ann} g_s^2}{2\pi^2} \left(\frac{\alpha_R}{0.025} \right)^2 M_R^3 \int_{y_{in}/y}^1 dz z^7 e^{-2zy} \end{aligned} \quad (4.4)$$

where the subindex “0” means that the solution is valid for $n_X \ll n_{eq}$, $y = M_X/T$ and we have used Eq. (2.22) and the expression for C_0 below this equation.

For the initial temperature we take $T_{in} = 6 \cdot 10^{-3} M_R$, according to Eq. (2.26), and $T_{fin} = T_h = 5.1 \cdot 10^{-6} M_R$ (2.29). Correspondingly $y_{in} = 1.7 \cdot 10^2 M_X/M_R$, and $y_{fin} = 2 \cdot 10^5 M_X/M_R$ and so $y_{fin}/y_{in} \approx 10^3$.

To check validity of this solution we have to compare $n_{X0}(y)$ to n_{eq} (4.3):

$$\begin{aligned} F_2(y) &\equiv \frac{n_{X0}(y)}{n_{eq}} = 5\sqrt{2} \cdot 10^{-7} \frac{\alpha^2 \beta_{ann} g_s}{\sqrt{\pi}} \\ &\quad \left(\frac{\alpha_R}{0.025} \right)^2 e^y y^{3/2} \int_{y_{in}/y}^1 dz z^7 e^{-2zy} \\ &= 8.7 \cdot 10^{-9} \left(\frac{M_R}{M_X} \right)^3 e^y y^{3/2} \int_{y_{in}/y}^1 dz z^7 e^{-2zy}, \end{aligned} \quad (4.5)$$

where we have taken $g_s = 2$, $\beta_{ann} = 10$ and lastly, according to the line below Eq. (2.26), $\alpha = 0.033$ and $\alpha_R = 0.025$.

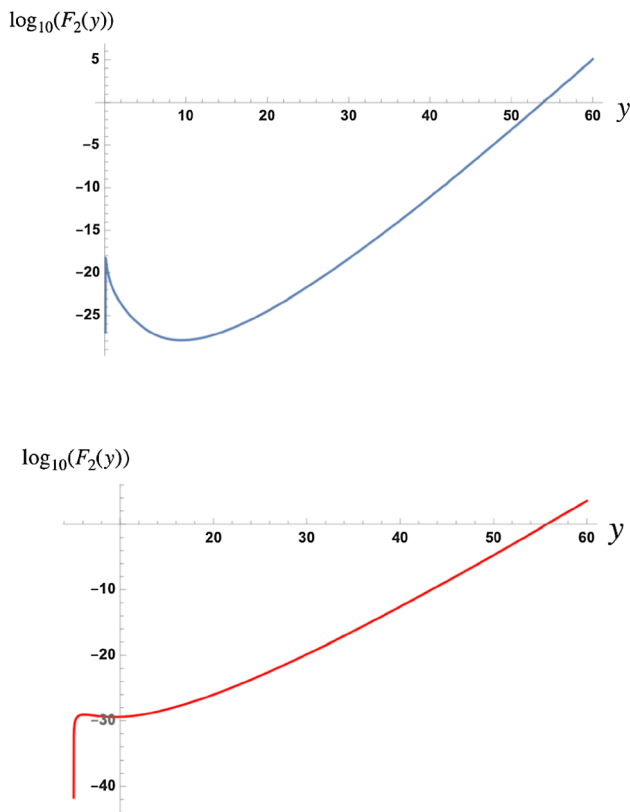


Fig. 2 Log of ratio of the calculated number density of X-particles to the equilibrium number density (4.5) calculated in the limit $n_X \ll n_{eq}$; left panel: $y_{in} = 0.1$ and right panel: $y_{in} = 5$

The ratio $F_2(y)$ is depicted in Figs. 2, 3 as function of y for different values of y_{in} . The ratio remains smaller than unity for sufficiently small $y < y_{max} = 50 - 150$ depending upon y_{in} . If $y_{fin} < y_{max}$, the assumption $n_X \ll n_{eq}$ is justified and the solution (4.4) is a good approximation to the exact solution. In the opposite case, when $y_{fin} > y_{max}$, we have to solve Eq. (4.1) numerically.

To solve the Eq. (4.1) it is convenient to introduce the new function according to:

$$n_X = g_s \left(\frac{a_{in}}{a(t)} \right)^3 M_X^3 z(t) = g_s M_X^3 \left(\frac{T_{in}}{T} \right)^{-8} z, \quad (4.6)$$

where $a(t)$ is the cosmological scale factor and a_{in} is its initial value at some time $t = t_{in}$, when X-particles became non-relativistic. In terms of z , Eq. (4.1) is reduced to:

$$\dot{z} = \langle \sigma_{ann} v \rangle M_X^3 \left(\frac{a_{in}}{a} \right)^3 (z_{eq}^2 - z^2). \quad (4.7)$$

Next, let us change the variables from t to $y = M_X/T$. Evidently $\dot{y} = -y(\dot{T}/T)$. Using time-temperature relation (2.22), we find

$$\frac{dz}{dt} = \frac{M_X^4}{4C_0 M_R^3 y^3} \frac{dz}{dy}. \quad (4.8)$$

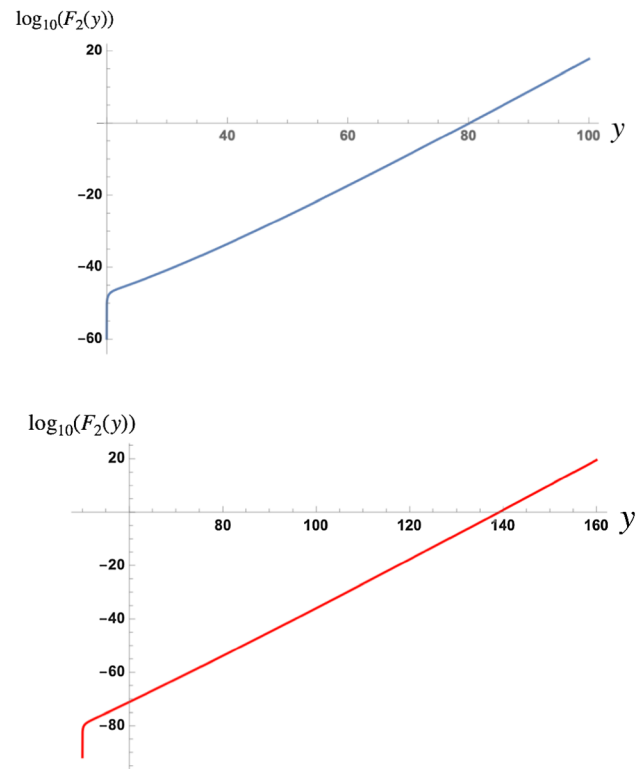


Fig. 3 Log of the ratio of the calculated number density of X-particles to the equilibrium number density (4.5) calculated in the limit $n_X \ll n_{eq}$; left panel: $y_{in} = 20$ and right panel: $y_{in} = 50$

Keeping in mind that

$$\left(\frac{a_{in}}{a} \right)^3 = \left(\frac{t_{in}}{t} \right)^2 = \left(\frac{y_{in}}{y} \right)^8, \quad (4.9)$$

we find finally:

$$\frac{dz}{dy} = 4\pi g_s C_0 \alpha^2 \beta_{ann} \mu^3 \frac{y_{in}^8}{y^6} \left(\frac{y^{13}}{8\pi^3 y_{in}^{16} e^{2y}} - z^2 \right), \quad (4.10)$$

where $\mu = M_R/M_X$.

With the values above values of α_R and α , see the discussion after Eq. (4.2), we find that the value of the coefficient in the r.h.s. of Eq. (4.10) is $4\pi g_s C_0 \alpha^2 \beta_{ann} = 1.4 \cdot 10^{-7}$.

Numerical solution of this equation indicates that $z(y)$ tends asymptotically at large y to a constant value z_{asym} . The energy density of X-particles is expressed through z_{asym} as follows. We assume that below $T = T_h$ the ratio of number density of X-particles to the number density of relativistic particles remains constant and hence is equal to the ratio n_X/n_{CMB} at the present time, where $n_{CMB} = 412/\text{cm}^3$ is the contemporary number density of photons in cosmic microwave background radiation. The number density of X-particles is expressed through z according to Eq. (4.6). Thus the asymptotic ratio of the number densities of X to the num-

ber density of relativistic particles is

$$F_{asym} = \frac{n_X(T_h)}{n_{rel}(T_h)} = \left[M_X^3 (y_{in}/y_h)^8 z_{asym} \right] \cdot \left[\pi^2 g_* T_h^3 / 90 \right]^{-1}. \quad (4.11)$$

We assume that $y_{in} \approx 10^2/\mu$, $y_{fin} = y_h = 2 \cdot 10^5/\mu$, according to the discussion after Eq. (4.4), and so $y_{fin}/y_{in} \approx 2 \cdot 10^3$. Hence the energy density of X-particles today would be equal to:

$$\rho_X^{(0)} = (412/\text{cm}^3) M_X F_{asym} = 3 \cdot 10^9 \mu^{-4} z_{asym} \frac{\text{keV}}{\text{cm}^3}, \quad (4.12)$$

where z_{asym} is the asymptotic value of $z(y)$ at large y but still smaller than y_h . The value of z_{asym} can be found from the numerical solution of Eq. (4.10). However, the solution demonstrates surprising feature: its derivative changes sign at $y \lesssim 10$, when $n_X \ll n_{eq}$, as it is seen from the value of F_2 presented in Figs. 2 and 3. Probably this evidently incorrect result for $z(y)$ originated from a very small coefficient in front of the brackets in Eq. (4.10).

The problem can be avoided if we introduce the new function $u(y)$ according to:

$$z(y) = \frac{u(y)}{(2\pi)^{3/2} y_{in}^{3/2} \exp(y_{in})}. \quad (4.13)$$

In terms of $u(y)$ kinetic equation takes the form:

$$\frac{du}{dy} = \frac{1.4 \cdot 10^{-7} \mu^3 y_{in}^{13/2}}{(2\pi)^{3/2} y^6 \exp(y_{in})} \left[\left(\frac{y}{y_{in}} \right)^{13} e^{2(y_{in}-y)} - u^2 \right]. \quad (4.14)$$

The numerical solution of this equation does not show any pathological features and may be trusted, so we express the contemporary energy of dark matter made of stable X-particles through the asymptotic value of $u(y)$ as

$$\rho_X^{(0)} = \frac{3 \cdot 10^9 \mu^{-4} u_{asym}}{(2\pi)^{3/2} y_{in}^{3/2} \exp(y_{in})} \frac{\text{keV}}{\text{cm}^3}. \quad (4.15)$$

Remind that $y_{in} \approx 100/\mu$ and presumably $\mu > 1$.

The asymptotic value u_{asym} is found from the numerical solution of Eq. (4.14) and is depicted in Figs. 4 and 5 for different values of μ .

The logarithm of the energy density of X-particles (4.15) with respect to the observed energy density of dark matter as a function of M_X is presented in Fig. 6. If $M_X \approx 5 \cdot 10^{12}$ GeV, X-particles may be viable candidates for the carriers of the cosmological dark matter.

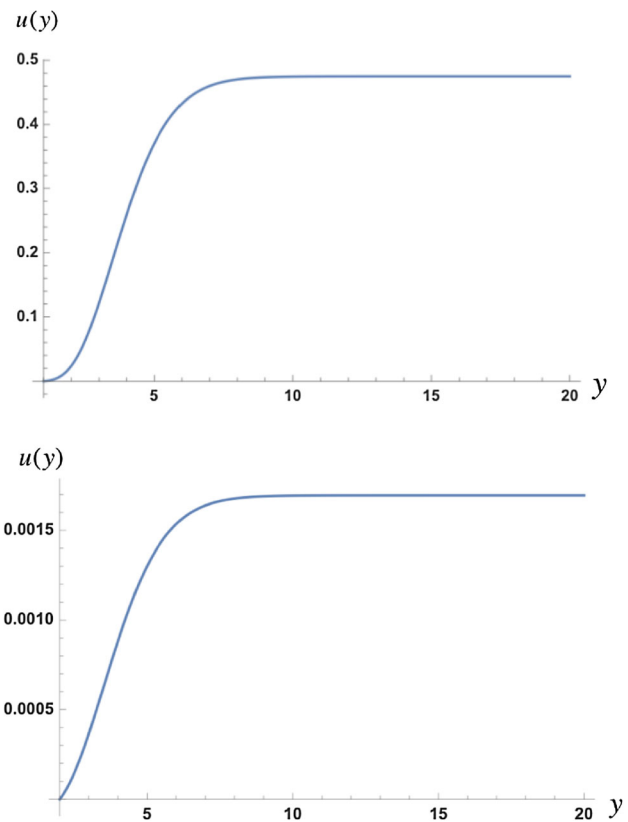


Fig. 4 Evolution of $u(y)$ for $\mu = 100$ (left) and $\mu = 50$ (right)

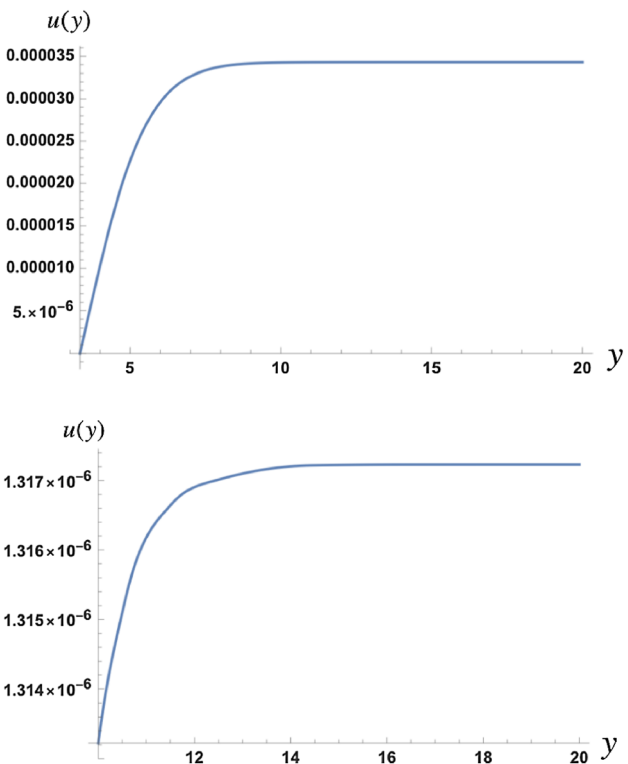


Fig. 5 Evolution of $u(y)$ for $\mu = 30$ (left) and $\mu = 20$ (right)

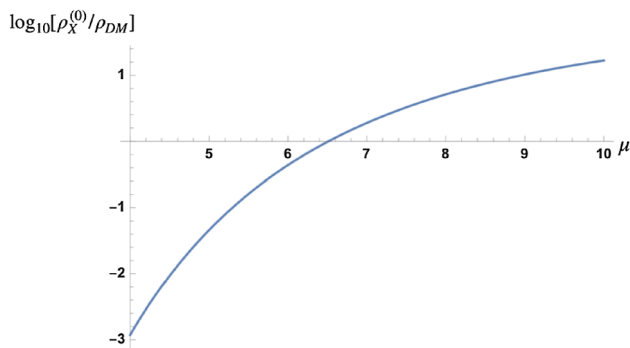


Fig. 6 Log of the ratio of the energy density of X -particles (4.15) to the observed energy density of dark matter as a function of $\mu = M_R/M_X$

5 Possible observations

This section is outside of the scope of our paper. It contains a discussion of some rather speculative possibilities of observation of the products of X -particle slow decay or enhanced annihilation in ultra high energy cosmic rays. More detailed study of the phenomena considered below demand separate work and the effects may be very weak or even non-existing. So the reader can skip this short section.

There are two possibilities to make X -particles visible: firstly, due to possible high density of $X\bar{X}$ -systems and, secondly, because of hypothetical instability of X -particles.

According to results of this and our previous papers [9, 11] the mass of dark matter particles, with the interaction strength typical for supersymmetric ones, can be in the range from 10^6 to 10^{13} GeV. It is tempting to find if and how they could be observed, except for their gravitational effects on galactic and cosmological scales.

The average cosmological energy/mass density of X -particles in the universe is approximately 1 keV/cm^3 , while in galaxies it is about 1 GeV/cm^3 . So their number densities should respectively be:

$$n_{\text{cosm}} = 10^{-12} M_6^{-1} / \text{cm}^3, \quad n_{\text{gal}} = 10^{-6} M_6^{-1} / \text{cm}^3, \quad (5.1)$$

where $M_6 = M_X / (10^6 \text{ GeV})$.

The characteristic annihilation time in a galaxy is:

$$\tau_{\text{gal}}^{\text{ann}} = 1 / [\sigma_{\text{ann}} v n_{\text{gal}}] \approx 10^{37} M_6^3 \text{ s}, \quad (5.2)$$

where we have taken $\sigma_{\text{ann}} v \approx 10^{-2} / M_X^2$.

The total energy flux from all annihilations in the Galaxy of the size $R_{\text{gal}} \approx 10 \text{ kpc} = 10^{22} \text{ cm}$ would be

$$\begin{aligned} L_{\text{gal}} &= n_{\text{gal}} E R_{\text{gal}} / \tau_{\text{gal}}^{\text{ann}} \approx 10^{-15} M_6^{-3} \text{ GeV/cm}^2/\text{s} \\ &= 3 \cdot 10^2 M_6^{-3} \text{ GeV/km}^2/\text{year}, \end{aligned} \quad (5.3)$$

with characteristic energy of the order of $E \sim M_X$.

The annihilation would be strongly enhanced in clusters (clumps) of dark matter [27], especially in neutralino

stars [28]. Based on the latter reference, for the annihilation cross-section $\sigma_{\text{ann}} v = 4 \cdot 10^{-42} M_6^{-2} \text{ cm}^2 \approx 10^{-31} M_6^{-2} \text{ cm}^3/\text{s}$, we can conclude that the observation of $X\bar{X}$ -annihilation from neutralino stars is not unrealistic.

Due to their huge mass relic X -particles might form gravitationally bound states and then annihilate like positronium. Instead of fine structure constant $\alpha = 1/137$ we must use the gravitational coupling constant $\alpha_G = (M_X/m_{\text{Pl}})^2$. In complete analogy with para-positronium decay the lifetime of such bound state with respect to annihilation would be

$$\tau_G \sim (\alpha_G^5 M_X)^{-1} \approx 5 \cdot 10^{23} M_{13}^{-11} \text{ s}, \quad (5.4)$$

where $M_{13} = M_X / (10^{13} \text{ GeV})$.

The flux of ultra-high energy cosmic rays (UHECR) with energy $\sim 10^{21} \text{ eV}$ produced by the population of the bound states of $X\bar{X}$, say, from the sphere of the radius of $R = 1 \text{ Gpc}$ would be:

$$F = n_{\text{gal}} R f / \tau_G = 2 \cdot 10^5 f M_{13}^{10} \text{ cm}^{-2} \text{ s}^{-1}, \quad (5.5)$$

where f is the fraction of bound states with respect to total number of X -particles.

Comparing this result with the data presented in Ref. [29] we can conclude that the flux of the UHECR produced in the decay of $X\bar{X}$ bound states would agree with the data if $f \sim 10^{-11}$.

Calculation of f is subject to many uncertainties and it is not the aim of the present work. It will be done elsewhere.

X -particle would be observable if they are unstable. Heavy X -particles would decay through formation of virtual black holes, according to the Zeldovich mechanism [30, 31]. If X -particles are composite states of three fundamental constituents, as proton made of three quarks, their life-time with respect to virtual BH stimulated decay would be

$$\tau_{X, \text{BH}} \sim \frac{m_{\text{Pl}}^4}{M_X^5} \sim 10^{-13} \text{ s} \left(\frac{10^{13} \text{ GeV}}{M_X} \right)^5. \quad (5.6)$$

To make the time $\tau_{X, \text{BH}}$ larger than the universe age $t_U \approx 4 \cdot 10^{17} \text{ s}$ we need $M_X < 10^7 \text{ GeV}$. In this case the products of the decays of X -particles with such masses could be observable in the flux of the cosmic rays with energy somewhat below 10^7 GeV .

The life-time may be further suppressed if we apply the conjecture of Ref. [32] which leads to a strong suppression of the decay through virtual black holes for spinning or electrically charged X -particles. However, this suppression does not operate for spinless neutral particles. Moreover, it would not be efficient enough to sufficiently suppress the decay probability of the superheavy particles of dark matter with masses of the order of 10^{13} GeV . The decay rate may be strongly diminished if X -particles consist of more than three fundamental constituents. For example, if X -particles consist of six fundamental constituents, then the decay life-time would

be

$$\tau'_{X,BH} \sim \frac{m_{Pl}^{10}}{M_X^{11}} \sim 10^{23} \text{ s} \left(\frac{10^{13} \text{ GeV}}{M_X} \right)^5. \quad (5.7)$$

This life-time is safely above the universe age $t_U \approx 4 \cdot 10^{17} \text{ s}$.

6 Conclusion and discussion

There is general agreement that the conventional Friedmann cosmology is incompatible with the existence of stable particles having interaction strength typical for supersymmetry and heavier than several TeV. A possible way to save life of such particles, we call them here X -particles, may be a modification of the standard cosmological expansion law in such a way that the density of such heavy relics would be significantly reduced. A natural way to realize such reduction is presented by the popular now Starobinsky inflationary model [113]. If the epoch of the domination of the curvature oscillations (the scalaron domination) lasted after freezing of massive species, their density with respect to the plasma entropy could be noticeably suppressed by production of radiation from the scalaron decay.

The concrete range of the allowed mass values depends upon the dominant decay mode of the scalaron. If the scalaron is minimally coupled to scalar particles X_S , the decay amplitude does not depend upon the scalar particle mass and leads to too high energy density of X -particles, if $M_{X_S} \ll M_R$. An acceptably low density of X_S can be achieved if $M_{X_S} \gtrsim M_R \approx 3 \cdot 10^{13} \text{ GeV}$.

If X -scalars are conformally coupled to curvature or X -particles are fermions, then the probability of the scalaron decay is proportional to M_X^2 . For sufficiently small M_X the production of X -particles would be quite weak, so that their cosmological energy density would be close to the observed density of dark matter if $M_X \sim 10^6 \text{ GeV}$ [11].

There is another complication due to conformal anomaly, which leads to efficient decay of scalaron into massless or light gauge bosons. There are some versions of supersymmetric theories where conformal anomaly is absent, which were considered in Ref. [11]. In the present work we have not impose this restriction and studied a model with full strength conformal anomaly. In this case the thermalization of the cosmological plasma started from the creation of gauge bosons and the reactions between them created all other particle species.

There are two possible processes through which X -particles could be produced: direct decay of the scalaron into a pair of $\bar{X}X$ and the thermal production of X 's in plasma. To restrict the density of X -particles produced by the direct decay the observed value M_X should be below 10^7 GeV . But in this case the thermal production of X 's would be too strong.

We can resolve this inconsistency if the direct decay of the scalaron into X -particles is suppressed and due to that a larger M_X is allowed, so the thermal production would not be dangerous. The direct decay can be very strongly suppressed if X -particles are Majorana fermions, which cannot be created by a scalar field in the lowest order of perturbation theory. It opens the possibility for X -particles to make proper amount of dark matter, if their mass is about $5 \cdot 10^{12} \text{ GeV}$.

Thus a supersymmetric type of dark matter particles seems to be possible if their mass is quite high from 10^6 up to $5 \cdot 10^{12} \text{ GeV}$, or even higher than the scalaron mass, $M_R = 3 \cdot 10^{13} \text{ GeV}$. There is no chance to discover these particles in accelerator experiments in foreseeable future, but they may be observable through cosmic rays from their annihilations in high density clumps of dark matter, or from annihilation in their gravitationally bound two-body states, or through the products of their decays, since they naturally should be unstable.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The following study is based on the theoretical model first presented in our previous work [18]. Since the model itself is theoretical in nature, and the proposed dark matter candidate cannot be detected by current accelerators, the data are absent.]

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