



Open-charm tetraquark X_c and open-bottom tetraquark X_b

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Abstract Motivated by the LHCb observation of exotic states $X_{0,1}(2900)$ with four open quark flavors in the $D^- K^+$ invariant mass distribution in the decay channel $B^\pm \rightarrow D^+ D^- K^\pm$, we study the spectrum and decay properties of the open charm tetraquarks. Using the two-body chromomagnetic interactions, we find that the two newly observed states can be interpreted as a radial excited tetraquark with $J^P = 0^+$ and an orbitally excited tetraquark with $J^P = 1^-$, respectively. We then explore the mass and decays of the other flavor-open tetraquarks made of $sud\bar{c}$ and $ds\bar{u}\bar{c}$, which are in the $\bar{6}$ or 15 representation of the flavor SU(3) group. We point that these two states can be found through the decays: $X_{ds\bar{u}\bar{c}}^{(0)} \rightarrow (D^- K^-, D_s^- \pi^-)$, and $X_{sud\bar{c}}^{(0)} \rightarrow D_s^- \pi^+$. We also apply our analysis to open bottom tetraquark X_b and predict their masses. The open-flavored X_b can be discovered through the following decays: $X_{ud\bar{s}\bar{b}} \rightarrow B^0 K^+$, $X_{ds\bar{u}\bar{b}}^{(0)} \rightarrow (B^0 K^-, B_s^0 \pi^-)$, and $X_{su\bar{d}\bar{b}}^{(0)} \rightarrow B_s^0 \pi^+$.

1 Introduction

Very recently, the LHCb collaboration has reported an intriguing and important discovery of two exotic structures with open quark flavors in the invariant mass distribution of $D^- K^+$ of the channel $B^\pm \rightarrow D^+ D^- K^\pm$ [1–3]. The relatively narrower one, named as $X_0(2900)$, has the mass and decay width as [3]

$$m_{X_0(2900)} = 2.866 \pm 0.007 \pm 0.002 \text{ GeV},$$

$$\Gamma_{X_0(2900)} = 57 \pm 12 \pm 4 \text{ MeV},$$

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while the broader one is called $X_1(2900)$ and has

$$m_{X_1(2900)} = 2.904 \pm 0.005 \pm 0.001 \text{ GeV},$$

$$\Gamma_{X_1(2900)} = 110 \pm 11 \pm 4 \text{ MeV}.$$

These two structures are 502 MeV and 540 MeV higher than the DK threshold, respectively. Both of them can strongly decay into $D^- K^+$ and thus have the minimum quark content $[ud\bar{s}\bar{c}]$. Once that this discovery is confirmed, it is anticipated that our knowledge of QCD color confinement will be greatly deepened.

In 2016 the D0 collaboration reported an open flavor state $X(5568)$ decaying into $B_s^0 \pi$ [4] but such a state is not confirmed by other experiments such as LHCb [5], CMS [6], CDF [7] and ATLAS [8]. Though most of experiments did not reveal the existence of the $X(5568)$, a lot of theoretical studies on the open flavor tetraquarks [9–25] have been simulated.

In Ref. [22], we pointed out the existence of the open charm X_c tetraquark states in 2016 and firstly proposed to hunt for the X_c states in B and B_c decays. Based on the two-body Coulomb and chromomagnetic interactions model, we calculated the masses of the X_c tetraquarks. The 0^+ and 1^+ ground-states composed of $[ud\bar{s}\bar{c}]$ are predicted to lie in the range 2.4–2.6 GeV having a limited phase space for decays into $D^- K^+$ which cannot be identified with new the $X_{0,1}(2900)$ states. But it is worth to investigate carefully the possible peaks in the invariant mass distribution of $D^- K^+$. In addition it is interesting to notice that the newly observed $X_{0,1}(2900)$ can be attributed to the orbitally and radially excited state. One main focus of this work is to explore this possibility.

In addition, the discovery of the $X_{0,1}(2900)$ is of great value to explore other related tetraquark states such as the ones are composed of $[us\bar{d}\bar{c}]$ and $[ds\bar{u}\bar{c}]$. In the flavor SU(3)

symmetry, the charmed tetraquarks are decomposed as the $\bar{6}$ or 15 representation. In the following we will carry out a calculation of the masses for these open-charm tetraquarks, and the corresponding open bottom multiplets X_b . We will also use flavor SU(3) symmetry to study related strong two body hadronic decays and give some relations of decay widths among different decay channels, which may provide some guidances for experimental searches.

The rest of this paper is organized as follows. In Sect. 2, the heavy tetraquarks are decomposed into different irreducible representations and the spectra of X_c and X_b tetraquarks is predicted. Using the SU(3) flavor symmetry, decay properties of X_c and X_b tetraquarks are given in Sect. 3. We also discuss the golden channels to hunt for the possible $X_{0,1}(2900)$ partners. A brief summary is given in the last section.

2 Spectra of heavy tetraquarks $X_{c,b}$

To start with, we classify heavy tetraquarks with open-charm (bottom) according to SU(3) representations. These tetraquarks can be denoted as X_Q (or $X_{qq'\bar{q}''\bar{Q}}$ when the flavor component is needed), where q, q' and q'' are light quarks, and $Q = c, b$ is a heavy quark. There are many applications of SU(3) flavor symmetry in Refs. [26–38]. Considering the fact that the light quarks belong to a triplet **3** representation and the heavy quark Q is a singlet in the flavor SU(3) symmetry, the heavy tetraquarks are classified into different irreducible representations as $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$. When the heavy tetraquarks with four different flavors are involved, one only needs to consider the $\bar{\mathbf{6}}$ and **15** representations. The observed states may belong to one of these two representations but a specific assignment requests more experimental and theoretical studies.

The $\bar{\mathbf{6}}$ representation will be denoted as $X_{[i,j]}^k$ ($i, j, k = 1, 2, 3$ corresponding to the u, d, s quark), where the indices i and j are antisymmetric. Their explicit expression are [17]

$$\begin{aligned} X_{[2,3]}^1 &= \frac{1}{\sqrt{2}} X'_{ds\bar{u}}, & X_{[3,1]}^2 &= \frac{1}{\sqrt{2}} X'_{s\bar{u}d}, \\ X_{[1,2]}^3 &= \frac{1}{\sqrt{2}} X'_{ud\bar{s}}, & X_{[1,2]}^1 &= X_{[2,3]}^3 = \frac{1}{2} Y'_{(u\bar{u}, s\bar{s})d}, \\ X_{[3,1]}^1 &= X_{[2,3]}^2 = \frac{1}{2} Y'_{(u\bar{u}, d\bar{d})s}, & X_{[1,2]}^2 &= X_{[3,1]}^3 = \frac{1}{2} Y'_{(d\bar{d}, s\bar{s})u}. \end{aligned} \tag{1}$$

We will use $X_{[i,j]}^k$ to abbreviate the **15** representation, where the indices i and j are symmetric [17]:

$$\begin{aligned} X_{[2,3]}^1 &= \frac{1}{\sqrt{2}} X_{ds\bar{u}}, & X_{[3,1]}^2 &= \frac{1}{\sqrt{2}} X_{s\bar{u}d}, \\ X_{[1,2]}^3 &= \frac{1}{\sqrt{2}} X_{ud\bar{s}}, & X_{[1,1]}^1 &= \left(\frac{Y_{\pi u}}{\sqrt{2}} + \frac{Y_{\eta u}}{\sqrt{6}} \right), \end{aligned}$$

$$\begin{aligned} X_{[1,2]}^1 &= \frac{1}{\sqrt{2}} \left(\frac{Y_{\pi d}}{\sqrt{2}} + \frac{Y_{\eta d}}{\sqrt{6}} \right), & X_{[1,3]}^1 &= \frac{1}{\sqrt{2}} \left(\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}} \right), \\ X_{[2,1]}^2 &= \frac{1}{\sqrt{2}} \left(-\frac{Y_{\pi u}}{\sqrt{2}} + \frac{Y_{\eta u}}{\sqrt{6}} \right), & X_{[2,2]}^2 &= \left(-\frac{Y_{\pi d}}{\sqrt{2}} + \frac{Y_{\eta d}}{\sqrt{6}} \right), \\ X_{[2,3]}^2 &= \frac{1}{\sqrt{2}} \left(-\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}} \right), & X_{[3,1]}^3 &= -\frac{Y_{\eta u}}{\sqrt{3}}, \\ X_{[3,2]}^3 &= -\frac{Y_{\eta d}}{\sqrt{3}}, & X_{[3,3]}^3 &= -\frac{Y_{\eta s}}{\sqrt{3}}, & X_{[2,2]}^1 &= Z_{d\bar{d}\bar{u}}, \\ X_{[3,3]}^1 &= Z_{s\bar{s}\bar{u}}, & X_{[1,1]}^2 &= Z_{u\bar{u}\bar{d}}, & X_{[3,3]}^2 &= Z_{s\bar{s}\bar{d}}, \\ X_{[1,1]}^3 &= Z_{u\bar{u}\bar{s}}, & X_{[2,2]}^3 &= Z_{d\bar{d}\bar{s}}. \end{aligned} \tag{2}$$

Note that the heavy quark c or b is not explicitly shown in the above. But one can easily add the heavy quark in the following application. The above SU(3) classification is applicable to the ground states, orbitally-excited and radially-excited tetraquarks. In the following we carry out a calculation of their corresponding masses using the two-body Coulomb and chromomagnetic interactions model.

Based on the diquark configuration proposed in Ref. [39], we assume that the open heavy flavor tetraquark is composed of a light diquark, a light quark, and a heavy flavor quark. Their mass spectra can be calculated using the two-body chromomagnetic interactions. Correspondingly, the effective Hamiltonian for a tetraquark state with spin and orbital interaction is written as [40–43],

$$\begin{aligned} H &= m_\delta + m_{q''} + m_Q + H_{SS}^\delta + H_{SS}^{\bar{q}''\bar{Q}} + H_{SS}^{\delta\bar{q}''} \\ &\quad + H_{SS}^{\delta\bar{Q}} + H_{SL} + H_{LL}, \end{aligned} \tag{3}$$

with the spin and orbital interactions

$$\begin{aligned} H_{SS}^\delta &= 2(\kappa_{qq'})_{\bar{\mathbf{3}}} (\mathbf{S}_q \cdot \mathbf{S}_{q'}), \\ H_{SS}^{\bar{q}''\bar{Q}} &= 2(\kappa_{Qq''})_{\bar{\mathbf{3}}} (\mathbf{S}_{\bar{Q}} \cdot \mathbf{S}_{q''}), \\ H_{SS}^{\delta\bar{q}''} &= 2\kappa_{q\bar{q}''} (\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}''}) + 2\kappa_{q'\bar{q}''} (\mathbf{S}_{q'} \cdot \mathbf{S}_{\bar{q}''}), \\ H_{SS}^{\delta\bar{Q}} &= 2\kappa_{q\bar{Q}} (\mathbf{S}_q \cdot \mathbf{S}_{\bar{Q}}) + 2\kappa_{q'\bar{Q}} (\mathbf{S}_{q'} \cdot \mathbf{S}_{\bar{Q}}), \\ H_{SL} &= 2A_\delta (\mathbf{S}_\delta \cdot \mathbf{L}) + 2A_{\bar{q}''\bar{Q}} (\mathbf{S}_{\bar{q}''\bar{Q}} \cdot \mathbf{L}), \\ H_{LL} &= B_Q \frac{L(L+1)}{2}. \end{aligned} \tag{4}$$

The parameters in the above formalism can be determined from various meson and baryon masses. Using the mass difference among hadrons with different spin and orbital quantum numbers, the chromomagnetic couplings can be fixed. According to the previous extractions in Refs. [40–46], we give a collection of the relevant chromomagnetic coupling parameters in the following. The chromomagnetic coupling constants are used as: $(\kappa_{qq})_{\bar{\mathbf{3}}} = 103 \text{ MeV}$, $(\kappa_{sq})_{\bar{\mathbf{3}}} = 64 \text{ MeV}$, $(\kappa_{cq})_{\bar{\mathbf{3}}} = 22 \text{ MeV}$, $(\kappa_{cs})_{\bar{\mathbf{3}}} = 25 \text{ MeV}$, $(\kappa_{ss})_{\bar{\mathbf{3}}} = 72 \text{ MeV}$, $(\kappa_{q\bar{q}})_0 = 315 \text{ MeV}$, $(\kappa_{s\bar{q}})_0 = 195 \text{ MeV}$, $(\kappa_{s\bar{s}})_0 = 121 \text{ MeV}$, $(\kappa_{c\bar{q}})_0 = 70 \text{ MeV}$ and $(\kappa_{c\bar{s}})_0 = 72 \text{ MeV}$. We will employ the relation $\kappa_{ij} = \frac{1}{4}(\kappa_{ij})_0$ for the quark–antiquark coupling,

which is derived from one gluon exchange model. The spin-orbit and orbital coupling constants can be extracted from the P-wave meson or baryons. We adopt $A_{\bar{s}\bar{c}} = A_\delta = 50 \text{ MeV}$ and $B_c = 495 \text{ MeV}$ [44], $A_{\bar{u}\bar{b}} = A_\delta = 5 \text{ MeV}$ and $B_b = 408 \text{ MeV}$ or $A_{\bar{s}\bar{b}} = A_\delta = 3 \text{ MeV}$ and $B_b = 423 \text{ MeV}$ [42,45].

Within the above chromomagnetic coupling parameters, we can further determine the effective quark masses in the two-body chromomagnetic interaction model. In principle, we need to consider the uncertainties of all the parameters in the two-body chromomagnetic interaction model at the same time, which we will discuss in future works. For discussions in the following, we will take the errors due to quark masses as an indication of possible errors for the mass spectra for illustration. For pseudoscalar and vector mesons, we have

$$m_H(q\bar{q}')(J^P) = m_q + m'_q + \kappa_{q\bar{q}'} \left(J(J+1) - \frac{3}{2} \right), \quad (5)$$

where q' can be either light quark or heavy quark. Inputting $m_{\pi^0} = 134.98 \text{ MeV}$, $m_{\pi^\pm} = 139.57 \text{ MeV}$, $m_{\rho(770)} = 769.0 \pm 0.9 \text{ MeV}$ [47], we obtained

$$m_q = 0.305 \pm 0.002 \text{ GeV}. \quad (6)$$

Inputting $m_{K^0} = 497.611 \pm 0.013 \text{ MeV}$, $m_{K^\pm} = 493.677 \pm 0.016 \text{ MeV}$, $m_{K^*(892)} = 895.55 \pm 0.20 \text{ MeV}$ [47], we obtained

$$m_s = 0.490 \pm 0.009 \text{ GeV}. \quad (7)$$

Inputting $m_{D^0} = 1869.65 \pm 0.05 \text{ MeV}$, $m_{D^\pm} = 1864.83 \pm 0.05 \text{ MeV}$, $m_{D^{*0}} = 2006.85 \pm 0.05 \text{ MeV}$, $m_{D^{*\pm}} = 2010.26 \pm 0.05 \text{ MeV}$ [47], we obtained

$$m_c = 1.670 \pm 0.006 \text{ GeV}. \quad (8)$$

Inputting $m_{B^0} = 5279.65 \pm 0.12 \text{ MeV}$, $m_{B^\pm} = 5279.34 \pm 0.12 \text{ MeV}$, $m_{B^*} = 5324.70 \pm 0.21 \text{ MeV}$ [47], we obtained

$$m_b = 5.008 \pm 0.001 \text{ GeV}. \quad (9)$$

The diquark mass satisfies the relation $m_{ss} - m_{sq} = m_{sq} - m_{qq}$ and we have $m_{qq} = 0.395 \text{ GeV}$, $m_{sq} = 0.590 \text{ GeV}$, and $m_{ss} = 0.785 \text{ GeV}$ [40,41,44].

The spectra of S-wave tetraquarks $X_c(1S)$ have been given in Ref. [22]. The 0^+ $[ud\bar{s}\bar{c}]$ ground-state was determined to have a mass 2.36 GeV, which is much lower than the new LHCb data. Thereby the identification of the observed 0^+ and 1^- states is likely to rely on the orbitally or radially excited states.

We now calculate the spectra of $X_c(1P)$ and $X_c(2S)$ with different light quark contents from orbital or radial excitations, and the results are tabulated in Tables 1 and 2, respectively. From the orbitally excited states in Table 1, one can see that the $X_{ud\bar{s}\bar{c}}$ in the 15 representation with 1^- has a mass around 2.91 GeV and can decay into $D^- K^+$. This could be a candidate to explain the newly $X_1(2900)$ states observed

by LHCb collaboration [1]. The $J^P = 1^- X_{ud\bar{s}\bar{c}}$ states with the mass around (2.88, 2.98, 3.00) GeV and the $J^P = 1^- X'_{ud\bar{s}\bar{c}}$ states with the mass around (2.81, 2.86) GeV are also interesting and can decay into $D^- K^+$, and thus future experiments are likely to discover them. In the table we also listed masses for states with 2^- and 3^- , but other orbitally excited states $X_c(1P)$ either do not have the quark content $[ud\bar{s}\bar{c}]$ or can not directly decay into $D^- K^+$ by the spin-parity constraint. We will discuss their decay patterns for experimental searches later.

To explain the $X_0(2900)$, one needs to find a 0^+ state with higher mass than the ground state. We find that $X_c(2S)$ has such a possibility. To calculate masses of radially excited hadron, it is convenient to construct the hadron Regge trajectories in (n, M^2) plane [48]

$$n = cM^2 + c_0, \quad (10)$$

where n is the radial quantum number, while M is the hadron mass. This relation is hold in most of hadron systems. c being the slope and c_0 being intercept, both of which are parameters and different for different hadron system. If we assume that the first radially excited $X'_{ud\bar{s}\bar{c}}$ state with 0^+ in the $\bar{6}$ representation may be identified as the newly $X_0(2900)$ states observed by LHCb collaboration [1]. Then we can fit the slope and intercept in Regge trajectory relation for open charm tetraquarks

$$c = (0.378 \pm 0.008) \text{ GeV}^{-2}, \quad c_0 = -1.11 \pm 0.04, \quad (11)$$

These values are close to the global fits of slope and intercept in heavy-light systems. In Ref. [48], $c = (0.362 \pm 0.011) \text{ GeV}^{-2}$, $c_0 = -0.322 \pm 0.090$ are fitted for $D(n^1 S_0)$ mesons and $c = (0.375 \pm 0.007) \text{ GeV}^{-2}$, $c_0 = -0.550 \pm 0.058$ are fitted for $D^*(n^3 S_1)$ mesons. Note that $n_r = n - 1$ is introduced in the Regge relation in Ref. [48] and thus the intercept $\beta_0 = c_0 - 1$ in Ref. [48]. The ground states of $X_c(1S)$ tetraquarks have been predicted in Ref. [22]. Consider that the slopes are very close between two similar systems but the intercepts may be different, thus we can use the slope in Eq. (11) and the masses of ground states in Ref. [22] to predict the radial excitation states. We give the results for the masses of radially excited $X_c(2S)$ tetraquarks in Table 2. From this table, one can see that the $J^P = 0^+ X'_{ud\bar{s}\bar{c}}$ state with the mass around 2.97 GeV is also interesting for experimental search. Other radially excited states $X_c(2S)$ either do not have the quark content $[ud\bar{s}\bar{c}]$ or can not directly decays into $D^- K^+$ by spin-parity constraint.

Our analysis can be extended to the $Q = b$ case. For bottom mesons, the slope and intercept in Regge trajectory relation are fitted as $c = (0.173 \pm 0.007) \text{ GeV}^{-2}$, $c_0 = -3.913 \pm 0.269$ are fitted for $B(n^1 S_0)$ mesons and $c = (0.176 \pm 0.006) \text{ GeV}^{-2}$, $c_0 = -4.082 \pm 0.243$ are fit-

ted for $B^*(n^3S_1)$ mesons. Thus we may employ the slope $c = (0.176 \pm 0.006) \text{ GeV}^{-2}$ and the spectra of $X_b(2S)$ can be obtained. In Table 1, we present the masses of P-wave $X_b(1P)$ tetraquark partners in both $\bar{\mathbf{6}}$ and $\mathbf{15}$ representation. In Table 2, we present the masses of S-wave $X_b(2S)$ tetraquark partners in both $\bar{\mathbf{6}}$ and $\mathbf{15}$ representation.

3 Two-body strong decay of $X_{c,b}$

We now study the possible strong decays of the $X_Q(1P)$ and $X_Q(2S)$ and focus on the $Q_i + P$ final states. The $1^- X_Q(1P)$ quantum field is labeled as X^μ , while the $0^+ X_Q(2S)$ is labeled as X . The Q_i is one of the heavy meson D_i and B_i mesons as $D_i = (D^0(u\bar{c}), D^-(d\bar{c}), D_s^-(s\bar{c}))$ and $B_i = (B^+(u\bar{b}), B^0(d\bar{b}), B_s^0(s\bar{b}))$. The P is a pseudo-scalar meson in the octet

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}. \tag{12}$$

Using heavy quark effective theory, we find that the interacting terms $\bar{Q}v \cdot AX$ and $\bar{Q}A_\mu X^\mu$ are responsible for the leading decays [17]. Here A is the axial-vector field, and v is the heavy quark velocity. Note that all the SU(3) flavor indices are contracted in above equation. Their flavor SU(3) transformation are

$$X_{jk}^i \rightarrow U_i^i X_{j'k'}^{i'} (U^\dagger)^{j'} j (U^\dagger)^{k'} k, \quad Q_i \rightarrow U_i^j Q_j \tag{13}$$

$$A_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \rightarrow UA_\mu U^\dagger,$$

where ξ^\dagger is defined as $\xi(x) = \sqrt{\Sigma(x)}$ and $\Sigma(x) = \exp(2i\Pi/\sqrt{2}f)$.

The $X_c \rightarrow D_i P$ decay amplitude can then be parameterized as

$$\mathcal{M}(X_c \rightarrow D_i P) = \beta' X_{[i,j]}^k \bar{D}_i \Pi_k^j + \beta X_{[i,j]}^k \bar{D}_i \Pi_k^j, \tag{14}$$

with β and β' being the nonperturbative amplitudes to be given latter. Similarly the $X_b \rightarrow B_i P$ decay amplitudes can be parameterized as

$$\mathcal{M}(X_b \rightarrow B_i P) = \alpha' X_{[i,j]}^k \bar{B}_i \Pi_k^j + \alpha X_{[i,j]}^k \bar{B}_i \Pi_k^j. \tag{15}$$

Results for the $X_c \rightarrow D_i P$ amplitudes are collected in Tables 3 and 4, while the results for the $X_b \rightarrow B_i P$ amplitudes can be obtained using the replacements $D^0 \rightarrow B^+$, $D^- \rightarrow B^0$, $D_s^- \rightarrow B_s^0$, $X_c \rightarrow X_b$, and $\beta^{(\prime)} \rightarrow \alpha^{(\prime)}$ from Tables 3 and 4.

It is interesting to note that one can also reconstruct $X_{0,1}$ in $X_{0,1} \rightarrow D^0 \bar{K}^0$, whose decay width is the same order of $X_{0,1} \rightarrow D^- K^+$. This serves as a confirmation of the

model. The other X_c tetraquark partners can be searched for using results in Tables 3 and 4. Of particular interests are the tetraquarks with four different quarks can be hunted by $X_{ds\bar{u}\bar{c}} \rightarrow D^- K^-, X'_{ds\bar{u}\bar{c}} \rightarrow D^- K^-, X'_{ds\bar{u}\bar{c}} \rightarrow D_s^- \pi^-, X_{sud\bar{c}} \rightarrow D_s^- \pi^+, X_{ds\bar{u}\bar{c}} \rightarrow D_s^- \pi^-, X'_{sud\bar{c}} \rightarrow D_s^- \pi^+$.

For $X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^- K^+$, we have the amplitude

$$\mathcal{M}(X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^- K^+) = -\frac{\beta'_c}{\sqrt{2}} \frac{1}{\sqrt{2}f_\pi} E_K \sqrt{m_X m_D}, \tag{16}$$

and the decay width

$$\Gamma(X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^- K^+) = \frac{\beta'_c{}^2}{32\pi} |\vec{p}_K| \frac{m_D}{m_X} \left(\frac{E_K}{f_\pi}\right)^2, \tag{17}$$

where the dimensionless coupling β'_c is parameterized as $\beta'_c = \frac{\sqrt{2}f_\pi}{E_K \sqrt{m_X m_D}} \beta'$ [17]. We have $|\vec{p}_K| = \frac{\sqrt{(m_X^2 - (m_D - m_K)^2)(m_X^2 - (m_D + m_K)^2)}}{2m_X}$ and $E_K = \sqrt{m_K^2 + |\vec{p}_K|^2}$.

We can estimate the decay width of X_0 as

$$\Gamma_{X_0} \approx \Gamma(X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^- K^+) + \Gamma(X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^0 K^0) \approx 2\Gamma(X'_{ud\bar{s}\bar{c}}(0^+) \rightarrow D^- K^+), \tag{18}$$

where the SU(3) symmetry breaking effects are neglected. Using the LHCb measurement $m_{X_0} = 2.866 \text{ GeV}$ and $\Gamma_{X_0} = 57 \text{ MeV}$, one can extract the dimensionless coupling as $\beta'_c \approx 0.37$.

For $X_{ud\bar{s}\bar{c}}(1^-) \rightarrow D^- K^+$, we have the amplitude

$$\mathcal{M}(X_{ud\bar{s}\bar{c}}(1^-)(p_X, \epsilon) \rightarrow D^-(p_D)K^+(p_K)) = \frac{\beta_c}{\sqrt{2}} \frac{1}{\sqrt{2}f_\pi} \epsilon \cdot (p_D - p_K) \sqrt{m_X m_D}, \tag{19}$$

and the decay width

$$\Gamma(X_{ud\bar{s}\bar{c}}(1^-)(p_X, \epsilon) \rightarrow D^-(p_D)K^+(p_K)) = \frac{\beta_c^2}{32\pi} |\vec{p}_{K^+}| \frac{m_D V_X}{m_X f_\pi^2}, \tag{20}$$

where the dimensionless coupling β_c is parameterized as $\beta_c = \frac{\sqrt{2}f_\pi}{\epsilon \cdot (p_D - p_K) \sqrt{m_X m_D}} \beta$, and $V_X = 4 \left(\frac{(m_D^2 - m_K^2 + m_X^2)^2}{4m_X^2} - m_D^2 \right)$. The decay width of X_1 is then given as

Table 1 Predictions of the masses (GeV) of orbitally excited $X_{c(b)}(1P)$ tetraquarks in both $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations. Since the isospin breaking effects are not taken into account, the states obtained by the $u \leftrightarrow d$ replacement have degenerate masses. Thus the first column of this table and Table 2 contains the states with the same mass. In the second col-

umn, different J^P numbers are listed for these particles. In the table two or more different masses appear in identical J^P for some states because of hyperfine splitting from spin-spin or spin-orbital coupling. The same reason also give more than one entries in Table 2. The mass denoted a “**” means two degenerate states. The uncertainty is from quark masses

$X_{c(b)}(1P)$ states	J^P	Mass (X_c)	Mass (X_b)
$X'_{ds\bar{u}}, X'_{su\bar{d}}, Y'_{(u\bar{u},d\bar{d})s}$	0^-	2.86 ± 0.01	6.20 ± 0.00
	1^-	$2.91 \pm 0.01, 2.92 \pm 0.01$	$6.20 \pm 0.00, 6.21 \pm 0.00$
	2^-	3.01 ± 0.01	6.23 ± 0.00
$X'_{ud\bar{s}}$	0^-	2.71 ± 0.02	6.16 ± 0.01
	1^-	$2.81 \pm 0.02, 2.86 \pm 0.02$	$6.12 \pm 0.01, 6.17 \pm 0.01$
	2^-	3.01 ± 0.02	6.18 ± 0.01
$Y'_{(u\bar{u},s\bar{s})d}, Y'_{(d\bar{d},s\bar{s})u}$	0^-	2.84 ± 0.02	6.18 ± 0.01
	1^-	$2.88 \pm 0.02, 2.89 \pm 0.02$	$6.16 \pm 0.01, 6.19 \pm 0.01$
	2^-	2.98 ± 0.02	6.20 ± 0.01
$X_{ds\bar{u}}, X_{su\bar{d}}, Y_{\pi s}$	0^-	$2.89 \pm 0.01, 2.98 \pm 0.01$	$6.23 \pm 0.00, 6.36 \pm 0.00$
	1^-	$2.93 \pm 0.01, 2.95 \pm 0.01, 3.01 \pm 0.01, 3.03 \pm 0.01$	$6.24^* \pm 0.00, 6.37^* \pm 0.00$
	2^-	$3.04 \pm 0.01, 3.11 \pm 0.01, 3.13 \pm 0.01$	$6.26 \pm 0.00, 6.39^* \pm 0.00$
	3^-	3.25 ± 0.01	6.42 ± 0.00
$X_{ud\bar{s}}, Z_{uu\bar{s}}, Z_{dd\bar{s}}$	0^-	$2.81 \pm 0.02, 2.90 \pm 0.02$	$6.25 \pm 0.01, 6.38 \pm 0.01$
	1^-	$2.88 \pm 0.02, 2.91 \pm 0.02, 2.98 \pm 0.02, 3.00 \pm 0.02$	$6.26 \pm 0.01, 6.27 \pm 0.01, 6.38 \pm 0.01, 6.42 \pm 0.01$
	2^-	$3.08 \pm 0.02, 3.11 \pm 0.02, 3.20 \pm 0.02$	$6.27 \pm 0.01, 6.39 \pm 0.01, 6.43 \pm 0.01$
	3^-	3.38 ± 0.02	6.45 ± 0.01
$Y_{\pi u}, Y_{\pi d}, Z_{uu\bar{d}}, Z_{dd\bar{u}}$	0^-	$2.65 \pm 0.01, 2.84 \pm 0.01$	$5.99 \pm 0.00, 6.21 \pm 0.00$
	1^-	$2.70 \pm 0.01, 2.72 \pm 0.01, 2.88 \pm 0.01, 2.86 \pm 0.01$	$6.00^* \pm 0.00, 6.22^* \pm 0.00$
	2^-	$2.80 \pm 0.01, 2.96 \pm 0.01, 2.98 \pm 0.01$	$6.02 \pm 0.00, 6.24^* \pm 0.00$
	3^-	3.10 ± 0.01	6.45 ± 0.00
$Y_{\eta u}, Y_{\eta d}$	0^-	$2.97 \pm 0.02, 3.06 \pm 0.02$	$6.30 \pm 0.01, 6.43 \pm 0.01$
	1^-	$3.02 \pm 0.02, 3.03 \pm 0.02, 3.09 \pm 0.02, 3.10 \pm 0.02$	$6.32^* \pm 0.01, 6.44^* \pm 0.01$
	2^-	$3.11 \pm 0.02, 3.18 \pm 0.02, 3.19 \pm 0.02$	$6.33 \pm 0.01, 6.46^* \pm 0.01$
	3^-	3.32 ± 0.02	6.49 ± 0.01
$Y_{\eta s}$	0^-	$3.20 \pm 0.02, 3.25 \pm 0.02$	$6.52 \pm 0.01, 6.60 \pm 0.01$
	1^-	$3.24^* \pm 0.02, 3.37 \pm 0.02, 3.38 \pm 0.02$	$6.53 \pm 0.01, 6.54 \pm 0.01, 6.62 \pm 0.01, 6.65 \pm 0.01$
	2^-	$3.32 \pm 0.02, 3.37 \pm 0.02, 3.38 \pm 0.02$	$6.54 \pm 0.01, 6.63 \pm 0.01, 6.66 \pm 0.01$
	3^-	3.50 ± 0.02	6.68 ± 0.01
$Z_{ss\bar{u}}, Z_{ss\bar{d}}$	0^-	$3.13 \pm 0.01, 3.16 \pm 0.01$	$6.46 \pm 0.00, 6.54 \pm 0.00$
	1^-	$3.17 \pm 0.01, 3.19 \pm 0.01, 3.21 \pm 0.01$	$6.47^* \pm 0.00, 6.55 \pm 0.00$
	2^-	$3.27 \pm 0.01, 3.29 \pm 0.01, 3.31 \pm 0.01$	$6.49 \pm 0.00, 6.57^* \pm 0.00$
	3^-	3.43 ± 0.01	6.60 ± 0.00

$$\begin{aligned}
 \Gamma_{X_1} &\approx \Gamma(X_{ud\bar{s}\bar{c}}(1^-) \rightarrow D^- K^+) \\
 &\quad + \Gamma(X_{ud\bar{s}\bar{c}}(1^-) \rightarrow D^0 K^0) \\
 &\approx 2\Gamma(X_{ud\bar{s}\bar{c}}(1^-) \rightarrow D^- K^+). \tag{21}
 \end{aligned}$$

Using the LHCb measurement $m_{X_1} = 2.904 \text{ GeV}$ and $\Gamma_{X_1} = 110 \text{ MeV}$, one can extract the dimensionless coupling $\beta_c \approx 0.30$. From the above calculation, one can find that $\beta_c \approx \beta'_c$.

In the following, we will give some relations of the decay widths of the new decay channels of $X_{0,1}$ and their counterparts.

From the flavor SU(3) amplitudes in Table 3, we have

$$\begin{aligned}
 \Gamma(X'_{ud\bar{s}} \rightarrow D^- K^+) &= \Gamma(X'_{ud\bar{s}} \rightarrow D^0 K^0) \\
 &= \Gamma(X'_{ds\bar{u}} \rightarrow D^- K^-) = \Gamma(X'_{ds\bar{u}} \rightarrow D_s^- \pi^-) \\
 &= \Gamma(X'_{su\bar{d}} \rightarrow D^0 \bar{K}^0) = \Gamma(X'_{su\bar{d}} \rightarrow D_s^- \pi^+). \tag{22}
 \end{aligned}$$

Table 2 Predictions of the masses (GeV) of radially excited $X_{c(b)}(2S)$ tetraquarks in both $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations. The uncertainty is from both the quark masses and slope parameter in Regge trajectories. The ground states of $X_c(1S)$ tetraquarks have been predicted in Ref. [22]

$X_{c(b)}(2S)$ states	J^P	Mass (X_c)	Mass (X_b)
$X'_{ds\bar{u}}, X'_{sud}, Y'_{(u\bar{u},d\bar{d})s}$	0^+	2.93 ± 0.02	6.27 ± 0.02
	1^+	2.97 ± 0.02	6.28 ± 0.02
	0^+	$2.866 \pm 0.007 \pm 0.002^a$	6.18 ± 0.03
$X'_{ud\bar{s}}$	0^+	2.91 ± 0.03	6.22 ± 0.03
	1^+	2.90 ± 0.03	6.32 ± 0.03
$Y'_{(u\bar{u},s\bar{s})d}, Y'_{(d\bar{d},s\bar{s})u}$	0^+	2.90 ± 0.03	6.32 ± 0.03
	1^+	2.94 ± 0.03	6.34 ± 0.03
	0^+	2.96 ± 0.02	6.30 ± 0.02
$X_{ds\bar{u}}, X_{sud}, Y_{\pi s}$	0^+	2.96 ± 0.02	6.30 ± 0.02
	1^+	$2.99 \pm 0.02, 3.07 \pm 0.02$	$6.31 \pm 0.02, 6.43 \pm 0.02$
	2^+	3.13 ± 0.02	6.45 ± 0.02
$X_{ud\bar{s}}, Z_{uu\bar{s}}, Z_{dd\bar{s}}$	0^+	2.97 ± 0.03	6.32 ± 0.03
	1^+	$3.00 \pm 0.03, 3.08 \pm 0.03$	$6.31 \pm 0.03, 6.43 \pm 0.03$
	2^+	3.14 ± 0.03	6.47 ± 0.03
$Y_{\pi u}, Y_{\pi d}, Z_{uu\bar{d}}, Z_{dd\bar{u}}$	0^+	2.77 ± 0.02	6.08 ± 0.02
	1^+	$2.79 \pm 0.02, 2.94 \pm 0.02$	$6.09 \pm 0.02, 6.29 \pm 0.02$
	2^+	3.01 ± 0.02	6.32 ± 0.02
$Y_{\eta u}, Y_{\eta d}$	0^+	3.02 ± 0.03	6.37 ± 0.03
	1^+	$3.05 \pm 0.03, 3.12 \pm 0.03$	$6.38 \pm 0.03, 6.49 \pm 0.03$
	2^+	3.19 ± 0.03	6.56 ± 0.03
$Y_{\eta s}$	0^+	3.20 ± 0.03	6.57 ± 0.03
	1^+	$3.23 \pm 0.03, 3.27 \pm 0.03$	$6.56 \pm 0.03, 6.64 \pm 0.03$
	2^+	3.34 ± 0.03	6.69 ± 0.03
$Z_{ss\bar{u}}, Z_{ss\bar{d}}$	0^+	3.16 ± 0.02	6.52 ± 0.02
	1^+	$3.19 \pm 0.02, 3.22 \pm 0.02$	$6.53 \pm 0.02, 6.60 \pm 0.02$
	2^+	3.29 ± 0.02	6.61 ± 0.02

^aWe take the mass of $X_0(2900)$ as an input parameter and the statistical and systematic errors will be combined for simplicity

Thus we can estimate the following decay widths for the open charm tetraquarks in $\bar{\mathbf{6}}$ representation

$$\Gamma_{X'_{ds\bar{u}c}} = \Gamma_{X'_{sud\bar{c}}} \approx 57 \text{ MeV.} \tag{23}$$

From the flavor SU(3) amplitudes in Table 4, we have

$$\begin{aligned} \Gamma(X_{ud\bar{s}} \rightarrow D^- K^+) &= \Gamma(X_{ud\bar{s}} \rightarrow D^0 K^0) \\ &= \Gamma(X_{sud} \rightarrow D_s^- \pi^+) \\ &= \Gamma(X_{ds\bar{u}} \rightarrow D_s^- \pi^-) = \Gamma(X_{ds\bar{u}} \rightarrow D^- K^-) \\ &= \Gamma(Y_{\pi s} \rightarrow D_s^- \pi^0) = \Gamma(Y_{\eta s} \rightarrow D_s^- \eta). \end{aligned} \tag{24}$$

$$\begin{aligned} 2\Gamma(X_{ud\bar{s}} \rightarrow D^- K^+) &= \Gamma(Z_{uu\bar{d}} \rightarrow D^0 \pi^+) \\ &= \Gamma(Z_{uu\bar{s}} \rightarrow D^0 K^+) = \Gamma(Z_{ss\bar{d}} \rightarrow D_s^- \bar{K}^0) \\ &= \Gamma(Z_{dd\bar{s}} \rightarrow D^- K^0) = \Gamma(Z_{dd\bar{u}} \rightarrow D^- \pi^-) \\ &= \Gamma(Z_{ss\bar{u}} \rightarrow D_s^- K^-). \end{aligned} \tag{25}$$

Thus we can estimate the following decay widths for the open charm tetraquarks in $\mathbf{15}$ representation

$$\begin{aligned} \Gamma_{Y_{\pi s\bar{c}}} &= \Gamma_{Y_{\eta s\bar{c}}} \approx 55 \text{ MeV,} \\ \Gamma_{X_{su\bar{d}}} &= \Gamma_{X_{ds\bar{u}}} = \Gamma_{Z_{uu\bar{d}}} = \Gamma_{Z_{uu\bar{s}}} = \Gamma_{Z_{dd\bar{s}}} \\ &= \Gamma_{Z_{dd\bar{u}}} = \Gamma_{Z_{ss\bar{u}}} = \Gamma_{Z_{ss\bar{d}}} \approx 110 \text{ MeV.} \end{aligned} \tag{26}$$

Both $X_0(2900)$ (as a 2^1S_0 $X'_{ud\bar{s}\bar{c}}$ state in the $\bar{\mathbf{6}}$ representation with $J^P = 0^+$) and $X_1(2900)$ (as a 1^3P_1 $X_{ud\bar{s}\bar{c}}$ state in the $\mathbf{15}$ representation with $J^P = 1^-$) can directly decay into $D^- K^+$. In principle, the S wave decay width is larger than the P wave decay width. At this stage, it remains puzzling that the $X_0(2900)$ has half of decay width of $X_1(2900)$. A plausible interpretation is that one of the two states may get mixed with other components, but a more conclusive result can be derived with more data on the decay patterns and their partners. We hope to have a more comprehensive analysis when more data is available.

As a straightforward extension, one can also investigate the X_b tetraquark decays. We explicitly give predictions of the masses and decay widths for $X_{b;0}$ and $X_{b;1}$, which are

Table 3 Decay amplitudes of $X_c \rightarrow D_i P$ for $\bar{6}$ representation tetraquarks containing $X_0(2900)$. The results can be easily applied to $X_b \rightarrow B_i P$ by $D^0 \rightarrow B^+, D^- \rightarrow B^0, D_s^- \rightarrow B_s^0, X_c \rightarrow X_b$, and $\beta' \rightarrow \alpha'$

Channel	Amplitude
$Y'_{(u\bar{u},d\bar{d})s} \rightarrow D^0 K^-$	$\frac{1}{2}\beta'$
$X'_{s\bar{u}d} \rightarrow D^0 \bar{K}^0$	$\frac{\beta'}{\sqrt{2}}$
$X'_{ud\bar{s}} \rightarrow D^0 K^0$	$\frac{\beta'}{\sqrt{2}}$
$Y'_{(d\bar{d},s\bar{s})u} \rightarrow D^0 \pi^0$	$-\frac{\beta'}{2\sqrt{2}}$
$Y'_{(d\bar{d},s\bar{s})u} \rightarrow D^0 \eta$	$-\frac{\beta'}{2\sqrt{6}}$
$Y'_{(u\bar{u},s\bar{s})d} \rightarrow \pi^- D^0$	$\frac{1}{2}\beta'$
$X'_{ds\bar{u}} \rightarrow K^- D^-$	$\frac{\beta'}{\sqrt{2}}$
$X'_{ud\bar{s}} \rightarrow D^- K^+$	$-\frac{\beta'}{\sqrt{2}}$
$Y'_{(d\bar{d},s\bar{s})u} \rightarrow D_s^- K^+$	$-\frac{1}{2}\beta'$
$Y'_{(u\bar{u},d\bar{d})s} \rightarrow D^- \bar{K}^0$	$\frac{1}{2}\beta'$
$Y'_{(u\bar{u},s\bar{s})d} \rightarrow D_s^- K^0$	$-\frac{1}{2}\beta'$
$Y'_{(u\bar{u},s\bar{s})d} \rightarrow D^- \eta$	$-\frac{1}{2}\sqrt{\frac{3}{2}}\beta'$
$Y'_{(d\bar{d},s\bar{s})u} \rightarrow D^- \pi^+$	$-\frac{1}{2}\beta'$
$Y'_{(u\bar{u},s\bar{s})d} \rightarrow D^- \pi^0$	$-\frac{\beta'}{2\sqrt{2}}$
$X'_{ds\bar{u}} \rightarrow D_s^- \pi^-$	$-\frac{\beta'}{\sqrt{2}}$
$X'_{s\bar{u}d} \rightarrow D_s^- \pi^+$	$-\frac{\beta'}{\sqrt{2}}$
$Y'_{(u\bar{u}-d\bar{d})s} \rightarrow D_s^- \eta$	$-\frac{\beta'}{\sqrt{6}}$

the partner of $X_0(2900)$ and $X_1(2900)$. As discussed before, the $X'_{ud\bar{s}\bar{c}}$ state with 0^+ and mass 2.86 GeV can be used to explain $X_0(2900)$ while the $X_{ud\bar{s}\bar{c}}$ state with 1^- and mass 2.91 GeV can be used to explain $X_1(2900)$. So one can obtain the masses of $X_{b;0}$ and $X_{b;1}$ with the $\bar{c} \rightarrow \bar{b}$ replacement from Tables 1 and 2. We have

$$m_{X_{b;0}} = 6.20 \text{ GeV}, \quad m_{X_{b;1}} = 6.27 \text{ GeV}. \tag{28}$$

Using the formulae in Eqs. (17) and (20), and the $\bar{c} \rightarrow \bar{b}$ replacement, we have $X_{b;0,1} \rightarrow B^0 K^+$ and $X_{b;0,1} \rightarrow B^+ \bar{K}^0$. Then we can estimate their decay widths

$$\Gamma_{X_{b;0}} \approx 64 \left(\frac{\beta'_c}{\alpha'_b}\right)^2 \text{ MeV}, \quad \Gamma_{X_{b;1}} \approx 131 \left(\frac{\beta_c}{\alpha_b}\right)^2 \text{ MeV}, \tag{29}$$

where $\frac{\beta'_c}{\alpha'_b} \approx \frac{\beta_c}{\alpha_b} \sim \mathcal{O}(1)$. We hope these two detectable $X_{0,1}$ partner states can be examined in $X_{b;0,1} \rightarrow B^0 K^+$ and $X_{b;0,1} \rightarrow B^+ \bar{K}^0$ by experiments.

Table 4 Decay amplitudes of $X_c \rightarrow D_i P$ for **15** representation tetraquarks containing $X_1(2900)$. The results can be easily applied to $X_b \rightarrow B_i P$ by $D^0 \rightarrow B^+, D^- \rightarrow B^0, D_s^- \rightarrow B_s^0, X_c \rightarrow X_b$, and $\beta \rightarrow \alpha$

Channel	Amplitude	Channel	Amplitude
$X_{ud\bar{s}} \rightarrow D^0 K^0$	$\frac{\beta}{\sqrt{2}}$	$X_{s\bar{u}d} \rightarrow D^0 \bar{K}^0$	$\frac{\beta}{\sqrt{2}}$
$Z_{uu\bar{d}} \rightarrow D^0 \pi^+$	β	$Z_{uu\bar{s}} \rightarrow D^0 K^+$	β
$X_{ud\bar{s}} \rightarrow K^+ D^-$	$\frac{\beta}{\sqrt{2}}$	$X_{ds\bar{u}} \rightarrow K^- D^-$	$\frac{\beta}{\sqrt{2}}$
$Y_{\eta d} \rightarrow D_s^- K^0$	$-\frac{\beta}{\sqrt{3}}$	$Z_{ss\bar{d}} \rightarrow D_s^- \bar{K}^0$	β
$Z_{dd\bar{s}} \rightarrow D^- K^0$	β	$X_{ds\bar{u}} \rightarrow D_s^- \pi^-$	$\frac{\beta}{\sqrt{2}}$
$X_{s\bar{u}d} \rightarrow D_s^- \pi^+$	$\frac{\beta}{\sqrt{2}}$	$Z_{dd\bar{u}} \rightarrow D^- \pi^-$	β
$Z_{ss\bar{u}} \rightarrow K^- D_s^-$	β	$Y_{\eta s} \rightarrow D^- \bar{K}^0$	$\frac{\beta}{2\sqrt{3}}$
$Y_{\pi s} \rightarrow D^- \bar{K}^0$	$-\frac{1}{2}\beta$	$Y_{\eta d} \rightarrow \pi^- D^0$	$\frac{\beta}{2\sqrt{3}}$
$Y_{\pi d} \rightarrow D^0 \pi^-$	$\frac{1}{2}\beta$	$Y_{\eta s} \rightarrow D^0 K^-$	$\frac{\beta}{2\sqrt{3}}$
$Y_{\pi s} \rightarrow D^0 K^-$	$\frac{1}{2}\beta$	$Y_{\eta u} \rightarrow D^0 \pi^0$	$-\frac{\beta}{2\sqrt{6}}$
$Y_{\eta u} \rightarrow D^0 \pi^0$	$\frac{\beta}{2\sqrt{3}}$	$Y_{\pi u} \rightarrow D^0 \pi^0$	$\frac{\beta}{2\sqrt{2}}$
$Y_{\pi u} \rightarrow D^0 \pi^0$	$\frac{1}{2}\beta$	$Y_{\pi u} \rightarrow D^0 \eta$	$-\frac{\beta}{2\sqrt{6}}$
$Y_{\pi u} \rightarrow D^0 \eta$	$\frac{\beta}{2\sqrt{3}}$	$Y_{\eta u} \rightarrow D^0 \eta$	$\frac{5\beta}{6\sqrt{2}}$
$Y_{\eta u} \rightarrow D^0 \eta$	$\frac{1}{6}\beta$	$Y_{\eta d} \rightarrow \eta D^-$	$\frac{5\beta}{6\sqrt{2}}$
$Y_{\eta d} \rightarrow \eta D^-$	$\frac{1}{6}\beta$	$Y_{\pi d} \rightarrow \eta D^-$	$\frac{\beta}{2\sqrt{6}}$
$Y_{\pi d} \rightarrow \eta D^-$	$-\frac{\beta}{2\sqrt{3}}$	$Y_{\eta d} \rightarrow \pi^0 D^-$	$\frac{\beta}{2\sqrt{6}}$
$Y_{\eta d} \rightarrow \pi^0 D^-$	$-\frac{\beta}{2\sqrt{3}}$	$Y_{\pi d} \rightarrow \pi^0 D^-$	$\frac{\beta}{2\sqrt{2}}$
$Y_{\pi d} \rightarrow \pi^0 D^-$	$\frac{1}{2}\beta$	$Y_{\eta u} \rightarrow K^+ D_s^-$	$-\frac{\beta}{\sqrt{3}}$
$Y_{\pi s} \rightarrow \pi^0 D_s^-$	$\frac{\beta}{\sqrt{2}}$	$Y_{\eta s} \rightarrow \eta D_s^-$	$\frac{\beta}{\sqrt{2}}$
$Y_{\eta u} \rightarrow D^- \pi^+$	$\frac{\beta}{2\sqrt{3}}$	$Y_{\pi u} \rightarrow D^- \pi^+$	$-\frac{1}{2}\beta$

4 Conclusion

In this paper, we have studied the spectra and the decay properties of open-charm tetraquarks X_c and open-bottom tetraquarks X_b . The newly $X_{0,1}(2900)$ observed by the LHCb collaboration can be interpreted as a radial excited tetraquark X_c composed of $[ud\bar{s}\bar{c}]$ with $J^P = 0^+$ and an orbitally excited tetraquark with $J^P = 1^-$, respectively. Using the flavor SU(3) symmetry, we made a detailed classification of all open charm tetraquarks, and then explored the mass and decays of the other flavor-open tetraquarks made of $su\bar{d}\bar{c}$ and $ds\bar{u}\bar{c}$. We pointed that these two states can be found through the decays: $X_{ds\bar{u}\bar{c}}^{(0)} \rightarrow (D^- K^-, D_s^- \pi^-)$, and $X_{su\bar{d}\bar{c}}^{(0)} \rightarrow D_s^- \pi^+$. We also applied our analysis to open bottom tetraquark X_b and predict their masses. The open-flavored X_b can be discovered through the following decays: $X_{ud\bar{s}\bar{b}} \rightarrow B^0 K^+, X_{ds\bar{u}\bar{b}}^{(0)} \rightarrow (B^0 K^-, B_s^0 \pi^-)$, and

$X_{s\bar{d}\bar{b}}^{(\prime)} \rightarrow B_s^0 \pi^+$. We hope that these theoretical proposals can be carried out in future experimental studies.

Note Added —When this manuscript is being prepared, a preprint [49] appears, in which the authors also explained these two X_c states. After we finished this manuscript, it was pointed out to us that a $D^* K^*$ bound state was predicted in Ref. [50].

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