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Full two-loop QCD corrections to the Higgs mass in the MSSM with heavy superpartners

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Abstract We improve the determination of the Higgs-boson mass in the MSSM with heavy superpartners, by computing the two-loop threshold corrections to the quartic Higgs coupling that involve both the strong and the electroweak gauge couplings. Combined with earlier results, this completes the calculation of the two-loop QCD corrections to the quartic coupling at the SUSY scale. We also compare different computations of the relation between the quartic coupling and the pole mass of the Higgs boson at the EW scale. We find that the numerical impact of the new corrections on the prediction for the Higgs mass is modest, but comparable to the accuracy of the Higgs-mass measurement at the LHC.

1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is one of the best-motivated extensions of the Standard Model (SM), and probably the most studied. The Higgs sector of the MSSM consists of two $SU(2)$ doublets, but the model allows for a so-called "decoupling limit" in which a combination of the two doublets has SM-like couplings to matter fermions and gauge bosons – so that its neutral scalar component h can be identified with the Higgs boson discovered at the LHC [1,2], which itself is broadly SM-like [3] – while the orthogonal combination of doublets is much heavier. An important aspect of the MSSM is the existence of relations between the quartic Higgs couplings and the electroweak (EW) gauge couplings. In the decoupling limit, these relations induce a tree-level prediction $(m_h^2)^{\rm tree} \approx m_Z^2 \cos^2 2\beta$

for the squared mass of the SM-like scalar, where m_Z is the Z-boson mass, and the angle β is related to the ratio of the vacuum expectation values (vevs) of the two Higgs doublets by $\tan \beta = v_2/v_1$, and determines their admixture into h. Consequently, in the MSSM, the tree-level contribution can only make up for at most half of the squared mass of the observed Higgs boson, $(m_h^2)^{\rm obs} \approx (125 \ {\rm GeV})^2$ [4]. The rest must arise from radiative corrections. It has been known since the early 1990s [5–10] that the most relevant corrections to the Higgs mass are those controlled by the top Yukawa coupling, $g_t \sim \mathcal{O}(1)$, which involve the top quark and its superpartners, the stop squarks. These corrections are enhanced by logarithms of the ratio between stop and top masses, and also show a significant dependence on the value of the left–right stop mixing parameter X_t. In particular, for values of $\tan \beta$ large enough to saturate the tree-level prediction, a Higgs mass around 125 GeV can be obtained with an average stop mass M_S of about 1–2 TeV when $X_t/M_S \approx 2$, whereas for vanishing X_t the stops need to be heavier than 10 TeV.

Over the years, the crucial role of the radiative corrections stimulated a wide effort to compute them with the highest possible precision, in order to keep the theoretical uncertainty of the Higgs-mass prediction under control. By now, that computation is indeed quite advanced: full one-loop corrections [11–16] and two-loop corrections in the limit of vanishing external momentum [17–30] are available, and the dominant momentum-dependent two-loop corrections [31–35] as well as the dominant three-loop corrections [36–42] have also been obtained.[1] However, when the SUSY scale M_S

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[1] We focus here on the MSSM with real parameters. Significant efforts have also been devoted to the Higgs-mass calculation in the presence of CP-violating phases [43–53], as well as in non-minimal SUSY extensions of the SM.

is significantly larger than the EW scale (which we can identify, e.g., with the top mass m_t), any fixed-order computation of m_h may become inadequate, because radiative corrections of order n in the loop expansion contain terms enhanced by as much as $\ln^n(M_S/m_t)$. In the presence of a significant hierarchy between the scales, the computation of the Higgs mass needs to be reorganized in an effective field theory (EFT) approach: the heavy particles are integrated out at the scale M_S, where they only affect the matching conditions for the couplings of the EFT valid below M_S; the appropriate renormalization group equations (RGEs) are then used to evolve those couplings between the SUSY scale and the EW scale, where the running couplings are related to physical observables such as the Higgs-boson mass and the masses of fermions and gauge bosons. In this approach, the computation is free of large logarithmic terms both at the SUSY scale and at the EW scale, while the effect of those terms is accounted for to all orders in the loop expansion by the evolution of the couplings between the two scales. More precisely, large corrections can be resummed to the (next-to)^n-leading-logarithmic (N^nLL) order by means of n-loop calculations at the SUSY and EW scales combined with $(n+1)$-loop RGEs.

The EFT approach to the computation of the MSSM Higgs mass dates back to the early 1990s [54–56], and it has also been exploited in the past [57–63] to determine analytically the coefficients of the logarithmic terms in the Higgs-mass corrections, by solving perturbatively the appropriate systems of boundary conditions and RGEs. In recent years, after the LHC results pushed the expectations for the SUSY scale into the TeV range, the realization that an accurate prediction for the Higgs mass in the MSSM cannot prescind from the resummation of the large logarithmic corrections brought the EFT computation under renewed focus [64–79]. In the simplest scenario in which all of the SUSY particles as well as the heavy Higgs doublet of the MSSM are clustered around a single scale M_S, so that the EFT valid below that scale is just the SM, the state of the art now includes: full one-loop and partial two-loop matching conditions for the quartic Higgs coupling at the SUSY scale, computed for arbitrary values of the relevant SUSY parameters [66,73]; full three-loop RGEs for all of the parameters of the SM Lagrangian [80–85]; full two-loop relations at the EW scale between the running SM parameters and a set of physical observables which include the pole Higgs mass [86–88]. The combination of these results allows for a full NLL resummation of the large logarithmic corrections to the Higgs mass, whereas the NNLL resummation can only be considered partial, because in refs. [66,73] the two-loop matching conditions for the quartic Higgs coupling were

computed in the "gaugeless limit" of vanishing EW gauge couplings.[2]

Beyond the pure EFT calculation of the Higgs mass, different "hybrid" approaches to combine the existing "diagrammatic" (i.e., fixed-order) calculations with a resummation of the logarithmic corrections have been proposed [64,70–72]. The aim is to include terms suppressed by powers of v^2/M_S^2 (where we denote by v the vev of a SM-like Higgs scalar) up to the perturbative order accounted for by the diagrammatic calculation. In the EFT calculation, those terms can be mapped to the effect of non-renormalizable, higher-dimensional operators, and they are neglected when the theory valid below the matching scale is taken to be the plain SM in the unbroken phase of the EW symmetry. To avoid double counting, the hybrid approaches require a careful subtraction of the terms that are accounted for by both the diagrammatic and the EFT calculations, and indeed a few successive adjustments [74,75,79] were necessary to obtain predictions for m_h that, in the limit of very heavy SUSY masses in which the $\mathcal{O}(v^2/M_S^2)$ terms are certainly negligible, show the expected agreement with the pure EFT calculation. The comparison between the predictions of the hybrid and pure EFT calculations, as well as a direct study [73] of the effects of non-renormalizable operators in the EFT, also show that the $\mathcal{O}(v^2/M_S^2)$ corrections are significantly suppressed for the values of M_S that are large enough to allow for $m_h \approx 125$ GeV. Other recent developments of the EFT approach include: the study of MSSM scenarios in which both Higgs doublets are light, so that the effective theory valid below the SUSY scale is a two-Higgs-doublet model (THDM) [68,69,77]; the application of functional techniques to the full one-loop matching of the MSSM onto the SM [92]; the calculation of one-loop matching conditions between the couplings of two generic renormalizable theories [93,94], which can then be adapted to SUSY (or non-SUSY) models other than the MSSM.

In this paper we focus again on the simplest EFT setup in which the theory valid below the SUSY scale is the SM, and we take a further step towards the full NNLL resummation of the large logarithmic corrections. In particular, we compute the two-loop threshold corrections to the quartic Higgs coupling that involve both the strong and the EW gauge couplings. Combined with the "gaugeless" results of Refs. [66,73], this completes the calculation of the two-loop threshold corrections that involve the strong gauge coupling. We also discuss the necessary inclusion of contributions beyond the gaugeless limit in the relation between the pole Higgs mass and the $\overline{\text{MS}}$-renormalized quartic Higgs cou-

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[2] A partial N^3LL resummation of the corrections involving only the highest powers of the strong gauge coupling is also available, combining the three-loop matching condition of Ref. [78] with SM results from refs. [85,89–91].

pling at the EW scale, and we compare the results of the full two-loop calculations of that relation given in Refs. [86,87], respectively. Finally, considering a representative scenario for the MSSM with heavy superpartners, we find that the numerical impact of the new corrections on the prediction for the Higgs mass is modest, but comparable to the accuracy of the Higgs-mass measurement at the LHC.

2 Two-loop matching of the quartic Higgs coupling

In this section we describe our calculation of the two-loop QCD contributions to the matching condition for the quartic Higgs coupling. We consider the setup in which all SUSY particles as well as a linear combination of the two Higgs doublets of the MSSM are integrated out at a common renormalization scale $Q \approx M_S$, so that the EFT valid below the matching scale is the SM. In our conventions the potential for the SM-like Higgs doublet H contains the quartic interaction term $\frac{\lambda}{2}|H|^4$, and the tree-level squared mass of its neutral scalar component is $(m_h^2)^{\text{tree}} = 2\lambda v^2$, with $v = \langle H^0 \rangle \approx 174$ GeV. Then the two-loop matching condition for the quartic coupling takes the form

$$\lambda(Q) = \frac{1}{4}\left[g^2(Q) + g'^2(Q)\right]c_{2\beta}^2(Q) + \Delta\lambda^{1\ell} + \Delta\lambda^{2\ell}, \quad (1)$$

where g and g' are the EW gauge couplings, β can be interpreted as the angle[3] that rotates the two original MSSM doublets into a light doublet H and a massive doublet A, and $\Delta\lambda^{n\ell}$ is the n-loop threshold correction to the quartic coupling arising from integrating out the heavy particles at the scale Q. The complete result for the one-loop correction $\Delta\lambda^{1\ell}$, valid for arbitrary values of all the relevant SUSY parameters, can be found in Refs. [66,73]. It is computed under the assumptions that λ, g and g' in Eq. (1) are $\overline{\text{MS}}$-renormalized parameters of the SM, and that β is defined beyond tree level as described in section 2.2 of Ref. [66], removing entirely the contributions of the off-diagonal wave-function renormalization (WFR) of the Higgs doublets. As to the two-loop correction $\Delta\lambda^{2\ell}$, Ref. [66] provided the contributions of $\mathcal{O}(g_t^4 g_s^2)$, where g_s is the strong gauge coupling, for arbitrary values of all the relevant SUSY parameters; Ref. [73] provided in addition the two-loop contributions involving only the third-family Yukawa couplings, g_t, g_b and g_τ, again for arbitrary SUSY parameters, and also discussed some subtleties in the derivation of the $\mathcal{O}(g_b^4 g_s^2)$ contributions from the known results for the $\mathcal{O}(g_t^4 g_s^2)$ ones. Altogether, the results of Refs. [66,73] amounted to a complete determination of $\Delta\lambda^{2\ell}$ in the limit of vanishing EW gauge (and first-two-generation Yukawa) couplings. In this paper

we take a step beyond this "gaugeless limit" and compute the remaining corrections that involve the strong gauge coupling, namely those of $\mathcal{O}(g_{t,b}^2 g^2 g_s^2)$, those of $\mathcal{O}(g_{t,b}^2 g'^2 g_s^2)$ and those involving only gauge couplings, i.e. of $\mathcal{O}(g^4 g_s^2)$ and $\mathcal{O}(g'^4 g_s^2)$.

We can decompose the "mixed" QCD–EW threshold correction to the quartic Higgs coupling into three terms:

$$\Delta\lambda^{2\ell,\,\text{QCD-EW}} = \Delta\lambda^{2\ell,\,\text{1PI}} + \Delta\lambda^{2\ell,\,\text{WFR}} + \Delta\lambda^{2\ell,\,\text{RS}}. \quad (2)$$

The first term on the r.h.s. of the equation above denotes the contributions of two-loop, one-particle-irreducible (1PI) diagrams with four external Higgs fields involving both strong-interaction vertices (namely squark–gluon, four-squark or quark–squark–gluino vertices) and D-term-induced quartic Higgs–squark vertices proportional to g^2 or to g'^2, and possibly also vertices controlled by the Yukawa couplings. This term can be computed with a relatively straightforward adaptation of the effective-potential approach developed in Refs. [66,73] for the corresponding calculation in the gaugeless limit. In contrast, the remaining terms on the r.h.s. of Eq. (2) require different approaches. The second term involves the two-loop, $\mathcal{O}(g_{t,b}^2 g_s^2)$ squark contributions to the WFR of the Higgs field, which multiply the tree-level quartic coupling, see Eq. (1), giving rise to $\mathcal{O}(g_{t,b}^2 g^2 g_s^2)$ and $\mathcal{O}(g_{t,b}^2 g'^2 g_s^2)$ corrections. This term requires a computation of the external-momentum dependence of the two-loop self-energy of the Higgs boson, analogous to the one performed in Refs. [31–35,53] for the "diagrammatic" case. Finally, the last term on the r.h.s. of Eq. (2) arises from the fact that, while our calculation of the matching condition for the quartic Higgs coupling λ is performed in the $\overline{\text{DR}}$ renormalization scheme assuming the field content of the MSSM, in the EFT valid below the SUSY scale – i.e., the SM – λ is interpreted as an $\overline{\text{MS}}$-renormalized quantity. Moreover, we find it convenient to use $\overline{\text{MS}}$-renormalized parameters of the SM also for the EW gauge couplings entering the tree-level part of the matching condition, see Eq. (1), and for the top Yukawa coupling entering the one-loop part (but not for the bottom Yukawa coupling, as discussed in Ref. [73]). The corrections arising from the change of renormalization scheme for λ can in turn be extracted from the two-loop self-energy diagrams for the Higgs boson, those arising from the change of scheme and model for the EW gauge couplings require a computation of the external-momentum dependence of the two-loop self-energy of the Z boson, while those arising from the change of scheme and model for the top Yukawa coupling are easier to obtain, being just the product of one-loop terms. In the rest of this section we will describe in more detail our computation of each of the three terms on the r.h.s. of Eq. (2).

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[3] Here and thereafter, we use the shortcuts $c_\phi \equiv \cos\phi$ and $s_\phi \equiv \sin\phi$ for a generic angle ϕ.

2.1 1PI contributions

The 1PI, two-loop contribution to the matching condition for the quartic Higgs coupling can be expressed as

$$\Delta\lambda^{2\ell,\text{1PI}} = \frac{1}{2}\left.\frac{\partial^4 \Delta V^{2\ell,\tilde{q}}}{\partial^2 H^\dagger \partial^2 H}\right|_{H=0}, \tag{3}$$

where $\Delta V^{2\ell,\tilde{q}}$ denotes the contribution to the MSSM scalar potential from two-loop diagrams involving the strong gauge interactions of the squarks, and the derivatives are computed at $H = 0$ because we perform the matching between the MSSM and the SM in the unbroken phase of the EW symmetry. Since the strong interactions do not mix different types of squarks, we will now describe the derivation of the stop contribution to $\Delta\lambda^{2\ell,\text{1PI}}$, and later describe how to translate it into the sbottom contribution and into the contributions of the squarks of the first two generations.

It is convenient to start from the well-known expression for the stop contribution of $\mathcal{O}(g_s^2)$ to the MSSM scalar potential in the broken phase of the EW symmetry [20],

$$\Delta V^{2\ell,\tilde{t}} = \kappa^2 g_s^2 C_F N_c \Big\{ 2 m_{\tilde{t}_1}^2 I(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2, 0)$$
$$+ 2 L(m_{\tilde{t}_1}^2, m_{\tilde{g}}^2, m_t^2) - 4 m_t m_{\tilde{g}} s_{2\theta_t} I(m_{\tilde{t}_1}^2, m_{\tilde{g}}^2, m_t^2)$$
$$+ \left(1 - \frac{s_{2\theta_t}^2}{2}\right) J(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) + \frac{s_{2\theta_t}^2}{2} J(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$
$$+ \left[m_{\tilde{t}_1} \leftrightarrow m_{\tilde{t}_2}, s_{2\theta_t} \rightarrow -s_{2\theta_t}\right] \Big\}, \tag{4}$$

where $\kappa = 1/(16\pi^2)$ is a loop factor, $C_F = 4/3$ and $N_c = 3$ are color factors, the loop integrals $I(x, y, z)$, $L(x, y, z)$ and $J(x, y)$ in Eq. (4) are defined, e.g., in appendix D of Ref. [95], $m_{\tilde{g}}$ stands for the gluino mass, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the two stop-mass eigenstates, and θ_t denotes the stop mixing angle. The latter is related to the top and stop masses and to the left–right stop mixing parameter by

$$s_{2\theta_t} = \frac{2 m_t X_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}. \tag{5}$$

We recall that $X_t = A_t - \mu \cot\beta$, where A_t is the trilinear soft SUSY-breaking Higgs-stop interaction term and μ is the Higgs-higgsino mass parameter in the superpotential (in fact, those two parameters enter our results only combined into X_t). To compute the fourth derivative of the effective potential entering Eq. (3) we express the stop masses and mixing angle as functions of a field-dependent top mass $m_t = \hat{g}_t |H|$, where by $\hat{g}_t$ we denote[4] a SM-like Yukawa coupling

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[4] We denote with a hat $\overline{\text{DR}}$-renormalized couplings of the MSSM, and without a hat $\overline{\text{MS}}$-renormalized couplings of the SM. However, in the two-loop part of the corrections the distinction between hatted and un-hatted couplings amounts to a higher-order effect, thus we will drop the hats there to reduce clutter.

related to its MSSM counterpart $\hat{y}_t$ by $\hat{g}_t = \hat{y}_t \sin\beta$, and of a field-dependent Z-boson mass $m_Z = \hat{g}_Z |H|$, where we define $\hat{g}_Z^2 = (\hat{g}^2 + \hat{g}'^2)/2$. We then obtain

$$\left.\frac{\partial^4 \Delta V^{2\ell,\tilde{t}}}{\partial^2 H^\dagger \partial^2 H}\right|_{H=0} = \Big[g_t^2 g_Z^2 \left(2 V_{tZ}^{(2)} + 12 m_t^2 V_{ttZ}^{(3)} + 4 m_t^4 V_{tttZ}^{(4)} \right.$$
$$\left. + 3 m_t^2 m_Z^2 V_{ttZZ}^{(4)} \right)$$
$$+ g_t^4 \left(2 V_{tt}^{(2)} + 4 m_t^2 V_{ttt}^{(3)} + m_t^4 V_{tttt}^{(4)} \right) \Big]_{m_t, m_Z \to 0}$$
$$+ \Big[t \longleftrightarrow Z \Big], \tag{6}$$

where the last term is obtained from the previous ones by swapping top and Z, and we used the shortcuts

$$V_{p_1\dots p_k}^{(k)} = \frac{d^k \Delta V^{2\ell,\tilde{t}}}{dm_{p_1}^2 \cdots dm_{p_k}^2}, \quad (p_i = t, Z). \tag{7}$$

The terms proportional to g_t^4 in the second line of Eq. (6) give rise to the $\mathcal{O}(g_t^4 g_s^2)$ contributions to $\Delta\lambda^{2\ell,\text{1PI}}$ already computed in Ref. [66], and we will not consider them further. We will focus instead on the mixed QCD–EW contributions to $\Delta\lambda^{2\ell,\text{1PI}}$ arising from the terms in Eq. (6) that are proportional to $g_t^2 g_Z^2$ and to g_Z^4. To obtain the derivatives with respect to m_t and m_Z of the stop masses and mixing entering $\Delta V^{2\ell,\tilde{t}}$ we exploit the relations

$$\frac{dm_{\tilde{t}_{1,2}}^2}{dm_t^2} = 1 \pm \frac{s_{2\theta_t} X_t}{2 m_t}, \quad \frac{ds_{2\theta_t}}{dm_t^2} = \frac{s_{2\theta_t} c_{2\theta_t}^2}{2 m_t^2}, \tag{8}$$

$$\frac{dm_{\tilde{t}_{1,2}}^2}{dm_Z^2} = \frac{c_{2\beta}}{2} \left[d_L^t + d_R^t \pm c_{2\theta_t} (d_L^t - d_R^t) \right],$$

$$\frac{ds_{2\theta_t}}{dm_Z^2} = -\frac{c_{2\beta}}{2} \left(d_L^t - d_R^t \right) \frac{s_{2\theta_t}^2 c_{2\theta_t}}{m_t X_t}, \tag{9}$$

where

$$d_L^t = \frac{1}{2} - \frac{2}{3} \sin^2\theta_W, \quad d_R^t = \frac{2}{3} \sin^2\theta_W, \tag{10}$$

θ_W being the Weinberg angle. After taking the required derivatives of $\Delta V^{2\ell,\tilde{t}}$ with respect to m_t^2 and m_Z^2, we use Eq. (5) to make the dependence of θ_t on m_t explicit; we expand the function $\Phi(m_{\tilde{t}_i}^2, m_{\tilde{g}}^2, m_t^2)$ entering the loop integrals (see appendix D of Ref. [95]) in powers of m_t^2; we take the limit $|H| \to 0$, leading to $m_t, m_Z \to 0$; and we identify $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ with the soft SUSY-breaking stop mass parameters m_{Q_3} and m_{U_3}. We remark that, differently from the $\mathcal{O}(g_t^4 g_s^2)$ contributions computed in Ref. [66] and the $\mathcal{O}(g_t^6)$ contributions computed in Ref. [73], the mixed QCD–EW contributions to $\Delta\lambda^{2\ell,\text{1PI}}$ computed here do not contain infrared divergences that need to be canceled out in the matching of the quartic Higgs coupling between MSSM and SM. Also, the terms proportional to $g_t^2 g_Z^2$ and to g_Z^4 in Eq. (6)

that involve more than two derivatives of the two-loop effective potential vanish directly when we take the limit $|H| \to 0$, thus the mixed QCD–EW contributions to $\Delta\lambda^{2\ell,1\text{PI}}$ can be related as usual to the corresponding two-loop corrections to the Higgs mass.

In order to obtain the contributions to $\Delta\lambda^{2\ell,1\text{PI}}$ from diagrams involving sbottoms, it is sufficient to perform the replacements $g_t \to g_b$, $X_t \to X_b$, $m_{U_3} \to m_{D_3}$ and $d_{L,R}^t \to d_{L,R}^b$ in the contributions from diagrams involving stops, with

$$d_L^b = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \quad d_R^b = -\frac{1}{3}\sin^2\theta_W. \tag{11}$$

Finally, we recall that our calculation neglects the Yukawa couplings of the first two generations. The contributions to $\Delta\lambda^{2\ell,1\text{PI}}$ from diagrams involving up-type (or down-type) squarks of the first two generations can be obtained by setting $g_t = 0$ (or $g_b = 0$) in the contributions from diagrams involving stops (or sbottoms), and replacing the soft SUSY-breaking stop (or sbottom) mass parameters with those of the appropriate generation.

The result for $\Delta\lambda^{2\ell,1\text{PI}}$ with full dependence on all of the input parameters is lengthy and not particularly illuminating, and we make it available upon request – together with all of the other corrections computed in this paper – in electronic form. We show here a simplified result valid in the limit in which all squark masses (m_{Q_i}, m_{U_i}, m_{D_i}, with $i = 1, 2, 3$) as well as the gluino mass $m_{\tilde{g}}$ are set equal to a common SUSY scale M_S. In units of $\kappa^2 g_s^2 C_F N_c$, we find

$$\Delta\lambda^{2\ell,1\text{PI}} = \frac{3}{4}\left(g^4 + \frac{11}{9}g'^4\right)c_{2\beta}^2 - \left(g^2 + g'^2\right)c_{2\beta}$$
$$\times\left[g_t^2\left(1 + \ln^2\frac{M_S^2}{Q^2} - 2\frac{X_t}{M_S}\ln\frac{M_S^2}{Q^2} - \frac{X_t^2}{M_S^2}\right) - (t \to b)\right], \tag{12}$$

where $(t \to b)$ denotes terms obtained from the previous ones within square brackets via the replacements $g_t \to g_b$ and $X_t \to X_b$. We remark in passing that there are no contributions of $\mathcal{O}(g^2 g'^2 g_s^2)$, because the corresponding diagrams involve the trace of the generators of the $SU(2)$ gauge group.

2.2 WFR contributions

Differently from the case of the "gaugeless" corrections computed in Refs. [66,73], where the quartic Higgs coupling can be considered vanishing at tree level, the mixed QCD–EW corrections include a contribution in which the tree-level coupling is combined with the two-loop WFR of the Higgs field. This reads

$$\Delta\lambda^{2\ell,\text{WFR}} = -\frac{1}{2}(g^2 + g'^2)c_{2\beta}\left(\frac{d\hat{\Pi}_{hh}^{2\ell,\tilde{q}}}{dp^2}\bigg|_{H=0} + \Delta_{\text{WFR}}\right), \tag{13}$$

where $\hat{\Pi}_{hh}^{2\ell,\tilde{q}}$ denotes the contribution to the renormalized Higgs-boson self-energy[5] from two-loop diagrams involving the strong gauge interactions of the squarks, and the derivative is taken with respect to the external momentum p^2. In this case the notation $H = 0$ means that, after having taken the derivative, we take the limit $v \to 0$. This implies $m_q, m_Z \to 0$ as well as $p^2 \to 0$ (because p^2 is ultimately set to $2\lambda v^2$). Finally, the shift Δ_{WFR} stems from the matching of the one-loop WFR, and will be discussed below.

The relevant two-loop self-energy diagrams are generated with FeynArts [96], using a modified version of the original MSSM model file [97] that implements the QCD interactions in the background field gauge. The color factors are simplified with a private package and the Dirac algebra is handled by TRACER [98]. In order to obtain a result valid in the limit $v \to 0$, we performed an asymptotic expansion of the self-energy in the heavy superparticle masses analogous to the one described in section 3 of Ref. [99]. As a useful cross-check of our result, we verified that it agrees with the one that can be obtained by taking appropriate limits in the explicit analytic formulae for the Higgs-boson self-energy given in Refs. [31–33].

Our result for the derivative of the two-loop self-energy with full dependence on all of the input parameters is in turn made available upon request. In the simplified scenario of degenerate superparticle masses we find, in units of $\kappa^2 g_s^2 C_F N_c$,

$$\frac{d\hat{\Pi}_{hh}^{2\ell,\tilde{q}}}{dp^2}\bigg|_{H=0} = g_t^2\left[\frac{13}{4} + 3\ln\frac{M_S^2}{Q^2} + \ln^2\frac{M_S^2}{Q^2}\right.$$
$$-\frac{2X_t}{3M_S}\left(4 + \ln\frac{M_S^2}{Q^2}\right) + \frac{7X_t^2}{6M_S^2}$$
$$\left.+ 2\left(\ln\frac{M_S^2}{Q^2} - \frac{X_t}{M_S}\right)\left(1 - \ln\frac{-p^2}{Q^2}\right)\right]$$
$$+(t \to b). \tag{14}$$

We remark that there are no contributions proportional to the EW gauge couplings. This is due to the fact that, in the limit of unbroken EW symmetry, the D-term-induced $\mathcal{O}(g^2)$ and $\mathcal{O}(g'^2)$ couplings of the Higgs boson to squarks enter only self-energy diagrams that do not depend on the external

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[5] For the self-energies of both the Higgs boson and the Z boson, we adopt in this paper the sign convention according to which $m_{\text{pole}}^2 = m_{\text{run}}^2 - \Pi(m^2)$. The WFR for the Higgs boson is then $Z_h = 1 - d\Pi_{hh}(p^2)/dp^2$. Note that this is the opposite of the sign convention adopted in Refs. [31–33,93].

momentum. We also remark that the derivative of the two-loop self-energy has logarithmic infra-red (IR) divergences in the limit $p^2 \to 0$, see the last term within square brackets in the third line of Eq. (14). These divergences cancel out in the matching of the Higgs-boson WFR between the MSSM and the SM. Indeed, in the limit $v \to 0$ the one-loop contribution of top and bottom quarks to the derivative of the self-energy, which is present both above and below the matching scale, reads

$$\frac{d\,\hat{\Pi}_{hh}^{1\ell,\,q}}{dp^2}\bigg|_{H=0} = \kappa\,N_c\left(g_t^2 + g_b^2\right)\left(1 - \ln\frac{-p^2}{Q^2}\right). \quad (15)$$

However, the top and bottom Yukawa couplings must be interpreted as the ones of the MSSM above the matching scale, and the ones of the SM below the matching scale. Thus, in the matching between the MSSM and the SM the derivative of the two-loop Higgs self-energy receives the shift

$$\Delta_{\mathrm{WFR}} = 2\,\kappa\,N_c\left(g_t^2\,\Delta g_t^{\tilde{t},\,g_s^2} + g_b^2\,\Delta g_b^{\tilde{b},\,g_s^2}\right)\left(1 - \ln\frac{-p^2}{Q^2}\right), \quad (16)$$

where $\Delta g_t^{\tilde{t},\,g_s^2}$ denotes the one-loop, $\mathcal{O}(g_s^2)$ contribution from diagrams involving stops to the difference between the MSSM coupling $\hat{g}_t$ and the SM coupling g_t,

$$\Delta g_t^{\tilde{t},\,g_s^2} = -\kappa\,g_s^2\,C_F\left[\ln\frac{m_{\tilde{g}}^2}{Q^2} + \widetilde{F}_6\left(\frac{m_{Q_3}}{m_{\tilde{g}}}\right)\right.$$
$$\left. + \widetilde{F}_6\left(\frac{m_{U_3}}{m_{\tilde{g}}}\right) - \frac{X_t}{m_{\tilde{g}}}\,\widetilde{F}_9\left(\frac{m_{Q_3}}{m_{\tilde{g}}}, \frac{m_{U_3}}{m_{\tilde{g}}}\right)\right], \quad (17)$$

and the analogous shift in the bottom Yukawa coupling, $\Delta g_b^{\tilde{b},\,g_s^2}$, can be obtained from Eq. (17) with the replacements $m_{U_3} \to m_{D_3}$ and $X_t \to X_b$. The loop functions $\widetilde{F}_6(x)$ and $\widetilde{F}_9(x, y)$ are defined in the appendix A of Ref. [66]. Using the limits $\widetilde{F}_6(1) = 0$ and $\widetilde{F}_9(1, 1) = 1$ it is easy to see that, for degenerate superparticle masses, the shifts in Eq. (16) cancel out entirely the terms within square brackets in the third line of Eq. (14). We have of course checked that the cancellation of the IR divergences in the matching holds even when we retain the full dependence on all of the relevant superparticle masses.

2.3 Contributions arising from the definitions of the couplings

The third contribution to the mixed QCD–EW correction to the quartic Higgs coupling in Eq. (2), which we denoted as $\Delta\lambda^{2\ell,\,\mathrm{RS}}$, collects in fact three separate contributions arising from differences in the renormalization scheme used for the couplings of the MSSM and for those of the EFT valid below the matching scale (i.e., the SM):

$$\Delta\lambda^{2\ell,\,\mathrm{RS}} = \Delta\lambda_\lambda^{2\ell,\,\mathrm{RS}} + \Delta\lambda_g^{2\ell,\,\mathrm{RS}} + \Delta\lambda_{g_t}^{2\ell,\,\mathrm{RS}}. \quad (18)$$

The first contribution in the equation above stems from the fact that supersymmetry provides a prediction for the $\overline{\mathrm{DR}}$-renormalized quartic Higgs coupling, whereas we interpret the parameter λ in the EFT as renormalized in the $\overline{\mathrm{MS}}$ scheme. The difference between $\lambda^{\overline{\mathrm{DR}}}$ and $\lambda^{\overline{\mathrm{MS}}}$ contains terms of $\mathcal{O}(\lambda\,g_t^2\,g_s^2)$ and $\mathcal{O}(\lambda\,g_b^2\,g_s^2)$, arising from the dependence on the regularization method of the two-loop quark–gluon contributions to the Higgs WFR, which translate to mixed QCD–EW terms when λ is replaced by its tree-level MSSM prediction. In the limit $v \to 0$ we find

$$\frac{d\,\hat{\Pi}_{hh}^{2\ell,\,q}}{dp^2}\bigg|_{H=0}^{\mathrm{DRED}} - \frac{d\,\hat{\Pi}_{hh}^{2\ell,\,q}}{dp^2}\bigg|_{H=0}^{\mathrm{DREG}} = 2\,\kappa^2\,g_s^2\,(g_t^2 + g_b^2)\,C_F\,N_c$$
$$\times\left(1 - \ln\frac{-p^2}{Q^2}\right) - \frac{1}{2}\kappa^2\,g_s^2\,(g_t^2 + g_b^2)\,C_F\,N_c, \quad (19)$$

where DRED and DREG stand for dimensional reduction and dimensional regularization, respectively. This is again in agreement with the result that can be obtained by taking the appropriate limits in the analytic formulae of Refs. [31–33]. The first term on the r.h.s. of Eq. (19) contains an IR divergence for $p^2 \to 0$, but that term cancels out in the matching between MSSM and SM when the top and bottom Yukawa couplings entering the one-loop quark contribution to the WFR, see Eq. (15), are translated from the $\overline{\mathrm{DR}}$ scheme to the $\overline{\mathrm{MS}}$ scheme according to

$$g_q^{\overline{\mathrm{DR}}} = g_q^{\overline{\mathrm{MS}}}\left(1 + \Delta g_q^{\mathrm{reg},\,g_s^2}\right), \quad \text{with} \quad \Delta g_q^{\mathrm{reg},\,g_s^2} = -\kappa\,g_s^2\,C_F. \quad (20)$$

The surviving term on the r.h.s. of Eq. (19) then leads to the following correction to the quartic Higgs coupling:

$$\Delta\lambda_\lambda^{2\ell,\,\mathrm{RS}} = \frac{1}{4}\kappa^2\,g_s^2\,C_F\,N_c\,(g_t^2 + g_b^2)\,(g^2 + g'^2)\,c_{2\beta}^2. \quad (21)$$

Supersymmetry connects the tree-level quartic Higgs coupling to the $\overline{\mathrm{DR}}$-renormalized EW gauge couplings of the MSSM. Since we choose instead to express the tree-level part of the matching condition for λ, see Eq. (1), in terms of $\overline{\mathrm{MS}}$-renormalized couplings of the SM, the threshold correction $\Delta\lambda^{2\ell}$ receives an additional shift, i.e. the second contribution in Eq. (18). The relation between the two sets of EW gauge couplings reads

$$\hat{g}^2 + \hat{g}'^2 = (g^2 + g'^2)\left(1 + \cdots + \frac{d\,\hat{\Pi}_{ZZ}^{2\ell,\,q}}{dp^2}\bigg|_{H=0}^{\mathrm{DRED}}\right.$$
$$\left. - \frac{d\,\hat{\Pi}_{ZZ}^{2\ell,\,q}}{dp^2}\bigg|_{H=0}^{\mathrm{DREG}} + \frac{d\,\hat{\Pi}_{ZZ}^{2\ell,\,\tilde{q}}}{dp^2}\bigg|_{H=0}\right), \quad (22)$$

where the ellipsis denotes one- and two-loop terms that are not of $\mathcal{O}(g^4 g_s^2)$ or $\mathcal{O}(g'^4 g_s^2)$. We denote by $\hat{\Pi}_{ZZ}^{2\ell,\,q}$ the two-loop quark–gluon contribution to the transverse part of the renormalized Z-boson self-energy, computed either in dimensional reduction or in dimensional regularization, and by $\hat{\Pi}_{ZZ}^{2\ell,\,\tilde{q}}$ the contribution from two-loop diagrams involving the strong gauge interactions of the squarks.[6] The notation $H = 0$ means again the limit $v \to 0$, which in this case can be obtained by Taylor expansion in m_q^2 and p^2, since both the squark contribution and the DRED–DREG difference of the quark contribution are free of IR divergences. In units of $\kappa^2 g_s^2 C_F N_c$, we find

$$
\Delta\lambda_g^{2\ell,\,\mathrm{RS}} = \frac{c_{2\beta}^2}{4} \left\{ -\frac{3\,g^4}{2} - \frac{11\,g'^4}{6} \right.
$$
$$
- \sum_{i=1}^{3} \left[g^4\, F(m_{Q_i}^2, m_{\tilde{g}}^2) \right.
$$
$$
+ \frac{g'^4}{9} \left(F(m_{Q_i}^2, m_{\tilde{g}}^2) + 8\, F(m_{U_i}^2, m_{\tilde{g}}^2) \right.
$$
$$
\left. \left. \left. + 2\, F(m_{D_i}^2, m_{\tilde{g}}^2) \right) \right] \right\}, \tag{23}
$$

where the two terms within curly brackets in the first line account for the $\overline{\mathrm{DR}}$–$\overline{\mathrm{MS}}$ conversion of g^2 and g'^2, respectively, whereas the remaining terms (where the sum runs over three squark generations) account for their MSSM–SM threshold correction. The function $F(m_{\tilde{q}}^2, m_{\tilde{g}}^2)$ is defined as

$$
F(m_{\tilde{q}}^2, m_{\tilde{g}}^2) = \frac{1}{12}\left(7 + \frac{8\,m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right) + \frac{2\,m_{\tilde{g}}^4}{3\,m_{\tilde{q}}^2\,(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)}\ln\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}
$$
$$
+ \left(1 - \frac{2\,m_{\tilde{g}}^2}{3\,m_{\tilde{q}}^2}\right)\ln\frac{m_{\tilde{q}}^2}{Q^2}. \tag{24}
$$

We checked that the explicit renormalization-scale dependence of the $\mathcal{O}(g^4 g_s^2)$ and $\mathcal{O}(g'^4 g_s^2)$ threshold corrections to g^2 and g'^2 is consistent with what can be inferred from the difference between their β-functions in the SM [100] and in the MSSM [101–103].

Finally, the third contribution in Eq. (18) arises from the fact that we choose to express the $\mathcal{O}(g^2 g_t^2)$ and $\mathcal{O}(g'^2 g_t^2)$ terms in the one-loop threshold correction to the quartic Higgs coupling in terms of the $\overline{\mathrm{MS}}$-renormalized top Yukawa coupling of the SM. The resulting shift in the two-loop correction reads

$$
\Delta\lambda_{g_t}^{2\ell,\,\mathrm{RS}} = 2\left(\Delta g_t^{\tilde{t},\,g_s^2} + \Delta g_q^{\mathrm{reg},\,g_s^2}\right)\Delta\lambda^{1\ell,\,\tilde{t}\mathrm{EW}}, \tag{25}
$$

---

[6] The fact that these mixed QCD–EW corrections should depend only on the quark and squark contributions to the gauge-boson self-energy can be easily inferred by considering the renormalization of the gauge couplings of the leptons.

where $\Delta g_t^{\tilde{t},\,g_s^2}$ and $\Delta g_q^{\mathrm{reg},\,g_s^2}$ are the one-loop, $\mathcal{O}(g_s^2)$ shifts given in Eqs. (17) and (20), respectively, and

$$
\Delta\lambda^{1\ell,\,\tilde{t}\mathrm{EW}} = \kappa\, N_c\, g_t^2\, c_{2\beta} \left\{ \frac{1}{2}\left(g^2 - \frac{g'^2}{3}\right)\ln\frac{m_{Q_3}^2}{Q^2} \right.
$$
$$
+ \frac{2\,g'^2}{3}\ln\frac{m_{U_3}^2}{Q^2} + \frac{X_t^2}{4\,m_{Q_3} m_{U_3}}
$$
$$
\times \left[g'^2\,\widetilde{F}_3(x_{QU}) + g^2\,\widetilde{F}_4(x_{QU}) \right.
$$
$$
\left. \left. - \frac{c_{2\beta}}{3}(g^2 + g'^2)\,\widetilde{F}_5(x_{QU}) \right] \right\}, \tag{26}
$$

where $x_{QU} = m_{Q_3}/m_{U_3}$ and the loop functions $\widetilde{F}_3(x)$, $\widetilde{F}_4(x)$ and $\widetilde{F}_5(x)$ are defined in the appendix A of Ref. [66]. Note that all three functions are equal to 1 for $x = 1$.

2.4 Combining all contributions

We now provide a result that combines all of the contributions discussed in the previous sections, valid in the limit of degenerate superparticle masses $m_{Q_i} = m_{U_i} = m_{D_i} = m_{\tilde{g}} = M_S$. In units of $\kappa^2 g_s^2 C_F N_c$, we find

$$
\Delta\lambda^{2\ell,\,\mathrm{QCD-EW}} = c_{2\beta}^2 \left\{ -\frac{1}{16}\left(g^4 + \frac{11}{9}g'^4\right)\left(1 + 4\ln\frac{M_S^2}{Q^2}\right) \right.
$$
$$
+ (g^2 + g'^2)\left\{ (g_t^2 + g_b^2)\left(-\frac{11}{8} - \frac{3}{2}\ln\frac{M_S^2}{Q^2} - \frac{1}{2}\ln^2\frac{M_S^2}{Q^2}\right) \right.
$$
$$
+ g_t^2\left[\frac{X_t}{3\,M_S}\left(4 + \ln\frac{M_S^2}{Q^2}\right) - \frac{X_t^2}{12\,M_S^2}\left(5 - 2\ln\frac{M_S^2}{Q^2}\right) - \frac{X_t^3}{6\,M_S^3}\right]
$$
$$
\left. \left. + g_b^2\left[\frac{X_b}{3\,M_S}\left(4 + \ln\frac{M_S^2}{Q^2}\right) - \frac{7\,X_b^2}{12\,M_S^2}\right]\right\}\right\}
$$
$$
+ c_{2\beta}(g^2 + g'^2)\left\{ g_t^2\left[-1 - \ln\frac{M_S^2}{Q^2} - 2\ln^2\frac{M_S^2}{Q^2}\right.\right.
$$
$$
\left. + 3\frac{X_t}{M_S}\ln\frac{M_S^2}{Q^2} + \frac{X_t^2}{2\,M_S^2}\left(1 - \ln\frac{M_S^2}{Q^2}\right) + \frac{X_t^3}{2\,M_S^3}\right]
$$
$$
\left. \left. + g_b^2\left[1 + \ln^2\frac{M_S^2}{Q^2} - 2\frac{X_b}{M_S}\ln\frac{M_S^2}{Q^2} - \frac{X_b^2}{M_S^2}\right]\right\}\right\}. \tag{27}
$$

As a non-trivial check of our final result, we verified that by taking the derivative of the r.h.s. of Eq. (1) with respect to $\ln Q^2$ we can recover the $\mathcal{O}(\lambda\, g_t^2\, g_s^2)$ and $\mathcal{O}(\lambda\, g_b^2\, g_s^2)$ terms of the β-function for the quartic Higgs coupling of the SM [104]:

$$
\frac{d\lambda}{d\ln Q^2} \supset 40\,\kappa^2\,\lambda\,g_s^2\,(g_t^2 + g_b^2). \tag{28}
$$

To this effect, we must combine the explicit scale dependence of our result for $\Delta\lambda^{2\ell,\,\mathrm{QCD-EW}}$ with the implicit scale dependence of the parameters that enter the tree-level and one-loop parts of the matching condition (for the squark contributions

to the latter, see the appendix of Ref. [73]). In particular, we need the terms that involve the strong gauge coupling in the two-loop β-functions of the EW gauge couplings [100],

$$\frac{dg^2}{d\ln Q^2} \supset 12\,\kappa^2\,g^4\,g_s^2, \quad \frac{dg'^2}{d\ln Q^2} \supset \frac{44}{3}\,\kappa^2\,g'^4\,g_s^2, \qquad (29)$$

in the two-loop β-function of $c_{2\beta}$ [105],

$$\frac{dc_{2\beta}}{d\ln Q^2} \supset (3\,\kappa + 16\,\kappa^2\,\hat{g}_s^{\,2})\left[\hat{g}_t^2 - \hat{g}_b^2 + c_{2\beta}\,(\hat{g}_t^2 + \hat{g}_b^2)\right], \tag{30}$$

and in the one-loop β-functions of the parameters that enter $\Delta\lambda^{1\ell}$,

$$\frac{d\,(m_{Q_i}^2,\,m_{U_i}^2,\,m_{D_i}^2)}{d\ln Q^2} \supset -\frac{16}{3}\,\kappa\,\hat{g}_s^{\,2}\,m_{\tilde{g}}^2,$$

$$\frac{d\,(X_t,\,X_b)}{d\ln Q^2} \supset \frac{16}{3}\,\kappa\,\hat{g}_s^{\,2}\,m_{\tilde{g}}, \tag{31}$$

$$\frac{dg_t^2}{d\ln Q^2} \supset -8\,\kappa\,g_t^2\,g_s^2, \quad \frac{d\hat{g}_b^2}{d\ln Q^2} \supset -\frac{16}{3}\,\kappa\,\hat{g}_b^2\,\hat{g}_s^{\,2}. \tag{32}$$

Note that in Eqs. (29)–(32) we distinguish the (hatted) $\overline{\rm DR}$-renormalized couplings of the MSSM from the (unhatted) $\overline{\rm MS}$-renormalized couplings of the SM (however, the distinction is irrelevant for the strong gauge coupling, which enters only the two-loop part of the calculation). Finally, to recover Eq. (28) we need to convert the remaining MSSM Yukawa couplings in the one-loop part of the derivative of λ into their SM counterparts, and exploit in the two-loop part the tree-level MSSM relation to replace $(g^2 + g'^2)\,c_{2\beta}^2$ with 4λ.

3 The EW-scale determination of the Higgs mass

A consistent determination of the Higgs mass in the EFT approach requires that the relation between the pole Higgs mass and the $\overline{\rm MS}$-renormalized quartic Higgs coupling at the EW scale be computed at the same perturbative order in the various SM couplings as the SUSY-scale threshold correction $\Delta\lambda$. In particular, the "gaugeless" calculation of Refs. [66,73] requires a full determination of the one-loop corrections to the Higgs mass at the EW scale, combined with the two-loop corrections obtained in the limit $g = g' = \lambda = 0$. Denoting the scale at which we perform the calculation of the Higgs mass as $Q_{\rm EW}$, this approximation implies

$$m_h^2 = \frac{\lambda(Q_{\rm EW})}{\sqrt{2}\,G_F}\left[1 - \delta^{1\ell}(Q_{\rm EW})\right]$$
$$+ 8\,\kappa^2\,C_F\,N_c\,g_t^2\,g_s^2\,m_t^2\left(3\,\ell_t^2 + \ell_t\right)$$

$$-\,2\,\kappa^2\,N_c\,g_t^4\,m_t^2\left(9\,\ell_t^2 - 3\,\ell_t + 2 + \frac{\pi^2}{3}\right), \tag{33}$$

where G_F is the Fermi constant, $\delta^{1\ell}$ is the one-loop correction first computed in Ref. [106], and $\ell_t = \ln(m_t^2/Q_{\rm EW})$. The two-loop terms in the second and third lines of Eq. (33) are taken from Ref. [107]. Note that the form of the $\mathcal{O}(g_t^4\,m_t^2)$ terms implies that the top-quark contribution to $\delta^{1\ell}$ is expressed in terms of $\overline{\rm MS}$-renormalized top and Higgs masses. Also, note that Eq. (33) omits for conciseness all corrections involving the bottom and tau Yukawa couplings, which in the SM are greatly suppressed with respect to their top-only counterparts.

In the SUSY-scale calculation of $\Delta\lambda^{2\ell}$ described in Sect. 2 we go beyond the gaugeless limit, and we include contributions that involve both the EW gauge couplings and the strong gauge coupling. Strictly speaking, to match the accuracy of that calculation at the EW scale we only need to replace the $\mathcal{O}(g_t^2\,g_s^2\,m_t^2)$ terms in Eq. (33) with the complete contributions arising from two-loop diagrams involving quarks and gluons, retaining the dependence on the external momentum in the Higgs self-energy. Explicit formulae for those contributions are provided in Ref. [108]. However, the full two-loop contributions to the relation between the pole Higgs mass, $\lambda(Q_{\rm EW})$ and G_F in the SM are also available [86–88]. These contributions can in principle be implemented in our EFT calculation, to prepare the ground for the eventual completion of the NNLL resummation of the large logarithmic corrections. We stress that the inclusion at the EW scale of two-loop corrections whose counterparts are still missing at the SUSY scale cannot be claimed to improve the overall accuracy of the calculation, but it does not degrade it either. Indeed, in the EFT approach the EW-scale and SUSY-scale sides of the calculation are separately free of log-enhanced terms, and the inclusion of additional pieces in only one side does not entail the risk of spoiling crucial cancellations between large corrections.

The results of the full two-loop calculation of Ref. [86] were made public in the form of interpolating formulae. In particular, the relation between the Higgs quartic coupling and the pole Higgs and top masses reads[7]

$$\lambda(Q_{\rm EW} = m_t) = 0.25208 + 0.00412\left(\frac{m_h}{\rm GeV} - 125.15\right)$$
$$- 0.00008\left(\frac{m_t}{\rm GeV} - 173.34\right) \pm 0.00060_{\rm th}, \tag{34}$$

where the remaining input parameters – namely, the gauge-boson masses, G_F and $g_s(m_Z)$ – are set to the central values listed in Ref. [86]. Equation (34) can be exploited to treat the measured value of the Higgs mass as an input parameter:

---

[7] Note that our normalization for λ differs from the one in Refs. [86–88] by a factor 2.
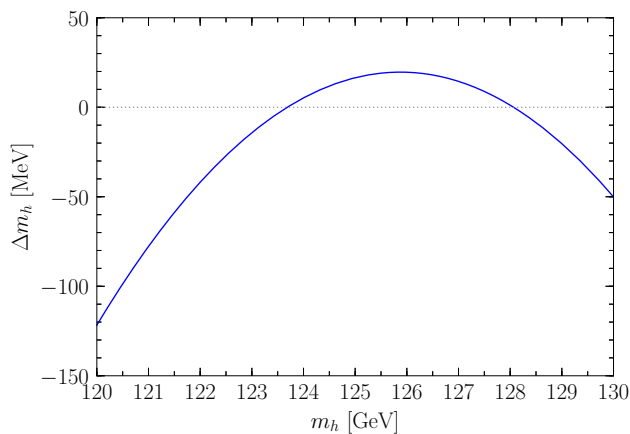
Fig. 1 Difference (in MeV) between the Higgs mass given as input to the code mr and the Higgs mass obtained by inserting in Eq. (34) the value of $\lambda(m_t)$ computed by mr, as a function of the input Higgs mass

it is then possible to evolve λ to the SUSY scale using the RGEs of the SM, and use the threshold condition in Eq. (1) to determine one of the MSSM parameters (e.g., a common mass scale for the stops, or the stop mixing parameter X_t, or $\tan\beta$) as a function of the others. In alternative, Eq. (34) can be inverted to predict the Higgs mass starting from a full set of MSSM parameters, using the value of $\lambda(m_t)$ obtained by evolving the $\lambda(M_S)$ computed in Eq. (1) down to the EW scale. In the latter approach, a phenomenological analysis of the MSSM may well encounter points of the parameter space in which the prediction for the Higgs mass is several GeV away from the measured value. It is then legitimate to wonder about the range of validity of the linear interpolation involved in Eq. (34), which was obtained in a pure-SM context with the (small) uncertainty of the Higgs mass measurement in mind.

To test Eq. (34), we compare its predictions against those of the independent two-loop calculation presented in Ref. [87]. The latter is made available in the public code mr [109], which computes the $\overline{\text{MS}}$-renormalized parameters of the SM Lagrangian from a set of physical observables that includes G_F and the pole masses of the Higgs and gauge bosons and of the top and bottom quarks. We start from an input value for the pole Higgs mass ranging between 120 GeV and 130 GeV, feed it into mr, then insert the value of $\lambda(m_t)$ computed by mr into Eq. (34) to obtain a new prediction for m_h according to the calculation of Ref. [86]. The remaining input parameters are fixed to the central values considered in Ref. [86]. In Fig. 1 we plot the difference between the initial and final values of m_h, which can be taken as a measure of the discrepancy between the two calculations, as a function of the initial value. In the vicinity of $m_h = 125$ GeV, where the interpolation involved in Eq. (34) can be expected to be accurate, the two values of m_h differ by about 20 MeV, i.e. by less than 0.02%. This is well within the theoretical uncertainty

estimated in the last term of Eq. (34), which implies a shift in m_h of about 150 MeV. Such a good numerical agreement is particularly remarkable in view of the fact that the calculations of Refs. [86,87] differ substantially in what concerns the renormalization of the Higgs vev and the corresponding treatment of the tadpole contributions (they also differ in the treatment of higher-order QCD corrections to the top Yukawa coupling). When we move away from the observed value of the Higgs mass, the discrepancy between the two calculations varies, reaching up to about 120 MeV for the lowest considered value $m_h = 120$ GeV. While such discrepancy remains within the theoretical uncertainty of Eq. (34), the behavior of the blue line in Fig. 1 suggests that, for values of m_h a few GeV away from the observed one, the linear interpolation loses accuracy and the full dependence on the value of the Higgs mass should be taken into account.

Finally, we performed an analogous test on the m_t dependence of Eq. (34), keeping the value of the pole Higgs mass that we feed into mr fixed to 125.15 GeV, and varying the pole top mass by ± 2 GeV around its central value of 173.34 GeV. We find that the difference between the initial value of m_h and the one obtained by inserting in Eq. (34) the value of $\lambda(m_t)$ computed by mr varies only by about 10 MeV in the considered range of m_t. This suggests that the linear interpolation of the dependence on the pole top mass in Eq. (34) is not problematic.

4 Impact of the mixed QCD–EW corrections

In this section we investigate the numerical impact of the mixed QCD–EW threshold corrections to the quartic Higgs coupling on the prediction for the Higgs mass in the MSSM with heavy superpartners. We use the code mr [109] to extract – at full two-loop accuracy – the $\overline{\text{MS}}$-renormalized parameters of the SM Lagrangian from a set of physical observables, and to evolve them up to the SUSY scale using the three-loop RGEs of the SM. As mentioned in the previous section, in our EFT approach the fact that we combine a full two-loop calculation at the EW scale with an incomplete two-loop calculation at the SUSY scale does not entail the risk of spoiling crucial cancellations between large corrections. Throughout the section we use the world average $m_t = 173.34$ GeV [110] for the pole top mass, and fix the remaining physical inputs (other than the Higgs mass) to their current PDG values [111], namely $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}, $m_Z = 91.1876$ GeV, $m_W = 80.385$ GeV, $m_b = 4.78$ GeV and $\alpha_s(m_Z) = 0.1181$.
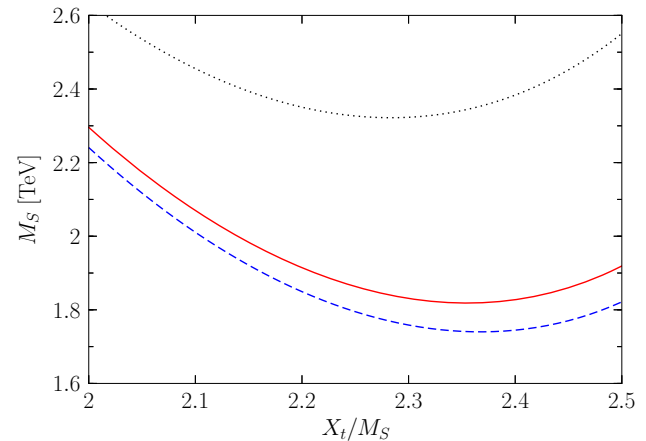
In order to obtain a prediction for the Higgs mass from a full set of MSSM parameters, we vary the value of the pole mass m_h that we give as input to mr until the value of the $\overline{\text{MS}}$-renormalized SM parameter $\lambda(Q)$ returned by the code at the SUSY scale $Q = M_S$ coincides with the MSSM prediction
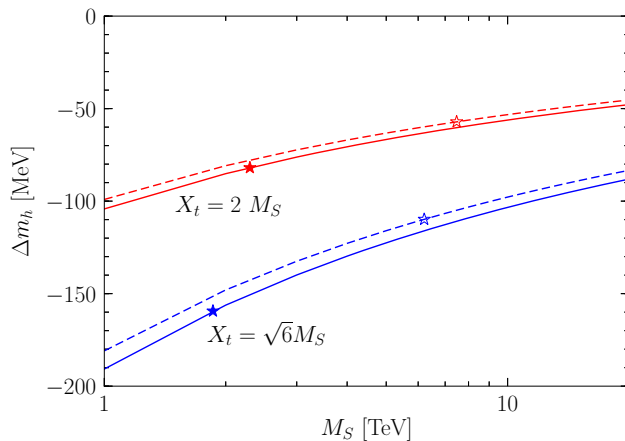
Fig. 2 Difference (in MeV) between the predictions for the Higgs mass obtained with and without the inclusion of the mixed QCD–EW corrections to the quartic Higgs coupling, as a function of a common SUSY scale M_S, for $X_t = \sqrt{6}\,M_S$ (lower, blue lines) or $X_t = 2\,M_S$ (upper, red lines), and $A_b = A_\tau = A_t$. In each set of lines the solid one is obtained with $\tan\beta = 20$ and the dashed one with $\tan\beta = 5$. The star on each line marks the value of M_S for which the improved calculation of $\Delta\lambda^{2\ell}$ leads to $m_h = 125.09$ GeV

Fig. 3 Values of the common SUSY scale M_S and of the stop mixing term X_t that lead to $m_h = 125.09$ GeV, for $\tan\beta = 20$ and $A_b = A_\tau = A_t$. The dotted line is obtained including only the one-loop threshold corrections to the Higgs quartic coupling, the dashed line includes the two-loop corrections in the gaugeless limit, and the solid line includes also the effect of the mixed QCD–EW corrections

of Eq. (1). In addition to the mixed QCD–EW corrections to the quartic Higgs coupling computed in this paper, we use the results of Refs. [66,73] for the full one-loop correction $\Delta\lambda^{1\ell}$ and for the "gaugeless" part of the two-loop correction $\Delta\lambda^{2\ell}$. We recall that, as discussed in Ref. [73], we must also convert the $\overline{\text{MS}}$-renormalized bottom Yukawa coupling $g_b(M_S)$ returned by mr into its $\overline{\text{DR}}$-renormalized MSSM counterpart, $\hat{g}_b(M_S)$, to avoid the occurrence of potentially large $\tan\beta$-enhanced terms in $\Delta\lambda^{2\ell}$. To this effect, we make use of Eqs. (8), (9), (12) and (13) of Ref. [73].

In Fig. 2 we show the difference (in MeV) between the EFT predictions for the Higgs mass obtained with and without the inclusion of the mixed QCD–EW corrections to the quartic Higgs coupling. We consider a simplified MSSM scenario in which the masses of all superparticles (sfermions, gauginos and higgsinos) as well as the mass of the heavy Higgs doublet are set equal to the common SUSY scale M_S, which we vary between 1 TeV and 20 TeV. The left–right stop mixing parameter is fixed either as $X_t = \sqrt{6}\,M_S$ (lower, blue lines) or $X_t = 2\,M_S$ (upper, red lines). In each set of lines the solid one is obtained with $\tan\beta = 20$ and the dashed one with $\tan\beta = 5$. The remaining input parameters are the trilinear Higgs–sfermion interaction terms for sbottoms and staus, which we fix as $A_b = A_\tau = A_t$. We remark that all of the MSSM parameters are interpreted as $\overline{\text{DR}}$-renormalized quantities expressed at the renormalization scale $Q = M_S$. The star on each line marks the value of M_S for which the improved calculation of $\Delta\lambda^{2\ell}$ (i.e., including the mixed QCD–EW corrections) leads to the observed value of the Higgs mass, $m_h = 125.09$ GeV [4].

Figure 2 shows that, in the considered scenario, the mixed QCD–EW corrections computed in this paper are fairly small, shifting the MSSM prediction for m_h downwards by $\mathcal{O}(100)$ MeV. The fact that the corrections are reduced (in absolute value) for larger values of M_S is partially due to the scale dependence of the relevant couplings: with the exception of g', they all decrease with increasing $Q = M_S$. The comparison between the solid and dashed lines in each set shows that the dependence of the mixed QCD–EW corrections on $\tan\beta$ is rather mild (in contrast, the overall prediction for m_h depends strongly on $\tan\beta$, as shown by the relative position of the stars on the solid and dashed lines). Finally, the comparison between the lower (blue) and upper (red) sets of lines shows that the mixed QCD–EW corrections depend rather strongly on the ratio $|X_t/M_S|$, with smaller ratios leading to smaller corrections. Indeed, we checked that for $|X_t/M_S| < 1$ the effect of the corrections can be at most of $\mathcal{O}(10)$ MeV in the considered scenario.

An alternative way to assess the effect of the newly-computed corrections consists in taking the measured value of the Higgs mass as an input parameter, and using the matching condition on the quartic Higgs coupling at the SUSY scale, Eq. (1), to constrain the MSSM parameters. In Fig. 3 we show the values of M_S and X_t that lead to $m_h = 125.09$ GeV, in the simplified scenario with degenerate superparticle and heavy-Higgs masses, for $\tan\beta = 20$ and $A_b = A_\tau = A_t$. We focus on values of the ratio X_t/M_S between 2 and 2.5, which allow for SUSY masses around 2 TeV (i.e., roughly at the limit of the HL-LHC reach [112]). Once again, all of the MSSM parameters are interpreted as $\overline{\text{DR}}$-renormalized quantities expressed at the renormalization scale $Q = M_S$. The (black) dotted line in Fig. 3 is obtained including only the

one-loop threshold corrections to the Higgs quartic coupling, the (blue) dashed line includes the two-loop corrections in the gaugeless limit, and the (red) solid line includes also the effect of the mixed QCD–EW corrections. Unsurprisingly, the comparison between the three lines shows that the mixed QCD–EW corrections are sub-dominant with respect to the two-loop corrections computed in the gaugeless limit. Nevertheless, they can shift the value of M_S that leads to the observed Higgs mass by $\mathcal{O}(100)$ GeV in the direction of heavier superparticles.

5 Conclusions

If the MSSM is realized in nature, both the measured value of the Higgs mass and the negative results of the searches for superparticles at the LHC suggest some degree of separation between the SUSY scale and the EW scale. In this scenario, the MSSM prediction for the Higgs mass is subject to potentially large logarithmic corrections, making a fixed-order calculation of m_h inadequate and calling for an all-orders resummation in the EFT approach.

In this paper we improved the EFT calculation of the Higgs mass in the MSSM, by computing the class of two-loop threshold corrections to the quartic Higgs coupling that involve both the strong and the EW gauge couplings. Combined with the $\mathcal{O}(g_t^4\, g_s^2)$ and $\mathcal{O}(g_b^4\, g_s^2)$ corrections previously provided in Refs. [66,73], this completes the calculation of the two-loop threshold corrections that involve the strong gauge coupling. Our calculation involves novel complications with respect to the case of the "gaugeless" two-loop corrections of Refs. [66,73]. While in the latter all two-loop diagrams could be computed in the effective potential approach (i.e., for vanishing external momenta), the mixed QCD–EW corrections include contributions from the $\mathcal{O}(p^2)$ parts of the two-loop self-energies of the Higgs and gauge bosons. We obtained results for the threshold corrections to the quartic Higgs coupling valid for generic values of all the relevant SUSY parameters, which we make available on request in electronic form. For the sake of illustration, in Sect. 2 we provided explicit formulae in the simplified limit of degenerate superparticle masses.

We remark that our calculation can be trivially adapted also to the split-SUSY scenario in which the gluino is much lighter than the squarks, by taking the limit of vanishing gluino mass in our full results. On the other hand, in scenarios in which the gluino is heavier than the squarks the two-loop corrections to the quartic Higgs coupling contain potentially large terms enhanced by powers of the ratios between the gluino mass and the squark masses. This is a well-known aspect of the $\overline{\text{DR}}$ renormalization of the squark masses and trilinear couplings [24,67,73,113], which could be addressed either by devising an "on-shell" scheme adapted to the heavy-

SUSY setup, or by building a tower of EFTs in which the gluino is independently decoupled at a higher scale than the squarks.

In the EFT approach, the inclusion of new threshold corrections to the quartic Higgs coupling at the SUSY scale mandates that corrections of the same perturbative order in the relevant couplings be included in the calculation of the pole Higgs mass at the EW scale. In the simplest heavy-SUSY setup in which the effective theory valid below the SUSY scale is just the SM, we can exploit the full two-loop calculations of the relation between m_h, $\lambda(Q_{\text{EW}})$ and G_F presented in Refs. [86–88]. In Sect. 3 we compared the results of two of those calculations, Refs. [86,87], discussing the range of validity of an interpolating formula provided in Ref. [86].

In Sect. 4 we investigated the numerical impact of the mixed QCD–EW corrections to the quartic Higgs coupling. We considered a simplified MSSM scenario with degenerate masses for all superparticles and for the heavy Higgs doublet, focusing on the region of the parameter space in which the prediction for the Higgs mass is close to the observed value and the stop squarks are in principle still accessible at the HL-LHC. We used the code mr [109], based on the calculation of Ref. [87], to extract all of the SM couplings from a set of physical observables and to evolve them up to the SUSY scale, where we compare the value of the quartic Higgs coupling with its MSSM prediction. We found that the impact of the newly-computed two-loop corrections on the prediction for the Higgs mass tends to be small, and it is certainly sub-dominant with respect to the impact of the "gaugeless" two-loop corrections. In the considered scenario, the mixed QCD–EW corrections can shift the prediction for the Higgs mass by $\mathcal{O}(100)$ MeV, and they can shift the values of the stop masses required to obtain the observed value of m_h by $\mathcal{O}(100)$ GeV.

We stress that the smallness of these effects is in fact a desirable feature of the EFT approach to the calculation of the Higgs mass. While the logarithmically enhanced corrections are accounted for by the evolution of the parameters between the matching scale and the EW scale, and high-precision calculations at the EW scale can be borrowed from the SM, the small impact of new two-loop corrections computed at the SUSY scale suggests that the uncertainty associated to uncomputed higher-order terms should be well under control in the considered scenario. On the other hand, we recall that the accuracy of the measurement of the Higgs mass at the LHC has already reached the level of 100–200 MeV [111] – i.e., it is comparable to the effects of the corrections discussed in this paper – and will improve further when more data are analyzed. If SUSY eventually shows up at the TeV scale, the mass and couplings of the SM-like Higgs boson will serve as precision observables to constrain MSSM parameters that might not be directly accessible by experiment. To this purpose, the accuracy of the theoretical predictions will have to

match the experimental one, making a full inclusion of two-loop effects in the Higgs-mass calculation unavoidable. Our results should be viewed as a necessary step in that direction.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Explicit formulae for the corrections computed in this paper are available on request in electronic form.]

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