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Letter

H_0 tension: clue to common nature of dark sector?

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Abstract The recently sharpened H_0 tension is argued not to be a result of data calibration or any other systematic feature, but an indication for the common nature of dark matter and dark energy. This conclusion is drawn within modified weak-field General Relativity where the accelerated expansion of the Universe and the dynamics of galaxy groups and clusters are described by the same parameter, the cosmological constant. The common nature of the dark sector hence will result in an intrinsic discrepancy/tension between the local and global determinations of the values of the Hubble constant.

1 Introduction

Recent measurements [1] increased the existing tension between the Hubble constant determinations from Planck satellite data [2] and lower redshift observations; the earlier studies and various approaches for resolving the tension are discussed in [1].

We will consider the H_0 tension within the approach of weak-field modified general relativity (GR) which enabled the common description of the dark matter and dark energy by means of the same value of the cosmological constant [3–5]. That approach is based on Newton's theorem on the equivalence of the gravity of the sphere and of a point situated in its center and provides a natural way for the weak-field modification of GR, so that dark energy is described by the Friedmann–Lemaître–Robertson–Walker (FLRW) equations, while the dark matter in galaxy groups and clusters is described by the weak-field GR.

It is a principal fact that by now both the strong-field GR has been tested by the discovery of gravitational waves, and the weak-field effects, such as at frame-dragging, have been traced by measurements of laser ranging satellites [6]. The weak-field modifications we are discussing are by now far from being tested at satellite measurements and therefore the dynamical features of the local Universe, including of the galactic dark halos [7], galaxy groups [5,8], can serve as unique probes for such weak-field modifications of GR. Among other modified gravity tests are the accurate measurements of gravitational lenses [9], along with the effects in the solar system [10] or traced from a large scale matter distribution [11].

Thus, we show that if the cosmological constant Λ describes both accelerated expansion and dark matter at galaxy cluster scales, then it will lead to the intrinsic discrepancy in the global and local values of the Hubble constant.

2 Newton's theorem and Λ

In [3] it is shown that weak-field GR can involve the cosmological constant Λ, so that the metric tensor components have the form

$$g_{00} = 1 - \frac{2Gm}{rc^2} - \frac{\Lambda r^2}{3}; \quad g_{rr} = \left(1 - \frac{2Gm}{rc^2} - \frac{\Lambda r^2}{3}\right)^{-1}. \tag{1}$$

This follows from the consideration of the general function for the force satisfying Newton's theorem on the identity of sphere's gravity and that of a point situated in its center and, crucially, then a shell's internal gravity is no more force-free [12]. Namely, the most general form of the function for the gravitational force which satisfies that theorem is

$$F(r) = C_1 r^{-2} + C_2 r, \tag{2}$$

where C_1 and C_2 are constants of integration; for derivation and discussion see [4,12]. The first term in Eq. (2) corresponds to the ordinary Newtonian law, and once the modified Newtonian law (for the potential) is taken as weak-field GR, one has Eq. (1), where the second constant, C_2, corresponds to Λ (up to a numerical coefficient and c^2) [3,4]. Namely, the second constant, Λ, on the one hand, acts as the

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cosmological constant in the cosmological solutions of Einstein equations; on the other hand, it enters in the low-energy limit of GR, which hence is attributed to the Hamiltonian dynamics of galaxy groups and clusters [5], instead of the commonly used Newtonian potential.

Within the isometry group representation the Lorentz group O(1,3) acts as stabilizer subgroup of the isometry group of 4D maximally symmetric Lorentzian geometries and, depending on the sign of Λ (+,-, 0), one has the non-relativistic limits [3]

$$\begin{aligned} &\Lambda > 0 : O(1,4) \to (O(3) \times O(1,1)) \ltimes R^6, \\ &\Lambda = 0 : IO(1,3) \to (O(3) \times R) \ltimes R^6, \\ &\Lambda < 0 : O(2,3) \to (O(3) \times O(2)) \ltimes R^6. \end{aligned} \tag{3}$$

The O(3) is the stabilizer group for the spatial geometry since for all three cases the spatial algebra is Euclidean,

$$E(3) = R^3 \rtimes O(3). \tag{4}$$

Thus, Newton's theorem in the language of group theory can be formulated as follows: each point of the spatial geometry admits the O(3) symmetry.

An important consequence of Eq. (2) is that the linear term (related to the C_2 constant) can produce a non-zero force inside the shell. This is a unique feature since pure Newtonian gravity according to Gauss' law cannot influence anything inside the shell. Furthermore, this mathematical feature of Eq. (2) can be considered to be in agreement with the observational indications that the properties of galactic disks are determined by halos; see [4].

3 Local and global Hubble flows with Λ

The Hubble–Lemaître law, one of established pillars of modern cosmology, is characterized by the Hubble constant H_0, which can be derived in various ways, depending on the observational dataset. Namely, the Planck satellite provided the data on the cosmic microwave background (CMB), which within the ΛCDM model led to the following *global* values: $H_0 = 67,66 \pm 0.42\,\mathrm{km\,s^{-1}Mpc^{-1}}$ and $\Lambda = 1.11\,10^{-52}\mathrm{m}^{-2}$ [13]. The recent analysis of Cepheid variables in the Large Magellanic Cloud (LMC) by the Hubble Space Telescope (HST) [1] led to the *local* value $H = 74.03 \pm 1.42\,\mathrm{km\,s^{-1}Mpc^{-1}}$. This discrepancy between the global and local values of the Hubble constant is the above-mentioned tension.

Our Universe is considered to be described by the FLRW metric,

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left(\frac{1}{1-kr^2}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right), \tag{5}$$

where, depending on the sign of the sectional curvature k, the spatial geometry can be spherical $k = 1$, Euclidean $k = 0$ or hyperbolic $k = -1$. Consequently, the 00-component of the Einstein equations for this metric is written as

$$H^2 = -\frac{k^2c^2}{a^2(t)} + \frac{\Lambda c^2}{3} + \frac{8\pi G\rho}{3}, \tag{6}$$

where $H = \dot{a}(t)/a(t)$ is the Hubble constant.

Here an important point is the following. The Hubble–Lemaître law originally was established for a sample of nearby galaxies, which are members of the Local Group. For them the empirical Hubble–Lemaître law seemed to confirm the FLRW equations, however, later it became clear that not only the galaxies have their peculiar velocities but the Local Group itself is gravitationally bounded to a larger configuration; see [14]. In other words, that law was observed at scales for which it should not be observed. Nevertheless, in spite of this apparent contradiction the local flow has been confirmed by observations: the detailed analysis of the nearby galaxy surveys reveal the local Hubble flow with $H_{\mathrm{loc}} = 78 \pm 2\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [15].

We will now show that, considering Eq. (1) as the weak-field limit of GR, it is possible to solve this tension. Namely, the global Hubble flow will be described by the cosmological constant of the FLRW metric, while the local flow by weak-field GR is given by Eq. (1).

Therefore, we are not allowed to use the FLRW metric on local scales since the Local Supercluster galaxies do not move by the FLRW geodesics. On the other hand due to the attractive nature of pure Newtonian gravity one cannot produce a repulsive force causing the local Hubble flow. However, if we consider the additional linear term of Eq. (2) the Λ-term can cause a repulsive acceleration, thus

$$a = -\frac{GM}{r^2} + \frac{\Lambda c^2 r}{3}. \tag{7}$$

It is simple to find the distance at which the acceleration of ordinary Newtonian term becomes subdominant with respect to the second term. In Table 1 the values for such distances are listed for different mass scales. For objects less massive than the Local Group (LG), that *critical distance* is located outside the object's boundary, which means that it cannot be observed. For LG, the critical distance is around 1.4 Mpc.

Table 1 Critical distance for different objects

Central object	Mass (kg)	Radius (m)
Earth	5.97×10^{24}	4.92×10^{16}
Sun	$2 \times 10^{30} = M_\odot$	3.42×10^{18}
Sgr A*	$4.3 \times 10^6 M_\odot$	5.56×10^{20}
Milky way	$1.5 \times 10^{12} M_\odot$	3.91×10^{22}
Local group	$2 \times 10^{12} M_\odot$	4.31×10^{22}

Here, it is worth to mention that, since we have used Eq. (1) according to Newton's theorem, this distance can be considered as the radius of a sphere for which the whole mass of LG is concentrated at its center. Thus, we conclude that, for the objects located outside this radius, we will be able to observe an outward acceleration. These results, obtained based on Newton's theorem, are in agreement with other analyses [18].

Meanwhile, considering the weak-field limit according to Eq. (1), one can obtain the analog of Eq. (6) for the non-relativistic case

$$H^2 = \frac{\Lambda c^2}{3} + \frac{8\pi G\rho}{3}. \tag{8}$$

In spite of the apparent similarity of Eqs. (8) and (6), there is an important difference between them. Indeed, in Eq. (8) $k = 0$ and ρ stands only for the matter density (baryonic and non-baryonic), while ρ in Eq. (6) includes the contribution also of the radiation density; in this context see the comparative discussion on FLRW and McCrea–Milne cosmologies in [19]. Thus, one can conclude that the H observed by HST on local scales is not the one obtained via Eq. (6) by considering the FLRW metric. It is a local effect, which can be described by Eq. (8). However, before considering the weak-field limit equations for local flow, first let us take a look at Eq. (1) itself. According to the principles of GR, the weak-field limit is defined when $\phi/c^2 \ll 1$, where ϕ is the weak-field potential. Now, by taking this into consideration, besides the Newtonian term a new limit is defined at large distances:

$$\frac{\Lambda r^2}{3} << 1, \quad r \simeq 1.46 \quad 10^{26} m = 5.33\text{Gpc}. \tag{9}$$

Considering the fact that the local Hubble flow is observed on scales of a few Mpc, we are allowed to use Eq. (8) to describe that flow. By taking cosmological parameters [13], Eq. (6) confirms that the total matter density in our Universe is $\rho = 2.68\,10^{-27}\text{kg m}^{-3}$. However, by substituting $H = 74.03 \pm 1.42\text{km s}^{-1}\text{Mpc}^{-1}$, the matter density which causes the observed local Hubble flow will be $\rho_{\text{loc}} = 4.37^{+0.40}_{-0.39}\,10^{-27}\text{kgm}^{-3}$.

Now, in order to complete our justification we need to check the mean density of the local astrophysical structures. From a hierarchical point of view the LG is located about 20 Mpc away from the Virgo cluster [16]. The Virgo cluster itself together with LG is in a larger Virgo supercluster [20], which itself is part of the Laniakea supercluster [17]. Considering the mass and their distances from LG, it is possible to find the distance where the densities of these objects become exactly equal to ρ_{loc}. These results are exhibited in Table 2.

Table 2 Distances of objects where the density is ρ_{loc}

Object	Mass (kg)	Distance from LG (Mpc)
Local group	$2 \times 10^{12} M_\odot$	1.95 ± 0.06
Virgo cluster	$1.2 \times 10^{15} M_\odot$	$3.45^{0.48}_{0.52}$
Virgo supercluster	$1.48 \times 10^{15} M_\odot$	$2.26^{0.51}_{0.56}$
Laniakea	$10^{17} M_\odot$	$5.00^{2.07}_{2.29}$

From these results it becomes clear that not only the error bars fully cover each other, but also the whole range of the local flow is covered by these values i.e. from 1.70 to 7.07 Mpc. Meanwhile, according to Eq. (7) the critical distance of the Virgo supercluster from LG roughly is 7.27 Mpc, which means that the objects beyond that distance are gravitationally bounded to the supercluster. Considering the upper limit of Table 2 it turns out that there is no overlapping between the bounded objects and those moving away according to Eq. (8). Furthermore, these values exactly coincide with the density of the Virgo cluster at distances in which the Virgo-centric flow changes to the FLRW linear Hubble–Lemaître law [18].

Thus, the H_0 tension is not a calibration discrepancy but a natural consequence of the presence of Λ in GR as well as weak-field limit equations. For the global value we have to consider Eq. (6) as the immediate consequence of FLRW metric and the cosmological parameters defined as

$$\Omega_k = -\frac{k^2 c^2}{a^2(t) H^2}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}, \quad \Omega_m = \frac{8\pi G\rho}{3H^2}. \tag{10}$$

The local value of H is obtained by weak-field limit equations and depends strictly on the local density of matter distribution.

Note that, besides the above-mentioned two evaluations of H, other independent measurements also confirm this discrepancy. Among such measurements are those of the Dark Energy Survey (DES) Collaboration, where the so-called inverse distance ladder method based on baryon acoustic oscillations (BAO) is used [21]. Considering the BAO as a standard ruler in cosmology, it turns out that its scale is roughly equal to 150 Mpc which clearly exceeds the typical distance of our local structures (the Virgo cluster etc.). Namely, the relevant SNe Ia are located at redshifts $0.018 < z < 0.85$ [21], which means that according to the Planck data [13] such objects are located at distances $80\text{Mpc} < r < 3\text{Gpc}$. Thus, by comparing these scales with the typical distance to our local structures, one concludes that the measured H for these observations should mainly be induced by cosmological parameters. This statement is justified by their measured value $H = 67.77 \pm 1.30\,\text{km s}^{-1}\text{Mpc}^{-1}$.

Other measurements, again using BAO, are those of [22], where like the DES survey, the distances are $1.8\text{Gpc} < r < 6.2\text{Gpc}$ and yield $H = 67.6^{+0.91}_{-0.87}\,\text{km s}^{-1}\text{Mpc}^{-1}$.

Thus, one can conclude that there are two different Hs, of two different scales, *local* and *global* ones. Consequently, the measurement of these two quantities will depend on scales attributed by the observations. Namely, for observations of local scales it is expected that one get the local H, while moving to cosmological scales, i.e. beyond the Virgo cluster, the measurements should yield the global H.

Note one more important point: although currently the numerical values of these two different Hs are close to each other, their physical content is totally different. Namely, this semi-coincidence is due to the fact that for the global case the density in Eq. (6) is the current mean density in the Universe. At earlier phases of the Universe the radiation density had a major contribution to the mean density,

$$\Omega_\rho = \Omega_m + \Omega_r. \tag{11}$$

Also, current observations [13] indicate a curvature of the Universe close to zero, $k = 0$, and hence Eq. (6) will be similar to the weak-field equation. In other words, while for the local flow – no matter in which era – the contribution of the matter density would have been the dominant one, for the global flow the contribution to the density in Eq. (6) was different for other cosmological eras where the radiation and k were not negligible.

Considering the FLRW metric's Hubble constant, i.e., the global H for different eras, one has

$$H(t) = H_0[\Omega_m a^{-3}(t) + \Omega_r a^{-4}(t) + \Omega_k a^{-2}(t) + \Omega_\Lambda]^{1/2}, \tag{12}$$

where H_0 is the current value of the global Hubble constant. In this sense, the above statement about the differences between Hs will also be true as the Universe tends to the de Sitter phase. In that case, all Ωs except Ω_Λ will gradually tend to zero. But again, for the local measures one still will have the same non-zero matter density.

4 Conclusions

The H_0 tension cannot be a result of data calibration/ systematic features, but it is a genuine indication for the common nature of the dark matter and dark energy. This conclusion is drawn above based on Newton's theorem and the resulting weak-field limit of general relativity, which includes the Λ constant. Within that approach, while the Friedmannian equations with the Λ term describe the accelerated Universe, the same Λ is responsible for the dynamics of galaxy groups and clusters. Correspondingly, the *global* Hubble constant derived from the CMB and the *local* one devised from galaxy surveys, including the Local Supercluster, have to differ.

Then the long known so-called local Hubble flow [15], i.e., when the galaxies within the Local Supercluster fit the Hubble–Lemaître law, that the galaxies themselves are not moving via geodesics of the FLRW metric, finds its natural explanation within the metric equation (1). In other words, the local value of H_0 has to take into account the contribution of the cosmological constant (as entering the weak-field GR) in the kinematics of the galaxies along with the observed value of the mean density of matter.

Accurate studies of the dynamics of galactic halos, groups and galaxy clusters, e.g. gravitational lensing, can be decisive for further probing of the described weak-field GR and the common nature of the dark sector.

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