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Electromagnetic dipole moments and time reversal violating interactions for high energy charged baryons in bent crystals at LHC

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Abstract Spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurements of constants determining T-odd, P-odd (CP) violating interactions and P-odd, T-even interactions of charm, beauty and strange charged baryons with electrons and nucleus (nucleons), similarly to the possibility of measuring their electric and magnetic moments. For a particle moving in a bent crystal new effects caused by non-elastic processes arise. In addition to the spin rotation around the effective magnetic field $\boldsymbol{B}$ direction (the bending axis direction), due to electromagnetic interactions and leading to changes in spin direction in the plane orthogonal to $\boldsymbol{B}$, the spin acquires projection along the $\boldsymbol{B}$ direction. Moreover, when spin rotates around the direction of the electric field $\boldsymbol{E}$ or the particle momentum $\boldsymbol{k}$, a spin component along $\boldsymbol{E}$ or $\boldsymbol{k}$, respectively, is induced.

1 Introduction

The spin rotation phenomenon for particles channelled in a bent crystal, which was theoretically predicted in [1] and observed in [2–4], gives the opportunity to measure the anomalous magnetic moment of high energy short-lived particles. The availability of beams with energies up to 7 TeV at the LHC and further growth of energy and luminosity at the FCC improve the possibility of using this phenomenon, along spin depolarization, for measuring anomalous magnetic moments of positively charged, as well as neutral and negatively charged short-lived baryons, and τ leptons [5–8]. The detailed analysis of conditions of the experiment aiming to measure the magnetic dipole moment (MDM) of Λ_c^+ and Ξ_c^+ charm baryons at the LHC was accomplished recently in [9,10]. The spin rotation phenomenon of the channelled particle also allows to obtain information about the electric dipole moment of short-lived baryons, which an elementary particle can take as a result of T-invariance violation (Fig. 1) [10,11].

Besides electromagnetic interaction a channelled particle moving in a crystal, experiences weak interaction with electrons and nuclei, as well as strong interaction with nuclei. When analyzing particle's spin rotation, which is caused by electric dipole moment interaction with electric field, one should consider both P-odd, T-even and P-odd, T-odd non-invariant spin rotations, resulting from weak interaction with electrons and nuclei [12,13].

This paper demonstrates that spin precession of channelled particles in bent crystals at the LHC gives unique possibility to measure constants determining T-odd, P-odd (CP) violating interactions and P-odd, T-even interactions of charm,beauty and strange baryons, similarly to possibility of measuring their electric and magnetic moments considered in [5–11]. It is also shown that for a particle moving in a bent crystal, in addition to the spin rotation around the effective magnetic field $\boldsymbol{B}$ direction (the bending axis direction) that leads to spin direction change in the plane orthogonal to field $\boldsymbol{B}$, the spin acquires projection in direction of $\boldsymbol{B}$, caused by non-elastic processes. Similarly, when spin rotates around the direction of electric field $\boldsymbol{E}$ or the particle momentum $\boldsymbol{k}$, a spin component parallel to $\boldsymbol{E}$ or $\boldsymbol{k}$, respectively, appears. The unit vectors along directions of $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{k}$ will be hereinafter denoted as $\boldsymbol{N}_{\mathrm{m}}$, $\boldsymbol{N}_{\mathrm{T}}$ and $\boldsymbol{n}$ respectively.

2 Spin rotation and particles scattering in a crystal

The spin rotation phenomenon for a high-energy particle moving in a bent crystal can be described by equations similar to those for motion of particles' spin in a storage ring with the inner target [12,13]. The theory, which describes motion

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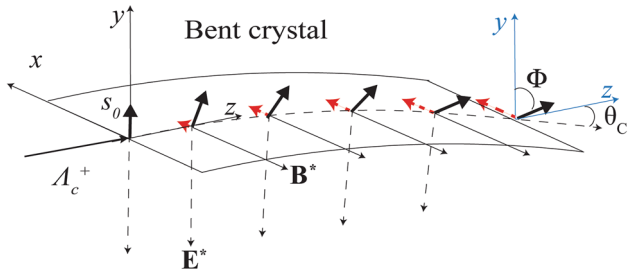

Fig. 1 Behavior of the spin rotation caused by magnetic moment and EDM. The figure is reprinted from figure 2 (right) in [11]. Black arrows represent spin rotation caused by magnetic dipole moment, red arrows represent spin rotation caused by electric dipole moment

of the particle spin in electromagnetic fields in the storage ring, was developed in many papers [14,18–23].

According to [14,18–23], the basic equation, which describes particle spin motion in electromagnetic fields, is the Thomas-Bargmann–Michel–Telegdi (T-BMT) equation. Refinement of the T-BMT equation allowing for the possible existence of non-zero EDM was made in [24] (see more details in [10,11,20]).

Now let us consider a particle, which moves in the electromagnetic field and possesses spin S. The term "particle spin" here means the expected value of the quantum mechanical spin operator. Motion of high-energy particles (Lorentz factor $\gamma \gg 1$) in non-magnetic crystal ($\boldsymbol{B} = 0$) will be considered. In this case spin motion is described by:

$$\frac{d\boldsymbol{S}}{dt} = [\boldsymbol{S} \times \boldsymbol{\Omega}], \tag{1}$$

$$\boldsymbol{\Omega} = -\frac{e(g-2)}{2mc}[\boldsymbol{\beta} \times \boldsymbol{E}], \tag{2}$$

where $\boldsymbol{S}$ is the spin vector, t is the time in the laboratory frame, m is the mass of the particle, e is its charge, $\boldsymbol{\beta} = \boldsymbol{v}/c$, where $\boldsymbol{v}$ denotes the particle velocity, $\boldsymbol{E}$ is the electric field at the point of particle location in the laboratory frame, and g is the gyromagnetic ratio. By definition, the particle magnetic moment is $\mu = (eg\hbar/2mc)S$. The T-BMT equation describes spin motion in the rest frame of the particle, wherein the spin is described by the three component vector $\boldsymbol{S}$. In practice the T-BMT equation describes the spin precession in the external electric and magnetic fields applied in typical present–day accelerators. However, it should be taken into account that particles in an accelerator or a bent crystal have some energy spread and move along different orbits. This necessitates to average the spin–dependent parameters of the particle over the phase space of the particle beam, requiring to introduce the beam polarization $\boldsymbol{\xi}$ [25]. That is why one always must bear in mind the distinction between the beam polarization $\boldsymbol{\xi}$ and the spin vector $\boldsymbol{S}$. A complete description of particle spin motion can be made by the use of spin density matrices equation (in more details see [13,26]).

If a particle possesses an intrinsic electric dipole moment, then the additional term, describing spin rotation induced by the EDM, should be added to (1),

$$\frac{d\boldsymbol{S}_{\mathrm{EDM}}}{dt} = \frac{D}{S\hbar}\left[\boldsymbol{S} \times \left\{\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\boldsymbol{E})\right\}\right], \tag{3}$$

where $D = ed$ is the electric dipole moment of the particle.

As a result, motion of particle spin due to the magnetic and electric dipole moments can be described by the following equation:

$$\frac{d\boldsymbol{S}}{dt} = -\frac{e(g-2)}{2mc}[\boldsymbol{S} \times [\boldsymbol{\beta} \times \boldsymbol{E}]] + \frac{D}{\hbar S}\left[\boldsymbol{S} \times \left\{\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\boldsymbol{E})\right\}\right]. \tag{4}$$

A high-energy particle moving in a crystal is scattered by atoms and, thus, interacts with electric and magnetic fields. However, electromagnetic interaction is not the only one influencing the scattering process. Particles also interact with electrons and nuclei via strong and weak interactions, this depending on the spin of the colliding particles.

The refractive index for a particle in matter formed by different scatterers has the form:

$$n = 1 + \frac{2\pi N}{k^2} f(0), \tag{5}$$

where N is the number of scatterers per cm^3, k is the wave number of the particle incident on the target and $f(0) \equiv f_{aa}(\boldsymbol{k}' - \boldsymbol{k} = 0)$ is the coherent elastic zero angle scattering amplitude. In this case the momentum of the scattered particle $\boldsymbol{p}' = \hbar\boldsymbol{k}'$ (where $\boldsymbol{k}'$ is a wave vector) equals to the initial momentum $\boldsymbol{p} = \hbar\boldsymbol{k}$. Let the quantum state of the atom (nucleus) to be characterized by stationary wave function Φ_a before interaction with the incident particle. After interaction the atom (nucleus) will stay in the same quantum state. If the energy of interaction between a particle and a scatterer depends on spin of the particle, then the scattering amplitude $f(\boldsymbol{k}' - \boldsymbol{k})$ also depends on spin. Consequently refractive index operator n depends on spin as well [13]. If matter is formed by different scatterers, then

$$n = 1 + \frac{2\pi}{k^2}\sum_j N_j f_j(0), \tag{6}$$

where N_j is the number of j-type scatterers per cm^3, and $f_j(0)$ is the amplitude of the particle coherent elastic zero-angle scattering by the j-type scatterer. The particle energy in the vacuum $E = \sqrt{\hbar^2k^2c^2 + m^2c^4}$ is not equal to the particle energy in the medium $E_{\mathrm{med}} = \sqrt{\hbar^2k^2n^2c^2 + m^2c^4}$ [13].

The energy conservation law immediately requires the particle in the medium to have the effective potential energy U_{eff}. This energy can be easily found from the following relation:

$$E = E_{\rm med} + U_{\rm eff}, \tag{7}$$

i.e.

$$\begin{aligned} U_{\rm eff} = E - E_{\rm med} &= -\frac{2\pi\hbar^2}{m\gamma} N f(E,0) \\ &= (2\pi)^3 N T_{aa}(\boldsymbol{k}' - \boldsymbol{k} = 0), \end{aligned} \tag{8}$$

$$\begin{aligned} f(E,0) &= -(2\pi)^2 \frac{E}{c^2\hbar^2} T_{aa}(\boldsymbol{k}' - \boldsymbol{k} = 0) \\ &= -(2\pi)^2 \frac{m\gamma}{\hbar^2} T_{aa}(\boldsymbol{k}' - \boldsymbol{k} = 0), \end{aligned} \tag{9}$$

where $T_{aa}(\boldsymbol{k}' - \boldsymbol{k} = 0)$ is the matrix element of T-operator describing elastic coherent zero-angle scattering.

Let us remind that T-operator is associated with the scattering matrix S [15,17]:

$$S_{ba} = \delta_{ba} - 2\pi i \delta(E_b - E_a) T_{ba}, \tag{10}$$

where E_a (E_b) is the energy of scattered particles before (after) the collision, and the matrix element T_{ba} corresponds to states a and b, which both have the same energy.

For the matter formed by different scatterers the effective potential energy reads as follows:

$$U_{\rm eff} = -\frac{2\pi\hbar^2}{m\gamma} \sum_j N_j f_j(E,0). \tag{11}$$

Due to periodic location of atoms in a crystal the effective potential energy of a particle moving in the crystal is a periodic function of particle's coordinates [13]:

$$U_{\rm eff}(\boldsymbol{r}) = \sum_{\boldsymbol{\tau}} U(\boldsymbol{\tau}) e^{i\boldsymbol{\tau}\boldsymbol{r}}, \tag{12}$$

where $\boldsymbol{\tau}$ is the reciprocal lattice vector of the crystal,

$$U(\boldsymbol{\tau}) = \frac{1}{V} \sum_j U_j(\boldsymbol{\tau}) e^{i\boldsymbol{\tau}\boldsymbol{r}_j}. \tag{13}$$

Here, V is the volume of the crystal elementary cell, $\boldsymbol{r_j}$ is the coordinate for the atom (nucleus) of type j in the crystal elementary cell, and

$$U_j(\boldsymbol{\tau}) = -\frac{2\pi\hbar^2}{m\gamma} F_j(\boldsymbol{\tau}). \tag{14}$$

According to (14) effective potential energy $U(\boldsymbol{\tau})$ is determined by amplitude $F_j(\boldsymbol{\tau}) = F_{jaa}(\boldsymbol{k}' - \boldsymbol{k} = \boldsymbol{\tau})$. In contrast to the case of chaotic matter, for which the effective potential energy is determined by the amplitude of elastic coherent scattering $f(\boldsymbol{k}' - \boldsymbol{k})$, in a crystal it is defined by the amplitude $F(\boldsymbol{\tau})$ (see [31]), which can be written as:

$$\begin{aligned} F_j(\boldsymbol{k}' - \boldsymbol{k}) = f_j(\boldsymbol{k}' - \boldsymbol{k}) - i\frac{k}{4\pi} \int f_j^*(\boldsymbol{k}'' \\ - \boldsymbol{k}') f_j(\boldsymbol{k}'' - \boldsymbol{k}) d\Omega_{k''}. \end{aligned} \tag{15}$$

where $d\Omega_{k''}$ means integration over all $\boldsymbol{k}''$ directions, with $|\boldsymbol{k}'| = |\boldsymbol{k}| = |\boldsymbol{k}''|$.

The amplitude $F(\boldsymbol{k}' - \boldsymbol{k})$ appears in (14) instead of the amplitude of elastic coherent scattering $f(\boldsymbol{k}' - \boldsymbol{k})$ due to the fact that the wave elastically scattered in a crystal is involved in formation of a coherent wave propagating through the crystal via rescattering by the periodically located centers. This is unlike scattering in amorphous matter.

3 Effective potential energy of a spin-particle moving close to the crystal planes (axes)

Elastic coherent scattering of a particle by an atom is caused by electromagnetic interaction of the particle with the atom electrons and nucleus, as well as weak and strong nuclear interaction with electrons and nucleus. The general expression for the amplitude of elastic scattering of a particle with spin $\frac{1}{2}$ by a spinless or unpolarized nuclei can be written as:

$$\begin{aligned} F(\boldsymbol{q}) = A_{\rm coul}(\boldsymbol{q}) + A_{\rm s}(\boldsymbol{q}) \\ + \big(B_{\rm magn}(\boldsymbol{q}) + B_{\rm s}(\boldsymbol{q})\big)\boldsymbol{\sigma}[\boldsymbol{n} \times \boldsymbol{q}] \\ + \big(B_{\rm we}(\boldsymbol{q}) + B_{\rm wnuc}(\boldsymbol{q})\big)\boldsymbol{\sigma}\boldsymbol{N}_{\rm w} \\ + \big(B_{\rm EDM}(\boldsymbol{q}) + B_{\rm Te}(\boldsymbol{q}) + B_{\rm Tnuc}(\boldsymbol{q})\big)\boldsymbol{\sigma}\boldsymbol{q}, \end{aligned} \tag{16}$$

Here, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and σ_x, σ_y, σ_z are the Pauli matrices, $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$, $A_{\rm coul}(\boldsymbol{q})$ is the spin-independent part of the elastic Coulomb scattering amplitude of a particle by an atom; and $A_{\rm s}(\boldsymbol{q})$ is the spin-independent part of the scattering amplitude, which is caused by strong interaction (the similar contribution caused by weak interaction it is negligibly small and hereinafter is omitted).

The spin-dependent amplitude is proportional to $B_{\rm magn}(\boldsymbol{q})$, determined by electromagnetic spin-orbit interaction, and $B_{\rm s}(\boldsymbol{q})$, responsible for the contribution of the spin-orbit strong interaction to a scattering process of a baryon by a nucleus.

The term proportional to the parity odd pseudo scalar $\boldsymbol{\sigma}\boldsymbol{N}_{\rm w}$ (unit vector $\boldsymbol{N}_{\rm w} = \frac{\boldsymbol{k}'+\boldsymbol{k}}{|\boldsymbol{k}'+\boldsymbol{k}|}$) includes two contributions: $B_{\rm we}(\boldsymbol{q})$, which describes elastic scattering caused by the parity violating weak interaction between the baryon and electrons; and $B_{\rm wnuc}(\boldsymbol{q})$, which describes elastic scattering caused by the parity violating weak interaction between the baryon and the nucleus.

Finally, the term proportional to the time (T) violating (CP non-invariant) pseudo scalar $\boldsymbol{\sigma}\boldsymbol{q}$ includes three contributions: $B_{\rm EDM}(\boldsymbol{q})$ describes elastic scattering of the baryon with EDM by the atom's Coulomb field; $B_{\rm Te}(\boldsymbol{q})$ describes possible short-range T-non-invariant interaction between the baryon and electrons; and $B_{\rm Tnuc}(\boldsymbol{q})$ describes scattering caused by T-non-invariant interaction between the baryon and nucleons.

Using amplitude $F(\boldsymbol{q})$, the potential energy $U_{\rm eff}(\boldsymbol{r})$ can be expressed as a sum of terms that describe contributions of different interactions:

$$U_{\rm eff}(\boldsymbol{r}) = U_{\rm coul}(\boldsymbol{r}) + U_{\rm s}(\boldsymbol{r}) + U_{\rm magn}(\boldsymbol{r}) + U_{\rm so}(\boldsymbol{r}) + U_{\rm w}(\boldsymbol{r}) + U_{\rm T}(\boldsymbol{r}), \quad (17)$$

where $U_{\rm coul}(\boldsymbol{r})$ is the Coulomb potential energy of interaction between baryon and crystal, $U_{\rm s}(\boldsymbol{r})$ describes spin independent contribution of nuclear interactions to the potential energy of interaction with crystal; $U_{\rm magn}(\boldsymbol{r})$ describes contribution to $U_{\rm eff}(\boldsymbol{r})$ caused by interaction between baryon magnetic moment and atoms electric field; $U_{\rm so}(\boldsymbol{r})$ is the contribution caused by spin-orbital nuclear interactions; $U_{\rm w}(\boldsymbol{r})$ describes the contribution caused by parity violating weak interactions; $U_{\rm T}(\boldsymbol{r})$ describes the contribution caused by T-violation interactions between baryon and crystal.

Further we will consider spin-rotation effect for a baryon moving in planar channeling conditions. According to the analysis given in appendices to this paper, contributions to potential energy $U_{\rm eff}(x)$ can be expressed as follows (x-axis is orthogonal to the chosen crystallographic plane),

(a) Potential energy $U_{\rm s}(x)$ (see Appendix A.2),

$$U_{\rm s}(x) = -\frac{2\pi\hbar^2}{m\gamma d_y d_z} N_{\rm nuc}(x) A_{\rm s}(0), \quad (18)$$

where $N_{\rm nuc}(x) = \iint N_{\rm nuc}(x, y', z')dy'dz'$ is the probability density function at point x.

(b) Effective potential energy determined by the anomalous magnetic moment (see Appendix A.3),

$$U_{\rm magn}(x) = -\frac{e\hbar}{2mc}\frac{g-2}{2}\boldsymbol{\sigma}\left[\boldsymbol{E}_{\rm plane}\times\boldsymbol{n}\right] - i\frac{1}{4d_y d_z mc^2}\frac{g-2}{2}\frac{\partial}{\partial x}\overline{\delta V^2(x)}\boldsymbol{\sigma}\boldsymbol{N} = -(\alpha_{\rm m} + i\delta_{\rm m})\boldsymbol{\sigma}\boldsymbol{N}, \quad (19)$$

where

$$\boldsymbol{N} = [\boldsymbol{n}_x \times \boldsymbol{n}], \quad (20)$$

$$\alpha_{\rm m} = \frac{e\hbar}{2mc}\frac{g-2}{2}E_x, \quad (21)$$

$$\delta_{\rm m} = \frac{1}{4d_y d_z mc^2}\frac{g-2}{2}\frac{\partial}{\partial x}\overline{\delta V^2(x)},$$

$$\overline{\delta V^2(x)} = \int\left\{\overline{\left[\int V_{\rm coul}(x, y, z)dz\right]^2} - \left[\overline{\int V_{\rm coul}(x, y, z)dz}\right]^2\right\}dy. \quad (22)$$

Here $\overline{\delta V^2(x)}$ is the mean square fluctuation of energy of Coulomb interaction between baryon and atom (A.31), and $\boldsymbol{n}_x$ is the unit vector along axis x.

(c) Effective potential energy determined by spin-orbit interaction (see Appendix A.4),

$$U_{\rm so} = -(\alpha_{\rm s} + i\delta_{\rm s})\boldsymbol{\sigma}\boldsymbol{N}, \quad (23)$$

where

$$\alpha_{\rm s} = -\frac{2\pi\hbar^2}{m\gamma d_y d_z}\frac{\partial N_{\rm nuc}}{\partial x}B'',$$

$$\delta_{\rm s} = \frac{2\pi\hbar^2}{m\gamma d_y d_z}B'\frac{\partial N_{\rm nuc}}{\partial x}, \quad (24)$$

with B' and B'' being real and imaginary parts of $B_{\rm s}(0)$.

(d) Effective potential energy $U_{\rm w}$ determined by P-odd and T-even interactions (see Appendix A.5),

$$U_{\rm w}(x) = U_{\rm we}(x) + U_{\rm wnuc}(x) = -(\alpha_{\rm w}(x) + i\delta_{\rm w}(x))\boldsymbol{\sigma}\boldsymbol{n},$$
$$\alpha_{\rm w}(x) = \alpha_{\rm we}(x) + \alpha_{\rm wnuc}(x),$$
$$\delta_{\rm w}(x) = \delta_{\rm we}(x) + \delta_{\rm wnuc}(x). \quad (25)$$

Using formulas adduced in Appendix A.4, $\alpha_{\rm w}(x)$ and $\delta_{\rm w}(x)$ read:

$$\alpha_{\rm w}(x) = \frac{2\pi\hbar^2}{m\gamma d_y d_z}(\tilde{B}'_{\rm we}(0)N_{\rm e}(x) + \tilde{B}'_{\rm wnuc}(0)N_{\rm nuc}(x)),$$
$$\delta_{\rm w}(x) = \frac{2\pi\hbar^2}{m\gamma d_y d_z}(\tilde{B}''_{\rm we}(0)N_{\rm e}(x) + \tilde{B}''_{\rm wnuc}(0)N_{\rm nuc}(x)). \quad (26)$$

(e) T-violation interactions lead to the following contribution to potential energy (see Appendix A.6):

$$U_{\rm T}(x) = U_{\rm EDM} + U_{\rm Te} + U_{\rm Tnuc} = -(\alpha_{\rm T}(x) + i\delta_{\rm T}(x))\boldsymbol{\sigma}\boldsymbol{N}_{\rm T}, \quad (27)$$

where

$$\alpha_{\rm T} = \alpha_{\rm EDM} + \alpha_{\rm Te} + \alpha_{\rm Tnuc},$$
$$\delta_{\rm T} = \delta_{\rm EDM} + \delta_{\rm Te} + \delta_{\rm Tnuc}.$$

Interaction energy between electric dipole moment and atoms' electric field is:

$$U_{\rm EDM} = -edE_{\rm plane}(x)\boldsymbol{\sigma}\boldsymbol{N}_{\rm T} - i\frac{d}{2d_y d_z\hbar c}\frac{\partial}{\partial x}\overline{\delta V^2(x)}\boldsymbol{\sigma}\boldsymbol{N}_{\rm T}, \quad (28)$$

where $E_{\rm plane}(x)$ is determined by the electric field of the plane, $\boldsymbol{E}_{\rm plane}(x) = E_x\boldsymbol{N}_{\rm T}$. The operator $U_{\rm EDM}$ can be expressed as:

$$U_{\rm EDM} = -(\alpha_{\rm EDM} + i\delta_{\rm EDM})\boldsymbol{\sigma}\boldsymbol{N}_{\rm T}. \quad (29)$$

Similar to $U_{\rm magn}$, energy $U_{\rm EDM}$ has non-zero both real and imaginary parts. The expression for $U_{\rm magn}$ converts to $U_{\rm EDM}$ by replacements $\frac{g-2}{2} \to 2\frac{d}{\lambda_{\rm c}}$ ($\lambda_{\rm c} = \frac{\hbar}{mc}$ is the Compton wave-length of the particle) and $\boldsymbol{N} \to \boldsymbol{N}_{\rm T}$. Therefore,

$$\frac{U_{\rm EDM}}{U_{\rm magn}} = \frac{4d}{\lambda_{\rm c}(g-2)} = \frac{ed}{\mu_A} = \frac{D}{\mu_A}. \quad (30)$$

Additions to $U_{\rm T}(x)$, which are caused by short-range T-violating interactions of the baryon with electrons and nuclei,

reads as follows:

$$U_{\mathrm{Te}}(x) = i\frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}_{\mathrm{Te}}(0)\frac{dN_{\mathrm{e}}(x)}{dx}\boldsymbol{\sigma}\boldsymbol{N}_{\mathrm{T}},$$

$$U_{\mathrm{Tnuc}}(x) = i\frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}_{\mathrm{Tnuc}}(0)\frac{dN_{\mathrm{nuc}}(x)}{dx}\boldsymbol{\sigma}\boldsymbol{N}_{\mathrm{T}},$$

$$N_{\mathrm{e(nuc)}}(x) = \int N_{\mathrm{e(nuc)}}(x,y,z)dydz. \tag{31}$$

Coefficients $\tilde{B}_{\mathrm{Te}}(0)$ and $\tilde{B}_{\mathrm{Tnuc}}(0)$ both are complex valued:

$$\tilde{B}_{\mathrm{Te(nuc)}}(0) = \tilde{B}'_{\mathrm{Te(nuc)}} + i\tilde{B}''_{\mathrm{Te(nuc)}}.$$

As a result we have:

$$U_{\mathrm{Te(nuc)}}(x) = -(\alpha_{\mathrm{Te(nuc)}} + i\delta_{\mathrm{Te(nuc)}})\boldsymbol{\sigma}\boldsymbol{N}_{\mathrm{T}}, \tag{32}$$

where

$$\alpha_{\mathrm{Te(nuc)}} = \frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}''_{\mathrm{Te(nuc)}}\frac{dN_{\mathrm{e(nuc)}}(x)}{dx},$$

$$\delta_{\mathrm{Te(nuc)}} = \frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}'_{\mathrm{Te(nuc)}}\frac{dN_{\mathrm{e(nuc)}}(x)}{dx}.$$

Thus in the experiment aimed to obtain the limit for the EDM value, the limits for the scattering amplitude, which is determined by T(CP)-noninvariant interactions of baryons with electrons and nuclei, will be obtained as well. It should be emphasized that studies of electric dipole moment and T-odd nuclear interactions in atoms and nuclei for nonrelativistic energies demonstrate that in the experiments to search for an EDM the contributions from all these interactions can be of the same order of magnitude [28]. The obtained values of these scattering amplitudes for different interaction types allows to derive values of corresponding constants, too. The simplest model enabling to describe T-odd nucleon-nucleon interaction is the Yukawa potential [29], which has the form $V_{\mathrm{T}} = -d_{\mathrm{T}}\boldsymbol{\sigma}\boldsymbol{r}\frac{e^{-\varkappa_{\mathrm{T}}r}}{r^2}$. Here d_{T} is the interaction constant, $\varkappa_{\mathrm{T}} \sim \frac{1}{M_{\mathrm{T}}}$, where M_{T} is the mass of heavy particles, exchange of which leads to interaction V_{T} [29]. Using this potential one can obtain the equations for $\alpha_{\mathrm{Te(nuc)}}$ with replacement in (A.29) of $V_{\mathrm{coul}} + V_{\mathrm{EDM}}$ by $V_{\mathrm{coul}} + V_{\mathrm{T}}$.

It should be noted that constant d_{T} for interaction between a heavy baryon and a nucleon can be greater than that for nucleon-nucleon interaction. This effect can be explained by the reasoning similar to that explaining the expected EDM growth for the heavy baryon. T-odd interaction mixing of stationary states with different parity for a heavy baryon is more effective then that for light baryons due to probably smaller spacing between energy levels corresponding to these states.

Let's note that since the scattering amplitude F is a complex value, thus the potential energy U is also a complex value. The real part of this energy describes changes in particles' energy as a result of interaction with matter, while the imaginary part describes absorption.

Every spin dependent contribution to U has the following structure:

$$A = -(\alpha + i\beta)\boldsymbol{\sigma}\boldsymbol{\zeta}, \tag{33}$$

where $\boldsymbol{\zeta}$ is the unit vector. Comparing this expression with the interaction energy between magnetic moment $\boldsymbol{\mu}$ and magnetic field $\boldsymbol{B}$:

$$U_{\mathrm{magn}} = -\mu\boldsymbol{\sigma}\boldsymbol{B}. \tag{34}$$

It can be seen that terms proportional to α in A causes spin rotation around $\boldsymbol{N}$. The imaginary part shows that absorption in matter depends on spin orientation with respect to $\boldsymbol{N}$. As a result a spin component directed along $\boldsymbol{N}$ appears (spin dichroism arises [13]).

The analogy between (33) and (34) leads us to the conclusion that a particle in matter is affected by the pseudomagnetic fields caused by strong and weak interactions (for low energy neutrons the effects determined by such fields were discovered and have been investigated for many years, see [13]).

Let's now consider a particle moving in a straight (unbent) crystal. The expression for U contains terms proportional to either the electric field projection onto x axis or derivatives of density of electrons $\frac{dN_{\mathrm{e}}(x)}{dx}$ and nuclei density $\frac{dN_{\mathrm{nuc}}(x)}{dx}$. As a result, particles moving between the planes are affected by pseudomagnetic fields, which change their sign due to transverse oscillations of the channelled particles. This leads to suppression of spin rotation in such fields (suppression decreases with growth of particle energy). The exceptions are the spin rotation and the spin dichroism effects caused by weak P-odd T-even interaction, those both increase with growth of crystal thickness; these effects also occur in amorphous medium [13].

4 P and CP violating spin rotation in bent crystals

Expressions for the interaction energy between a baryon and a crystal plane (axis), obtained above, allow us to find the equation describing evolution of the particle polarization vector in a bent crystal. These equations differ from those describing spin evolution in external electromagnetic fields in vacuum, by the presence of additional contributions from P and T (CP) noninvariant interactions between electrons and nuclei. Moreover, a new effect, which is caused by non-elastic processes, arises: along with the spin precession around vectors $\boldsymbol{N}_{\mathrm{m}}$, $\boldsymbol{N}_{\mathbf{T}}$ and $\boldsymbol{n}$, spin components parallel to these vectors appear, thus spin dichroism occurs.

Equations, which describe spin rotation in this case, can be obtained by the following approach [13]. Spin wave function $|\Psi(t)\rangle$ meets the equation as follows:

$$ih\frac{\partial|\Psi(t)\rangle}{\partial t} = U_{\text{eff}}|\Psi(t)\rangle. \tag{35}$$

Baryon polarization vector $\boldsymbol{\xi}$ can be found via $|\Psi(t)\rangle$:

$$\boldsymbol{\xi} = \frac{\langle\Psi(t)|\boldsymbol{\sigma}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}. \tag{36}$$

Thus the equation for spin rotation of a particle ($\gamma \gg 1$), which moves in a bent crystal, reads as follows:

$$\begin{aligned}\frac{d\boldsymbol{\xi}}{dt} = &-\frac{e(g-2)}{2mc}[\boldsymbol{\xi}\times[\boldsymbol{n}\times\boldsymbol{E}]] - \frac{2}{\hbar}\delta_{\text{m}}\{\boldsymbol{N}_{\text{m}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{m}}\boldsymbol{\xi})\}\\ &-\frac{2}{\hbar}\alpha_{\text{so}}[\boldsymbol{\xi}\times\boldsymbol{N}_{\text{m}}] - \frac{2}{\hbar}\delta_{\text{so}}\{\boldsymbol{N}_{\text{m}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{m}}\boldsymbol{\xi})\}\\ &+\frac{2ed}{\hbar}[\boldsymbol{\xi}\times\boldsymbol{E}] + \frac{2}{\hbar}\delta_{\text{EDM}}\{\boldsymbol{N}_{\text{T}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{T}}\boldsymbol{\xi})\}\\ &+\frac{2}{\hbar}(\alpha_{\text{Te}} + \alpha_{\text{Tnuc}})[\boldsymbol{\xi}\times\boldsymbol{N}_{\text{T}}]\\ &+\frac{2}{\hbar}(\delta_{\text{Te}} + \delta_{\text{Tnuc}})\{\boldsymbol{N}_{\text{T}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{T}}\boldsymbol{\xi})\}\\ &+\frac{2}{\hbar}\alpha_{\text{w}}[\boldsymbol{\xi}\times\boldsymbol{n}] - \frac{2}{\hbar}\delta_{\text{w}}\{\boldsymbol{n} - \boldsymbol{\xi}(\boldsymbol{\xi}\boldsymbol{n})\}.\end{aligned} \tag{37}$$

Let us note that vector $[\boldsymbol{n}\times\boldsymbol{E}]$ is parallel to vector $\boldsymbol{N}_{\text{m}} = [\boldsymbol{n}\times\boldsymbol{n}_x]$ and $\boldsymbol{N}_{\text{m}} = -\boldsymbol{N}$ (see (19), (23) and (A.28)), vector $\boldsymbol{E}$ is parallel to $\boldsymbol{N}_{\text{T}} = \boldsymbol{n}_x, \boldsymbol{n}_x \perp \boldsymbol{n}$. Equation (37) can be also expressed as:

$$\begin{aligned}\frac{d\boldsymbol{\xi}}{dt} = &-\left(\frac{e(g-2)}{2mc}E_x(x) + \frac{2}{\hbar}\alpha_{\text{so}}(x)\right)[\boldsymbol{\xi}\times\boldsymbol{N}_{\text{m}}]\\ &-\frac{2}{\hbar}\big(\delta_{\text{m}}(x) + \delta_{\text{so}}(x)\big)\big\{\boldsymbol{N}_{\text{m}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{m}}\boldsymbol{\xi})\big\}\\ &+\frac{2}{\hbar}\big(edE_x(x) + \alpha_{\text{Te}}(x) + \alpha_{\text{Tnuc}}(x)\big)[\boldsymbol{\xi}\times\boldsymbol{N}_{\text{T}}]\\ &+\frac{2}{\hbar}\big(\delta_{\text{EDM}}(x) + \delta_{\text{Te}}(x) + \delta_{\text{Tnuc}}(x)\big)\big\{\boldsymbol{N}_{\text{T}} - \boldsymbol{\xi}(\boldsymbol{N}_{\text{T}}\boldsymbol{\xi})\big\}\\ &+\frac{2}{\hbar}\alpha_{\text{w}}[\boldsymbol{\xi}\times\boldsymbol{n}] - \frac{2}{\hbar}\delta_{\text{w}}\big\{\boldsymbol{n} - \boldsymbol{\xi}(\boldsymbol{\xi}\boldsymbol{n})\big\}.\end{aligned} \tag{38}$$

For further comparison let us use the rearranged Eq. (4). The electric field $\boldsymbol{E}$ can be decomposed as $\boldsymbol{E} = \boldsymbol{E}_\perp + \boldsymbol{E}_\parallel$, where $\boldsymbol{E}_\perp$ is an electric field component perpendicular to the particle velocity, $\boldsymbol{E}_\parallel$ is an electric field component parallel to the particle velocity. In the considered case of high energies ($\gamma \gg 1$) contribution of $\boldsymbol{E}_\parallel$ component to spin evolution (according to (4)) is noticeably suppressed. This contribution in the second term is proportional to $1 - \beta^2 = 1/\gamma^2 \ll 1$.

Following [16] let us introduce polarization vector $\boldsymbol{\xi} = \frac{\boldsymbol{S}}{S}$. Equations for baryons with $S = \frac{1}{2}$ are considered further. For high-energy particles $\boldsymbol{\beta}$ can be considered equal to unit vector $\boldsymbol{n}$. As a result, Eq. (4) can be rewritten as:

$$\begin{aligned}\frac{d\boldsymbol{\xi}}{dt} = &-\frac{e(g-2)}{2mc}\left[\boldsymbol{\xi}\times[\boldsymbol{n}\times\boldsymbol{E}_\perp]\right]\\ &+\frac{2D}{\hbar}\left[\boldsymbol{\xi}\times\boldsymbol{E}_\perp\right].\end{aligned} \tag{39}$$

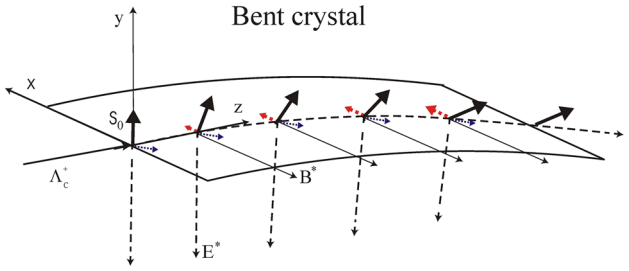

Fig. 2 Spin rotation caused by magnetic moment and T-reversal violation interactions (including EDM). Black arrows represent spin rotation about effective magnetic field (about bent axis, direction $\boldsymbol{N}_{\text{m}}$), red arrows represent spin component caused by rotation of EDM about direction $\boldsymbol{N}_{\text{T}}$, purple arrows represent the new effect – appearance of the spin component directed along $\boldsymbol{N}_{\text{m}}$ owing to the spin dichroism (spin rotation and dichroism in direction $\boldsymbol{N}_{\text{T}}$ owing to T-reversal violation and P-violating interactions, is not shown here for simplicity)

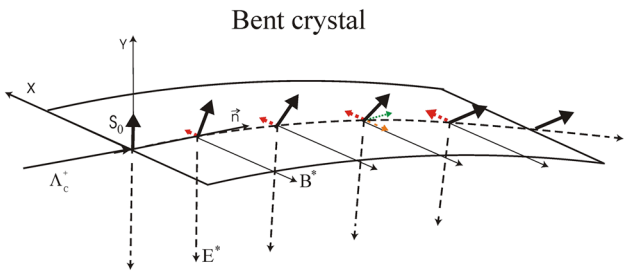

Fig. 3 Black and red arrows show spin rotation caused by magnetic moment, T-reversal violation interactions (including EDM), P-violation spin rotation about direction $\boldsymbol{n}$ (orange arrow) and spin component in direction $\boldsymbol{n}$ caused by spin dichroism (green arrow). Spin components caused by spin dichroism in direction $\boldsymbol{N}_{\text{m}}$ and direction $\boldsymbol{N}_{\text{T}}$ are not shown for simplicity

It can be seen that in comparison with (39) expressions (37, 38) contain new terms. According to (38), baryon spin rotates around three axes [30]: effective magnetic field direction $\boldsymbol{N}_{\text{m}}||[\boldsymbol{n}\times\boldsymbol{E}]$, electric field direction $\boldsymbol{N}_{\text{T}}||\boldsymbol{E}$ and momentum direction $\boldsymbol{n}$.

Non-elastic processes in crystals result in the new effect: terms proportional to δ lead to appearance of the polarization vector components in directions defined by vectors $\boldsymbol{N}_{\text{m}}$, $\boldsymbol{N}_{\text{T}}$ and $\boldsymbol{n}$. Let's pay attention to the fact that appearance of the spin component directed along effective magnetic field B^* ($\boldsymbol{N}_{\text{m}}$ direction) is caused by both spin rotation around direction of the electric field $\boldsymbol{E}$ ($\boldsymbol{N}_{\text{T}}$ direction), due to T-noninvariant violation, and spin dichroism processes, due to non-elastic processes between the magnetic moment and atoms of bent crystal. It can be seen that appearance of such spin component imitates the result of the T-noninvariant rotation (Figs. 2, 3).

From Eq. (38) the result, which was obtained in [30], can be derived. Namely, when an unpolarized beam enters a crystal, the beam polarization in direction of vectors $\boldsymbol{N}_{\text{m}}$, $\boldsymbol{N}_{\text{T}}$ and $\boldsymbol{n}$ arises.

Contributions to Eq. (38), which are caused by the interaction between a baryon and nuclei, depend on distribution of nuclei density $N_{\rm nuc}(x)$ (see terms proportional to $\alpha_{\rm so}(x)$, $\delta_{\rm so}(x)$, $\alpha_{\rm Tnuc}(x)$, $\delta_{\rm Tnuc}(x)$). As a result, for positively charged particles, moving in the channel along the trajectories located in the center of the channel, such contributions are suppressed.

Thus, according to (38), when conducting and interpreting experiments aimed for EDM, it should be considered that measurement of spin rotation provides information about the sum of contributions to T-noninvariant rotation. The mentioned rotation is determined by both EDM and short-range CP-noninvariant interactions.

Let's evaluate the most important new effects described by the Eq. (38) and consider the contribution to spin rotation caused by spin dichroism in direction of $\boldsymbol{N}_{\rm m}$. According to (19) coefficient $\delta_{\rm m}$ reads as follows:

$$\begin{aligned}\delta_{\rm m} &= \frac{1}{4d_y d_z mc^2}\frac{g-2}{2}\frac{\partial}{\partial x}\overline{\delta V^2(x)}\\ &= \frac{1}{4d_y d_z}mc^2\frac{g-2}{2}\frac{\partial}{\partial x}\\ &\times\int\left\{\overline{\left[\int V_{\rm coul}(x,y,z)dz\right]^2}\right.\\ &\left.-\left[\int\overline{V_{\rm coul}(x,y,z)}dz\right]^2\right\}dy,\end{aligned}\tag{40}$$

where $V_{\rm coul}(x,y,z) = \sum_i V_{\rm e}(x-x_i, y-y_i, z-z_i) - V_{\rm nuc}(x-\eta_{fx}, y-\eta_{fy}, z-\eta_{fz})$, x_i, y_i, z_i are the coordinates of the i-th electron in atom, $\eta_{fx}, \eta_{fy}, \eta_{fz}$ are the coordinates of the atom nucleus. Let us choose the position of equilibrium point for the oscillating nucleus as the origin of coordinates. The overline denotes averaging of electrons' and nuclei' positions over electron density distribution and nuclei oscillations; in other words, averaging with wave-functions of atoms in crystal. By means of these functions, the density distribution can be expressed as follows:

$$N(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots \boldsymbol{r}_z, \boldsymbol{\eta}) = N_{\rm e}(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots \boldsymbol{r}_z, \boldsymbol{\eta})N_{\rm nuc}(\boldsymbol{\eta}),\tag{41}$$

where $N_{\rm e}$ is the density distribution of electrons in atom, $N_{\rm nuc}$ is the density distribution of nucleus oscillations.

Let's introduce the function $W(x,y) = \int V(x,y,z)dz$. From (40) we have:

$$\begin{aligned}W(x,y) &= \sum_i\int V_{\rm e}(x-x_i, y-y_i, \xi)d\xi\\ &\quad -\int V_{\rm nuc}(x-\eta_x, y-\eta_y, \xi)d\xi\\ &= \sum_i W_{\rm e}(x-x_i, y-y_i) - W_{\rm nuc}(x-\eta_x, y-\eta_y),\end{aligned}\tag{42}$$

$$\begin{aligned}\overline{W^2(x,y)} &= \int\left[\sum_i W_{\rm e}(\boldsymbol{\rho}-\boldsymbol{\rho}_i) - W_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp)\right]^2\\ &\times N_{\rm e}(\boldsymbol{\rho}_1-\boldsymbol{\eta}_1, \ldots \boldsymbol{\rho}_z-\boldsymbol{\eta}_\perp)N_{\rm nuc}(\boldsymbol{\eta}_\perp)d^2\rho_1 d^2\rho_z d^2\boldsymbol{\eta}_\perp,\end{aligned}\tag{43}$$

where $\boldsymbol{\rho} = (x,y)$, $\boldsymbol{\eta}_\perp = (\eta_x, \eta_y)$, Z is the number of electrons in atom. In other words:

$$\begin{aligned}\overline{W^2(\boldsymbol{\rho})} &= \int\left\{\left[\sum_i W_{\rm e}(\boldsymbol{\rho}-\boldsymbol{\rho}_i)\right]^2\right.\\ &\left.-2\sum W_{\rm e}(\boldsymbol{\rho}-\boldsymbol{\rho}_i)W_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp) + W^2_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp)\right\}\\ &\times N_{\rm e}(\boldsymbol{\rho}_1-\boldsymbol{\eta}_1, \ldots \boldsymbol{\rho}_z-\boldsymbol{\eta}_\perp)N_{\rm nuc}(\boldsymbol{\eta}_\perp)d^2\rho_1 d^2\rho_z d^2\boldsymbol{\eta}_\perp.\end{aligned}\tag{44}$$

The result of averaging $\overline{W^2(\boldsymbol{\rho})}$ includes two contributions: that for density distribution of a single electron in atom and one dependent on coordinates of two electrons in the atom, which describes pair correlations in electrons positions in the atom. However, the influence of pair correlations will be ignored during the estimations. As a result the expression (44) can be represented as follows:

$$\begin{aligned}\overline{W^2(\boldsymbol{\rho})} &= \int d^2\eta_\perp\left\{Z\left[\langle W^2_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle_{\rm e} - \langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle^2_{\rm e}\right]\right.\\ &+Z^2\langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle^2_{\rm e} - 2Z\langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle W_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp)\\ &\left.+W^2_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp)\right\},\end{aligned}\tag{45}$$

where

$$\begin{aligned}\langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle_{\rm e} &= \int W_{\rm e}(\boldsymbol{\rho}-\boldsymbol{\rho}')N_{\rm e}(\boldsymbol{\rho}'-\boldsymbol{\eta}_\perp)d^2\rho',\\ \langle W^2_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle_{\rm e} &= \int W^2_{\rm e}(\boldsymbol{\rho}-\boldsymbol{\rho}')N_{\rm e}(\boldsymbol{\rho}'-\boldsymbol{\eta}_\perp)d^2\rho'.\end{aligned}\tag{46}$$

That means

$$\begin{aligned}\overline{W^2(\boldsymbol{\rho})} &= \int d^2\boldsymbol{\eta}_\perp\left\{Z[\langle W^2_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle_{\rm e} - \langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle^2_{\rm e}]+\right.\\ &\left.+(Z\langle W_{\rm e}(\boldsymbol{\rho},\boldsymbol{\eta}_\perp)\rangle_{\rm e} - W_{\rm nuc}(\boldsymbol{\rho}-\boldsymbol{\eta}_\perp))^2\right\}N_{\rm nuc}(\boldsymbol{\eta}_\perp).\end{aligned}\tag{47}$$

According to (40, 42–44) the function

$$\int\left[\overline{W^2(\boldsymbol{\rho})} - \overline{W(\boldsymbol{\rho})}^2\right]dy$$

determines the expression for $\delta_{\rm m}$. It should be noted that when fluctuations of Coulomb interaction of a baryon with atoms, caused by nuclei oscillations, are neglected, only those fluctuations, which are determined by distribution of electrons' coordinates in the atom, remain.

As a result, the following equation for $\delta_{\rm m}$ can be obtained:

$$\delta_{\rm m} = \frac{1}{4d_y d_z} mc^2 \frac{g-2}{2} \frac{\partial}{\partial x} \int \left\{ \overline{W^2(x,y)} - \overline{W(x,y)}^2 \right\} dy, \tag{48}$$

where

$$\overline{W(\boldsymbol{\rho})} = \int \left\{ Z \overline{W_{\rm e}(\boldsymbol{\rho}, \boldsymbol{\eta}_\perp)}^e - W_{\rm nuc}(\boldsymbol{\rho} - \boldsymbol{\eta}_\perp) \right\} \times N_{\rm nuc}(\boldsymbol{\eta}_\perp) d^2 \boldsymbol{\eta}_\perp,$$

$$\overline{W_{\rm e}(\boldsymbol{\rho}, \boldsymbol{\eta}_\perp)}^e = \int W_{\rm e}(\boldsymbol{\rho} - \boldsymbol{\rho}') N_{\rm e}(\boldsymbol{\rho}' - \boldsymbol{\eta}_\perp) d^2 \rho'.$$

Let us neglect nuclei oscillation to estimate the $\delta_{\rm m}$ value. In this case contribution to the meansquare fluctuation of the energy of Coulomb interaction between a baryon and an atom is caused by fluctuations of positions of electrons in the atom. As a result the expression for $\delta_{\rm m}$ reads as follows:

$$\delta_{\rm m} = \frac{1}{4d_y d_z} mc^2 \frac{g-2}{2} \times \frac{\partial}{\partial x} \int \left\{ z[\langle W_{\rm e}^2(x,y) \rangle_{\rm e} - \langle W_{\rm e}(x,y) \rangle_{\rm e}^2] \right\} dy, \tag{49}$$

where function $W_{\rm e}(x,y) = \int V(x-x', y-y', z) dz$. In expression (49) averaging of electrons distribution in the atom is conducted over variables x', y' (see explanations for expression (45)).

The shielded Coulomb potential is used to estimate fluctuations of Coulomb interaction of a baryon with atoms. Let's suppose that electrons are distributed uniformly over the area, which size is determined by the shielding radius. In this case, the following estimation for $\delta_{\rm m}$ can be obtained: $\delta_{\rm m} \sim 10^8 \div 10^9$ s^{-1} depending on the position of the baryon trajectory in the planar channel. According to [10,11] the expected sensitivity for the EDM measurement experiments is $ed \sim 10^{-17} e$ cm . Spin rotation frequency $\Omega_{\rm EDM} = \frac{2edE}{\hbar}$. The field E affecting baryons in a bent crystal can be obtained from the expression $E = \frac{m\gamma c^2}{eR}$, where R is the radius of the crystal curvature. Therefore $\Omega_{\rm EDM} = 2\frac{d}{R}\frac{W}{\hbar}$, where W is the baryon energy. For $R = 30$ m, $d \sim 10^{-17}$ cm and $W = 1$ TeV we have $\Omega_{\rm EDM} \simeq 10^7$ s^{-1}. As a result, the non-elastic processes, which are caused by magnetic moment scattering, can imitate the EDM contribution. Surely, more detailed computer simulation is needed.

The contributions of P-odd and T-even rotation to the general spin rotation effect can be evaluated by the following way. Precession frequency $\Omega_{\rm w}$ is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated by Fermi theory [27] for the energies, which are equal or smaller than those necessary for W and Z bosons production:

$$B'_{\rm we(nuc)} \sim G_{\rm F} k = \frac{10^{-5}}{m_{\rm p}^2} k = 10^{-5} \frac{\hbar}{m_{\rm p} c} \frac{m}{m_{\rm p}} \gamma = 10^{-5} \lambda_{\rm cp} \frac{m}{m_{\rm p}} \gamma, \tag{50}$$

where $G_{\rm F}$ is the Fermi constant, $m_{\rm p}$ is the proton mass, $\lambda_{\rm cp}$ is the proton Compton wavelength. For particles with energy from hundreds of GeV to TeV, $B'_{\rm we(nuc)} \sim G_{\rm F} k = 10^{-16}$ cm.

For different particle trajectories in a bent crystal the value of precession frequency $\Omega_{\rm w}$ could vary in the range $\Omega_{\rm w} \simeq 10^3 \div 10^4$ s^{-1}. Therefore, when a particle passes 10 cm in a crystal, its spin undergoes additional rotation around momentum direction at angle $\vartheta_{\rm p} \simeq 10^{-6} \div 10^{-7}$rad. For a heavy baryon this effect grows as a result of the mechanism similar to that causing the EDM growth (see the explanation for the growth of constant $d_{\rm T}$ hereinabove).

Absorption caused by parity violating weak interaction also contributes to change of spin direction (see in (37,38) the terms proportional to $\delta_{\rm w}$). This rotation is caused by the imaginary part of the weak scattering amplitude and is proportional to the difference of total scattering cross-sections $\sigma_{\uparrow\uparrow}$ and $\sigma_{\downarrow\uparrow}$ [30]. This difference is proportional to the factor, determined by interference of Coulomb and weak interactions for baryon scattering by an electron, as well as by interference of strong (Coulomb) and weak interactions for baryon scattering by nuclei [30]:

$$\sigma_{\uparrow\uparrow(\downarrow\uparrow)} = \int |f_{\rm coul(nuc)} + B_{0\rm w} \pm B_{\rm w}|^2 d\Omega, \tag{51}$$

$$\sigma_{\uparrow\uparrow} - \sigma_{\downarrow\uparrow} = 2 \int \left[(f_{\rm coul(nuc)} + B_{0\rm w}) B^* + (f_{\rm coul(nuc)} + B_{0\rm w})^* B \right] d\Omega. \tag{52}$$

When baryon trajectory passes in the area, where collisions with nuclei are important (this occurs in the vicinity of potential barrier for positively charged particles), the value $\delta_{\rm w} \sim 10^6 \div 10^7$ s^{-1}. Similar to the real part $B'_{\rm we(nuc)}$ for the case of heavy baryons the difference in cross-sections grows. Multiple scattering also contributes to spin rotation and depolarization [8,13,26,30,31]. Particularly, due to interference of magnetic, weak and Coulomb interactions, the root-mean-square scattering angle appears changed and dependent on spin orientation with respect to vectors $\boldsymbol{N}_{\rm m}$, $\boldsymbol{N}_{\mathbf{T}}$ and $\boldsymbol{n}$. Moreover, study of spin characteristics of particles, which are scattered on different angles by axes (planes), even when the crystal is unbent, gives the opportunity to derive the scattering amplitude (16) both for charged and neutral short-lived particles [30].

When measuring MDM and T-odd spin rotation in a bent crystal, one can eliminate parity violating rotation by the following way. Turning the crystal 180° around the direction of incident baryon momentum leaves P-odd spin rotation

unchanged, while the sign of MDM and T-odd spin rotations changes due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions the angle of rotation, which does not depend on P-odd effect, could be obtained. Summation of rotation angle measurements provides opportunity to measure P-odd spin rotation angle and suppress the contribution of MDM and T-odd spin rotation. Such measurement is similar to the one proposed in [10] to control systematic uncertainties: two crystals (with up and down bending) should be used to induce opposite spin precession to channelled baryons. Separation of contributions caused by MDM and T-odd spin rotation becomes possible when comparing experimental results for two different initial orientations of polarization vector $\boldsymbol{\xi}$. Namely, $\boldsymbol{\xi} \parallel \boldsymbol{N}_{\mathrm{m}}$ and $\boldsymbol{\xi} \parallel \boldsymbol{N}_{\mathbf{T}}$, i.e. the initial $\boldsymbol{\xi}$ is parallel to the bending axis of the crystal or to $\boldsymbol{E}$ direction.

5 Conclusion

Besides electromagnetic interaction, channelled particles moving in a crystal experience weak interaction with electrons and nuclei, as well as strong interaction with nuclei. When analyzing particle's spin rotation, which is caused by electric dipole moment interaction with electric field, one should consider non-invariant spin rotations both P-odd, T-even and P-odd, T-odd, resulting from weak interaction with electrons and nuclei. As demonstrated hereinabove, spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurement of constants determining T-odd, P-odd (CP) violating interactions and P-odd, T-even interactions of baryons with electrons and nucleus (nucleons), similarly to the possibility of measuring electric and magnetic moments of charm, beauty and strange charged baryons. For a particle moving in a bent crystal a new effect, which is caused by non-elastic processes, arises: in addition to the spin precession around three directions $\boldsymbol{N}_{\mathrm{m}}$, $\boldsymbol{N}_{\mathrm{T}}$, $\boldsymbol{n}$, the spin dichroism effect causes the appearance of the spin components in directions of $\boldsymbol{N}_{\mathrm{m}}$, $\boldsymbol{N}_{\mathrm{T}}$, $\boldsymbol{n}$. To separate P-noninvariant rotation from the MDM- and EDM-induced (T-odd) spin rotations the method of turning crystal by 180° or the one proposed in [10], using two crystals with up and down bending to control systematic uncertainties, can be used.To separate contributions caused by MDM and T-odd interactions, two crystals placed perpendicular to each other can be used.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the data used are taken from publications of other authors, which are properly cited within the manuscript.]

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A Appendices

A.1 Scattering amplitude

According to the analysis provided in section (2), coherent rescattering of the particle by periodically spaced atoms in crystal leads to elastic scattering of waves in crystal which doesn't cause absorption of these waves (unlike in amorphous matter). As a result effective potential energy of the particle in crystal is determined not by the amplitude of the elastic coherent scattering f, as in amorphous matter, but by some more complicated amplitude F which imaginary part does not contain contribution of elastic coherent scattering section. Features of the amplitude F are considered in this section. Following (15) we have:

$$F_j(0) = f_j(0) - i\frac{k}{4\pi}\int f_j^*(\boldsymbol{k}'' - \boldsymbol{k}')\, f_j(\boldsymbol{k}'' - \boldsymbol{k}) d\Omega_{k''}. \tag{A.1}$$

The integral in (A.1) is equal to the total cross-section of the elastic coherent scattering by a nucleus (atom). According to the optical theorem:

$$Imf_j(0) = \frac{k}{4\pi}\sigma_{\mathrm{tot}} = \frac{k}{4\pi}\sigma_{\mathrm{elast}} + \frac{k}{4\pi}\sigma_{\mathrm{non-elast}}. \tag{A.2}$$

In contrast to the case of matter with chaotically distributed scatterers, the amplitude $F_j(0)$ in crystal is expressed as follows:

$$F_j(0) = \tilde{f}_j(0),\ \tilde{f}_j(0) = f_j(0) - \frac{k}{4\pi}\sigma_{\mathrm{elast}}. \tag{A.3}$$

In other words, the cross-section of elastic coherent scattering in crystal does not contribute to the imaginary part of amplitude $F_j(0)$. This imaginary part is solely determined by the cross-section of non-elastic processes:

$$F_j(0) = ReF_j(0) + iImF_j(0) = ReF_j(0) + i\frac{k}{4\pi}\sigma_{\mathrm{non-elast}}. \tag{A.4}$$

The nonzero-angle scattering possesses similar features. This fact becomes clear when one uses the equality, which is correct for elastic scattering [16]:

$$Imf_{\rm elast}(\boldsymbol{k}'-\boldsymbol{k}) = \frac{k}{4\pi}\int f^{*}_{\rm elast}(\boldsymbol{k}''-\boldsymbol{k}')f_{\rm elast}(\boldsymbol{k}''-\boldsymbol{k})d\Omega_{k''}, \tag{A.5}$$

and subtracts the elastic scattering contribution from the imaginary part of $f(\boldsymbol{k}'-\boldsymbol{k})$ using (15).

The same result can be obtained by considering the interaction with the scatterer in terms of the perturbation theory. In case when the first Born approximation is used, the scattering amplitude $f^{(1)}(\boldsymbol{k}'-\boldsymbol{k})$ has zero imaginary part:

$$Imf^{(1)}_{aa}(\boldsymbol{k}'-\boldsymbol{k}) = 0. \tag{A.6}$$

The non-zero imaginary part arises when using the second-order Born approximation.

Let us remind that T-operator, which determines the scattering amplitude (see (9)), satisfies the following equation [15,17]:

$$T = V + V\frac{1}{E - H_0 + i\eta}T, \tag{A.7}$$

where V is the interaction energy, H_0 is the Hamilton operator of colliding systems located at large distance from each other.

As a result, for the elastic coherent scattering amplitude f_{aa} with the accuracy up to the second order terms over the interaction energy, one gets:

$$f_{aa}(\boldsymbol{k}'-\boldsymbol{k}) = -(2\pi)^2\frac{m\gamma}{\hbar^2}\left(\langle\Phi_{\boldsymbol{k}'a}|V|\Phi_{\boldsymbol{k}a}\rangle + \langle\Phi_{\boldsymbol{k}'a}|V\frac{1}{E_a(\boldsymbol{k}) - H_0 + i\eta}V|\Phi_{\boldsymbol{k}a}\rangle\right), \tag{A.8}$$

where $\Phi_{\boldsymbol{k}a}$ is an eigenfunction of Hamilton operator H_0,

$$\Phi_{\boldsymbol{k}a} = \frac{1}{(2\pi)^{3/2}}e^{i\boldsymbol{k}\boldsymbol{r}}\Phi_a, \tag{A.9}$$

Φ_a is the wave function of a scatterer stationary state and $H_0\Phi_{\boldsymbol{k}a} = E_a(\boldsymbol{k})\Phi_{\boldsymbol{k}a}$. Using the completeness of function $\Phi_{\boldsymbol{k}a}$ and replacing "1" in (A.8) by

$$\sum_{\boldsymbol{k}''b}|\Phi_{\boldsymbol{k}''b}\rangle\langle\Phi_{\boldsymbol{k}''b}| = 1, \tag{A.10}$$

one obtains the sum over the intermediate states b, which includes states with $b = a$. This term contains the following expression:

$$\frac{1}{E_a(\boldsymbol{k}) - E_a(\boldsymbol{k}'') + i\eta} = P\frac{1}{E_a(\boldsymbol{k}) - E_a(\boldsymbol{k}'')} - i\pi\delta(E_a(\boldsymbol{k}) - E_a(\boldsymbol{k}'')). \tag{A.11}$$

The "P" symbol means principal-value integrals. This real part contribution is small as compared to the first Born approximation, therefore, it will not be further considered. The imaginary term in (A.11), which is proportional to the δ function, leads to occurrence of an imaginary part in amplitude $f(\boldsymbol{k}'-\boldsymbol{k})$. Substitution of the expression with δ-function into (A.8) makes it obvious that the term in the sum, in which $b = a$, is equivalent to the term subtracted from the amplitude $f_{aa}(\boldsymbol{k}'-\boldsymbol{k})$ in (15). As a result only the terms caused by non-elastic processes and reactions with $b \neq a$ make contributions to the imaginary part of the amplitude in crystal. To simplify the consideration below, when expressing F, the contribution from the elastic coherent scattering to the imaginary part of the amplitude f_{aa} will not be considered and the second term in (15) also will not be written explicitly.

A.2 Effective potential energy of a spin-particle moving close to the crystal planes (axes)

Suppose a high energy particle enters into a crystal at a small angle to the crystallographic planes (axes), close to the Lindhard angle. This motion is determined by the plane (axis) potential $U_{\rm eff}(x)$ ($U_{\rm eff}(\boldsymbol{\rho})$), which can be obtained from $U_{\rm eff}(\boldsymbol{r})$ by averaging over the distribution of atoms (nuclei) in a crystal plane (axis). Similar result can be obtained by removing from the sum (12) either all the terms with $\tau_y \neq 0$, $\tau_z \neq 0$ for the case of planes or $\tau_z \neq 0$ for the case of axes.

As a consequence for the potential of periodically placed axes one can write:

$$U_{\rm eff}(\boldsymbol{\rho}) = \sum_{\boldsymbol{\tau}_\perp}U(\boldsymbol{\tau}_\perp, \boldsymbol{\tau}_z = 0)e^{i\boldsymbol{\tau}_\perp\boldsymbol{\rho}}, \tag{A.12}$$

where the z axis of the coordinate system is directed along the crystallographic axis.

For periodically placed planes the potential reads as follows:

$$U_{\rm eff}(x) = \sum_{\boldsymbol{\tau}_x}U(\tau_x, \tau_y = 0, \tau_z = 0)e^{i\tau_x x}, \tag{A.13}$$

with y, z planes of the coordinate system parallel to the crystallographic planes of chosen family. Let's remind that according to (13–14) the magnitude $U(\boldsymbol{\tau})$ is expressed in terms of amplitude $F(\boldsymbol{\tau})$.

Let's express the amplitude $F(\boldsymbol{q})$ as Fourier transformation of function $F(\boldsymbol{r})$:

$$F(\boldsymbol{q}) = \int F(\boldsymbol{r}')e^{-i\boldsymbol{q}\boldsymbol{r}'}d^3r'. \tag{A.14}$$

We perform summation of τ_x and $\boldsymbol{\tau}_\perp$ in (A.12, A.13) using the following expression:

$$\sum_{\tau_x}e^{i\tau_x x} = d_x\sum_{l}\delta(x - X_l), \tag{A.15}$$

where d_x is the lattice period along axis x; X_l are coordinates of plane l, and

$$\sum_{\tau_x,\tau_y} e^{i\tau_\perp \rho} = d_x d_y \sum_l \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_l), \tag{A.16}$$

where $\boldsymbol{\rho_l}$ is a coordinate of the axis, located in point $\boldsymbol{\rho_l}$; d_x, d_y are the lattice periods along axes x and y, respectively.

As a result, one obtains the following expression for the effective interaction potential energy between the incident particle and the plane (axis) (the lattice is assumed to consist of atoms of one kind):

$$\begin{aligned} U(x) &= -\sum_{\tau_x} \frac{2\pi\hbar^2}{m\gamma V} F(q_x = \tau_x, q_y = q_z = 0) e^{i\tau_x x} \\ &= -\frac{2\pi\hbar^2}{m\gamma V d_y d_z} F(x, q_y = q_z = 0), \end{aligned} \tag{A.17}$$

$$\begin{aligned} U(\boldsymbol{\rho}) &= -\frac{2\pi\hbar^2}{m\gamma V} \sum_{\tau_x,\tau_y} F(q_x = \tau_x, q_y = \tau_y, q_z = 0) e^{i\tau_\perp \rho} \\ &= -\frac{2\pi\hbar^2}{m\gamma d_z} F(\boldsymbol{\rho}, q_z = 0), \end{aligned} \tag{A.18}$$

where d_z is the lattice period along axis z.

According to (A.12, A.13, A.17, A.18) the contributions to the effective potential energy are caused by interactions of different types including short-range and long-range interactions. In the presence of several types of interaction, to describe their different contributions to the scattering amplitude, it is convenient to separate scattering caused only by long-range interactions and to present amplitude in the following form:

$$f(\boldsymbol{q}) = f_{\text{long}}(\boldsymbol{q}) + f_{\text{short-long}}(\boldsymbol{q}), \tag{A.19}$$

where $f_{\text{long}}(\boldsymbol{q})$ is a scattering amplitude determined by the long-range Coulomb and magnetic interactions (assuming that the short-range interactions are absent), $f_{\text{short-long}}(\boldsymbol{q})$ is the scattering amplitude determined by the short-range interactions (for calculation of this amplitude the waves scattered by long-range interactions were used as the incident waves). For general scattering theory in the presence of several interactions see, for example, [15,17].

When several types of interactions influence on the scattering amplitude, it can be easily studied with the help of perturbation theory. Let interaction energy V be a sum of several interactions: $V = \sum_i V_i$. Then at the first Born approximation the scattering amplitude is a sum of the scattering amplitudes $f_1(V_i)$, each caused by the particular interaction: $f = \sum_i f_1(V_i)$. But at the second Born approximation the additional term f_2, which is determined by the following expression, appears in the scattering amplitude (see [15–17]):

$$f_2 = V \frac{1}{E - H_0 - i\eta} V = \sum_p V_p \frac{1}{E - H_0 - i\eta} \sum_l V_l. \tag{A.20}$$

As one can see, (A.20) contains interference of contributions to f proportional to $V_p V_l$.

Let us now consider how the different terms included in (16) contribute to the effective potential energy of particle interaction with the crystal.

The Coulomb amplitude, described by the first term in (16), leads to the conventional expression for potential energy of interaction between a charged particle and a plane (axis).

The second term $A_s(\boldsymbol{q})$ is caused by the short-range interaction. Amplitude $A_s(\boldsymbol{q})$ can be written as:

$$A_s(\boldsymbol{q}) = A_{\text{nuc}}(\boldsymbol{q}) \Phi_{\text{osc}}(\boldsymbol{q}), \tag{A.21}$$

where $A_{\text{nuc}}(\boldsymbol{q})$ is the spin independent part of the amplitude of elastic scattering by a resting nucleus, $\Phi_{\text{osc}}(\boldsymbol{q})$ is the form-factor caused by nucleus oscillations in crystal.

Owing to the short-range kind of strong interactions, amplitude $A_{\text{nuc}}(\boldsymbol{q})$ is equal to zero-angle scattering amplitude $A(0)$ within the range of scattering angles $\vartheta \leq \frac{1}{kR_{\text{osc}}} \ll 1$.

Form-factor $\Phi_{\text{osc}}(\boldsymbol{q})$ has the form [16]:

$$\begin{aligned} \Phi_{\text{osc}}(\boldsymbol{q}) &= \sum_n \rho_n \langle \varphi_n(r) | e^{-i\boldsymbol{q}\boldsymbol{r}} | \varphi_n(r) \rangle \\ &= \int e^{-i\boldsymbol{q}\boldsymbol{r}} N_{\text{nuc}}(\boldsymbol{r}) d^3 r, \end{aligned} \tag{A.22}$$

where $\varphi_n(r)$ is the wave function describing vibrational state of nuclei in crystal, summation $\sum_n \rho_n$ means statistical averaging with the Gibbs distribution over the vibrational states of a nucleus in a crystal. The squared form-factor $\Phi_{\text{osc}}(\boldsymbol{q})$ is equal to Debye-Waller factor, and $N_{\text{nuc}}(\boldsymbol{r})$ is the density of probability to find the vibrating nuclei in point $\boldsymbol{r}$, $\int N_{\text{nuc}}(\boldsymbol{r}) d^3 r = 1$.

As a result, according to (A.17), this contribution to the effective potential energy of a plane can be written as follows:

$$U_{\text{nuc}}(x) = -\frac{2\pi\hbar^2}{m\gamma d_y d_z} N_{\text{nuc}}(x) A_{\text{nuc}}(0), \tag{A.23}$$

where $N_{\text{nuc}}(x) = \iint N_{\text{nuc}}(x, y', z') dy' dz'$ is the probability density to find the vibrating nuclei in point x (in direction orthogonal to the chosen crystallographic plane).

Similarly, for the axis we have:

$$U(\boldsymbol{\rho}) = -\frac{2\pi\hbar^2}{m\gamma d_z} N_{\text{nuc}}(\boldsymbol{\rho}) A_{\text{nuc}}(0), \tag{A.24}$$

where $N_{\text{nuc}}(\boldsymbol{\rho}) = \int N_{\text{nuc}}(\boldsymbol{\rho}, z') dz'$.

A.3 Effective potential energy determined by the anomalous magnetic moment

According to (16) the scattering amplitude, which is determined by baryon's anomalous magnetic moment, has the form:

$$F_{\text{magn}}(q) = B_{\text{magn}}(q)\boldsymbol{\sigma}[\boldsymbol{n}\times\boldsymbol{q}]. \tag{A.25}$$

Defining the scattering amplitude at the first step one could solely consider magnetic scattering and its interference with Coulomb scattering (see (A.19)), and at the second step add the term caused by interference between magnetic and nuclear interactions.

For first step the perturbation theory can be used. In the first order the interference of the magnetic moment scattering by the Coulomb field with the Coulomb scattering of baryon electric charge by the Coulomb field is absent. The amplitude $F^{(1)}$ reads as follows:

$$F^{(1)}_{\text{magn}}(\boldsymbol{q}) = i f_{\text{coul}}(\boldsymbol{q})\frac{\hbar}{mc}\frac{g-2}{2}\frac{1}{2}\boldsymbol{\sigma}[\boldsymbol{n}\times\boldsymbol{q}], \tag{A.26}$$

where $f_{\text{coul}}(\boldsymbol{q})$ is the amplitude of Coulomb scattering of a baryon by an atom in the first Born approximation, m is the baryon mass.

It should be noted that the coefficient, by which $\boldsymbol{\sigma}$ is multiplied, in the expression for amplitude $F^{(1)}_{\text{magn}}(\boldsymbol{q})$ is purely imaginary. After substitution of (A.26) into (A.17) and summation over τ_x one obtains the expression for effective interaction energy as follows:

$$U_{\text{magn}}(x) = -\frac{e\hbar}{2mc}\frac{g-2}{2}\boldsymbol{\sigma}[\boldsymbol{E}_{\text{plane}}(x)\times\boldsymbol{n}], \tag{A.27}$$

where $\boldsymbol{E}_{\text{plane}}(x)$ denotes the electric field, produced by the crystallographic plane in point x. In axis case $U_{\text{magn}}(\boldsymbol{\rho})$ can be obtained by replacement of x by $\boldsymbol{\rho}$ in (A.27) and $\boldsymbol{E}_{\text{plane}}(x)$ by $\boldsymbol{E}_{\text{axis}}(\boldsymbol{\rho})$, respectively.

Using (A.27) and Heisenberg equations for spin operator, the motion equation for ether spin or vector polarization (1), (2) can be obtained for $B = 0$ and $\gamma \gg 1$.

The effective interaction energy (A.27) can be rewritten as follows:

$$U_{\text{magn}} = -\frac{e\hbar}{2mc}\frac{g-2}{2}E_{\text{plane}\,x}(x)\boldsymbol{\sigma}\boldsymbol{N}, \tag{A.28}$$

where $\boldsymbol{N} = [\boldsymbol{n}_x\times\boldsymbol{n}]$ is the unit vector, $\boldsymbol{n}_x \perp \boldsymbol{n}$, unit vector $\boldsymbol{n}$ is parallel to the crystallographic plane.

The term for the effective potential energy, which is in front of $\boldsymbol{\sigma}$ in expression (A.28) is purely real. However, the coefficient in the expression for scattering amplitude $F(\boldsymbol{q})$, by which $\boldsymbol{\sigma}$ is multiplied, has non-zero both real and imaginary parts. Due to this fact, the effective potential energy U also has non-zero both real and imaginary parts.

In the second order of perturbation theory this coefficient in amplitude $F(\boldsymbol{q})$ is not purely imaginary as well. By means of (15), (A.8)–(A.11) the following expression for contribution $\tilde{F}^{(2)}(q)$ to amplitude $F(\boldsymbol{q})$ can be obtained:

$$\begin{aligned}
\tilde{F}^{(2)}(\boldsymbol{q}=\boldsymbol{\tau}) &= i\frac{k}{4\pi\hbar^2c^2}\\
&\times\left\{\langle\Phi_a|\iint e^{-i\boldsymbol{\tau}\boldsymbol{r}_\perp}\left[\int V(\boldsymbol{r}_\perp,z)dz\right]^2 d^2r_\perp|\Phi_a\rangle\right.\\
&\left.-\iint e^{-i\boldsymbol{\tau}\boldsymbol{r}_\perp}\left[\int\langle\Phi_a|V(\boldsymbol{r}_\perp,z)|\Phi_a\rangle dz\right]^2 d^2r_\perp\right\}\\
&= i\frac{k}{4\pi\hbar^2c^2}\iint e^{-i\boldsymbol{\tau}\boldsymbol{r}_\perp}\left\{\langle\Phi_a|\left[\int V(\boldsymbol{r}_\perp,z)dz\right]^2|\Phi_a\rangle\right.\\
&\left.-\left[\int\langle\Phi_a|V(\boldsymbol{r}_\perp,z)|\Phi_a\rangle dz\right]^2\right\}d^2r_\perp\\
&= i\frac{k}{4\pi\hbar^2c^2}\iint e^{-i\boldsymbol{\tau}\boldsymbol{r}_\perp}\left\{\overline{\left[\int V(\boldsymbol{r}_\perp,z)dz\right]^2}\right.\\
&\left.-\left[\overline{\int V(\boldsymbol{r}_\perp,z)dz}\right]^2\right\}d^2r_\perp,
\end{aligned} \tag{A.29}$$

where $V(\boldsymbol{r}_\perp,z) = V_{\text{coul}}(\boldsymbol{r}_\perp,z) + V_{\text{magn}}(\boldsymbol{r}_\perp,z)$, z axis of the coordinate system is directed along the unit vector $\boldsymbol{n}$, $V_{\text{magn}}(\boldsymbol{r}_\perp,z) = -\mu_a\boldsymbol{\sigma}[\boldsymbol{E}(\boldsymbol{r}_\perp,z)\times\boldsymbol{n}]$, μ_a is the anomalous magnetic moment of the particle $\mu_a = \frac{e\hbar}{2mc}(\frac{g-2}{2})$.

When deriving (A.29), it was considered that the particle energy is much greater than the electrons' binding energy in atoms and the atoms' binding energy in crystal. As a result, it is at first possible to examine scattering by electrons and nuclei, which rest in points $\boldsymbol{r}_i$, and then to average the result over the electrons and nuclei positions with wave functions $|\Phi_a\rangle$ (impulse approximation, for example see [15]). The overline in (A.29) and hereinafter denotes such kind of averaging. The contribution caused by interference between magnetic and nuclear scattering, and the contributions determined by the squared magnetic moment of a particle should complete the expression mentioned above. For positively charged particles, moving far from the top of the potential barrier, the contribution caused by interactions with nuclei is suppressed and will be omitted in consideration hereinafter. Contributions proportional to the particle's squared magnetic moment are smaller then those caused by the interference between magnetic and Coulomb scattering and will, thus, be also omitted. After substitution of (A.29) into (A.17) and summation over τ_x the following expression for the contribution to the effective potential energy caused by amplitude $\tilde{F}_{\text{magn}}(\boldsymbol{\tau})$ can be obtained:

$$U^{(2)}_{\text{magn}}(x) = -i\frac{1}{4d_y\,d_z\,mc^2}\left(\frac{g-2}{2}\right)\frac{\partial}{\partial x}\overline{\delta V^2(x)}\boldsymbol{\sigma}\boldsymbol{N}, \tag{A.30}$$

where $\boldsymbol{N} = [\boldsymbol{n}_x\times\boldsymbol{n}]$, $\boldsymbol{n}_x \perp \boldsymbol{n}$, $\boldsymbol{n}_x$ is the unit vector along axis x,

$$\overline{\delta V^2(x)} = \int \left\{ \overline{\left[\int V_{\rm coul}(x,y,z)dz\right]^2} - \left[\overline{\int V_{\rm coul}(x,y,z)dz}\right]^2 \right\} dy. \quad (A.31)$$

Similarly for the case of axial channeling it can be obtained:

$$U^{(2)}_{\rm magn}(\boldsymbol{\rho}) = -i\frac{1}{d_z mc^2}\frac{g-2}{2}\boldsymbol{\sigma}[\nabla_\rho \overline{\delta V^2(\boldsymbol{\rho})} \times \boldsymbol{n}]. \quad (A.32)$$

For the axisymmetric case:

$$U^{(2)}_{\rm magn}(\boldsymbol{\rho}) = -i\frac{1}{4d_z mc^2}\frac{g-2}{2}\frac{\partial}{\partial \boldsymbol{\rho}}\overline{\delta V^2(\rho)}[\boldsymbol{n}_\rho \times \boldsymbol{n}], \quad (A.33)$$

where $\overline{\delta V^2(\boldsymbol{\rho})} = \overline{\left[\int V_{\rm coul}(\boldsymbol{\rho},z)dz\right]^2} - \left[\overline{\int V_{\rm coul}(\boldsymbol{\rho},z)dz}\right]^2$, $\boldsymbol{n}_\rho = \frac{\boldsymbol{\rho}}{\rho}$ is the unit vector, $\boldsymbol{n}_\rho \perp \boldsymbol{n}$.

In the planar channelling case $U_{\rm magn}$ is determined by the expression (19).

A.4 Effective potential energy U determined by spin-orbit interaction

According to (16) the part of the scattering amplitude caused by strong spin-orbit interaction has the form:

$$F_{\rm so}(\boldsymbol{q}=\boldsymbol{\tau}) = B_{\rm s}(\boldsymbol{\tau})\boldsymbol{\sigma}[\boldsymbol{n}\times\boldsymbol{\tau}]. \quad (A.34)$$

The coefficient $B_{\rm s}(\boldsymbol{\tau})$ can be expressed similar to (A.21) as follows:

$$B_{\rm s}(\boldsymbol{\tau}) = B_{\rm s}(\boldsymbol{\tau})\Phi_{\rm osc}(\boldsymbol{\tau}), \quad (A.35)$$

where $B_{\rm snuc}(\boldsymbol{\tau})$ describes scattering by a resting nucleus, $\Phi_{\rm osc}(\boldsymbol{\tau})$ is the form-factor determined by nucleus oscillations in crystal.

In the considered case, similar to the approach used when deriving (A.23), the short-range character of the nuclear forces and small (as compared to the amplitude of nucleus oscillations) nucleus radius enables assumption $B_{\rm nuc}(\boldsymbol{\tau}) \approx B_{\rm nuc}(0)$. It is important that the coefficient $B_{\rm nuc}(0)$ has non-zero both real and imaginary parts:

$$B_{\rm nuc}(0) = B'_{\rm nuc} + iB''_{\rm nuc}. \quad (A.36)$$

This is similar to the case of amplitude, which describes scattering of the magnetic moment by the atom (nucleus). To obtain the expression for the effective potential energy the summation over τ_x should be conducted in (A.17). The resulted expression is similar to that for $U_{\rm magn}$. For example, for the crystal plane case see expression (23) for $U_{\rm so}$. Let us remind that the contribution determined by elastic scattering, which is described by the second term in (A.1), is negligibly small in comparison with non-elastic contributions to the amplitude and, therefore, can be omitted.

A.5 Effective potential energy U determined by P-odd and T-even interactions

The next group of terms, which are proportional to $B_{\rm w}$, is determined by weak P-odd and T-even interactions. According to (16) the corresponding terms in the scattering amplitude can be written as:

$$F_{\rm w}(\boldsymbol{q}) = \big(B_{\rm we}(\boldsymbol{q}) + B_{\rm wnuc}(\boldsymbol{q})\big)\boldsymbol{\sigma}\boldsymbol{N}_{\rm w}. \quad (A.37)$$

Contribution $B_{\rm we}(\boldsymbol{q})$ caused by the parity violating weak interaction between a baryon and electrons can be expressed as follows:

$$B_{\rm we}(\boldsymbol{q}) = \tilde{B}_{\rm we}(\boldsymbol{q})\Phi_{\rm e}(\boldsymbol{q}), \quad (A.38)$$

where $\tilde{B}_{\rm we}$ is the coefficient defining the amplitude of baryon elastic scattering by a resting electron $f_{\rm we}(\boldsymbol{q}) = \tilde{B}_{\rm we}\boldsymbol{\sigma}\boldsymbol{N}_{\rm w}$, $\Phi_{\rm e}(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\boldsymbol{r}}N_{\rm e}(\boldsymbol{r})d^3r$, $\int N_{\rm e}(\boldsymbol{r})d^3r = Z$, Z is the nucleus charge. Minor corrections caused by the thermal oscillations of atoms' centers of gravity will not be considered below. To take them into consideration one should multiply $\Phi_{\rm e}(\boldsymbol{q})$ by $\Phi_{\rm osc}(\boldsymbol{q})$, which is the form-factor defined by oscillations of atoms nucleus.

Term $B_{\rm wnuc}(\boldsymbol{q})$ (see (A.37)), which is caused by parity violating weak interaction between a baryon and a nucleus, reads as follows:

$$B_{\rm wnuc}(\boldsymbol{q}) = \tilde{B}_{\rm wnuc}(\boldsymbol{q})\Phi_{\rm osc}(\boldsymbol{q}), \quad (A.39)$$

where $\tilde{B}_{\rm wnuc}$ is the coefficient defining the amplitude of a baryon elastic scattering by a resting nucleus $f_{\rm wnuc} = \tilde{B}_{\rm wnuc}\boldsymbol{\sigma}\boldsymbol{N}_{\rm w}$.

Due to the short-range character of P-violating interactions, when angle $\vartheta \simeq \frac{\tau}{k} \ll 1$, coefficients $\tilde{B}_{\rm we}(\boldsymbol{q}) \simeq \tilde{B}_{\rm we}(0)$ and $\tilde{B}_{\rm wnuc}(\boldsymbol{q}) \simeq \tilde{B}_{\rm wnuc}(0)$, $\boldsymbol{N}_{\rm w} \simeq \boldsymbol{n}$. As a result, the following expressions can be obtained for effective potential energy $U_{\rm w}$ of P-violating interaction of a baryon with a crystal plane (axis):

$$U_{\rm w} = U_{\rm we} + U_{\rm wnuc} \quad (A.40)$$

(a) for the case of plane:

$$U_{\rm we}(x) = -\frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}_{\rm we}(0)N_{\rm e}(x)\boldsymbol{\sigma}\boldsymbol{n},$$
$$U_{\rm wnuc}(x) = -\frac{2\pi\hbar^2}{m\gamma d_y d_z}\tilde{B}_{\rm wnuc}(0)N_{\rm nuc}(x)\boldsymbol{\sigma}\boldsymbol{n},$$
$$N_{\rm e(nuc)}(x) = \int N_{\rm e(nuc)}(x,y,z)dydz. \quad (A.41)$$

(b) for the case of axis:

$$U_{\rm we}(\boldsymbol{\rho}) = -\frac{2\pi\hbar^2}{m\gamma d_z}\{\tilde{B}_{\rm we}(0)N_{\rm e}(\boldsymbol{\rho}) + \tilde{B}_{\rm wnuc}(0)N_{\rm nuc}(\boldsymbol{\rho})\}\boldsymbol{\sigma n},$$
$$N_{\rm e(nuc)}(\boldsymbol{\rho}) = \int N_{\rm e(nuc)}(\boldsymbol{\rho}, z)dz. \quad \text{(A.42)}$$

As a result for planar channelling case see expression (25).

A.6 Effective potential energy U determined by the electric dipole moment and other T-nonivariant interactions

Let us consider now the electric dipole moment and other T-nonivariant contributions to the spin rotation. According to (16) the corresponding terms in the scattering amplitude can be written as:

$$F_{\rm T}(\boldsymbol{q}) = (B_{\rm EDM}(\boldsymbol{q}) + B_{\rm Te}(\boldsymbol{q}) + B_{\rm Tnuc}(\boldsymbol{q}))\boldsymbol{\sigma q}. \quad \text{(A.43)}$$

Let's consider the term $F_{\rm EDM}(\boldsymbol{q}) = B_{\rm EDM}(\boldsymbol{q})\boldsymbol{\sigma q}$. The coefficient $B_{\rm EDM}(\boldsymbol{q})$ has non-zero both real and imaginary parts $B_{\rm EDM}(\boldsymbol{q}) = B'_{\rm EDM} + iB''_{\rm EDM}$. By the approach used for deriving $F_{\rm magn}(\boldsymbol{q})$, for $F_{EDM}(\boldsymbol{q})$ one can obtain:

$$F_{\rm EDM}(\boldsymbol{q}) = -i\frac{m\gamma d}{2\pi\hbar^2}V_{\rm coul}(\boldsymbol{q})\boldsymbol{\sigma q} + \frac{k}{4\pi\hbar^2c^2} \times \iint e^{-i\boldsymbol{q}_\perp\boldsymbol{r}_\perp}\left\{\overline{\left[\int V(\boldsymbol{r}_\perp, z)dz\right]^2} - \left[\int \overline{V(\boldsymbol{r}_\perp, z)}dz\right]^2\right\}d^2r_\perp, \quad \text{(A.44)}$$

where $V(\boldsymbol{r}) = V_{\rm coul}(\boldsymbol{r}) + V_{\rm EDM}(\boldsymbol{r})$, $V_{\rm EDM} = -D\boldsymbol{\sigma E}$ is the energy of interaction between electric dipole moment D and electric field $\boldsymbol{E}$, $D = ed$, e is the electric charge of the particle.

Using (A.17) one can obtain expressions (28) and (29) for the potential energy of interaction between a particle and a crystal plane.

Let's remind that amplitude $F_{\rm T}(\boldsymbol{q})$ contains terms both caused by the EDM and determined by the short-range T-noninvariant interactions of a baryon with electrons and nuclei $B_{\rm Te}(\boldsymbol{q})$ and $B_{\rm Tnuc}(\boldsymbol{q})$. Contributions caused by these terms should also be added to the effective potential energy of the interaction between the baryon and nuclei of the crystal $U_{\rm T}(x)$:

$$U_{\rm T}(x) = U_{\rm EDM} + U_{\rm Te} + U_{\rm Tnuc} = -(\alpha_{\rm T}(x) + i\delta_{\rm T}(x))\boldsymbol{\sigma} N_{\rm T}, \quad \text{(A.45)}$$

where $\alpha_{\rm T} = \alpha_{\rm EDM} + \alpha_{\rm Te} + \alpha_{\rm Tnuc}$, $\delta_{\rm T} = \delta_{\rm EDM} + \delta_{\rm Te} + \delta_{\rm Tnuc}$, unit vector $\boldsymbol{N}_{\rm T}$ is orthogonal to the plane.

Expressions for coefficients $\alpha_{\rm Te(nuc)}$ and $\delta_{\rm Te(nuc)}$ can be evaluated in terms of scattering amplitude by the following way. Let's define the form-factor determined by electrons distribution in atom and nucleus oscillations.

$$B_{\rm Te}(\boldsymbol{q}) = \tilde{B}_{\rm Te}(\boldsymbol{q})\Phi_{\rm e}(\boldsymbol{q}),$$
$$B_{\rm Tnuc}(\boldsymbol{q}) = \tilde{B}_{\rm Tnuc}(\boldsymbol{q})\Phi_{\rm osc}(\boldsymbol{q}), \quad \text{(A.46)}$$

where $\Phi_{\rm e}(\boldsymbol{q}) = \int e^{-i\boldsymbol{qr}}N_{\rm e}(\boldsymbol{r})d^3r$, $N_{\rm e}(\boldsymbol{r})$ is electrons distribution density in atom, $\int N_{\rm e}(\boldsymbol{r})d^3r = Z$, Z is the nucleus charge, $\Phi_{\rm osc}(\boldsymbol{q})$ is determined by (A.22), $\tilde{B}_{\rm Te}$ is the coefficient defining amplitude of baryon scattering by resting electron $f_{\rm Te} = \tilde{B}_{\rm Te}(\boldsymbol{q})\boldsymbol{\sigma q}$, $\tilde{B}_{\rm nuc}(\boldsymbol{q})$ is the coefficient defining amplitude of baryon scattering by a resting nucleus $f_{\rm Tnuc} = \tilde{B}_{\rm nuc}(\boldsymbol{q})\boldsymbol{\sigma q}$. Let's remind that in compliance with (15) the contribution caused by elastic coherent scattering should be subtracted from the amplitude $B_{\rm T}$. However, at high energies this contribution is negligibly small in comparison with non-elastic contributions to the amplitude and, therefore, can be omitted.

Due to the short-range character of T-noninvariant interactions at angle $\vartheta \simeq \frac{\tau}{k} \ll 1$ coefficients $\tilde{B}_{\rm Te}(\boldsymbol{q}) \simeq \tilde{B}_{\rm Te}(0)$ and $\tilde{B}_{\rm nuc}(\boldsymbol{q}) \simeq \tilde{B}_{\rm nuc}(0)$. As a result, expression (31), (32) can be obtained.

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