



Study of the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ in the OKA experiment

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Abstract A high statistics data sample of the decays of K^+ mesons to three charged particles was accumulated by the OKA experiment in 2012 and 2013. This allowed to select a clean sample of about 450 events with $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ decays with the energy of the photon in the kaon rest frame greater than 30 MeV. The measured branching fraction of the $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$, with $E_\gamma^* > 30$ MeV is equal to $(0.71 \pm 0.05) \times 10^{-5}$. The measured differential branching fraction over photon energy is compared with the prediction of the chiral perturbation theory to $\mathcal{O}(p^4)$. A search for an up-down asymmetry of the photon with respect to the hadronic system decay plane is also performed.

1 Introduction

The present experimental status of $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ is rather meagre. It was observed in one experiment on statistics of 7 events [1]. The photon energies in these events were low, that did not allow to search for deviations from a simple QED process of photon emission. The measured value of the $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ branching ratio [2] is $(1.04 \pm 0.31) \times 10^{-4}$ for $E_\gamma^* > 5$ MeV. In the present analysis we have a possibility for more detailed study of this decay using larger data sample collected by the OKA experiment. This decay has certain interest for the theory, in partic-

ular, for the chiral perturbation theory [3–6] (CHPT), which gives appropriate framework for analysis of such decays. To the lowest order in an expansion in momenta and meson masses, this radiative decay is completely determined [7] by the non-radiative amplitude for $K^+ \rightarrow \pi^+\pi^-\pi^+$. However, at next-to-leading order, a full-fledged CHPT calculation of nonleptonic weak amplitudes of $\mathcal{O}(p^4)$ is required. Such a calculation is done in [8]. It is interesting to confront these results to the experimental data. In a decay in the beauty sector $B^+ \rightarrow K^+\pi^+\pi^-\gamma$ the LHCb experiment has found significant up-down asymmetry of the photon with respect to the hadronic system decay plane [9]. This observable was proposed in [10], it is both P and T-odd. We perform an analogous study for the radiative kaon decay $K \rightarrow 3\pi\gamma$ to search for new physics effects.

2 Separated kaon beam and OKA experiment

The OKA experiment makes use of a secondary hadron beam at the U-70 Proton Synchrotron of NRC “Kurchatov Institute” – IHEP, Protvino, with enhanced fraction of kaons obtained by RF-separation with Panofsky scheme [11]. The OKA setup, Fig. 1, is a double magnetic spectrometer complemented by electromagnetic and hadron calorimeters and a Decay Volume. The first magnetic spectrometer, consisting of the magnet M_1 and surrounding 1 mm pitch PC’s ($BPC_{(1Y)}$, $BPC_{(2Y,2X)}$, $BPC_{(3X,3Y)}$, $BPC_{(4X,4Y)}$) serves for the beam momentum measurement. It is supplemented by

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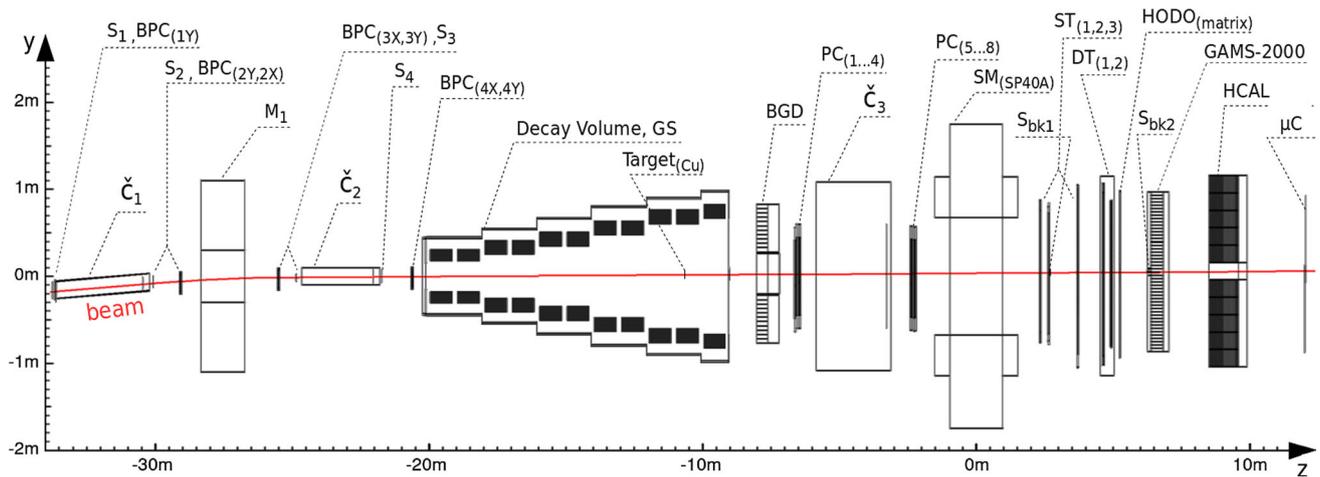


Fig. 1 Schematic elevation view of the OKA setup, reproduced from [13], see the text for details

two gas threshold Cherenkov counters \check{C}_1, \check{C}_2 for kaon identification and by beam scintillation counters S_1, S_2, S_3, S_4 . The 11 m long Decay Volume (DV) filled with helium contains 11 rings of guard system (GS). To reinforce GS, a gamma detector (BGD), made of lead glass blocks located behind the DV is used as a veto at large angles, while low angle particles pass through a central opening. The wide aperture spectrometric magnet, $SM_{(SP40A)}$, with a field integral of ~ 1 Tm serves as a spectrometer for the charged decay products together with corresponding tracking chambers: 2 mm PC's ($PC_{1,\dots,8}$), 9 mm diameter straw tubes $ST_{(1,2,3)}$ and 30 mm diameter drift tubes $DT_{1,2}$. The matrix hodoscope $HODO_{(matrix)}$ is composed of 252 scintillator tiles with WLS+SiPM readout. It is used in the trigger, improves time resolution and links x - y projections of a track. Two scintillation counters S_{bk1}, S_{bk2} serve to suppress undecayed beam particles. At the end of the OKA setup there are two calorimeters: electromagnetic (GAMS-2000) made of lead glass blocks and a hadron one ($HCAL_{(GDA)}$) – 100 iron-scintillator sandwiches. Finally, four partially overlapping muon counters are located downstream the HCAL. The data acquisition system of the OKA setup [12] operates at ~ 25 kHz event rate with the mean event size of ~ 4 kByte. The details of the setup and the beam can be found elsewhere [13].

3 The data and the analysis procedure

The study of the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ is performed with the data set accumulated in 2012 and 2013 runs with 17.7 GeV/c beam momentum. Two main triggers were used, both include the same component which selects beam kaons decaying inside the OKA setup. It requires a coincidence of 4 beam scintillation counters, a combination of two Cerenkov's (\check{C}_1 sees pions, \check{C}_2 pions and kaons) and, finally, anticoincidence of two scintillation counters S_{bk1}, S_{bk2} , located on the

beam axis after the magnet to suppress undecayed beam particles $Tr_{Kdecay} = S_1 \cdot S_2 \cdot S_3 \cdot S_4 \cdot \check{C}_1 \cdot \check{C}_2 \cdot \bar{S}_{bk}$. Then the first trigger requires an energy deposition in GAMS-2000 e.m. calorimeter higher than 2.5 GeV to suppress the dominating $K^+ \rightarrow \mu^+\nu$ decay: $Tr_{GAMS} = Tr_{Kdecay} \cdot (E_{GAMS} > 2.5)$. The second one, $Tr_{HODO} = Tr_{Kdecay} \cdot (2 \leq Mult \leq 4)$, includes additionally a requirement on multiplicity in the Matrix hodoscope, which allows to efficiently select events with 3 charged particles. The beam intensity, counted by the coincidence of 4 counters ($S_1 \cdot S_2 \cdot S_3 \cdot S_4$) was $\sim 2 \times 10^6$ per spill, the fraction of kaons in the beam was $\sim 12.5\%$, i.e. the kaon intensity was ~ 250 k/spill. The total number of kaons entering the DV corresponds to $\sim 3.4 \times 10^{10}$.

The Monte Carlo statistics was generated with Geant3 [14] program which includes a realistic description of the setup. The Monte Carlo events are passed through full OKA reconstruction procedures. Monte Carlo sample for the signal events is using as the input the weight, proportional to the square of the absolute value of the leading order matrix element for the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$, given in [7]. For the estimation of the background contribution to the selected data set, a sample of the Monte Carlo events with six main decay channels of charged kaon ($\pi^+\pi^0, \pi^+\pi^0\pi^0, \pi^+\pi^-\pi^+, \mu^+\nu, \pi^0\mu^+\nu, \pi^0e^+\nu$) mixed accordingly to the branching fractions, with the total statistics about equal to that of the recorded data sample is used. The weights for the 3-body decays are calculated from the data, presented in PDG [2]. The same sample serves for the $K \rightarrow 3\pi$ normalization channel efficiency calculations.

3.1 Event selection

To select $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ decay channel in off-line analysis a set of requirements is applied:

- the momentum of the beam track is required to be measured;
- the number of the secondary charged tracks is required to be equal to three and the net charge of them is equal to +1;
- the decay vertex should have good χ^2 and should be with a margin inside the DV;
- the charged tracks are not identified as electrons in the GAMS-2000 electromagnetic calorimeter;
- the missing mass squared to each positive pion $M_{miss}^2(\pi^+) = (P_{K^+} - P_{\pi^+})^2$ should be greater than 0.07 GeV^2 ;
- the event should contain one and only one photon with the energy $E_\gamma > 0.5 \text{ GeV}$;
- the invariant mass of the photon with each pion ($M(\pi\gamma)$) should be greater than 0.17 GeV ;
- the square of the transverse momentum of the $3\pi\gamma$ system is less than 0.001 GeV^2 ;
- the ratio of the momentum of the $3\pi\gamma$ system to the momentum of the beam track should be within the range $0.95\text{--}1.05$.

The cut on the square of the missing mass to each positive pion $M_{miss}^2(\pi^+)$ is used for the suppression of the background from the decay $K^+ \rightarrow \pi^+\pi^0$ with $\pi^0 \rightarrow e^+e^-\gamma$. The main source of the background for the $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ decay is the decay of kaon to three charged pions when the pions produce hadron showers in the electromagnetic calorimeter and, because of fluctuations, a part of a shower is not associated with a charged track by the reconstruction program. To suppress such kind of background a cut on $M(\pi\gamma)$ is done. The reconstructed momentum of the $3\pi\gamma$ system should be equal to the momentum of the beam particle. This motivates the last two cuts listed above. For the illustration, the distributions of the square of the transverse momentum of the $3\pi\gamma$ system and the ratio of absolute value of its momentum to the momentum of the beam particle after all the previous cuts is shown in Figs. 2 and 3. The invariant mass distribution of the $3\pi\gamma$ system after application of all the cuts listed above is shown in Fig. 4. We see clear separation of the signal, peaking around the nominal value of the kaon PDG [2] mass and the background concentrating at higher masses. The number of events in the signal region is about 450. As almost all the photons in the selected $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ events have the energy in the kaon rest frame greater than 0.03 GeV , in the following analysis we apply an additional explicit cut $E_\gamma^* > 0.03 \text{ GeV}$.

3.2 Measurement of the branching ratio of the $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ decay

The invariant mass distribution of the $3\pi\gamma$ system in the signal region is shown in Fig. 5. The Monte Carlo distributions for the main sources of the background and the signal events

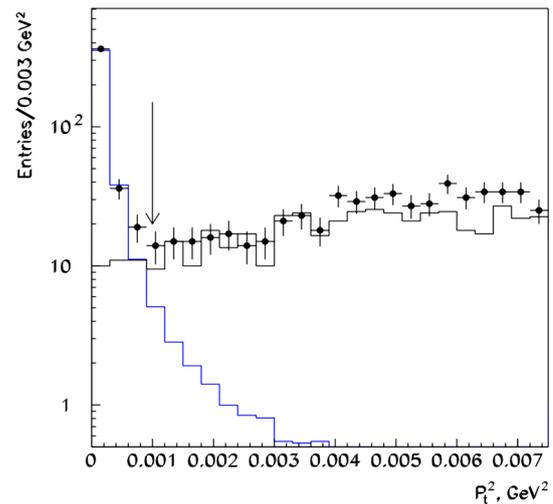


Fig. 2 The distribution of the square of transverse momentum of the $3\pi\gamma$ system for the data (black circles with errors), main sources of the background (black histogram) and the MC signal (blue histogram). Vertical arrow corresponds to the cut position

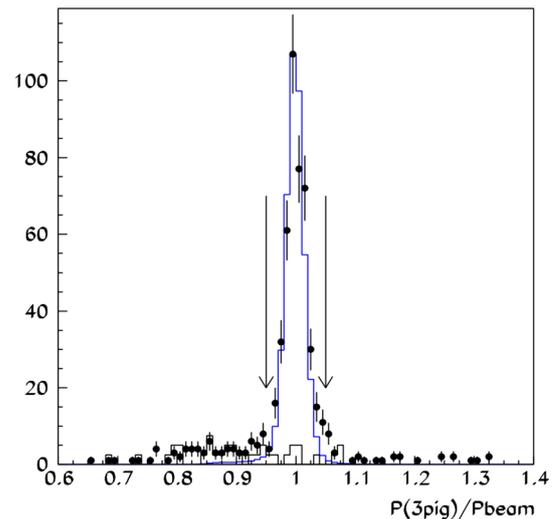


Fig. 3 The ratio of the absolute value of the momentum of the $3\pi\gamma$ system to the momentum of the beam particle for the data (black circles with errors), main sources of the background (black histogram) and the MC signal (blue histogram). Vertical arrows show the selected signal region

are shown together with the data. We define the mass range of $0.486\text{--}0.504 \text{ GeV}$ as a signal region. The number of the decays of $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ is determined as the difference of the number of the data events in the signal region and the expected number of events from the background. The expected background contribution to the signal region is determined using Monte Carlo events for six main channels of the kaon decays described in the above section. To determine the branching fraction of the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ we use the decay $K^+ \rightarrow \pi^+\pi^-\pi^+$ as the normalization channel. It is triggered with the same multiplicity trigger

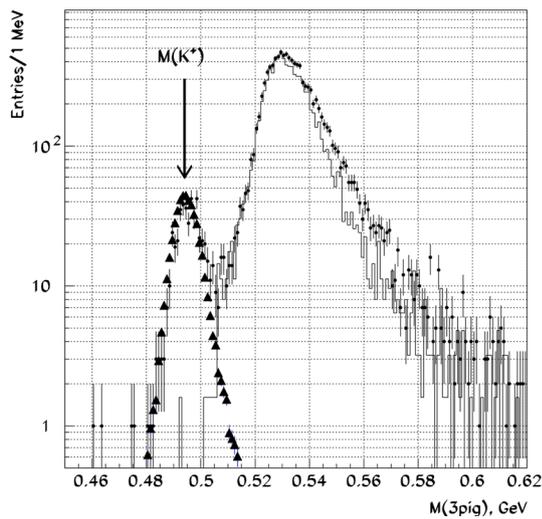


Fig. 4 The invariant mass distribution of the $3\pi\gamma$ system for the data (black circles), main sources of background (histogram) and the MC signal (black triangles)

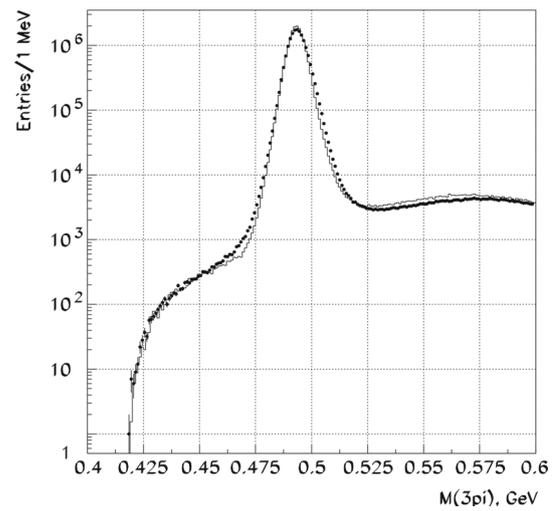


Fig. 6 The invariant mass distribution of three charged pions for the data (black circles) and for the Monte Carlo for the six main channels of the charged kaon decay (histogram)

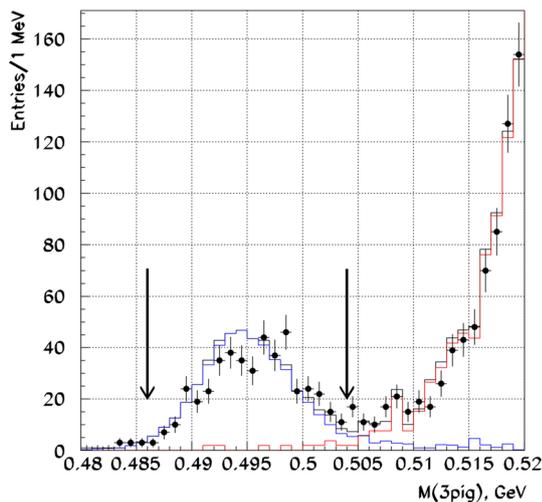


Fig. 5 The invariant mass distribution of the $3\pi\gamma$ system for the data (black circles), the signal decays (blue histogram), the main sources of the background (red histogram) and the sum of the contributions of the signal decays and the background (black histogram). The arrows show the signal region used in the analysis

(Tr_{HODO}) as the signal and selected with the same criteria as for the pions of the $3\pi\gamma$ decay, thus we expect the cancellation of many systematic effects related to pions in the ratio of the branching fractions $\text{Br}(K^+ \rightarrow \pi^+\pi^-\pi^+\gamma)/\text{Br}(K^+ \rightarrow \pi^+\pi^-\pi^+)$. The invariant mass spectrum of the system of three charged pions for the same selection criteria as for the charged part of the $3\pi\gamma$ decay is shown in Fig. 6.

The branching fraction of the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ is determined in the following way:

$$\text{Br}(3\pi\gamma) = \text{Br}(3\pi)_{PDG} \times \epsilon(3\pi) \times N(3\pi\gamma)_D / \epsilon(3\pi\gamma) / N(3\pi)_D,$$

where $\text{Br}(3\pi)_{PDG}$ is the branching fraction of the decay $K^+ \rightarrow \pi^+\pi^-\pi^+$ from PDG [2]; $\epsilon(3\pi)$ and $\epsilon(3\pi\gamma)$ are the reconstruction efficiencies for the decays $K^+ \rightarrow \pi^+\pi^-\pi^+$ and $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$, determined from the Monte Carlo; $N(3\pi\gamma)_D$ and $N(3\pi)_D$ are the numbers of the decays of $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ and $K^+ \rightarrow \pi^+\pi^-\pi^+$ in the data. Efficiencies $\epsilon(3\pi)$ and $\epsilon(3\pi\gamma)$, determined from the Monte Carlo, are 0.12 ± 0.002 and 0.024 ± 0.001 , respectively. The obtained result is

$$\text{Br}(3\pi\gamma) = (0.71 \pm 0.04(\text{stat.})) \times 10^{-5}.$$

The main source of the systematic error is the uncertainty in the estimate of the background contribution to the signal region (see Figs. 4 and 5). For the estimation of this uncertainty we varied the normalization of the background distribution. This leads to the systematic uncertainty of about 0.027×10^{-5} in the branching fraction. Another source of systematic error is cut's related. To estimate that, the cuts were varied within experimental resolution around nominal value and the branching was recalculated. The maximal change in the branching was considered as the systematic error. Adding all the systematic errors in quadrature gives a total systematic error of 0.03×10^{-5} . The final result is:

$$\text{Br}(3\pi\gamma) = (0.71 \pm 0.04(\text{stat.}) \pm 0.03(\text{syst.})) \times 10^{-5}, \\ E_\gamma^* > 0.03 \text{ GeV}.$$

The theory prediction in the framework of the CHPT is $\text{Br}_{th} = (0.665 \pm 0.005) \times 10^{-5}$ [8].

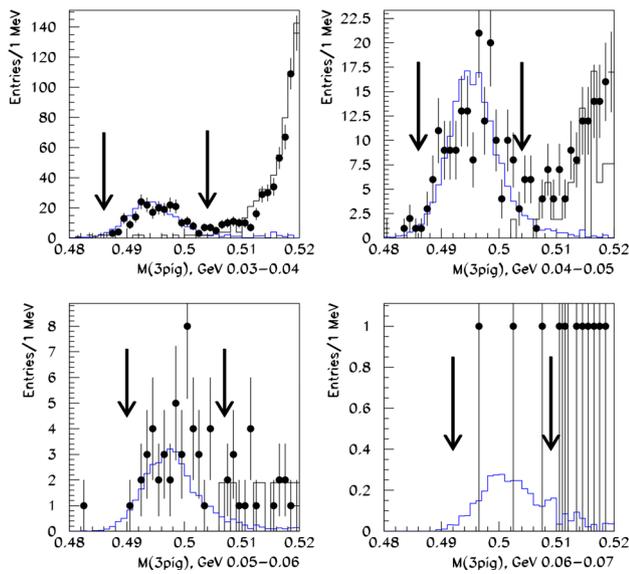


Fig. 7 Invariant mass distributions of the $3\pi\gamma$ system for the data (black circles), signal decays (blue histogram) and main sources of the background (black histogram) for the different energy ranges of the photon in the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$. The top left distribution is for the photon energy range 0.03–0.04 GeV, the top right one is for the range 0.04–0.05 GeV, the bottom left one is for the 0.05–0.06 GeV and the last one is for the 0.06–0.07 GeV. The arrows show the signal regions used in the analysis

3.3 Measurement of the differential branching fraction over photon energy

To perform a measurement of the differential branching fraction over photon energy, we split the data sample into four parts with photon energies in the rest frame of the kaon lying in the ranges 0.03–0.04, 0.04–0.05, 0.05–0.06 and 0.06–0.07 GeV. After that we apply the procedure of the previous section to each of the four data samples. The $3\pi\gamma$ invariant mass spectra for the listed above ranges of the photon energy are shown in Fig. 7 together with expected signal and background contributions.

The obtained values of the branching fractions for the given energy ranges are listed in the Table 1 and shown in the Fig. 8. For comparison, we also give the $O(p^4)$ CHPT predictions from [8].

Table 1 Values of the branching fractions for the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ in the intervals of the photon energy in the kaon rest frame. The uncertainties of the data given in the table are the statistical ones

Energy interval (GeV)	Branching fraction (data)	Branching fraction (CHPT [8])
0.03–0.04	$(5.17 \pm 0.34) \times 10^{-6}$	$(4.93 \pm 0.05) \times 10^{-6}$
0.04–0.05	$(1.55 \pm 0.12) \times 10^{-6}$	$(1.44 \pm 0.01) \times 10^{-6}$
0.05–0.06	$(0.35 \pm 0.05) \times 10^{-6}$	$(0.269 \pm 0.003) \times 10^{-6}$
0.06–0.07	$(0.11 \pm 0.06) \times 10^{-7}$	$(0.136 \pm 0.002) \times 10^{-7}$

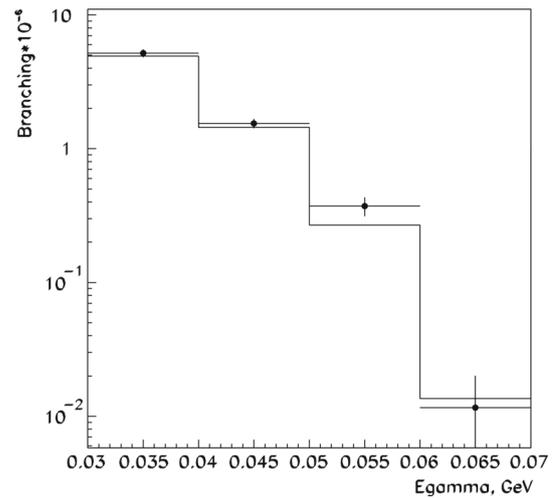


Fig. 8 Measured branching fractions for the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ in the photon energy ranges in the kaon rest frame for the data (black circles) and the CHPT prediction from [8] (histogram)

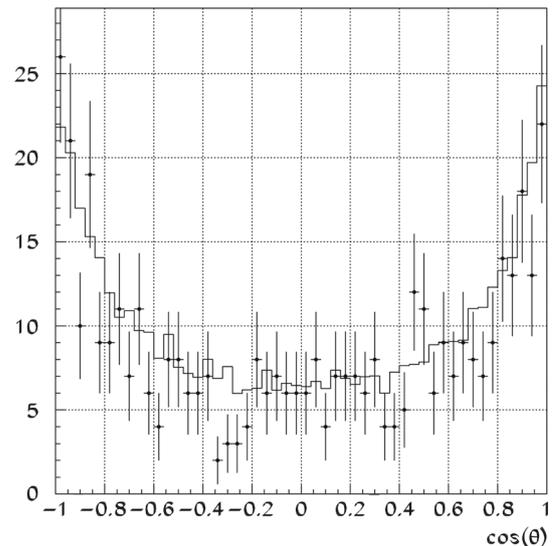


Fig. 9 The distribution (black circles) of the cosine of the angle of the photon direction with respect to the hadronic system decay plane (see the text). The histogram shows the Monte Carlo prediction for the decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ with the matrix element from [8]

4 Search for a photon up-down asymmetry

In Fig. 9 we show the distribution of the cosine of the angle of the photon direction with respect to the pion system decay plane in the 3-pion rest frame:

$$\cos(\theta) = n_\gamma \cdot [p_f(\pi) \times p_s(\pi)] / |[p_f(\pi) \times p_s(\pi)]|,$$

where n_γ is the unit vector of photon direction in the 3-pion rest frame, $[p_f(\pi) \times p_s(\pi)]$ is the vector product of the momenta of the fastest and slowest pions in the same frame. For comparison, we show the same distribution for the Monte Carlo signal events with the matrix element from [8]. The observed asymmetry in the data is:

$$A = (N(\cos\theta > 0) - N(\cos\theta < 0)) / N_{total} \\ = 0.03 \pm 0.05(stat.) \pm 0.03(syst.).$$

The main source of the systematic error comes from the experimental resolution in $\cos(\theta)$. The alternative choice to define the decay plane is to use the fastest and slowest π^+ . This results in $A^+ = -0.04 \pm 0.05(stat.) \pm 0.03(syst.)$. In both cases the observed asymmetry is consistent with zero within errors.

Conclusions

The decay $K^+ \rightarrow \pi^+\pi^-\pi^+\gamma$ is studied on statistics of 450 events. The measured branching fraction is $Br(3\pi\gamma) = (0.71 \pm 0.05) \times 10^{-5}$ for $E_\gamma^* > 0.03$ GeV. The differential branching fraction over photon energy is also determined. The measured branching fractions agree well within the errors with the calculations in the framework of the chiral perturbation theory. The measured up-down asymmetry of the photon with respect to the decay plane of the hadronic system is 0.03 ± 0.06 . No sign of P and T-odd effects is observed within present statistics.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: OKA data will be released after the intended period of operation, and corresponding publication of results.]

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