

Sterile neutrinos in string derived models

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Received: 27 August 2018 / Accepted: 14 October 2018 / Published online: 27 October 2018
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Abstract The MiniBooNE collaboration recently reported further evidence for the existence of sterile neutrinos, implying substantial mixing with the left-handed active neutrinos and at a comparable mass scale. I argue that while sterile neutrinos may arise naturally in large volume string models, they prove more of a challenge in heterotic-string models that replicate the Grand Unified Theory structure of the Standard Model matter states. Sterile neutrinos in heterotic-string models may imply the existence of an additional Abelian gauge symmetry of order 10–100 TeV.

1 Introduction

The MiniBooNE collaboration recently reported further evidence for the existence of sterile neutrinos [1]. The MiniBooNE data strengthens the results of the LSND collaboration that were obtained more than two decades ago [2–7]. If substantiated by further experimental data, these results will have profound implications on string phenomenology. While sterile neutrinos may arise naturally in large volume scenarios, they present more of a challenge in heterotic-string models that replicate the matter structure of Grand Unified Theories (GUTs). It should, however, be stressed that the experimental and phenomenological analysis in this regard is far from settled [8], and highlights the urgency for further experimental data to resolve the question.

Possible existence of sterile neutrinos has been studied in the Beyond the Standard Model field theory constructions [9–24]. Neutrino masses in string inspired top–down scenarios is reviewed in [25]. In this paper I examine the potential implications of sterile neutrinos in string derived models. The discussion for the most part will be qualitative and more elaborate investigations are relegated for future work.

Field theory extensions of the Standard Model often give rise to sterile neutrinos and by implications to neutrino

masses and mixing. The renormalisable Standard Model itself does not accommodate neutrino masses, which are mandated by the observation of neutrino oscillations [26,27]. However, as the sterile neutrinos are neutral under the Standard Model gauge group, one can write for them a large Majorana mass term, which produces the so-called seesaw mechanism [28–31]. At low scales there remain three active neutrinos, which mostly consist of the Standard Model left-handed neutrinos, whereas the right-handed neutrinos mass is of the order of the seesaw scale. The seesaw mechanism naturally explains the suppression of the left-handed neutrino masses, compared to the mass scale of the charged Standard Model particles.

This explanation of the light neutrino masses and oscillations also fits beautifully in $SO(10)$ -GUT embedding of the Standard Model, in which each generation, augmented by a right-handed neutrino, fits in the chiral **16** representation of $SO(10)$. The structure of the Standard Model matter charges then possess a robust mathematical underpinning. The seesaw mass scale is tied to the GUT symmetry breaking scale and therefore has a natural origin. Furthermore, GUT mass relations between the neutral leptons and charged fermions make the seesaw mechanism a necessity, rather than a nicety. Thus, within $SO(10)$ -GUTs the neutrino spectrum generically consists solely of three active neutrinos. Extending the $SO(10)$ symmetry to E_6 may naturally give rise to sterile neutrinos, depending on the symmetry breaking structures [9]. Furthermore, the existence of sterile neutrinos in this scheme may hinge on the existence of an Abelian symmetry, beyond the Standard Model, and that remains unbroken down to low scales.

An alternative to the seesaw mechanism is the possibility that neutrinos obtain their mass via electroweak symmetry breaking. In this case sterile neutrinos naturally arise as right-handed neutrinos and mirrors the mass generation of the Standard Model charged sector. One then needs to explain the, at least, nine orders of magnitude suppression of the neutrino Yukawa coupling compared to the Yukawa cou-

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plings in the charged sector. In this case one also abandons the GUT scenario, which is motivated by Standard Model matter charges.

In this paper, I focus for the most part on examination of sterile neutrinos in string GUT models. I argue that in the generic model sterile neutrinos do not arise and discuss the conditions that may allow them to appear. String GUT models contain right-handed neutrino states that arise from the $\mathbf{16}$ spinorial representation of $SO(10)$. While the $SO(10)$ gauge group is broken directly at the string level, remnants of the GUT symmetry give rise to mass relations between the up quark mass matrix and the neutrino Dirac mass matrix. Suppression of the active neutrino masses therefore necessitates employing the seesaw mechanism at a high scale, which gives the right-handed neutrinos mass of the order of the seesaw scale. Heuristically, we might associate the existence of light sterile neutrinos with an unbroken Abelian gauge symmetry below the string scale. The caveat with string derived GUT constructions is that obtaining such extra $U(1)$ symmetries, that may remain viable below the string scale, is not straightforward. The reason is that the typical E_6 $U(1)$ symmetries, that are amply discussed in the string inspired literature, are generically anomalous in the string derived models [32]. A highly non-trivial construction that enables an additional E_6 Z' to remain unbroken below the string scale was presented in Ref. [33], and utilises self-duality under the spinor-vector duality symmetry of Refs. [34–36].

2 Sterile neutrinos in large volume scenarios

In this section I briefly elaborate why sterile neutrinos may arise naturally in large volume scenarios. Neutrino masses in these scenarios, in which the fundamental scale of quantum gravity may be as low as the TeV scale, were discussed in e.g. Refs. [37–39]. In this case the $SO(10)$ GUT embedding of the Standard Model states must be abandoned. As discussed above the reason is the $SO(10)$ mass relations that necessitate a high seesaw scale. In large volume scenarios the seesaw scale is low, indicating that the origin of the right-handed neutrino states differs from that of the other Standard Model fields. This is achieved if the Standard Model particles are confined to a brane, whereas the right-handed neutrinos can propagate in the bulk [37–39]. The neutrino mass terms arise from the higher dimensional kinetic terms that produce Dirac mass terms. The Yukawa couplings of the left- and right-handed neutrinos are then suppressed by the volume of the extra dimensions. Considering a five dimensional theory, (x^μ, y) with $\mu = 0, \dots, 3$ and a compactified circle y with radius R . The right-handed neutrino is bulk fermion state, while the lepton and Higgs doublets are confined to the brane. The bulk Dirac spinor is decomposed in the Weyl

basis $\Psi = (v_R, \bar{v}_R^c)$ and takes the usual Fourier expansion

$$v_R^{(c)}(x, y) = \sum_n \frac{1}{\sqrt{2\pi r}} v_R^{(c)}(x) e^{iny/r}. \quad (2.1)$$

The four dimensional model then contains the usual tower of Kaluza–Klein states with Dirac masses n/r and a free action for the lepton doublet, which is localized on the wall. The leading interaction term between the walls fields and the bulk fermion is

$$S^{\text{int}} = \int d^4x \lambda l(x) h^*(x) v_R(x, y=0) \quad (2.2)$$

with λ being a dimensionless parameter. The Yukawa coupling λ is rescaled like the graviton and dilaton coupling to all brane fields. The effective Yukawa on the four dimensional brane is given by

$$\lambda_{(4)} = \frac{\lambda}{\sqrt{r^n M_*^n}}, \quad (2.3)$$

which leads to very strong suppression of the Dirac mass even for $\lambda \sim 1$:

$$m = \frac{v}{\sqrt{2}} \lambda_{(4)} = \frac{\lambda v}{\sqrt{2}} \frac{M_*}{M_{\text{Pl}}} \simeq \lambda \frac{M_*}{1 \text{ TeV}} \cdot 5 \cdot 10^{-5} \text{ eV}, \quad (2.4)$$

where v is the electroweak VEV. The brane left-handed neutrino couples to the tower of bulk Kaluza–Klein modes. The resulting mass matrix for every neutrino species is given by [37–39]

$$\mathbf{M} = \begin{pmatrix} m & 0 & 0 & 0 & \dots \\ m & 1/r & 0 & 0 & \dots \\ m & 0 & 2/r & 0 & \dots \\ m & 0 & 0 & 3/r & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (2.5)$$

In the limit $m = 0$, the mass matrix (2.5) has one vanishing eigenvalue and the usual ladder of KK masses. In this limit the left-handed neutrino is decoupled from the Kaluza–Klein tower of states. When m is finite, the left-handed neutrino mixes with the other states and the mixing angle θ_k between the left-handed neutrino and k th Kaluza–Klein state is given by

$$\theta_k \simeq \frac{mr}{|k|} \quad (2.6)$$

The suppression of $\lambda_{(4)}$, similar to the suppression of the gravitational couplings, results in the suppression of neutrino masses. However, for our purpose here it is noted that

the tower of Kaluza–Klein states acts naturally as sterile neutrinos and may lead to observable effects through their interactions with the left-handed neutrino masses [37–39]. String derived brane constructions that may be used toward realising the large volume scenarios were explored (see e.g. [40,41] and references therein).

3 Sterile neutrinos in string GUT constructions

In this section I explore possible existence of sterile neutrinos in string GUT constructions, which are string models that retain the embedding of the Standard Model states in $SO(10)$ and E_6 representations. While the GUT symmetries are broken in these models directly at the string level, the matter states are obtained from GUT representations that are decomposed under the final unbroken GUT subgroup. Consequently, the embedding preserves the weak hypercharge $U(1)$ GUT charges, and the canonical $\sin^2 \theta_W(M_{\text{GUT}}) = 3/8$ normalisation. String GUT models may, in general, be obtained from perturbative and nonperturbative constructions. The concrete class of models that are investigated here are perturbative heterotic string models in the free fermionic formulation [42–44]. These string vacua correspond to $Z_2 \times Z_2$ toroidal orbifold compactifications [45–49]. The free fermionic representation is constructed at enhanced symmetry point in the toroidal moduli space, and deformation away from the free fermionic point are obtained by adding world-sheet Thirring interactions among the world-sheet fermions [50]. This construction produces a large space of phenomenological three generation models [51,52,54–67] that can be used to explore the physics of the Standard Model and its extensions. Details of the free fermionic formulation and of the phenomenological three generation models are given in the references and will not be repeated. Only the features relevant for the question of sterile neutrinos and neutrino masses will be highlighted here.

In the free fermionic formulation of the light-cone heterotic-string in four dimensions there are 20 left-moving, and 44 right-moving, worldsheet real fermions. The 64 free fermions are denoted by

$$\{\psi^{1,2}, (\chi, y, \omega)^{1,\dots,6} | (\bar{y}, \bar{\omega})^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\},$$

where 32 of the right-moving real fermions are grouped into 16 complex fermions that produce the Cartan generators of the rank 16 gauge group. Here $\bar{\phi}^{1,\dots,8}$ are the Cartan generators of the rank eight hidden sector gauge group and $\bar{\psi}^{1,\dots,5}$ are the Cartan generators of the $SO(10)$ GUT group. The complex worldsheet fermions $\bar{\eta}^{1,2,3}$ generate three Abelian currents, $U(1)_{1,2,3}$, in the Cartan subalgebra of the observable gauge group with $U(1)_\zeta$ being their linear combination

$$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3. \quad (3.1)$$

Models in the free fermionic formulation are constructed by specifying a set of boundary condition basis vectors that specify the transformation properties of the 64 worldsheet fermions around the noncontractible loops of the worldsheet torus, $B = \{v_1, v_2, v_3, \dots\}$, and the one-loop Generalised GSO (GGSO) phases in the partition function $c^{[v_i]}_{[v_j]}$ [42–44]. The basis vectors spans an additive group Ξ , that contains all possible linear combinations of the basis vectors $\Xi = \sum_k n_k v_k$, where $n_k = 0, \dots, N_{v_k} - 1$, and N_{v_k} denote the order of each of the basis vectors. The physical representations in the Hilbert space of a given sector $\xi \in \Xi$ are obtained by acting on the vacuum with fermionic and bosonic oscillators and by applying the GGSO projections. The $U(1)$ charges of the physical states with respect to the Cartan generators of the four dimensional gauge group are given by

$$Q(f) = \frac{1}{2} \xi(f) + F_\xi(f),$$

where $\xi(f)$ is the boundary condition of the complex worldsheet fermion f in the sector ξ , and $F_\xi(f)$ is a fermion number operator [42–44]. The phenomenological properties of the string model are extracted by calculating tree-level and higher order terms in the superpotential and by analysing its flat directions [68–71].

The free fermionic formulation of the heterotic-string produced a large space of phenomenological models that share the underlying $Z_2 \times Z_2$ orbifold structure, with differing unbroken $SO(10)$ subgroups. The construction of the models can be viewed in two stages. The first consist of the basis vectors that preserve the $SO(10)$ symmetry. The models at this stage possess (2,0) worldsheet supersymmetry, with $N = 1$ spacetime supersymmetry, and a number of vectorial and spinorial representations of $SO(10)$. The second stage of construction consist of the inclusion of the basis vectors that break the $SO(10)$ gauge group to a subgroup. The $SO(10)$ breaking vectors correspond to Wilson line breaking in the corresponding $Z_2 \times Z_2$ orbifold models. The $SO(10)$ preserving basis vectors are typically denoted by $\{b_i\}$ $i = 1, 2, 3, \dots$, whereas those that break the $SO(10)$ symmetry are denoted by small Greek letters $\{\alpha, \beta, \gamma, \dots\}$. In all these models the unbroken $SO(10)$ subgroup contains an unbroken combination of the $SO(10)$ Cartan generators, beyond the Standard Model gauge group. This additional combination must therefore be broken in the effective field theory low energy limit of the string models. The only available fields in the string models to achieve this breaking are fields that arise from the spinorial representation of $SO(10)$ and its conjugate. The case of the Standard-like models also contains exotic states that can be employed toward that end. This distinction will not be important in the following. The weak hypercharge combination and the additional unbroken

$U(1)$ are given by

$$U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L, \quad (3.2)$$

$$U(1)_{Z'} = U(1)_C - U(1)_L, \quad (3.3)$$

where $U(1)_C$ and $U(1)_L$ are¹ defined in terms of the worldsheet charges by

$$Q_C = Q(\bar{\psi}^1) + Q(\bar{\psi}^2) + Q(\bar{\psi}^3) \\ \text{and } Q_L = Q(\bar{\psi}^4) + Q(\bar{\psi}^5). \quad (3.4)$$

The string models contain additional unbroken $U(1)$ symmetries and an unbroken hidden sector gauge group, the details of which are not crucial for the general discussion here. Aside from the linear combination given by $U(1)_\zeta$ in Eq. (3.1), which arises from the breaking $E_6 \rightarrow SO(10) \times U(1)_\zeta$. This breaking pattern is a generic consequence of the breaking of the worldsheet supersymmetry from $(2, 2)$ to $(2, 0)$ and is of vital importance in the ensuing discussion. The point is that a generic consequence of this breaking is that $U(1)_\zeta$ is anomalous in most of the phenomenological free fermionic string vacua, and therefore cannot remain unbroken below the string scale. The sole example in which it is anomaly free is the string model of Ref. [33].

The entire spectrum of specific free fermionic string models is typically derived by applying the generalised GGSO projections. Here I focus on the generic features of the spectrum relevant for the question of sterile neutrinos. The three chiral generations are obtained in these models from the three twisted sectors of the $Z_2 \times Z_2$ orbifold, which are denoted in the free fermionic models as $B_{1,2,3}$. The Standard Model electroweak Higgs doublet may be obtained from the untwisted-sector or the twisted sectors. In either case fermion mass terms are obtained from coupling of the chiral generations to the Higgs doublets and quasi-realistic mass spectrum may be generated. Additionally, the models contain numerous Standard Model and $SO(10)$ singlet states that may transform under the hidden gauge group and are charged under the additional $U(1)$ symmetries. Typically, the models also contain a number of states that are singlets of the entire four dimensional gauge group. In some models the number of such singlets is correlated with the total number of generations and anti-generations.

In all the free fermionic models the breaking of the $U(1)$ symmetry in Eq. (3.3) requires the existence of heavy Higgs fields \mathcal{N} and $\bar{\mathcal{N}}$, that are obtained from the spinorial $\mathbf{16}$ and $\bar{\mathbf{16}}$ representations of $SO(10)$. In models with an intermediate non-Abelian gauge symmetry, like the flipped $SU(5)$ (FSU5) [51,64,65], the Pati–Salam (PS) [54,55,60–62], the left–right symmetric (LRS) [58,59,67], and the $SU(6) \times SU(2)$ (SU62) [63] models, the heavy Higgs fields break the non-

Abelian gauge symmetry, whereas in the standard-like models (SLM) [52,53,66] they only break the extra $U(1)$ Eq. (3.3). In the case of the FSU5 and SU62 models the breaking is constrained to be of the order of the GUT scale, whereas in the other cases it may be lower. The VEV of the $\bar{\mathcal{N}}$ field also generate the Majorana mass term of the right-handed neutrino in these models.

The structure of the neutrino mass matrix is quite generic in models inspired from the free fermionic models. The terms in the superpotential, in term of component fields, that generate the neutrino mass matrix are (see e.g. [72]),

$$L_i N_j \bar{h}, \quad N_i \bar{\mathcal{N}} \phi_j, \quad \phi_i \phi_j \phi_k, \quad (3.5)$$

where N_i , L_i and ϕ_i , with $i, j, k = 1, 2, 3$ the right-handed neutrinos; are the chiral lepton doublets; and three $SO(10)$ singlet fields, respectively; \bar{h} is the electroweak Higgs doublet and $\bar{\mathcal{N}}$ is the component of the heavy Higgs field that breaks $U(1)_{Z'}$ in Eq. (3.3). All these states appear in the string models, possibly as components of larger representation in as, e.g., the PS and SU62 models. Generally, the neutrino seesaw matrix has the form

$$(v_i, N_i, \phi_i) \begin{pmatrix} 0 & (M_D)_{ij} & 0 \\ (M_D)_{ij} & 0 & \langle \bar{\mathcal{N}} \rangle_{ij} \\ 0 & \langle \bar{\mathcal{N}} \rangle_{ij} & \langle \phi \rangle_{ij} \end{pmatrix} \begin{pmatrix} v_j \\ N_j \\ \phi_j \end{pmatrix}, \quad (3.6)$$

where M_D is the Dirac mass matrix arising from the first term in Eq. (3.5). The Dirac mass matrix is proportional to the up-quark mass matrix due to the underlying $SO(10)$ symmetry [72]. Taking the mass matrices to be diagonal the mass eigenstates are primarily v_i , N_i and ϕ_i with negligible mixing and with the eigenvalues

$$m_{v_j} \sim \left(\frac{k M_u^j}{\langle \bar{\mathcal{N}} \rangle} \right)^2 \langle \phi \rangle, \quad m_{N_j}, m_\phi \sim \langle \bar{\mathcal{N}} \rangle. \quad (3.7)$$

where k is a renormalisation factor due to RGE evolution. The important point to consider is whether any of the Standard Model singlet fields may serve the role of a sterile neutrino, i.e. it has to remain light and mix with the active neutrinos. The obvious observation is that in general the answer is negative. While the string models contain numerous Standard Model and $SO(10)$ singlet fields, they appear mostly as vector-like fields and would therefore receive heavy mass along supersymmetric flat directions. From Eq. (3.7) it is seen that the mass eigenvalues correspond to three light active neutrinos and six massive states with masses of the order of the seesaw scale. Therefore, there are no sterile neutrinos in these cases.

The question of light neutrino masses was analysed in some detail in concrete string derived models in Refs. [73–76]. Mass terms in the superpotential are obtained from renormalisable and nonrenormalisable terms by calculating correlators between vertex operators [68–71]. The models

¹ $U(1)_C = \frac{3}{2}U(1)_{B-L}$; $U(1)_L = 2U(1)_{T_{3R}}$.

typically contain an anomalous $U(1)$ gauge symmetry that breaks supersymmetry at the string scale and destabilises the vacuum. The vacuum is stabilised by assigning Vacuum Expectation Values to Standard Model singlet fields in the string massless spectrum, along supersymmetric F - and D -flat directions. Some of the nonrenormalisable operators then become renormalisable operators in the effective low energy field theory below the string scale. In this process many of the Standard Model singlets receive heavy mass and decouple from the low energy spectrum. The seesaw mass matrix in Eq. (3.6) requires that some of the singlet fields obtain intermediate mass. As the singlet fields are typically vector-like this presents a major difficulty in generating the seesaw mechanism in the string models, let alone allowing for the existence of sterile neutrinos. A priori it is not apparent that any of the non-chiral singlets can remain light. I focus here on the analysis performed in Refs. [74–76] for the model in Refs. [56,57]. Details of the model and its spectrum are given in Refs. [56,57].

The set of fields in the model of Refs. [56,57] that enter the seesaw mass matrix includes the three right-handed neutrinos, N_i ; the three left-handed neutrinos, L_i ; and the set of Standard Model singlets. These include: $SO(10)$ singlets with $U(1)$ and hidden charges, $\{\Phi_{45}, \Phi_{1,2,3}^\pm, \Phi_{13}, \Phi_{23}, \Phi_{12}, T_i, V_i\} \oplus \text{h.c.}$; a set of $SU(3) \times SU(2) \times U(1)_Y$ singlets, $H_{13-14,17-20,23-26}$, with $U(1)_{Z'}$ charge; three entirely neutral singlets $\xi_{1,2,3}$.

In Refs. [74–76] a detailed study of the superpotential, with nonrenormalisable terms up to order $N = 10$, and of the supersymmetric flat directions was presented. The resulting neutrino mass matrix takes the approximate form [74–76]

where

$$\begin{aligned} r &\sim 10^{-6}\text{GeV} & v &\sim 10^2\text{GeV} & w &\sim 10^6\text{GeV} \\ q &\sim 10^7\text{GeV} & u &\sim 10^8\text{GeV} & x &\sim 10^9\text{GeV} \\ z &\sim 10^{10}\text{GeV} & p &\sim 10^{11}\text{GeV} & y &\sim 10^{13}\text{GeV} \end{aligned}$$

It is important to emphasise that this solution is not aimed to produce a realistic mass and mixing spectrum for the fermionic fields, but merely to explore its qualitative features. The mass matrix (3.8) has the mass eigenvalues $\{1.7 \times 10^{13}, 1.7 \times 10^{13}, 2 \times 10^{11}, 1 \times 10^{10}, 9 \times 10^9, 1 \times 10^9, 1 \times 10^9, 5 \times 10^6, 101, 101, 17.5, 17.5, 0.02, 2.4^{-8}\}\text{GeV}$. The lightest eigenvalue, of order 10eV, is predominantly L_3 . The lightest singlet states, with order 10% mixing with L_2 , are two nearly degenerate combinations of $\sim 70\%$ mixture of Φ_1^- and Φ_3^- , and mass of order 17.5 GeV. The remaining spectrum is readily analyzed and contains mixtures of the right-handed neutrinos and the $SO(10)$ singlets, with heavier masses. The main demonstration from the analysis is the lesson that although a simple and elegant mechanism for the neutrino spectrum can be motivated from string theory in the form of (3.6), generating it from string models is not straightforward. The main difficulty is to understand how the singlet masses can be protected from being too massive. This problem already appears when trying to generate a seesaw mechanism for the three active neutrinos, let alone for any additional sterile neutrinos at a comparable mass scale.

Several comments are in order. The picture described above is generic in the string GUT models. The common features include the existence of an anomalous $U(1)$ symmetry and proliferation of Standard Model singlets in the

$$\begin{array}{c} \begin{array}{cccccccccccccccc} & L_3 & L_2 & L_1 & N_3 & N_2 & N_1 & H_{23} & H_{25} & \Phi_{13} & \Phi_{45} & \bar{\Phi}_1^- & \bar{\Phi}_3^- & \Phi_2^+ & \bar{\Phi}_2^+ \\ \begin{array}{l} L_3 \\ L_2 \\ L_1 \\ \\ N_3 \\ N_2 \\ N_1 \\ \\ H_{23} \\ H_{25} \\ \Phi_{13} \\ \Phi_{45} \\ \Phi_1^- \\ \Phi_3^- \\ \Phi_2^+ \\ \bar{\Phi}_2^+ \end{array} & \left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \\ 0 & r & r & 0 & 0 & 0 & 0 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & z & 0 & u & z & u & u & z & u & u \\ r & r & v & 0 & 0 & 0 & 0 & w & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & z & 0 & p & 0 & x & p & x & x & x & x & x \\ 0 & 0 & 0 & x & 0 & w & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u & 0 & x & 0 & q & x & y & y & 0 & y & y \\ 0 & 0 & 0 & 0 & z & 0 & p & 0 & x & p & x & x & x & x & x \\ 0 & 0 & 0 & 0 & u & 0 & x & 0 & y & x & q & q & q & q & q \\ 0 & 0 & 0 & 0 & u & 0 & x & 0 & y & x & q & q & q & q & q \\ 0 & 0 & 0 & 0 & z & 0 & x & 0 & 0 & x & q & q & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & 0 & x & 0 & y & x & q & q & x & q & q \end{array} \right) \end{array} \end{array}$$

massless spectrum that nevertheless gain heavy or intermediate mass in the effective field theory limit. Generation of quasi-realistic fermion mass matrices typically requires elaborate solutions of the flat directions [77]. This in turn generate intermediate mass terms for the Standard Model singlet fields, and therefore eliminates them from being candidate sterile neutrino states. We may also consider the possibility of generating Majorana mass terms for the right-handed neutrinos from the nonrenormalisable terms

$$N_i N_j \overline{\mathcal{N}} \overline{\mathcal{N}}, \quad (3.8)$$

where $\overline{\mathcal{N}}$ obtains a VEV that breaks the $U(1)_{Z'}$ in Eq. (3.3). In the SLM model of Refs. [56, 57] this requires the utilization of the exotic H fields that carry $-1/2Q(Z')$ with respect to the charge of the right-handed neutrino, as the $\overline{\mathcal{N}}$ state is not obtained in the massless spectrum. The needed terms are of the form $NNHHHH\phi^n$. In the model of Refs. [56, 57] such terms were not found up to $N = 14$ and hence cannot induce the seesaw mechanism. In the FSU5 [51, 64, 65], PS [54, 55, 60–62], LRS [58, 59, 67] and SU62 [63] models the $\overline{\mathcal{N}}$ states do arise in the spectrum and may generate terms of the form of Eq. (3.8). However, while this may contribute to the implementation of the seesaw mechanism, its role in allowing for light sterile neutrinos in the string derived models is yet to be examined.

From the discussion above it is clear that existence of sterile neutrinos at a mass scale of the order of the active neutrinos and with substantial mixing with them, is rather problematic in string derived GUT models. This situation is, however, not without hope. A model that may present an alternative scenario is the string derived Z' model of Ref. [33], where the unbroken $SO(10)$ subgroup is $SO(6) \times SO(4)$. The model is self dual under the spinor–vector duality of Refs. [34–36]. As a result the chiral spectrum in this models forms complete E_6 representation, and consequently the $U(1)_\zeta$ combination of Eq. (3.1) is anomaly free and may remain unbroken below the string scale. The gauge symmetry, however, is not enhanced, and spacetime vector bosons arise in this model solely from the untwisted sector. The entire massless spectrum of the model, and the charges under the gauge group, are given in Ref. [33]. Tables 1 and 2 provide a glossary of the states in the model and their charges under the $SU(4) \times SO(4) \times U(1)_\zeta$ gauge group. We note in particular the existence of the seven S and four \overline{S} states that are singlets of $SO(10)$ and are charged under $U(1)_\zeta$. There are therefore three chiral states that are singlets of $SO(10)$ and charged under $U(1)_\zeta$. They are left-over components from the 27 and $\overline{27}$ representations of E_6 .

The string model is obtained by trawling a self-dual model under the spinor–vector duality at the $SO(10)$ level, i.e. prior to breaking the $SO(10)$ symmetry to the Pati–Salam subgroup. Self-duality under the spinor–vector duality plays a key in the construction of the string model with anomaly free $U(1)_\zeta$, together with E_6 embedding of the $U(1)_\zeta$ charges.

In this respect it is noted that $U(1)_\zeta$ is anomaly free in the LRS heterotic-string models as well [58, 59, 67]. However, in the LRS models the $U(1)_\zeta$ charges of the Standard Model states do not possess the E_6 embedding. The VEV of \mathcal{N} and $\overline{\mathcal{N}}$ leaves unbroken the same $U(1)$ combination of $U(1)_C$, $U(1)_L$ and $U(1)_\zeta$, but in the case of the LRS models its breaking at low scales would lead to contradiction with $\sin^2 \theta_W(M_Z)$ and $\alpha_s(M_Z)$ [78]. An alternative to the construction of Ref. [33] is the construction proposed in Ref. [79] that essentially amounts to an alternative E_6 breaking pattern, as follows. The contributions to the observable gauge symmetry in the free fermionic models may come from two sectors. The first is the untwisted sector and the second is the x -sector that enhances the ten dimensional observable $SO(16)$ to E_8 . In most of the phenomenological free fermionic models the spacetime vector bosons from the x -sector are projected out, which results in the symmetry breaking pattern $E_6 \rightarrow SO(10) \times U(1)_\zeta$, and $U(1)_\zeta$ becoming anomalous. In Ref. [79] an alternative symmetry breaking pattern is proposed that retains the spacetime vector bosons from the x -sector and may allow for $U(1)_\zeta$ to remain anomaly free. An explicit string derived model that realises this symmetry breaking pattern is the three generation $SU(6) \times SU(2)$ GUT model of Ref. [63]. In this model $U(1)_\zeta$ is anomaly free by virtue of its embedding in the GUT group. It should be remarked that the precise combination of $U(1)_{1,2,3}$ that forms $U(1)_\zeta$ may differ from the one in Eq. (3.1), up to signs. This is the case, for example, in the model of Ref. [63]. The important properties of $U(1)_\zeta$ are that: it is the family universal combination; it is anomaly free; and the charges of the Standard Model states admit the E_6 embedding. The models of Refs. [33] and [63] demonstrate how this is achieved, either by exploiting the self-duality under the spinor–vector duality, as in the model Ref. [33], or embedding $U(1)_\zeta$ in a non-Abelian group, as in the model of Ref. [63].

In either of these cases the massless string spectrum contains the fields required to break the GUT symmetry to the Standard Model. I focus here on the model of Ref. [33]. The observable and hidden gauge groups at the string scale are produced by untwisted sector states and are given by:

$$\begin{aligned} \text{observable} : & SO(6) \times SO(4) \times U(1)_1 \times U(1)_2 \times U(1)_3 \\ \text{hidden} : & SO(4)^2 \times O(8) \end{aligned}$$

Additional spacetime vector bosons that may enhance the observable and hidden gauge symmetries are projected out in this model due to the choice of GGSO projection coefficients. There are two anomalous $U(1)$ s in the string model with

$$\text{Tr}U(1)_1 = 36 \quad \text{and} \quad \text{Tr}U(1)_3 = -36. \quad (3.9)$$

Therefore, the E_6 combination in Eq. (3.1) is anomaly free and can be as a component of an extra Z' below the string

Table 1 Observable sector field notation and associated states in [33]

Symbol	Fields in [33]	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_\zeta$
F_L	F_{1L}, F_{2L}, F_{3L}	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$+\frac{1}{2}$
F_R	F_{1R}	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$-\frac{1}{2}$
\bar{F}_R	$\bar{F}_{1R}, \bar{F}_{2R}, \bar{F}_{3R}, \bar{F}_{4R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$+\frac{1}{2}$
h	h_1, h_2, h_3	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	-1
Δ	D_1, \dots, D_7	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	-1
$\bar{\Delta}$	$\bar{D}_1, \bar{D}_2, \bar{D}_3, \bar{D}_6$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$+1$
S	$\Phi_{12}, \Phi_{13}, \Phi_{23}, \chi_1^+, \chi_2^+, \chi_3^+, \chi_5^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+2$
\bar{S}	$\bar{\Phi}_{12}, \bar{\Phi}_{13}, \bar{\Phi}_{23}, \bar{\chi}_4^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-2
ϕ	ϕ_1, ϕ_2	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+1$
$\bar{\phi}$	$\bar{\phi}_1, \bar{\phi}_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-1
ζ	$\Phi_{12}^-, \Phi_{13}^-, \Phi_{23}^-, \bar{\Phi}_{12}^-, \bar{\Phi}_{13}^-, \bar{\Phi}_{23}^-$ $\chi_1^-, \chi_2^-, \chi_3^-, \bar{\chi}_4^-, \chi_5^-$ $\zeta_i, \bar{\zeta}_i, i = 1, \dots, 9$ $\Phi_i, i = 1, \dots, 6$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0

scale. As noted from Table 2 the model contains hidden sector vector-like states, that include: four bidoublets denoted by H^\pm with $Q_\zeta = \pm 1$ charges; 12 neutral bidoublets denoted by H ; and five states that transform in the 8 representation of the hidden $SO(8)$ gauge group with $Q_\zeta = 0$. The observable $SO(6) \times SO(4)$ gauge symmetry in the model is broken by the VEVs of the heavy Higgs fields $\mathcal{H} = F_R$ and $\bar{\mathcal{H}}$, which is a linear combination of the four \bar{F}_R fields. The decomposition of these fields in terms of the Standard Model gauge group factors is given by:

$$\begin{aligned} \bar{\mathcal{H}}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) &\rightarrow u_H^c \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3} \right) + d_H^c \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3} \right) \\ &\quad + \bar{\mathcal{N}}(\mathbf{1}, \mathbf{1}, 0) + e_H^c(\mathbf{1}, \mathbf{1}, -1) \\ \mathcal{H}(\mathbf{4}, \mathbf{1}, \mathbf{2}) &\rightarrow u_H \left(\mathbf{3}, \mathbf{1}, -\frac{2}{3} \right) + d_H \left(\mathbf{3}, \mathbf{1}, \frac{1}{3} \right) \\ &\quad + \mathcal{N}(\mathbf{1}, \mathbf{1}, 0) + e_H(\mathbf{1}, \mathbf{1}, 1) \end{aligned}$$

The VEVs along the \mathcal{N} and $\bar{\mathcal{N}}$ directions leave the unbroken combination

$$U(1)_{Z'} = \frac{1}{5}(U(1)_C - U(1)_L) - U(1)_\zeta \notin SO(10), \quad (3.10)$$

that may remain unbroken below the string scale provided that $U(1)_{Z'}$ is anomaly free. The cancellation of the $U(1)_{Z'}$ anomalies requires the presence of the vector-like quarks $\{D_i, \bar{D}_i\}$ and leptons $\{H_i, \bar{H}_i\}$, that arise from the vectorial 10 representation of $SO(10)$, as well as the $SO(10)$ singlets S_i in the 27 of E_6 . The spectrum below the Pati–Salam breaking scale is displayed schematically in Table 3. The three right-handed neutrino N_L^i states become massive at the $SU(2)_R$ breaking scale, which generates the seesaw mechanism via either Eq. (3.6) or via the nonrenormalisable term in Eq. (3.8). I assume here that the spectrum is

Table 2 Hidden sector field notation and associated states in [33]

Symbol	Fields in [33]	$SU(2)^4 \times SO(8)$	$U(1)_\zeta$
H^+	H_{12}^3	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	$+1$
	H_{34}^2	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	$+1$
H^-	H_{12}^2	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	-1
	H_{34}^3	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	-1
H	H_{12}^1	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	0
	$H_{13}^i, i = 1, 2, 3$	$(\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	0
	$H_{14}^i, i = 1, 2, 3$	$(\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})$	0
	H_{23}^1	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	0
	H_{24}^1	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1})$	0
	$H_{34}^i, i = 1, 4, 5$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	0
Z	$Z_i, i = 1, \dots$	$(\mathbf{1}, \mathbf{1}, \mathbf{8})$	0

supersymmetric below the $SU(2)_R$ breaking scale, and allow for the possibility that the spectrum contains an additional pair of vector-like electroweak Higgs doublets. Additionally, the existence of light states ζ_i , that are neutral under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ low scale gauge group, is allowed. The states ϕ and $\bar{\phi}$ are exotic Wilsonian states [80], that match the $\phi_{1,2}$ and $\bar{\phi}_{1,2}$ in Table 1. The $U(1)_{Z'}$ gauge symmetry can be broken at low scales by the VEV of the $SO(10)$ singlets S_i and/or $\phi_{1,2}$.

The extended set of fields appearing in the seesaw mass matrix in this model are $\{L_i, N_i, H_i, S_i, h, \bar{h}, \phi, \bar{\phi}, \zeta_i\}$, and we wish to examine scenarios leading to a sterile neutrino with substantial mixing with the active neutrino. The allowed renormalisable couplings among these fields, contributing to the seesaw mass matrix, in the model are

Table 3 Spectrum and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ quantum numbers, with $i = 1, 2, 3$ for the three light generations. The charges are displayed in the normalisation used in free fermionic heterotic-string models

Field	$SU(3)_C$	$\times SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q_L^i	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
u_L^i	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
d_L^i	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
e_L^i	1	1	+1	$-\frac{2}{5}$
L_L^i	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
\bar{D}^i	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
H^i	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
\bar{H}^i	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
S^i	1	1	0	-2
h	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
\bar{h}	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
ϕ	1	1	0	-1
$\bar{\phi}$	1	1	0	+1
ζ^i	1	1	0	0

$$L_i N_j \bar{h}, L_i N_j \bar{H}_k, N_i \zeta_j \bar{N}, H_i \bar{H}_j S_k, H_i \bar{h} S_k, \bar{H}_i h \zeta_j, h \bar{h} \zeta_i, \phi \bar{\phi} \zeta_i, \bar{\phi} \phi S_i, \zeta_i \zeta_j \zeta_k. \quad (3.11)$$

as well as the nonrenormalisable coupling in Eq. (3.8). Restricting for simplicity solely to the set of chiral fields under the $U(1)_{Z'}$ gauge symmetry generates the seesaw mass matrix in Eq. (3.12), per generation,

$$\begin{pmatrix} L_i & S_i & H_i & \bar{H}_i & N_i \\ L_i & \begin{pmatrix} 0 & 0 & 0 & \lambda n & \lambda v \\ 0 & 0 & \lambda v_2 & \lambda v_3 & 0 \\ 0 & \lambda v_2 & 0 & z' & 0 \\ \lambda n & \lambda v_3 & z' & 0 & 0 \\ \lambda v & 0 & 0 & 0 & \frac{\bar{N}^2}{M} \end{pmatrix} \end{pmatrix},$$

which is produced by the relevant couplings in Eqs. (3.11, 3.8). The analysis here is for only for illustration and performed for a single generation. The seesaw mass matrix in Eq. (3.12) contains VEVs that break three distinct symmetries: the VEVs that break the $SU(2)_R$ symmetry, \bar{N} and n ; the VEVs that break the $U(1)_{Z'}$ symmetry denoted by z' in (3.12); and the VEVs that break the electroweak symmetry denoted by v, v_2 and v_3 in Eq. (3.12). Here v is the VEV that produces the Dirac mass terms that couples between the left- and right-handed neutrinos, and v_2 and v_3 are VEVs along the lepton doublets H and \bar{H} . The notation in Eq. (3.12) expresses generically the dependence of some mass entries on unknown Yukawa couplings. The aim here is not a detailed numerical analysis, but merely to provide an exam-

ple that demonstrates the possibility of sterile neutrinos in string GUT construction. The mass matrix of the light eigenvalues can be obtained by defining the seesaw mass matrix Eq. (3.12) in the form

$$\mathbf{M} = \begin{pmatrix} 0 & \mathbf{B} \\ \mathbf{B}^T & \mathbf{J} \end{pmatrix}, \quad (3.12)$$

with

$$\mathbf{J} = \begin{pmatrix} 0 & z' & 0 \\ z' & 0 & 0 \\ 0 & 0 & \frac{\bar{N}^2}{M} \end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix} 0 & \lambda n & \lambda v \\ \lambda v_2 & \lambda v_3 & 0 \end{pmatrix},$$

where $M \approx 10^{18}$ GeV is related to the heterotic-string scale. The light eigenvalues and eigenvectors are approximately given by those of the matrix

$$\mathbf{B} \mathbf{J}^{-1} \mathbf{B}^T \approx \begin{pmatrix} \frac{M(\lambda v)^2}{\bar{N}^2} & \frac{(\lambda n)(\lambda v_2)}{z'} \\ \frac{(\lambda n)(\lambda v_2)}{z'} & \frac{2(\lambda v_2)(\lambda v_3)}{z'} \end{pmatrix}, \quad (3.13)$$

where the λ coupling appearing is schematic. Taking the parameters in Eq. (3.12) to be given by:

$$\begin{aligned} \lambda v &= 1 \text{ GeV}; \\ \lambda v_2 &= 5 \times 10^{-4} \text{ GeV} \approx m_e; \\ \lambda v_3 &= 5 \times 10^{-4} \text{ GeV}; \approx m_e; \\ \lambda n &= 5 \times 10^{-4} \text{ GeV}; \approx m_e; \\ z' &= 5 \times 10^4 \text{ GeV} = 50 \text{ TeV}; \\ \bar{N} &= 5 \times 10^{14} \text{ GeV}, \end{aligned} \quad (3.14)$$

produces two light eigenvalues with $m_1 = 10^{-2}$ eV and $m_2 = 10^{-3}$ eV and three massive states with $m_3 = m_4 = 50$ TeV and $m_5 = 2.5 \times 10^{11}$ GeV. The heavy eigenstate is the right-handed neutrino. The two intermediate states are equal mixtures of the H_i and \bar{H}_i and the two light eigenstates are mixtures of the left-handed neutrino and S_i , with mixing angle $\sin \theta \approx 0.98$. This is of course only an illustrative scenario and many other possibilities exist. However, it does demonstrate the possibility of producing sterile neutrinos in heterotic-string constructions. And more importantly, it suggests that the existence of a sterile neutrino in this constructions may be correlated with a new Abelian gauge symmetry not far removed from scales currently probed in collider experiments.

4 Conclusions

The results of the MiniBooNE experiment provide evidence for the existence of sterile neutrinos in nature with substantial mixing with the active neutrinos. These results have profound

implications for attempts to unify the Standard Model with gravity. String theory provides a concrete framework where these implications can be studied.

In this paper I argued that sterile neutrinos arise naturally in large volume scenarios and by implication in string constructions in which the gravitational scale may be as low as the TeV scale. Of course, the precise nature of the sterile neutrinos in these constructions needs to be scrutinised, but it is clear that the sterile neutrinos in these models can affect various experimental observables that are being explored in contemporary experiments, e.g. lepton universality [37–39]. I further argued that obtaining sterile neutrinos in string GUT constructions proves more of a challenge, and investigated this question in perturbative heterotic-string models in the free fermionic formulation. I argued that generic phenomenological models in this category would not lead to sterile neutrinos at a mass scale comparable to the active neutrinos and with substantial mixing with them. The main reason being that the Standard Model singlet states in these models arise in vector-like representation with respect to the unbroken four dimensional gauge group at the string scale. A way to guarantee that some singlets remain light and can act as sterile neutrinos is if they are chiral with respect to an additional Abelian gauge symmetry, which is broken at low or intermediate scales. The construction of string models that allow for the required extra Abelian symmetry is, however, highly non-trivial and I discussed how it is realised in some free fermionic heterotic-string models. The results of the Mini-BooNE experiment and its successors may therefore provide vital guidance in the quest for unification of gravity and the gauge interactions.

Acknowledgements I would like to thank the Simons Center for Geometry and Physics at State University of New York in Stony Brook for hospitality while part of this work was conducted.

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