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Analysis of the doubly heavy baryon states and pentaquark states with QCD sum rules

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Abstract In this article, we study the doubly heavy baryon states and pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 7 and 13 respectively in a consistent way. In calculations, we separate the contributions of the negative parity and positive parity hadron states unambiguously, and study the masses and pole residues of the doubly heavy baryon states and pentaquark states in details. The present predictions can be confronted to the experimental data in the future.

1 Introduction

In 2017, the LHCb collaboration observed the doubly charmed baryon state Ξ_{cc}^{++} in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum, and obtained the mass $M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14\,\text{MeV}$ [1]. The observation of the Ξ_{cc}^{++} provides the crucial experimental input on the strong correlation between the two charm quarks, which may shed light on the spectroscopy of the doubly charmed baryon states, tetraquark states and pentaquark states. The attractive interaction induced by one-gluon exchange favors formation of the diquarks in color antitriplet [2,3], the favored configurations are the scalar ($C\gamma_5$) and axialvector ($C\gamma_\mu$) diquark states from the QCD sum rules [4–9]. For the heavy-heavy quark systems QQ, only the axialvector diquarks $\varepsilon^{ijk} Q_j^T C\gamma_\mu Q_k$ and tensor diquarks $\varepsilon^{ijk} Q_j^T C\sigma_{\mu\nu} Q_k$ survive due to the Fermi-Dirac statistics, the axialvector diquarks $\varepsilon^{ijk} Q_j^T C\gamma_\mu Q_k$ are more stable than the tensor diquarks $\varepsilon^{ijk} Q_j^T C\sigma_{\mu\nu} Q_k$, we can take the axialvector diquarks $\varepsilon^{ijk} Q_j^T C\gamma_\mu Q_k$ as basic constituents to study doubly heavy baryon states [10–20], tetraquark states [21–25] and pentaquark states with the QCD sum rules. The doubly heavy pentaquark states have not been studied with the QCD sum rules. In Ref. [26], the mass spectrum of the doubly heavy pentaquark states are studied in a color-magnetic interaction model.

In 2015, the LHCb collaboration studied the $\Lambda_b^0 \to J/\psi K^- p$ decays, and performed the amplitude analysis on all relevant masses and decay angles of the six-dimensional data using the helicity formalism and Breit–Wigner amplitudes to describe all resonances, and observed two exotic states $P_c(4380)$ and $P_c(4450)$ in the $J/\psi p$ invariant mass distributions with the significances of more than 9 standard deviations [27]. The $P_c(4380)$ and $P_c(4450)$ are excellent candidates for the hidden-charm pentaquark states [28–43]. Up to now, no experimental candidates for the doubly charmed or doubly bottom pentaquark states have been observed.

In Refs. [18–20,44–46], we separate the contributions of the positive parity and negative parity baryon states explicitly, and study the heavy, doubly-heavy and triply-heavy baryon states with the QCD sum rules in a systematic way, the truncations of the operator product expansion are shown explicitly in Table 1. We carry out the operator product expansion up to the vacuum condensates of dimension 4 for the positive parity doubly heavy baryon states [18,19], another detailed studied including the contributions of the higher dimensional vacuum condensates are still needed. While in Refs. [10–12], the contributions of the positive parity and negative parity doubly heavy baryon states are not separated explicitly.

In Refs. [39–43], we construct the diquark–diquark–antiquark type interpolating currents to study the $J^P = \frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, $\frac{5}{2}^\pm$ hidden-charm pentaquark states with the QCD sum rules in a systematic way by taking into account the vacuum condensates up to dimension 10 in the operator product expansion and separating the contributions of the positive parity and negative parity pentaquark states explicitly. In calculations, we take the energy scale formula $\mu = \sqrt{M_P^2 - (2\mathbb{M}_c)^2}$ with the effective c-quark mass $\mathbb{M}_c$ to determine the ideal energy scales of the QCD spectral densities.

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Table 1 The truncations in the operator product expansion (OPE) in the QCD sum rules for the heavy, doubly-heavy and triply-heavy baryon states

Baryons (parity)	OPE	References
Qqq' (±)	6	[19,20,44,45]
QQq (+)	4	[18,19]
QQq (−)	5	[20]
QQQ' (±)	4	[46]

In Ref. [47], we construct the diquark–diquark–antiquark type current to study the ground state triply-charmed pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10, and take the energy scale formula $\mu = \sqrt{M_P^2 - (3\mathbb{M}_c)^2}$ to determine the optimal energy scales of the QCD spectral densities.

In Ref. [48], we study the diquark–diquark–antiquark type charmed pentaquark states with $J^P = \frac{3}{2}^\pm$ with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 13 in a consistent way to explore the possible assignments of the new excited Ω_c states as the pentaquark states. The new excited Ω_c states may also be the P-wave excitations of the ground state Ω_c [49–54]. In Ref. [55], we study the $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Sigma_c^*$ pentaquark molecular states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 13 in a consistent way to explore the possible assignments of the $P_c(4380)$ and $P_c(4450)$ as the pentaquark molecular states. In calculations, we observe that the vacuum condensates of dimensions 11 and 13 play an important role in obtaining stable QCD sum rules [48,55].

In this article, we extend our previous works [18–20,24, 25,39,46–48,55] to study the doubly heavy baryon states and pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 7 and 13 respectively in a consistent way.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the doubly heavy baryon states and pentaquark states in Sect. 2; in Sect. 3, we present the numerical results and discussions; and Sect. 4 is reserved for our conclusion.

2 QCD sum rules for the doubly heavy baryon states and pentaquark states

In the following, we write down the two-point correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules,

$$\Pi(p) = i\int d^4x e^{ip\cdot x}\left\langle 0\left|T\left\{J(x)\bar{J}(0)\right\}\right|0\right\rangle,$$

$$\Pi_{\mu\nu}(p) = i\int d^4x e^{ip\cdot x}\left\langle 0\left|T\left\{J_\mu(x)\bar{J}_\nu(0)\right\}\right|0\right\rangle,$$

$$\Pi_{\mu\nu\alpha\beta}(p) = i\int d^4x e^{ip\cdot x}\left\langle 0\left|T\left\{J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(0)\right\}\right|0\right\rangle, \quad (1)$$

where the currents $J(x) = J^{QQq}(x)$, $J^{QQs}(x)$, $J^{QQud\bar{q}}(x)$, $J_\mu(x) = J_\mu^{QQq}(x)$, $J_\mu^{QQs}(x)$, $J_\mu^{QQud\bar{q}}(x)$, $J_{\mu\nu}(x) = J_{\mu\nu}^{QQud\bar{q}}(x)$,

$$\begin{aligned}
J^{QQq}(x) &= \varepsilon^{ijk} Q_i^T(x)C\gamma_\mu Q_j(x)\gamma_5\gamma^\mu q_k(x),\\
J^{QQs}(x) &= \varepsilon^{ijk} Q_i^T(x)C\gamma_\mu Q_j(x)\gamma_5\gamma^\mu s_k(x),\\
J_\mu^{QQq}(x) &= \varepsilon^{ijk} Q_i^T(x)C\gamma_\mu Q_j(x)\, q_k(x),\\
J_\mu^{QQs}(x) &= \varepsilon^{ijk} Q_i^T(x)C\gamma_\mu Q_j(x)\, s_k(x),\\
J^{QQud\bar{q}}(x) &= \varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn} Q_j^T(x)C\gamma_\mu Q_k(x) u_m^T(x)\\
&\quad\times C\gamma_5 d_n(x)\gamma_5\gamma^\mu C\bar{q}_a^T(x),\\
J_\mu^{QQud\bar{q}}(x) &= \varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn} Q_j^T(x)C\gamma_\mu Q_k(x) u_m^T(x)\\
&\quad\times C\gamma_5 d_n(x) C\bar{q}_a^T(x),\\
J_{\mu\nu}^{QQud\bar{q}}(x) &= \varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn} Q_j^T(x)C\gamma_\mu Q_k(x) u_m^T(x)\\
&\quad\times C\gamma_\nu d_n(x) C\bar{q}_a^T(x), \qquad (2)
\end{aligned}$$

the i, j, k, l, m, n, a are color indices, $Q = b, c$, $q = u, d$. In this article, we take the doubly heavy diquarks $\varepsilon^{ijk} Q_j^T C\gamma_\mu Q_k$ as basic constituents to construct the currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ to interpolate the doubly heavy baryon states (Ξ_{QQ}, Ξ_{QQ}^*, Ω_{QQ}, Ω_{QQ}^*) and pentaquark states with the spin $J = \frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$, respectively. In Refs. [22,24,25], we take the doubly heavy diquarks $\varepsilon^{ijk} Q_j^T C\gamma_\mu Q_k$ as basic constituents to study the doubly heavy tetraquark states. Recently, Azizi, Sarac and Sundu studied the meson-baryon type (or the color singlet-singlet type) doubly heavy pentaquark states with a hidden-charm (or hidden-bottom) quark pair using the QCD sum rules [56,57], while in the present work, we study the diquark–diquark–antiquark type doubly heavy pentaquark states with two charmed (or bottom) quarks. In this article, we choose the famous Ioffe currents, for more general currents interpolating the doubly heavy baryon states, one can consult Refs. [14–17], the simple Ioffe currents have shortcomings, more experimental data are still needed to select the best parameters in the more general currents.

The three quark currents $J(0)$ and $J_\mu(0)$ couple potentially to the $\frac{1}{2}^+$ and $\frac{1}{2}^-$, $\frac{3}{2}^+$ doubly heavy baryon states $B_{\frac{1}{2}}^+$ and $B_{\frac{1}{2}}^-$, $B_{\frac{3}{2}}^+$, respectively,

$$\begin{aligned}
\left\langle 0|J(0)|B_{\frac{1}{2}}^+(p)\right\rangle &= \lambda_{\frac{1}{2}}^+ U^+(p,s),\\
\left\langle 0|J_\mu(0)|B_{\frac{1}{2}}^-(p)\right\rangle &= f_{\frac{1}{2}}^-\, p_\mu U^-(p,s),\\
\left\langle 0|J_\mu(0)|B_{\frac{3}{2}}^+(p)\right\rangle &= \lambda_{\frac{3}{2}}^+ U_\mu^+(p,s), \qquad (3)
\end{aligned}$$

the currents $J(0)$ and $J_\mu(0)$ also couple potentially to the $\frac{1}{2}^-$ and $\frac{1}{2}^+$, $\frac{3}{2}^-$ doubly heavy baryon states $B^-_{\frac{1}{2}}$ and $B^+_{\frac{1}{2}}$, $B^-_{\frac{3}{2}}$, respectively,

$$\begin{aligned}
\left\langle 0|J(0)|B^-_{\frac{1}{2}}(p)\right\rangle &= \lambda^-_{\frac{1}{2}}\, i\gamma_5 U^-(p,s),\\
\left\langle 0|J_\mu(0)|B^+_{\frac{1}{2}}(p)\right\rangle &= f^+_{\frac{1}{2}}\, p_\mu i\gamma_5\, U^+(p,s),\\
\left\langle 0|J_\mu(0)|B^-_{\frac{3}{2}}(p)\right\rangle &= \lambda^-_{\frac{3}{2}}\, i\gamma_5 U^-_\mu(p,s),
\end{aligned}\tag{4}$$

because multiplying $i\gamma_5$ to the currents $J(x)$ and $J_\mu(x)$ changes their parity [18–20,44–46,58–60]. The five quark currents $J(0)$, $J_\mu(0)$ and $J_{\mu\nu}$ couple potentially to the $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{1}{2}^-$, $\frac{3}{2}^+$, $\frac{5}{2}^-$ doubly heavy pentaquark states $P^-_{\frac{1}{2}}$, $P^+_{\frac{1}{2}}$, $P^-_{\frac{3}{2}}$ and $P^-_{\frac{1}{2}}$, $P^+_{\frac{3}{2}}$, $P^-_{\frac{5}{2}}$, respectively,

$$\begin{aligned}
\left\langle 0|J(0)|P^-_{\frac{1}{2}}(p)\right\rangle &= \lambda^-_{\frac{1}{2}}\, U^-(p,s),\\
\left\langle 0|J_\mu(0)|P^+_{\frac{1}{2}}(p)\right\rangle &= f^+_{\frac{1}{2}}\, p_\mu\, U^+(p,s),\\
\left\langle 0|J_\mu(0)|P^-_{\frac{3}{2}}(p)\right\rangle &= \lambda^-_{\frac{3}{2}}\, U^-_\mu(p,s),\\
\left\langle 0|J_{\mu\nu}(0)|P^-_{\frac{1}{2}}(p)\right\rangle &= g^-_{\frac{1}{2}}\, p_\mu p_\nu\, U^-(p,s),\\
\left\langle 0|J_{\mu\nu}(0)|P^+_{\frac{3}{2}}(p)\right\rangle &= f^+_{\frac{3}{2}}\, \left[p_\mu U^+_\nu(p,s)+p_\nu U^+_\mu(p,s)\right],\\
\left\langle 0|J_{\mu\nu}(0)|P^-_{\frac{5}{2}}(p)\right\rangle &= \sqrt{2}\lambda^-_{\frac{5}{2}}\, U^-_{\mu\nu}(p,s),
\end{aligned}\tag{5}$$

the currents $J(0)$, $J_\mu(0)$ and $J_{\mu\nu}$ also couple potentially to the $\frac{1}{2}^+$, $\frac{1}{2}^-$, $\frac{3}{2}^+$ and $\frac{1}{2}^+$, $\frac{3}{2}^-$, $\frac{5}{2}^+$ doubly heavy pentaquark states $P^+_{\frac{1}{2}}$, $P^-_{\frac{1}{2}}$, $P^+_{\frac{3}{2}}$ and $P^+_{\frac{1}{2}}$, $P^-_{\frac{3}{2}}$, $P^+_{\frac{5}{2}}$, respectively,

$$\begin{aligned}
\left\langle 0|J(0)|P^+_{\frac{1}{2}}(p)\right\rangle &= \lambda^+_{\frac{1}{2}}\, i\gamma_5 U^+(p,s),\\
\left\langle 0|J_\mu(0)|P^-_{\frac{1}{2}}(p)\right\rangle &= f^-_{\frac{1}{2}}\, p_\mu\, i\gamma_5 U^-(p,s),\\
\left\langle 0|J_\mu(0)|P^+_{\frac{3}{2}}(p)\right\rangle &= \lambda^+_{\frac{3}{2}}\, i\gamma_5 U^+_\mu(p,s),\\
\left\langle 0|J_{\mu\nu}(0)|P^+_{\frac{1}{2}}(p)\right\rangle &= g^+_{\frac{1}{2}}\, p_\mu p_\nu\, i\gamma_5 U^+(p,s),\\
\left\langle 0|J_{\mu\nu}(0)|P^-_{\frac{3}{2}}(p)\right\rangle &= f^-_{\frac{3}{2}}\, i\gamma_5 \left[p_\mu U^-_\nu(p,s)+p_\nu U^-_\mu(p,s)\right],\\
\left\langle 0|J_{\mu\nu}(0)|P^+_{\frac{5}{2}}(p)\right\rangle &= \sqrt{2}\lambda^+_{\frac{5}{2}}\, i\gamma_5 U^+_{\mu\nu}(p,s),
\end{aligned}\tag{6}$$

because multiplying $i\gamma_5$ to the five quark currents $J(x)$, $J_\mu(x)$, $J_{\mu\nu}(x)$ also changes their parity [39–43]. The $\lambda^\pm_{\frac{1}{2}/\frac{3}{2}/\frac{5}{2}}$, $f^\pm_{\frac{1}{2}/\frac{3}{2}}$ and $g^\pm_{\frac{1}{2}}$ are the pole residues or the current-hadron coupling constants. The spinors $U^\pm(p,s)$ satisfy the Dirac equations $(\not{p}-M_\pm)U^\pm(p)=0$, while the spinors $U^\pm_\mu(p,s)$ and $U^\pm_{\mu\nu}(p,s)$ satisfy the Rarita–Schwinger equations $(\not{p}-M_\pm)U^\pm_\mu(p)=0$ and $(\not{p}-M_\pm)U^\pm_{\mu\nu}(p)=0$, and the relations $\gamma^\mu U^\pm_\mu(p,s)=0$, $p^\mu U^\pm_\mu(p,s)=0$, $\gamma^\mu U^\pm_{\mu\nu}(p,s)=0$, $p^\mu U^\pm_{\mu\nu}(p,s)=0$, $U^\pm_{\mu\nu}(p,s)=U^\pm_{\nu\mu}(p,s)$, respectively.

At the phenomenological side, we insert a complete set of intermediate doubly heavy baryon states or pentaquark states with the same quantum numbers as the current operators $J(x)$, $i\gamma_5 J(x)$, $J_\mu(x)$, $i\gamma_5 J_\mu(x)$, $J_{\mu\nu}(x)$ and $i\gamma_5 J_{\mu\nu}(x)$ into the correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ to obtain the hadronic representation [61,62]. After isolating the pole terms of the lowest states, we obtain the complex expressions:

$$\Pi(p) = \lambda^{+2}_{\frac{1}{2}}\frac{\not{p}+M_+}{M_+^2-p^2}+\lambda^{-2}_{\frac{1}{2}}\frac{\not{p}-M_-}{M_-^2-p^2}+\cdots,\tag{7}$$

$$\begin{aligned}
\Pi_{\mu\nu}(p) &= \lambda^{+2}_{\frac{3}{2}}\frac{\not{p}+M_+}{M_+^2-p^2}\left(-g_{\mu\nu}+\frac{\gamma_\mu\gamma_\nu}{3}\right.\\
&\quad\left.+\frac{2p_\mu p_\nu}{3p^2}-\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3\sqrt{p^2}}\right)\\
&\quad+\lambda^{-2}_{\frac{3}{2}}\frac{\not{p}-M_-}{M_-^2-p^2}\left(-g_{\mu\nu}+\frac{\gamma_\mu\gamma_\nu}{3}\right.\\
&\quad\left.+\frac{2p_\mu p_\nu}{3p^2}-\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3\sqrt{p^2}}\right)\\
&\quad+f^{-2}_{\frac{1}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}p_\mu p_\nu\\
&\quad+f^{+2}_{\frac{1}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}p_\mu p_\nu+\cdots,
\end{aligned}\tag{8}$$

for the doubly heavy baryon states, and

$$\Pi(p) = \lambda^{-2}_{\frac{1}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}+\lambda^{+2}_{\frac{1}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}+\cdots,\tag{9}$$

$$\begin{aligned}
\Pi_{\mu\nu}(p) &= \lambda^{-2}_{\frac{3}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}\left(-g_{\mu\nu}+\frac{\gamma_\mu\gamma_\nu}{3}\right.\\
&\quad\left.+\frac{2p_\mu p_\nu}{3p^2}-\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3\sqrt{p^2}}\right)\\
&\quad+\lambda^{+2}_{\frac{3}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}\left(-g_{\mu\nu}+\frac{\gamma_\mu\gamma_\nu}{3}+\frac{2p_\mu p_\nu}{3p^2}\right.\\
&\quad\left.-\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3\sqrt{p^2}}\right)+f^{+2}_{\frac{1}{2}}\frac{\not{p}+M_+}{M_+^2-p^2}p_\mu p_\nu\\
&\quad+f^{-2}_{\frac{1}{2}}\frac{\not{p}-M_-}{M_-^2-p^2}p_\mu p_\nu+\cdots,
\end{aligned}\tag{10}$$

$$\begin{aligned}
\Pi_{\mu\nu\alpha\beta}(p) &= 2\lambda^{-2}_{\frac{5}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}\left[\frac{\widetilde{g}_{\mu\alpha}\widetilde{g}_{\nu\beta}+\widetilde{g}_{\mu\beta}\widetilde{g}_{\nu\alpha}}{2}-\frac{\widetilde{g}_{\mu\nu}\widetilde{g}_{\alpha\beta}}{5}\right.\\
&\quad-\frac{1}{10}\left(\gamma_\mu\gamma_\alpha+\frac{\gamma_\mu p_\alpha-\gamma_\alpha p_\mu}{\sqrt{p^2}}-\frac{p_\mu p_\alpha}{p^2}\right)\widetilde{g}_{\nu\beta}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{10}\left(\gamma_\nu\gamma_\alpha+\frac{\gamma_\nu p_\alpha-\gamma_\alpha p_\nu}{\sqrt{p^2}}-\frac{p_\nu p_\alpha}{p^2}\right)\widetilde{g}_{\mu\beta}+\cdots\Bigg]\\
&+2\lambda_{\frac{5}{2}}^{+2}\frac{\not{p}-M_+}{M_+^2-p^2}\left[\frac{\widetilde{g}_{\mu\alpha}\widetilde{g}_{\nu\beta}+\widetilde{g}_{\mu\beta}\widetilde{g}_{\nu\alpha}}{2}-\frac{\widetilde{g}_{\mu\nu}\widetilde{g}_{\alpha\beta}}{5}\right.\\
&-\frac{1}{10}\left(\gamma_\mu\gamma_\alpha+\frac{\gamma_\mu p_\alpha-\gamma_\alpha p_\mu}{\sqrt{p^2}}-\frac{p_\mu p_\alpha}{p^2}\right)\widetilde{g}_{\nu\beta}\\
&-\frac{1}{10}\left(\gamma_\nu\gamma_\alpha+\frac{\gamma_\nu p_\alpha-\gamma_\alpha p_\nu}{\sqrt{p^2}}-\frac{p_\nu p_\alpha}{p^2}\right)\widetilde{g}_{\mu\beta}+\cdots\Bigg]\\
&+f_{\frac{3}{2}}^{+2}\frac{\not{p}+M_+}{M_+^2-p^2}\left[p_\mu p_\alpha\left(-g_{\nu\beta}+\frac{\gamma_\nu\gamma_\beta}{3}\right.\right.\\
&\left.\left.+\frac{2p_\nu p_\beta}{3p^2}-\frac{p_\nu\gamma_\beta-p_\beta\gamma_\nu}{3\sqrt{p^2}}\right)+\cdots\right]\\
&+f_{\frac{3}{2}}^{-2}\frac{\not{p}-M_-}{M_-^2-p^2}\left[p_\mu p_\alpha\left(-g_{\nu\beta}+\frac{\gamma_\nu\gamma_\beta}{3}\right.\right.\\
&\left.\left.+\frac{2p_\nu p_\beta}{3p^2}-\frac{p_\nu\gamma_\beta-p_\beta\gamma_\nu}{3\sqrt{p^2}}\right)+\cdots\right]\\
&+g_{\frac{1}{2}}^{-2}\frac{\not{p}+M_-}{M_-^2-p^2}p_\mu p_\nu p_\alpha p_\beta\\
&+g_{\frac{1}{2}}^{+2}\frac{\not{p}-M_+}{M_+^2-p^2}p_\mu p_\nu p_\alpha p_\beta+\cdots\,,
\end{aligned}
\tag{11}
$$

for the doubly heavy pentaquark states, where $\widetilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}$.

We can rewrite the correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ into the following form according to Lorentz covariance,

$$
\begin{aligned}
\Pi_{\mu\nu}(p)=&\,\Pi_{\frac{3}{2}}(p^2)\left(-g_{\mu\nu}\right)\\
&+\Pi^1_{\frac{3}{2}}(p^2)\,\gamma_\mu\gamma_\nu+\Pi^2_{\frac{3}{2}}(p^2)\left(p_\mu\gamma_\nu-p_\nu\gamma_\mu\right)\\
&+\Pi_{\frac{1}{2},\frac{3}{2}}(p^2)\,p_\mu p_\nu,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\Pi_{\mu\nu\alpha\beta}(p)=&\,\Pi_{\frac{5}{2}}(p^2)\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)\\
&+\Pi^1_{\frac{5}{2}}(p^2)\,g_{\mu\nu}g_{\alpha\beta}+\Pi^2_{\frac{5}{2}}(p^2)\left(g_{\mu\nu}p_\alpha p_\beta+g_{\alpha\beta}p_\mu p_\nu\right)\\
&+\Pi^3_{\frac{5}{2}}(p^2)\left(g_{\mu\alpha}\gamma_\nu\gamma_\beta+g_{\mu\beta}\gamma_\nu\gamma_\alpha+g_{\nu\alpha}\gamma_\mu\gamma_\beta\right.\\
&\left.+g_{\nu\beta}\gamma_\mu\gamma_\alpha\right)+\Pi^4_{\frac{5}{2}}(p^2)\left[g_{\nu\beta}\left(\gamma_\mu p_\alpha-\gamma_\alpha p_\mu\right)\right.\\
&+g_{\nu\alpha}\left(\gamma_\mu p_\beta-\gamma_\beta p_\mu\right)+g_{\mu\beta}\left(\gamma_\nu p_\alpha-\gamma_\alpha p_\nu\right)\\
&\left.+g_{\mu\alpha}\left(\gamma_\nu p_\beta-\gamma_\beta p_\nu\right)\right]+\Pi^1_{\frac{3}{2},\frac{5}{2}}(p^2)\\
&\times\left(g_{\mu\alpha}p_\nu p_\beta+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta+g_{\nu\beta}p_\mu p_\alpha\right)\\
&+\Pi^2_{\frac{3}{2},\frac{5}{2}}(p^2)\left(\gamma_\mu\gamma_\alpha p_\nu p_\beta+\gamma_\mu\gamma_\beta p_\nu p_\alpha+\gamma_\nu\gamma_\alpha p_\mu p_\beta\right.\\
&\left.+\gamma_\nu\gamma_\beta p_\mu p_\alpha\right)+\Pi^3_{\frac{3}{2},\frac{5}{2}}(p^2)\left[\left(\gamma_\mu p_\alpha-\gamma_\alpha p_\mu\right)p_\nu p_\beta\right.\\
&+\left(\gamma_\mu p_\beta-\gamma_\beta p_\mu\right)p_\nu p_\alpha+\left(\gamma_\nu p_\alpha-\gamma_\alpha p_\nu\right)p_\mu p_\beta\\
&\left.+\left(\gamma_\nu p_\beta-\gamma_\beta p_\nu\right)p_\mu p_\alpha\right]+\Pi_{\frac{1}{2},\frac{3}{2},\frac{5}{2}}(p^2)\,p_\mu p_\nu p_\alpha p_\beta,
\end{aligned}
\tag{13}
$$

the subscripts $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$ in the components $\Pi_{\frac{3}{2}}(p^2)$, $\Pi^1_{\frac{3}{2}}(p^2)$, $\Pi^2_{\frac{3}{2}}(p^2)$, $\Pi_{\frac{1}{2},\frac{3}{2}}(p^2)$, $\Pi_{\frac{5}{2}}(p^2)$, $\Pi^1_{\frac{5}{2}}(p^2)$, $\Pi^2_{\frac{5}{2}}(p^2)$, $\Pi^3_{\frac{5}{2}}(p^2)$, $\Pi^4_{\frac{5}{2}}(p^2)$, $\Pi^1_{\frac{3}{2},\frac{5}{2}}(p^2)$, $\Pi^2_{\frac{3}{2},\frac{5}{2}}(p^2)$, $\Pi^3_{\frac{3}{2},\frac{5}{2}}(p^2)$ and $\Pi_{\frac{1}{2},\frac{3}{2},\frac{5}{2}}(p^2)$ denote the spins of the pentaquark states, which means that the pentaquark states with $J=\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$ have contributions. The components $\Pi_{\frac{1}{2},\frac{3}{2}}(p^2)$, $\Pi^1_{\frac{3}{2},\frac{5}{2}}(p^2)$, $\Pi^2_{\frac{3}{2},\frac{5}{2}}(p^2)$, $\Pi^3_{\frac{3}{2},\frac{5}{2}}(p^2)$ and $\Pi_{\frac{1}{2},\frac{3}{2},\frac{5}{2}}(p^2)$ receive contributions from more than one pentaquark state, so they can be neglected. We can rewrite $\gamma_\mu\gamma_\nu=g_{\mu\nu}-i\sigma_{\mu\nu}$, then the components $\Pi^1_{\frac{3}{2}}(p^2)$, $\Pi^2_{\frac{3}{2}}(p^2)$, $\Pi^3_{\frac{5}{2}}(p^2)$ and $\Pi^4_{\frac{5}{2}}(p^2)$ are associated with tensor structures which are antisymmetric in the Lorentz indexes μ, ν, α or β. In calculations, we observe that such antisymmetric properties lead to smaller intervals of dimensions of the vacuum condensates, therefore worse QCD sum rules, so the components $\Pi^1_{\frac{3}{2}}(p^2)$, $\Pi^2_{\frac{3}{2}}(p^2)$, $\Pi^3_{\frac{5}{2}}(p^2)$ and $\Pi^4_{\frac{5}{2}}(p^2)$ can also be neglected. If we take the replacement $J_{\mu\nu}(x)\to\widehat{J}_{\mu\nu}(x)=J_{\mu\nu}(x)-\frac{1}{4}g_{\mu\nu}J_\alpha{}^\alpha(x)$ to subtract the contributions of the $J=\frac{1}{2}$ pentaquark states, a lots of terms $\propto g_{\mu\nu}$, $g_{\alpha\beta}$ disappear at the QCD side, and result in smaller intervals of dimensions of the vacuum condensates, so the components $\Pi^1_{\frac{5}{2}}(p^2)$ and $\Pi^2_{\frac{5}{2}}(p^2)$ are not the optimal choices to study the $J=\frac{5}{2}$ pentaquark states. Now only the components $\Pi_{\frac{3}{2}}(p^2)$ and $\Pi_{\frac{5}{2}}(p^2)$ are left. We can obtain definite conclusion by studying the QCD sum rules based on the components $\Pi^1_{\frac{3}{2}}(p^2)$, $\Pi^2_{\frac{3}{2}}(p^2)$, $\Pi^1_{\frac{5}{2}}(p^2)$, $\Pi^2_{\frac{5}{2}}(p^2)$, $\Pi^3_{\frac{5}{2}}(p^2)$ and $\Pi^4_{\frac{5}{2}}(p^2)$, this may be our next work.

In this article, we choose the tensor structures 1, $g_{\mu\nu}$ and $g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}$ for the correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ respectively to study the $J^P=\frac{1}{2}^+$, $\frac{3}{2}^+$ doubly heavy baryon states and the $J^P=\frac{1}{2}^-$, $\frac{3}{2}^-$ and $\frac{5}{2}^-$ doubly heavy pentaquark states to avoid contaminations,

$$
\begin{aligned}
\Pi(p)=&\,\Pi_{\frac{1}{2}}(p^2)+\cdots\,,\\
\Pi_{\mu\nu}(p)=&\,\Pi_{\frac{3}{2}}(p^2)\left(-g_{\mu\nu}\right)+\cdots\,,\\
\Pi_{\mu\nu\alpha\beta}(p)=&\,\Pi_{\frac{5}{2}}(p^2)\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)+\cdots\,.
\end{aligned}
\tag{14}
$$

Now we obtain the hadron spectral densities at phenomenological side through the dispersion relation,

$$
\begin{aligned}
\frac{\mathrm{Im}\Pi_j(s)}{\pi}=&\,\not{p}\left[\lambda_j^{+2}\delta\left(s-M_+^2\right)+\lambda_j^{-2}\delta\left(s-M_-^2\right)\right]\\
&+M_+\lambda_j^{+2}\delta\left(s-M_+^2\right)-M_-\lambda_j^{-2}\delta\left(s-M_-^2\right),\\
=&\,\not{p}\,\rho^1_{j,H}(s)+\rho^0_{j,H}(s),
\end{aligned}
\tag{15}
$$

where $j=\frac{1}{2},\,\frac{3}{2}$ for the doubly heavy baryon states, and

$$\begin{aligned}\frac{\mathrm{Im}\Pi_j(s)}{\pi}&=\not{p}\left[\lambda_j^{-2}\delta\left(s-M_-^2\right)+\lambda_j^{+2}\delta\left(s-M_+^2\right)\right]\\&\quad+M_-\lambda_j^{-2}\delta\left(s-M_-^2\right)-M_+\lambda_j^{+2}\delta\left(s-M_+^2\right),\\&=\not{p}\,\rho^1_{j,H}(s)+\rho^0_{j,H}(s),\end{aligned}\tag{16}$$

where $j=\frac{1}{2},\,\frac{3}{2},\,\frac{5}{2}$ for the doubly heavy pentaquark states, we introduce the subscript H to denote the hadron side. Then we introduce the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules at the phenomenological side (or the hadron side),

$$\begin{aligned}&\int_{4m_Q^2}^{s_0}ds\left[\sqrt{s}\rho^1_{j,H}(s)+\rho^0_{j,H}(s)\right]\exp\left(-\frac{s}{T^2}\right)\\&\quad=2M_+\lambda_j^{+2}\exp\left(-\frac{M_+^2}{T^2}\right),\end{aligned}\tag{17}$$

with $j=\frac{1}{2},\,\frac{3}{2}$ for the doubly heavy baryon states,

$$\begin{aligned}&\int_{4m_Q^2}^{s_0}ds\left[\sqrt{s}\rho^1_{j,H}(s)+\rho^0_{j,H}(s)\right]\exp\left(-\frac{s}{T^2}\right)\\&\quad=2M_-\lambda_j^{-2}\exp\left(-\frac{M_-^2}{T^2}\right),\end{aligned}\tag{18}$$

with $j=\frac{1}{2},\,\frac{3}{2},\,\frac{5}{2}$ for the doubly heavy pentaquark states, where the s_0 are the continuum threshold parameters and the T^2 are the Borel parameters. We separate the contributions of the negative parity hadron states from that of the positive parity hadron states unambiguously. In Eqs. (17–18), we choose the special combinations introduced in Refs. [39–43] to obtain the QCD sum rules, which differ from the non-covariant approach in Refs. [18–20,44–46,60].

We carry out the operator product expansion for the correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ up to the vacuum condensates of dimension 7 for the doubly heavy baryon states and dimension 13 for the doubly heavy pentaquark states, and assume vacuum saturation for the higher dimensional vacuum condensates. In calculations, we take the full light quark and heavy quark propagators,

$$\begin{aligned}S^{ij}(x)&=\frac{i\delta_{ij}\not{x}}{2\pi^2x^4}-\frac{\delta_{ij}\langle\bar{q}q\rangle}{12}-\frac{\delta_{ij}x^2\langle\bar{q}g_s\sigma Gq\rangle}{192}\\&\quad-\frac{ig_sG^a_{\alpha\beta}t^a_{ij}(\not{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\not{x})}{32\pi^2x^2}-\frac{\delta_{ij}x^4\langle\bar{q}q\rangle\langle g_s^2GG\rangle}{27648}\\&\quad-\frac{1}{8}\langle\bar{q}_j\sigma^{\mu\nu}q_i\rangle\sigma_{\mu\nu}+\cdots,\end{aligned}\tag{19}$$

$$\begin{aligned}S_s^{ij}(x)&=\frac{i\delta_{ij}\not{x}}{2\pi^2x^4}-\frac{\delta_{ij}m_s}{4\pi^2x^2}-\frac{\delta_{ij}\langle\bar{s}s\rangle}{12}+\frac{i\delta_{ij}\not{x}m_s\langle\bar{s}s\rangle}{48}\\&\quad-\frac{\delta_{ij}x^2\langle\bar{s}g_s\sigma Gs\rangle}{192}+\frac{i\delta_{ij}x^2\not{x}m_s\langle\bar{s}g_s\sigma Gs\rangle}{1152}\\&\quad-\frac{ig_sG^a_{\alpha\beta}t^a_{ij}(\not{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\not{x})}{32\pi^2x^2}-\frac{\delta_{ij}x^4\langle\bar{s}s\rangle\langle g_s^2GG\rangle}{27648}\\&\quad-\frac{1}{8}\langle\bar{s}_j\sigma^{\mu\nu}s_i\rangle\sigma_{\mu\nu}+\cdots,\end{aligned}\tag{20}$$

$$\begin{aligned}S_Q^{ij}(x)&=\frac{i}{(2\pi)^4}\int d^4ke^{-ik\cdot x}\left\{\frac{\delta_{ij}}{\not{k}-m_Q}\right.\\&\quad-\frac{g_sG^n_{\alpha\beta}t^n_{ij}}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma^{\alpha\beta}}{(k^2-m_Q^2)^2}\\&\quad\left.-\frac{g_s^2(t^at^b)_{ij}G^a_{\alpha\beta}G^b_{\mu\nu}(f^{\alpha\beta\mu\nu}+f^{\alpha\mu\beta\nu}+f^{\alpha\mu\nu\beta})}{4(k^2-m_Q^2)^5}+\cdots\right\},\\f^{\alpha\beta\mu\nu}&=(\not{k}+m_Q)\gamma^\alpha(\not{k}+m_Q)\gamma^\beta\\&\quad\times(\not{k}+m_Q)\gamma^\mu(\not{k}+m_Q)\gamma^\nu(\not{k}+m_Q),\end{aligned}\tag{21}$$

and $t^n=\frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix [62,63]. In Eqs. (19, 20), we retain the term $\langle\bar{q}_j\sigma_{\mu\nu}q_i\rangle$ ($\langle\bar{s}_j\sigma_{\mu\nu}s_i\rangle$) comes from the Fierz re-arrangement of the $\langle q_i\bar{q}_j\rangle$ ($\langle s_i\bar{s}_j\rangle$) to absorb the gluons emitted from other quark lines to extract the mixed condensate $\langle\bar{q}g_s\sigma Gq\rangle$ ($\langle\bar{s}g_s\sigma Gs\rangle$).

For the correlation functions $\Pi(p)$ and $\Pi_{\mu\nu}(p)$ of the doubly heavy three-quark currents, there are two heavy quark propagators and a light quark propagator, if each heavy quark line emits a gluon and each light quark line contributes a quark pair, we obtain a operator $GG\bar{q}q$ (or $GG\bar{s}s$), which is of dimension 7, for example,

$$\begin{aligned}\Pi(p)&=-2i\,\varepsilon^{ijk}\varepsilon^{i'j'k'}\int d^4xe^{ip\cdot x}\,\gamma_5\gamma^\mu S^{ii'}(x)\gamma^\nu\gamma_5\,\mathrm{Tr}\\&\quad\times\left[\gamma_\mu S_Q^{kk'}(x)\gamma_\nu CS_Q^{Tjj'}(x)C\right]\sim\langle G^c_{\rho\sigma}G^b_{\alpha\beta}\rangle\langle\bar{q}q\rangle,\end{aligned}\tag{22}$$

with the simple replacements $S^{ii'}(x)\to-\frac{1}{12}\delta^{ii'}\langle\bar{q}q\rangle$, $S_Q^{kk'}(x)\to G^c_{\rho\sigma}t^c_{kk'}$, $S_Q^{Tjj'}(x)\to G^b_{\alpha\beta}t^b_{jj'}$, we should take into account the vacuum condensates at least up to dimension 7. For the correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ of the doubly heavy five-quark currents, there are two heavy quark propagators and three light quark propagators, if each heavy quark line emits a gluon and each light quark line contributes a quark pair, we obtain a operator $GG\bar{u}u\bar{d}d\bar{q}q$, which is of dimension 13, for example,

$$\begin{aligned}\Pi(p)&=-2i\,\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}\varepsilon^{i'l'a'}\varepsilon^{i'j'k'}\varepsilon^{l'm'n'}\\&\quad\times\int d^4xe^{ip\cdot x}\,\gamma_5\gamma^\mu CS^{Ta'a}(-x)C\gamma^\nu\gamma_5\\&\quad\times\mathrm{Tr}\left[\gamma_\mu S_Q^{kk'}(x)\gamma_\nu CS_Q^{Tjj'}(x)C\right]\\&\quad\times\mathrm{Tr}\left[\gamma_5S^{nn'}(x)\gamma_5CS^{Tmm'}(x)C\right]\\&\sim\left\langle G^c_{\rho\sigma}G^b_{\alpha\beta}\right\rangle\langle\bar{q}q\rangle\langle\bar{q}q\rangle\langle\bar{q}q\rangle,\end{aligned}\tag{23}$$

with the simple replacements $S^{Ta'a}(-x)\to-\frac{1}{12}\delta^{aa'}\langle\bar{q}q\rangle$, $S^{nn'}(x)\to-\frac{1}{12}\delta^{nn'}\langle\bar{q}q\rangle$, $S^{Tmm'}(x)\to-\frac{1}{12}\delta^{mm'}\langle\bar{q}q\rangle$,

$S_Q^{kk'}(x) \to G^c_{\rho\sigma} t^c_{kk'}$, $S_Q^{Tjj'}(x) \to G^b_{\alpha\beta} t^b_{jj'}$, we should take into account the vacuum condensates at least up to dimension 13. We can carry out the operator product expansion by taking into account the vacuum condensates beyond dimension 7 or 13, however, it is a very difficult work.

The higher dimensional vacuum condensates play an important role in determining the Borel windows, as there appear terms of the orders $\mathcal{O}\left(\frac{1}{T^2}\right)$, $\mathcal{O}\left(\frac{1}{T^4}\right)$, $\mathcal{O}\left(\frac{1}{T^6}\right)$ in the QCD spectral densities, which manifest themselves at small values of the Borel parameter T^2, we have to choose large values of the T^2 to warrant convergence of the operator product expansion and appearance of the Borel platforms. In this article, we take the truncations $n \leq 7(13)$ and $k \leq 1$ in a consistent way, the operators of the orders $\mathcal{O}(\alpha_s^k)$ with $k > 1$ are discarded. For technical details in calculations, one can consult Refs. [64–70].

Once the analytical QCD spectral densities are obtained, we can take the quark–hadron duality below the continuum thresholds s_0 and introduce the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the following QCD sum rules:

$$2M_+\lambda_j^{+2}\exp\left(-\frac{M_+^2}{T^2}\right) = \int_{4m_Q^2}^{s_0} ds\left[\sqrt{s}\rho^1_{j,QCD}(s)+\rho^0_{j,QCD}(s)\right]\exp\left(-\frac{s}{T^2}\right), \tag{24}$$

$$2M_-\lambda_j^{-2}\exp\left(-\frac{M_-^2}{T^2}\right) = \int_{4m_Q^2}^{s_0} ds\left[\sqrt{s}\rho^1_{j,QCD}(s)+\rho^0_{j,QCD}(s)\right]\exp\left(-\frac{s}{T^2}\right), \tag{25}$$

where

$$\rho^{1/0}_{j,QCD}(s) = \rho_0^{1/0}(s)+\rho_3^{1/0}(s)+\rho_4^{1/0}(s) +\rho_5^{1/0}(s)+\rho_7^{1/0}(s),$$

with $j=\frac{1}{2}, \frac{3}{2}$ for the doubly heavy baryon states,

$$\rho^{1/0}_{j,QCD}(s) = \rho_0^{1/0}(s)+\rho_3^{1/0}(s)+\rho_4^{1/0}(s)+\rho_5^{1/0}(s) +\rho_6^{1/0}(s)+\rho_7^{1/0}(s)+\rho_8^{1/0}(s)+\rho_9^{1/0}(s) +\rho_{10}^{1/0}(s)+\rho_{11}^{1/0}(s)+\rho_{13}^{1/0}(s),$$

with $j=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ for the doubly heavy pentaquark states, the explicit expressions of the QCD spectral densities are given in the appendix.

We derive Eqs. (24, 25) with respect to $\tau=\frac{1}{T^2}$, then eliminate the pole residues $\lambda_j^{\pm}$ and obtain the QCD sum rules for the masses of the doubly heavy baryon states and pentaquark states,

$$M_\pm^2 = \frac{-\frac{d}{d\tau}\int_{4m_Q^2}^{s_0} ds\left[\sqrt{s}\,\rho^1_{j,QCD}(s)+\rho^0_{j,QCD}(s)\right]\exp(-\tau s)}{\int_{4m_Q^2}^{s_0} ds\left[\sqrt{s}\,\rho^1_{j,QCD}(s)+\rho_{j,QCD}(s)\right]\exp(-\tau s)}. \tag{26}$$

3 Numerical results and discussions

We take the standard values of the vacuum condensates $\langle\bar{q}q\rangle=-(0.24\pm0.01\,\text{GeV})^3$, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, $m_0^2=(0.8\pm0.1)\,\text{GeV}^2$, $\langle\bar{s}s\rangle=(0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle=m_0^2\langle\bar{s}s\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle=(0.33\,\text{GeV})^4$ at the energy scale $\mu=1\,\text{GeV}$ [61,62,71], and choose the $\overline{MS}$ masses $m_c(m_c)=(1.275\pm0.025)\,\text{GeV}$, $m_b(m_b)=(4.18\pm0.03)\,\text{GeV}$, $m_s(\mu=2\,\text{GeV})=0.095^{+0.009}_{-0.003}\,\text{GeV}$ from the Particle Data Group [72], and set $m_u=m_d=0$. Furthermore, we take into account the energy-scale dependence of the input parameters,

$$\begin{aligned}
\langle\bar{q}q\rangle(\mu) &= \langle\bar{q}q\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\
\langle\bar{s}s\rangle(\mu) &= \langle\bar{s}s\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\
\langle\bar{q}g_s\sigma Gq\rangle(\mu) &= \langle\bar{q}g_s\sigma Gq\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\
\langle\bar{s}g_s\sigma Gs\rangle(\mu) &= \langle\bar{s}g_s\sigma Gs\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}, \\
m_b(\mu) &= m_b(m_b)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{33-2n_f}}, \\
m_s(\mu) &= m_s(2\text{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{12}{33-2n_f}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2 t-\log t-1)+b_0 b_2}{b_0^4 t^2}\right],
\end{aligned} \tag{27}$$

where $t=\log\frac{\mu^2}{\Lambda^2}$, $b_0=\frac{33-2n_f}{12\pi}$, $b_1=\frac{153-19n_f}{24\pi^2}$, $b_2=\frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda=210\,\text{MeV}$, $292\,\text{MeV}$ and $332\,\text{MeV}$ for the flavors $n_f=5$, 4 and 3, respectively [72–74], and evolve all the input parameters to the optimal energy scales μ to extract the masses of the doubly heavy baryon states and pentaquark states.

In the article, we study the doubly heavy baryon states and pentaquark states, the two heavy quarks form a diquark state $\varepsilon^{ijk}Q_j^T C\gamma_\mu Q_k$, which serves as a static well potential and combines with a light quark state in color triplet to form a compact baryon state or combine with a light diquark and a light antiquark in color antitriplet to form a compact pentaquark state. While in the hidden-charm or hidden-bottom pentaquark states, the heavy quark Q serves as a static well potential and combines with the light quark to form a heavy diquark in color antitriplet, the heavy antiquark $\bar{Q}$ serves as another static well potential and combines with the light diquark in color antitriplet to form a heavy triquark in color triplet, then the heavy diquark and heavy triquark combine together to form a hidden-charm or hidden-bottom pentaquark state. The quark structures of the doubly heavy pentaquark states and hidden-charm or hidden-bottom pentaquark states are quite different.

The doubly heavy (or hidden-charm, hidden-bottom) tetraquark states X, Y, Z and pentaquark states P are characterized by the effective heavy quark masses $\mathbb{M}_Q$ and the virtuality $V=\sqrt{M^2_{X/Y/Z/P}-(2\mathbb{M}_Q)^2}$. In Refs. [64–70], we study the acceptable energy scales of the QCD spectral densities for the hidden-charm (or hidden-bottom) tetraquark states and molecular states in the QCD sum rules in details for the first time, and suggest an energy scale formula $\mu=V=\sqrt{M^2_{X/Y/Z}-(2\mathbb{M}_Q)^2}$ to determine the optimal energy scales. The energy scale formula also works well in studying the hidden-charm pentaquark states [39–43]. The updated values are $\mathbb{M}_c=1.82\,\text{GeV}$ and $\mathbb{M}_b=5.17\,\text{GeV}$ for the hidden-charm and hidden-bottom tetraquark states, respectively [75,76].

It is not necessary for the $\mathbb{M}_Q$ in the doubly heavy tetraquark states and pentaquark states to have the same values as the ones in the hidden-charm or hidden-bottom tetraquark states and pentaquark states. In Refs. [24,25], we observe that if we choose a slightly different value $\mathbb{M}_c=1.84\,\text{GeV}$ for the doubly charmed tetraquark states, the criteria of the QCD sum rules can be satisfied more easily, while the value $\mathbb{M}_b=5.17\,\text{GeV}$ survives for the doubly bottom tetraquark states. In this article, we take the energy scale formula as a constraint to study the doubly heavy pentaquark states.

At the phenomenological side, we exclude the contaminations of the higher resonances by setting in the truncations s_0,

$$\int_{4m_Q^2}^{s_0} ds\left[\sqrt{s}\rho^1_{j,H}(s)+\rho^0_{j,H}(s)\right]\exp\left(-\frac{s}{T^2}\right).$$

At the QCD side, there are terms of the Dirac δ function type, $\delta(s-\overline{m}_Q^2)$ and $\delta(s-\widetilde{m}_Q^2)$, which are associated with the higher dimensional vacuum condensates,

$$\int_{4m_Q^2}^{s_0} ds\,\delta(s-\overline{m}_Q^2)\exp\left(-\frac{s}{T^2}\right)$$
$$=\int_{4m_Q^2}^{\infty} ds\,\delta(s-\overline{m}_Q^2)\exp\left(-\frac{s}{T^2}\right)=\exp\left(-\frac{\overline{m}_Q^2}{T^2}\right),$$

$$\int_{4m_Q^2}^{s_0} ds\,\delta(s-\widetilde{m}_Q^2)\exp\left(-\frac{s}{T^2}\right)$$
$$=\int_{4m_Q^2}^{\infty} ds\,\delta(s-\widetilde{m}_Q^2)\exp\left(-\frac{s}{T^2}\right)=\exp\left(-\frac{\widetilde{m}_Q^2}{T^2}\right), \tag{28}$$

the upper bounds of the integrals are arbitrary for getting the same values, there may be some uncertainties originating from the higher resonances, as the truncations s_0 cannot exclude the contaminations of the higher resonances rigourously. Firstly, we need good convergent behaviors in the operator product expansion to obtain solid predictions.

Now we write down the definition for the contributions of the different terms in the operator product expansion,

$$D(n)=\frac{\int_{4m_Q^2}^{s_0} ds\,\rho_n(s)\,\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_Q^2}^{s_0} ds\,\rho(s)\,\exp\left(-\frac{s}{T^2}\right)}, \tag{29}$$

in stead of

$$D(n)=\frac{\int_{4m_Q^2}^{\infty} ds\,\rho_n(s)\,\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_Q^2}^{\infty} ds\,\rho(s)\,\exp\left(-\frac{s}{T^2}\right)}, \tag{30}$$

where the $\rho_n(s)$ are the QCD spectral densities for the vacuum condensates of dimension n, and the total spectral densities $\rho(s)=\sqrt{s}\rho^1_{QCD}(s)+\rho^0_{QCD}(s)$. The definition in Eq. (29) warrants the contributions of the higher dimensional vacuum condensates play a less important role if the operator product expansion is well convergent.

The contributions of the perturbative terms $D(0)$ are usually small for the tetraquark states and pentaquark states, we approximate the continuum contributions as $\rho(s)\Theta(s-s_0)$, and define the pole contributions (PC) or ground state contributions as

$$\text{PC}=\frac{\int_{4m_Q^2}^{s_0} ds\,\rho(s)\,\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_Q^2}^{\infty} ds\,\rho(s)\,\exp\left(-\frac{s}{T^2}\right)}. \tag{31}$$

If the pole dominance is satisfied at the phenomenological side, the uncertainties originate from the higher dimensional vacuum condensates are greatly suppressed. So the QCD sum rules must satisfy the two criteria, pole dominance at the phenomenological side and convergence of the operator product expansion at the QCD side.

For the lowest doubly charmed baryon state Ξ_{cc}, $M_{\Xi_{cc}^{++}}=3621.40\pm0.72\pm0.27\pm0.14\,\text{MeV}$ [1], which is smaller than $2\mathbb{M}_c=3.68\,\text{GeV}$, the energy scale formula $\mu=\sqrt{M^2_{\Xi_{cc}^{++}}-(2\mathbb{M}_c)^2}$ is failed to work, we can choose the typical energy scale $\mu=1\,\text{GeV}$ for the doubly charmed baryon states Ξ_{cc}, Ξ_{cc}^*, Ω_{cc}, Ω_{cc}^*, $2m_c(1\,\text{GeV})=2.70-2.84\,\text{GeV}<M_{\eta_c}<M_{\Xi_{cc}^{++}}$, the $\overline{MS}$ masses $m_c(1\,\text{GeV})=1.35-1.42\,\text{GeV}$,

Table 2 The energy scales μ, Borel parameters T^2, continuum threshold parameters s_0, pole contributions (pole) and contributions of the highest vacuum condensates for the doubly heavy baryon states and pentaquark states

	J^P	μ (GeV)	T^2 (GeV^2)	$\sqrt{s_0}$ (GeV)	Pole	$\|D(n=7/13)\|$
ccq	$\frac{1}{2}^+$	1.0	2.0–2.6	4.15 ± 0.10	(60–86)%	$\ll 1\%$
ccq	$\frac{3}{2}^+$	1.0	2.2–2.8	4.25 ± 0.10	(60–85)%	$\ll 1\%$
ccs	$\frac{1}{2}^+$	1.0	2.2–2.8	4.35 ± 0.10	(64–87)%	$\ll 1\%$
ccs	$\frac{3}{2}^+$	1.0	2.4–3.0	4.45 ± 0.10	(65–87)%	$\ll 1\%$
bbq	$\frac{1}{2}^+$	2.2	6.8–7.6	10.75 ± 0.10	(55–73)%	$\ll 1\%$
bbq	$\frac{3}{2}^+$	2.2	7.1–7.9	10.80 ± 0.10	(55–73)%	$\ll 1\%$
bbs	$\frac{1}{2}^+$	2.2	7.4–8.2	10.90 ± 0.10	(55–72)%	$\ll 1\%$
bbs	$\frac{3}{2}^+$	2.2	7.7–8.5	10.95 ± 0.10	(55–72)%	$\ll 1\%$
$ccud\bar{q}$	$\frac{1}{2}^-$	2.0	2.8–3.2	4.80 ± 0.10	(41–64)%	(1–2)%
$ccud\bar{q}$	$\frac{3}{2}^-$	2.2	3.0–3.4	4.90 ± 0.10	(41–63)%	$\sim 1\%$
$ccud\bar{q}$	$\frac{5}{2}^-$	2.4	3.2–3.6	5.00 ± 0.10	(40–61)%	$\leq 1\%$
$bbud\bar{q}$	$\frac{1}{2}^-$	2.9	6.9–7.7	11.40 ± 0.10	(40–60)%	(6–14)%
$bbud\bar{q}$	$\frac{3}{2}^-$	2.9	6.9–7.7	11.40 ± 0.10	(40–60)%	(7–16)%
$bbud\bar{q}$	$\frac{5}{2}^-$	3.1	7.2–8.0	11.50 ± 0.10	(41–60)%	(5–10)%

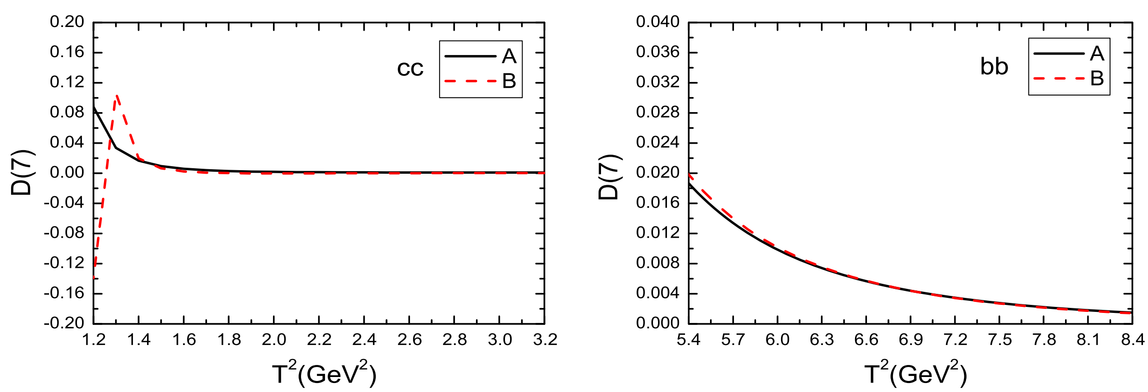

Fig. 1 The contributions of the vacuum condensates of dimension 7 in the operator product expansion, where A and B denote the Ξ_{QQ} baryon states with $J=\frac{1}{2}$ and $\frac{3}{2}$, respectively

the integrals $\int_{4m_c^2}^{s_0} ds\, \rho_{QCD}(s)\, \exp\left(-\frac{s}{T^2}\right)$ make sense for the conventional charmonium states and doubly charmed baryon states. In calculations, we observe that $2m_b(2.2\,\text{GeV}) = 9.28{-}9.44\,\text{GeV}$, while the bottomonium masses $M_{\eta_b} = 9.399\,\text{GeV}$, $M_\Upsilon = 9.46\,\text{GeV}$ [72], the energy scale $\mu = 2.2\,\text{GeV}$ is the lowest energy scale or marginal energy scale for the integrals $\int_{4m_b^2}^{s_0} ds\, \rho_{QCD}(s)\, \exp\left(-\frac{s}{T^2}\right)$ for the conventional bottomonium states. In this article, we choose $\mu = 2.2\,\text{GeV}$ for the doubly bottom baryon states Ξ_{bb}, Ξ^*_{bb}, Ω_{bb} and Ω^*_{bb}, which works well.

In Refs. [18–20,39–46], we separate the contributions of the positive parity and negative parity hadron states explicitly, and study the heavy, doubly-heavy, triply-heavy baryon states and the hidden-charm pentaquark states with the QCD sum rules in a systematic way. In calculations, we observe that the continuum threshold parameters $\sqrt{s_0} = M_{\rm gr}+(0.6{-}0.8)\,\text{GeV}$ works well, where the subscript gr denotes the ground states. In this article, we can take the continuum threshold parameters as $\sqrt{s_0} < M_{B/P} + 0.8\,\text{GeV}$.

In this article, we choose the Borel parameters T^2 and continuum threshold parameters s_0 to satisfy the following four criteria:

C1. Pole dominance at the phenomenological side;
C2. Convergence of the operator product expansion;
C3. Appearance of the Borel platforms;
C4. Satisfying the energy scale formula only for the doubly heavy pentaquark states,

by try and error.

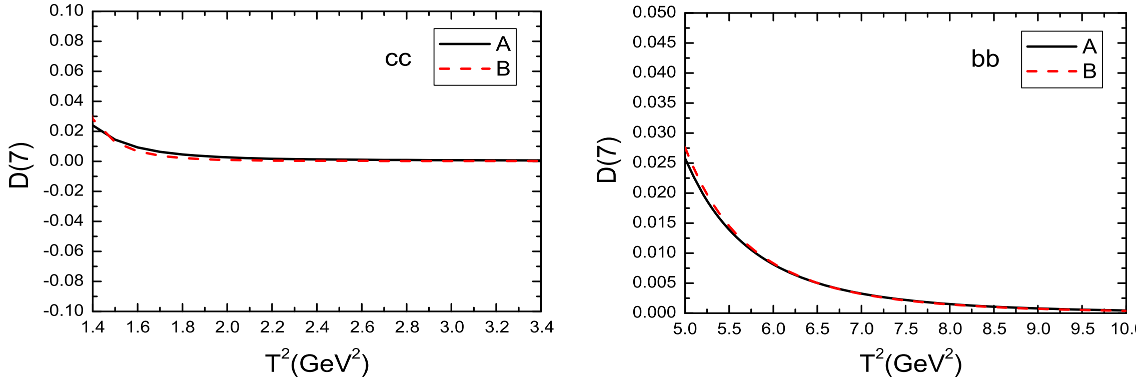

Fig. 2 The contributions of the vacuum condensates of dimension 7 in the operator product expansion, where A and B denote the Ω_{QQ} baryon states with $J = \frac{1}{2}$ and $\frac{3}{2}$, respectively

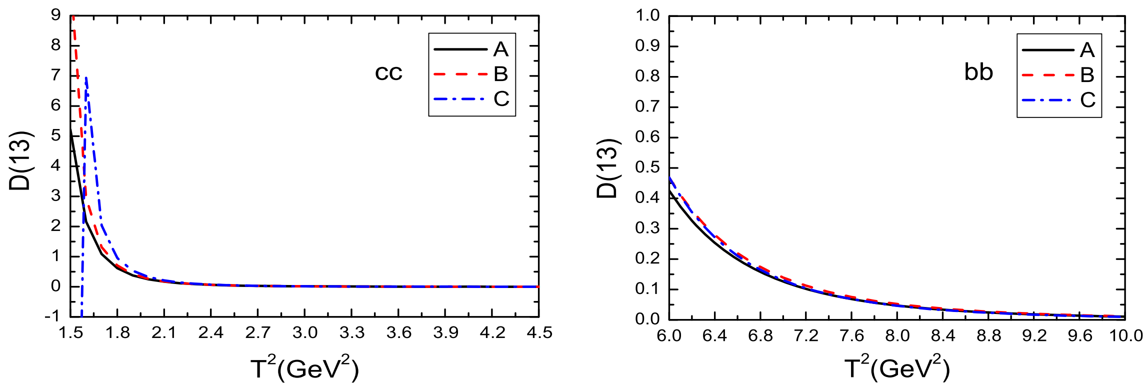

Fig. 3 The contributions of the vacuum condensates of dimension 13 in the operator product expansion, where A, B and C denote the doubly heavy pentaquark states with $J = \frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$, respectively

The resulting Borel parameters or Borel windows T^2, continuum threshold parameters s_0, energy scales of the QCD spectral densities, pole contributions of the ground states and contributions of the highest dimensional vacuum condensates are shown explicitly in Table 2. From the table, we can see that the pole dominance at the phenomenological side and the convergence of the operator product expansion at the QCD side are satisfied, or the criteria **C1** and **C2** are satisfied.

In Figs. 1, 2 and 3, we plot the contributions of the vacuum condensates of dimension 7 and 13 with variations of the Borel parameters T^2 for the doubly heavy baryon states and pentaquark states, respectively. From the figures, we can see explicitly that the contributions of the highest dimensional vacuum condensates are tiny in the Borel windows for the doubly heavy baryon states and doubly-charmed pentaquark states, while the contributions of the vacuum condensates of dimension 13 for the doubly bottom pentaquark states are somewhat larger, the smallest contributions are about $(5-7)\%$ in the Borel windows, the operator product expansion is still convergent. In fact, for the doubly bottom pentaquark states, the $D(13)$ decrease monotonously and quickly with the increase of the Borel parameter T^2, a slight larger

Table 3 The masses and pole residues of the doubly heavy baryon states and pentaquark states

	J^P	M (GeV)	λ (10^{-1} GeV^3)	λ (10^{-3} GeV^6)
ccq	$\frac{1}{2}^+$	$3.63^{+0.08}_{-0.07}$	$1.02^{+0.14}_{-0.10}$	
ccq	$\frac{3}{2}^+$	$3.75^{+0.07}_{-0.07}$	$0.65^{+0.07}_{-0.07}$	
ccs	$\frac{1}{2}^+$	$3.75^{+0.08}_{-0.09}$	$1.28^{+0.18}_{-0.17}$	
ccs	$\frac{3}{2}^+$	$3.85^{+0.08}_{-0.08}$	$0.81^{+0.09}_{-0.09}$	
bbq	$\frac{1}{2}^+$	$10.22^{+0.07}_{-0.07}$	$2.73^{+0.36}_{-0.31}$	
bbq	$\frac{3}{2}^+$	$10.27^{+0.07}_{-0.07}$	$1.65^{+0.21}_{-0.19}$	
bbs	$\frac{1}{2}^+$	$10.33^{+0.07}_{-0.08}$	$3.27^{+0.44}_{-0.41}$	
bbs	$\frac{3}{2}^+$	$10.37^{+0.07}_{-0.08}$	$1.97^{+0.26}_{-0.23}$	
$ccud\bar{q}$	$\frac{1}{2}^-$	$4.21^{+0.10}_{-0.11}$		$2.51^{+0.46}_{-0.39}$
$ccud\bar{q}$	$\frac{3}{2}^-$	$4.27^{+0.11}_{-0.10}$		$1.65^{+0.30}_{-0.25}$
$ccud\bar{q}$	$\frac{5}{2}^-$	$4.37^{+0.11}_{-0.11}$		$1.34^{+0.22}_{-0.20}$
$bbud\bar{q}$	$\frac{1}{2}^-$	$10.75^{+0.12}_{-0.12}$		$7.53^{+1.52}_{-1.39}$
$bbud\bar{q}$	$\frac{3}{2}^-$	$10.76^{+0.11}_{-0.13}$		$4.27^{+0.85}_{-0.78}$
$bbud\bar{q}$	$\frac{5}{2}^-$	$10.82^{+0.12}_{-0.13}$		$3.87^{+0.74}_{-0.68}$

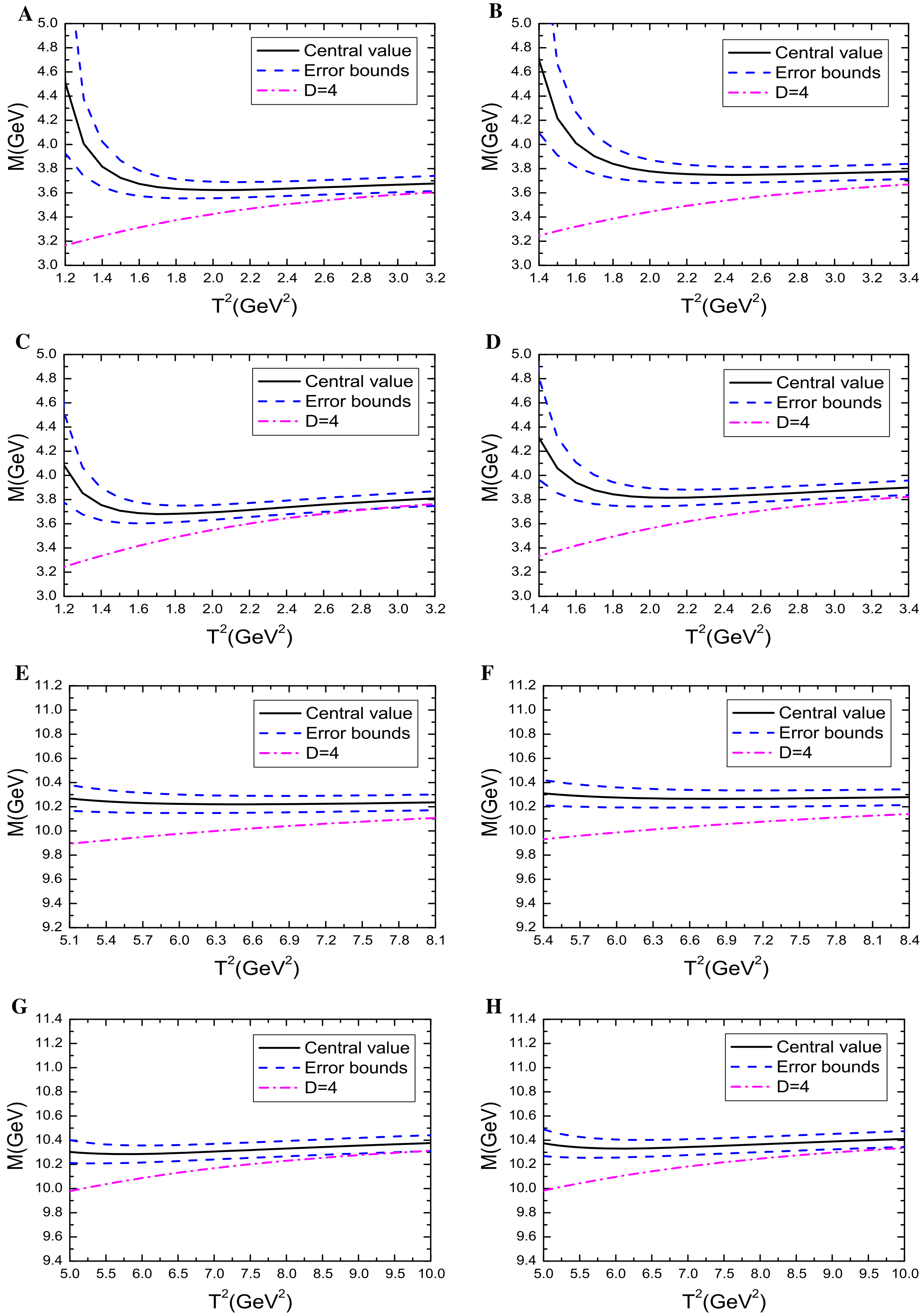

Fig. 4 The masses of the doubly heavy baryon states with variations of the Borel parameters T^2, where the A, B, C, D, E, F, G and H denote the Ξ_{cc}, Ξ^*_{cc}, Ω_{cc}, Ω^*_{cc}, Ξ_{bb}, Ξ^*_{bb}, Ω_{bb} and Ω^*_{bb}, respectively, the $D=4$ denotes the predictions based on the truncations of the operator product expansion up to the vacuum condensates of dimension 4

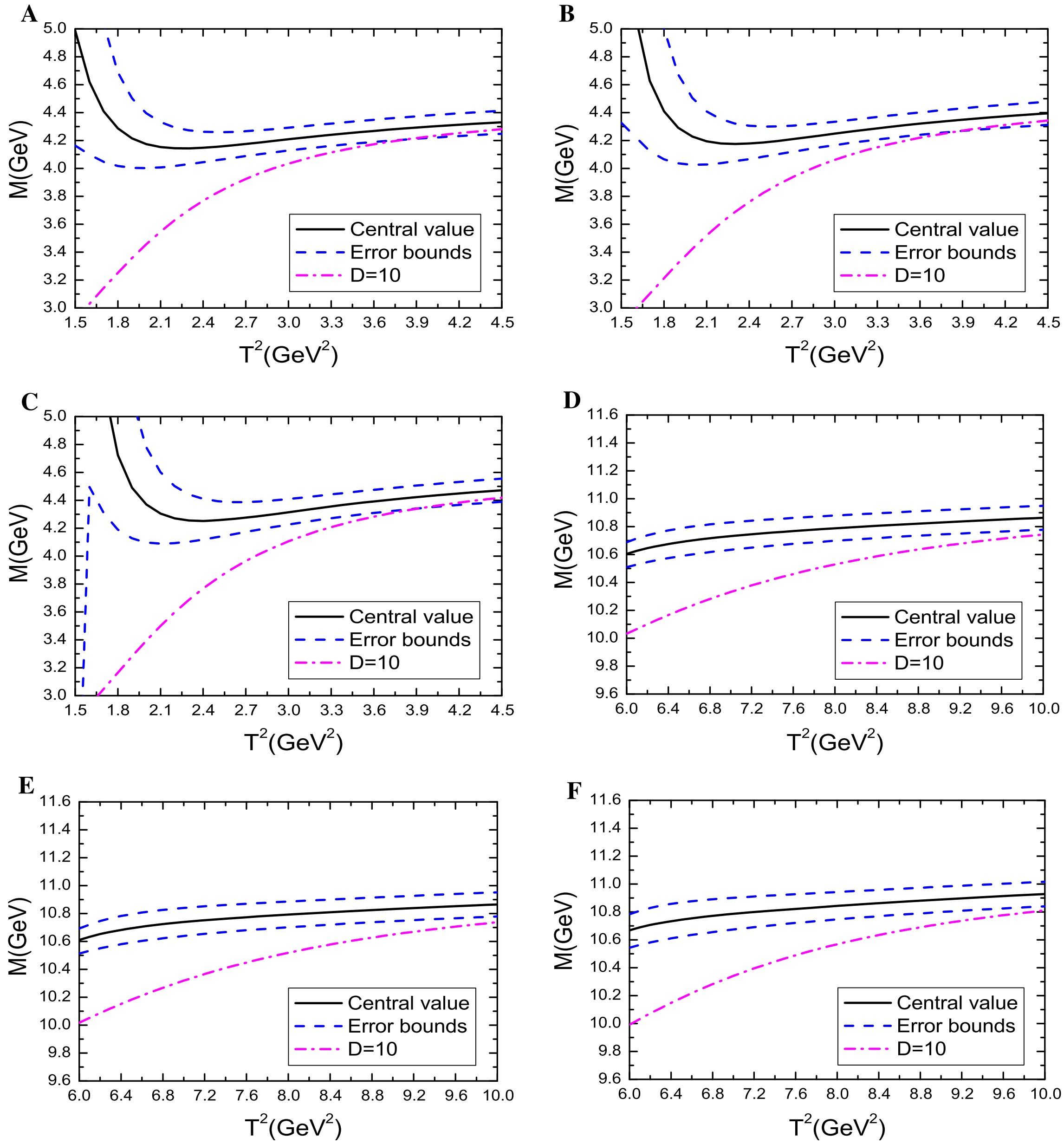

Fig. 5 The masses of the doubly heavy pentaquark states with variations of the Borel parameters T^2, where the A, B, C, D, E and F denote the $P_{cc,\frac{1}{2}}$, $P_{cc,\frac{3}{2}}$, $P_{cc,\frac{5}{2}}$, $P_{bb,\frac{1}{2}}$, $P_{bb,\frac{3}{2}}$ and $P_{bb,\frac{5}{2}}$, respectively, the $D = 10$ denotes the predictions based on the truncations of the operator product expansion up to the vacuum condensates of dimension 10

Borel parameter can lead to much smaller contribution. In calculations, we observe that the predicted doubly heavy pentaquark masses are rather stable with variations of the Borel parameters at the region $T^2 \geq T^2_{min}$, where the min denotes the minimal values, the predictions survive for larger Borel parameters, the somewhat large contributions $D(13)$ cannot impair the predictive ability.

We take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the doubly heavy baryon states and pentaquark states, which are shown explicitly in Table 3 and Figs. 4, 5, 6 and 7. From Table 3, we can see that the criterion **C4** is satisfied for the doubly heavy pentaquark states. In this article, we choose the effective heavy quark masses $\mathbb{M}_c = 1.84\,\text{GeV}$ and $\mathbb{M}_b = 5.17\,\text{GeV}$ for the doubly heavy tetraquark states [24,25], if we choose slightly larger mass $\mathbb{M}_b = 5.18\,\text{GeV}$, the energy scale formula is satisfied even better.

In Figs. 4, 5, 6 and 7, we plot the masses and pole residues at much larger ranges of the Borel parameters than the Borel windows. From the figures, we can see that there appear

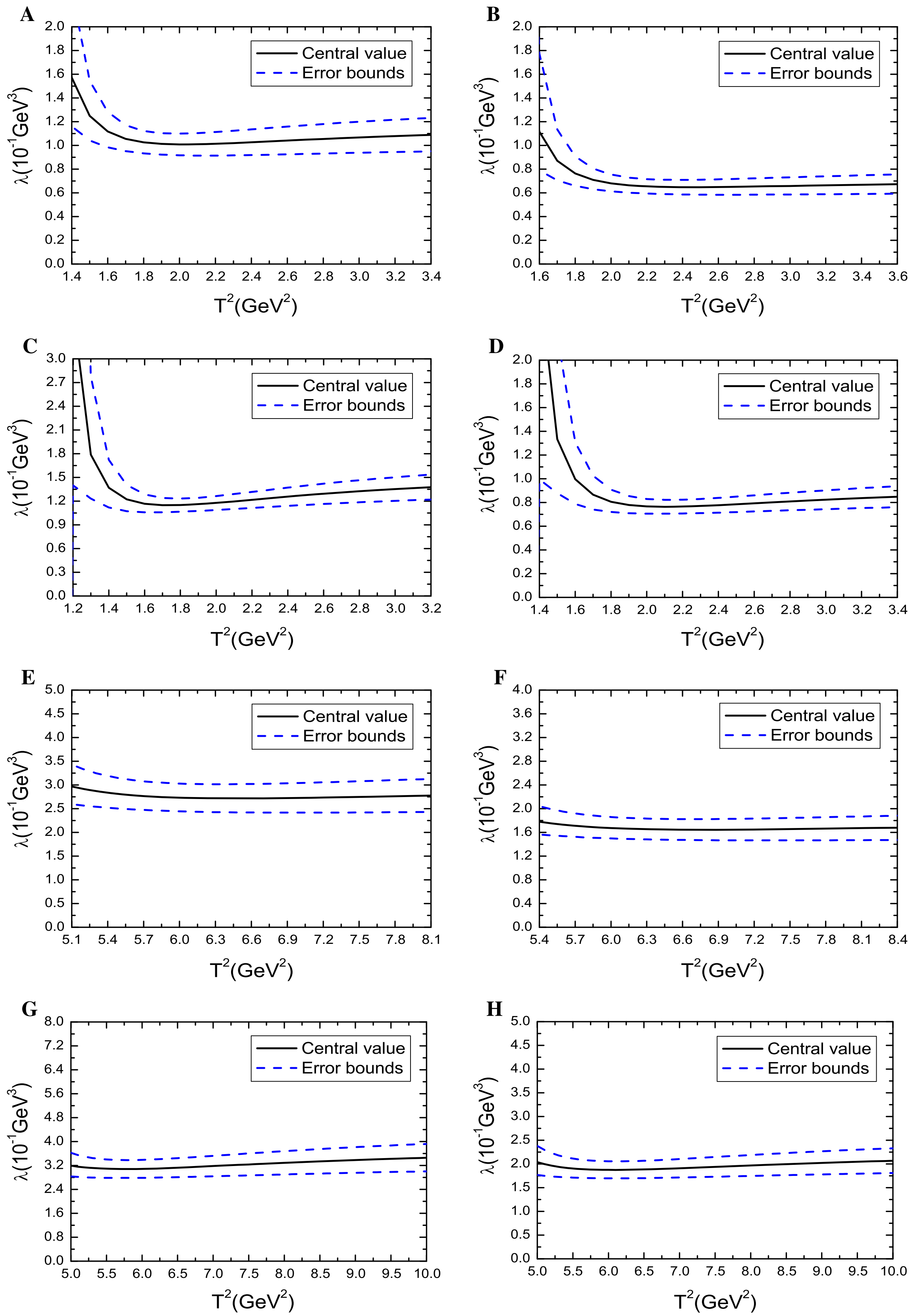

Fig. 6 The pole residues of the doubly heavy baryon states with variations of the Borel parameters T^2, where the A, B, C, D, E, F, G and H denote the Ξ_{cc}, Ξ^*_{cc}, Ω_{cc}, Ω^*_{cc}, Ξ_{bb}, Ξ^*_{bb}, Ω_{bb} and Ω^*_{bb}, respectively

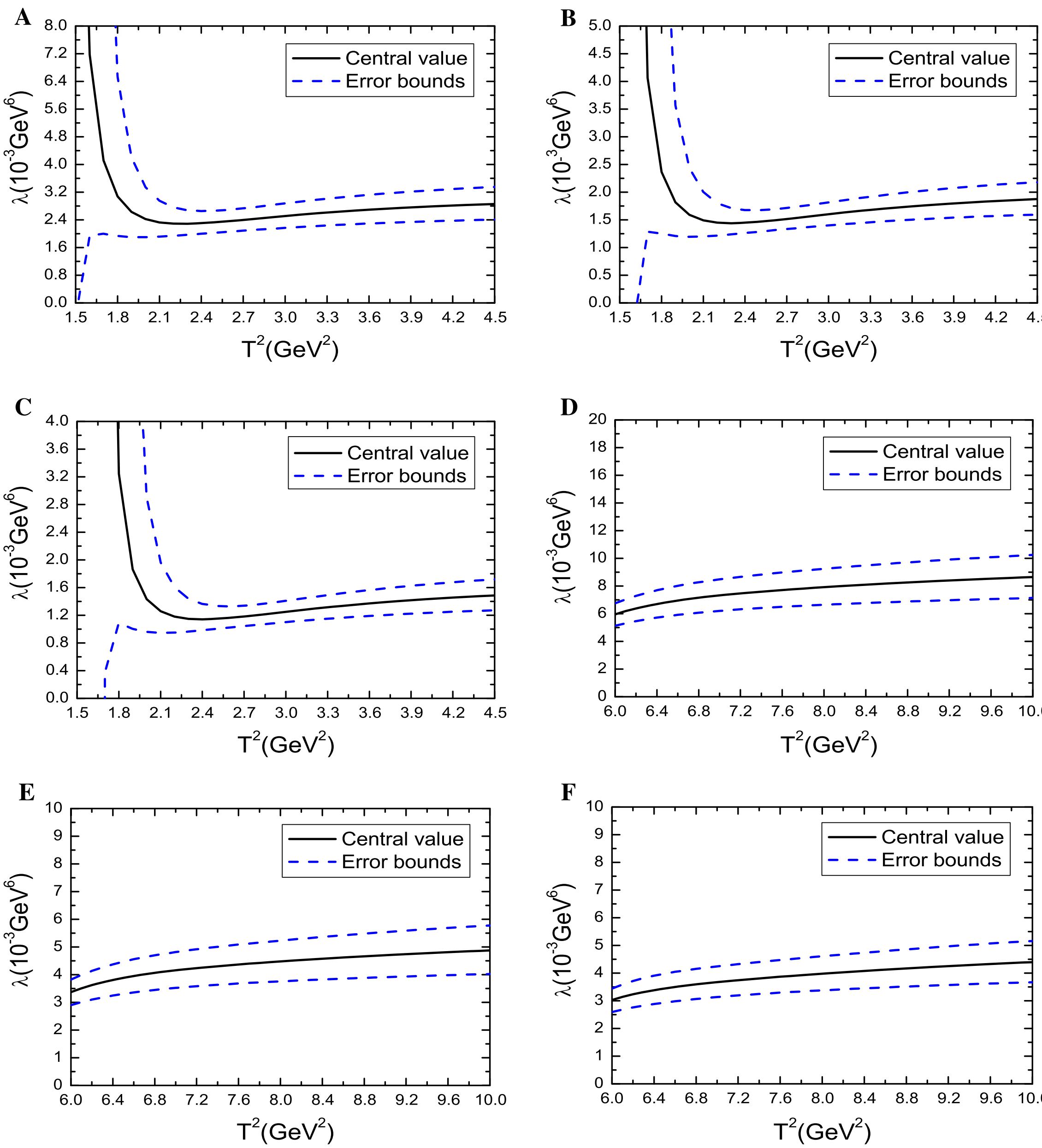

Fig. 7 The pole residues of the doubly heavy pentaquark states with variations of the Borel parameters T^2, where the A, B, C, D, E and F denote the $P_{cc,\frac{1}{2}}$, $P_{cc,\frac{3}{2}}$, $P_{cc,\frac{5}{2}}$, $P_{bb,\frac{1}{2}}$, $P_{bb,\frac{3}{2}}$ and $P_{bb,\frac{5}{2}}$, respectively

Borel platforms in the Borel windows, the criterion **C3** is also satisfied. In Figs. 4 and 5, we also plot the masses with the truncations of the operator product expansion up to the vacuum condensates of dimension 4 for the doubly heavy baryon states and of dimension 10 for the doubly heavy pentaquark states. From the figures, we can see that without including the vacuum condensates of dimensions 5, 7 and 11, 13 for the doubly heavy baryon states and pentaquark states, respectively, we cannot obtain very stable QCD sum rules with respect to variations of the Borel parameters, the higher dimensional vacuum condensates play an important role in determining the Borel platforms.

For the doubly heavy baryon states, the criteria **C1**, **C2** and **C3** are satisfied, for the doubly heavy pentaquark states, the criteria **C1**, **C2**, **C3** and **C4** are satisfied, we expect to make reliable predictions, which can be confronted to the experimental data in the future.

In the present work, we obtain the mass $M = 4.21^{+0.10}_{-0.11}$ GeV for the doubly charmed pentaquark state $ccud\bar{q}$ with $J^P = \frac{1}{2}^-$. While in Ref. [77], Yan et al. obtain the masses of the meson-baryon type doubly charmed pentaquark states with $J^P = \frac{1}{2}^-$ below 4.2 GeV based on the unitarized coupled-channel approach, which are in qualitative agreement with the present predictions. In Refs. [56,57], Azizi, Sarac and Sundu study the meson-baryon type hidden-charm (hidden-bottom) pentaquark states with $J^P = \frac{3}{2}^\pm$ and $\frac{5}{2}^\pm$ based on the QCD sum rules, the predicted masses 4.30 ± 0.10 GeV and 4.20 ± 0.15 GeV ($10.96^{+0.84}_{-0.88}$ GeV and $10.98^{+0.82}_{-0.82}$ GeV) for the hidden-charm (hidden-bottom) pentaquark states with $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^-$ respectively are compatible with the present calculations in magnitude, but differ from the present calculations quantitatively. We should bear in mind that they are quite different pentaquark states.

4 Conclusion

In this article, we study the doubly heavy baryon states and pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 7 and 13 respectively in a consistent way. In calculations, we separate the contributions of the negative parity and positive parity hadron states unambiguously, and study the masses and pole residues of the doubly heavy baryon states and pentaquark states in details, and obtain very stable QCD sum rules in the Borel windows. The prediction $M_{\Xi_{cc}} = 3.63^{+0.08}_{-0.07}$ GeV is in excellent agreement with the LHCb data $M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV, other predictions can be confronted to the experimental data in the future.

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Appendix

The explicit expressions of the QCD spectral densities:
For the Ω_{QQ} states,

$$\rho_0^1(s) = \frac{3}{8\pi^4}\int dydz\, yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(5s-3\overline{m}_Q^2\right) + \frac{3m_Q^2}{8\pi^4}\int dydz\,(1-y-z)\left(s-\overline{m}_Q^2\right),$$

$$\rho_3^1(s) = \frac{3m_s\langle\bar{s}s\rangle}{2\pi^2}\int dy\, y\,(1-y)\left[1+\frac{s}{2}\delta\left(s-\widetilde{m}_Q^2\right)\right],$$

$$\begin{aligned}\rho_4^1(s) =& -\frac{m_Q^2}{6\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z\,(1-y-z)}{y^2}\left(1+\frac{s}{2T^2}\right)\delta\left(s-\overline{m}_Q^2\right) \\ &-\frac{m_Q^4}{24\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^3}\delta\left(s-\overline{m}_Q^2\right) \\ &+\frac{m_Q^2}{8\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^2}\delta\left(s-\overline{m}_Q^2\right) \\ &+\frac{3}{16\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, z\left[1+\frac{s}{3}\delta\left(s-\overline{m}_Q^2\right)\right] \\ &+\frac{m_Q^2}{16\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y}\,\delta\left(s-\overline{m}_Q^2\right),\end{aligned}$$

$$\rho_5^1(s) = -\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{2\pi^2}\int dy\, y\,(1-y)\left(1+\frac{3s}{4T^2}+\frac{s^2}{4T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right),$$

$$\begin{aligned}\rho_7^1(s) =& -\frac{m_s m_Q^2\langle\bar{s}s\rangle}{18T^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1-y}{y^2}\,s\,\delta\left(s-\widetilde{m}_Q^2\right) \\ &-\frac{m_s m_Q^4\langle\bar{s}s\rangle}{36T^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^3}\,\delta\left(s-\widetilde{m}_Q^2\right) \\ &+\frac{m_s m_Q^2\langle\bar{s}s\rangle}{12T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\,\delta\left(s-\widetilde{m}_Q^2\right),\end{aligned}$$

$$\rho_0^0(s) = \frac{3m_s}{8\pi^4}\int dydz\, yz\,\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) + \frac{3m_s m_Q^2}{4\pi^4}\int dydz\,\left(s-\overline{m}_Q^2\right),$$

$$\rho_3^0(s) = -\frac{3\langle\bar{s}s\rangle}{2\pi^2}\int dy\, y\,(1-y)\, s,$$

$$\begin{aligned}\rho_4^0(s) =& -\frac{m_s m_Q^2}{24\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z}{y^2}\left(1+\frac{s}{T^2}\right)\delta\left(s-\overline{m}_Q^2\right) \\ &-\frac{m_s m_Q^4}{12\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y^3}\delta\left(s-\overline{m}_Q^2\right) \\ &+\frac{m_s m_Q^2}{4\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y^2}\delta\left(s-\overline{m}_Q^2\right) \\ &-\frac{m_s}{16\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\left[1+\frac{s}{2}\,\delta\left(s-\overline{m}_Q^2\right)\right],\end{aligned}$$

$$\rho_5^0(s) = \frac{3\langle\bar{s}g_s\sigma Gs\rangle}{4\pi^2}\int dy\, y\,(1-y)\left[1+\left(s+\frac{s^2}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)\right] - \frac{\langle\bar{s}g_s\sigma Gs\rangle}{8\pi^2}\int dy\left[1+\frac{3s}{2}\delta\left(s-\widetilde{m}_Q^2\right)\right],$$

$$\begin{aligned}\rho_7^0(s) =& \frac{m_Q^2\langle\bar{s}s\rangle}{18T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1-y}{y^2}\,s\,\delta\left(s-\widetilde{m}_Q^2\right) \\ &+\frac{m_Q^4\langle\bar{s}s\rangle}{9T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right) \\ &-\frac{m_Q^2\langle\bar{s}s\rangle}{3T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right) \\ &+\frac{\langle\bar{s}s\rangle}{24}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\ &-\frac{\langle\bar{s}s\rangle}{12}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, y\,(1-y) \\ &\left(1+\frac{s}{T^2}+\frac{s^2}{2T^4}+\frac{s^3}{2T^6}\right)\delta\left(s-\widetilde{m}_Q^2\right).\end{aligned}$$

For the Ω_{QQ}^* states,

$$\rho_0^1(s) = \frac{3}{16\pi^4}\int dydz\, yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) + \frac{3m_Q^2}{16\pi^4}\int dydz\,(1-y-z)\left(s-\overline{m}_Q^2\right),$$

$$\rho_3^1(s) = \frac{m_s\langle\bar{s}s\rangle}{4\pi^2}\int dy\, y\,(1-y)\left[1+s\,\delta\left(s-\widetilde{m}_Q^2\right)\right],$$

$$\rho_4^1(s) = -\frac{m_Q^2}{48\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \int dydz\,\frac{z\,(1-y-z)}{y^2}\left(1+\frac{s}{T^2}\right)\delta\left(s-\overline{m}_Q^2\right)$$
$$-\frac{m_Q^4}{48\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^3}\delta\left(s-\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2}{16\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^2}\delta\left(s-\overline{m}_Q^2\right)$$
$$-\frac{1}{48\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,(1-y-z)\left[1+\frac{s}{4}\delta\left(s-\overline{m}_Q^2\right)\right],$$

$$\rho_5^1(s) = -\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{16\pi^2}\int dy\, y\,(1-y)\left(1+\frac{4s}{3}+\frac{2s^2}{3T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right),$$

$$\rho_7^1(s) = \frac{m_s m_Q^2\langle\bar{s}s\rangle}{72T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1-y}{y^2}\left(1-\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)$$
$$-\frac{m_s m_Q^4\langle\bar{s}s\rangle}{72T^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right)$$
$$+\frac{m_s m_Q^2\langle\bar{s}s\rangle}{24T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right)$$
$$-\frac{m_s\langle\bar{s}s\rangle}{144T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\left(1+\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right),$$

$$\rho_0^0(s) = \frac{3m_s}{32\pi^4}\int dydz\, yz\left(s-\overline{m}_Q^2\right)\left(3s-\overline{m}_Q^2\right)$$
$$+\frac{3m_s m_Q^2}{16\pi^4}\int dydz\left(s-\overline{m}_Q^2\right),$$

$$\rho_3^0(s) = -\frac{\langle\bar{s}s\rangle}{2\pi^2}\int dy\, y\,(1-y)\; s,$$

$$\rho_4^0(s) = -\frac{m_s m_Q^2}{48\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z}{y^2}\,s\,\delta\left(s-\overline{m}_Q^2\right)$$
$$-\frac{m_s m_Q^4}{48\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y^3}\delta\left(s-\overline{m}_Q^2\right)$$
$$+\frac{m_s m_Q^2}{16\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y^2}\delta\left(s-\overline{m}_Q^2\right)$$
$$-\frac{m_s}{64\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\left[1+\frac{s}{3}\delta\left(s-\overline{m}_Q^2\right)\right],$$

$$\rho_5^0(s) = \frac{\langle\bar{s}g_s\sigma Gs\rangle}{16\pi^2}\int dy\, y\,(1-y)\left[3+\left(4s+\frac{2s^2}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)\right],$$

$$\rho_7^0(s) = -\frac{m_Q^2\langle\bar{s}s\rangle}{36T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1-y}{y^2}\left(1-\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)$$
$$+\frac{m_Q^4\langle\bar{s}s\rangle}{36T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right)$$
$$-\frac{m_Q^2\langle\bar{s}s\rangle}{12T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right)$$
$$+\frac{\langle\bar{s}s\rangle}{72}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\left(1+\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)$$
$$-\frac{\langle\bar{s}s\rangle}{72T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, y\,(1-y)\left(s+\frac{s^2}{T^2}+\frac{s^3}{T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right).$$

For the $QQcd\bar{q}$ states with $J^P=\frac{1}{2}^-$,

$$\rho_0^1(s) = \frac{1}{61440\pi^8}\int dydz\, yz\,(1-y-z)^4 \times\left(s-\overline{m}_Q^2\right)^4\left(8s-3\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2}{24576\pi^8}\int dydz\,(1-y-z)^4\left(s-\overline{m}_Q^2\right)^4,$$

$$\rho_4^1(s) = -\frac{m_Q^2}{9216\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\times\int dydz\,\frac{z\,(1-y-z)^4}{y^2}\left(s-\overline{m}_Q^2\right)\left(5s-3\overline{m}_Q^2\right)$$
$$-\frac{m_Q^4}{9216\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{(1-y-z)^4}{y^3}\left(s-\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2}{6144\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{(1-y-z)^4}{y^2}\left(s-\overline{m}_Q^2\right)^2$$
$$+\frac{1}{1024\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, yz\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^2\left(2s-\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2}{2048\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^2$$
$$+\frac{1}{6144\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, z\,(1-y-z)^3\left(s-\overline{m}_Q^2\right)^2\left(2s-\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2}{6144\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{(1-y-z)^3}{y}\left(s-\overline{m}_Q^2\right)^2,$$

$$\rho_6^1(s) = \frac{\langle\bar{q}q\rangle^2}{24\pi^4}\int dydz\, yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(5s-3\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2\langle\bar{q}q\rangle^2}{24\pi^4}\int dydz\,(1-y-z)\left(s-\overline{m}_Q^2\right),$$

$$\rho_8^1(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{24\pi^4}\int dydz\, yz\left(4s-3\overline{m}_Q^2\right)$$
$$-\frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{48\pi^4}\int dydz,$$

$$\rho_{10}^1(s) = \left[\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{64\pi^4}+\frac{\langle\bar{q}q\rangle^2}{72\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\right]\times\int dy\, y\,(1-y)\left[1+\frac{s}{2}\delta\left(s-\widetilde{m}_Q^2\right)\right]$$
$$-\frac{m_Q^2\langle\bar{q}q\rangle^2}{54\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\times\frac{z\,(1-y-z)}{y^2}\left(1+\frac{s}{2T^2}\right)\delta\left(s-\overline{m}_Q^2\right)$$
$$-\frac{m_Q^4\langle\bar{q}q\rangle^2}{216\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^3}\delta\left(s-\overline{m}_Q^2\right)$$
$$+\frac{m_Q^2\langle\bar{q}q\rangle^2}{72\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1-y-z}{y^2}\delta\left(s-\overline{m}_Q^2\right)$$
$$+\frac{1}{96\pi^2}\left[\langle\bar{q}q\rangle^2\left\langle\frac{\alpha_s GG}{\pi}\right\rangle+\frac{11\langle\bar{q}g_s\sigma Gq\rangle^2}{64\pi^2}\right]$$

$$
\begin{aligned}
&\times \int dydz\, z \left[1+\frac{s}{3}\delta\left(s-\overline{m}_Q^2\right)\right] \\
&+\frac{m_Q^2}{288\pi^2}\left[\langle\bar{q}q\rangle^2\left\langle\frac{\alpha_s GG}{\pi}\right\rangle+\frac{11\langle\bar{q}g_s\sigma Gq\rangle^2}{64\pi^2}\right] \\
&\times \int dydz\, \frac{1}{y}\delta\left(s-\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\rho_3^0(s) =& -\frac{\langle\bar{q}q\rangle}{768\pi^6}\int dydz\, yz\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^3\left(3s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle}{192\pi^6}\int dydz\, (1-y-z)^2\left(s-\overline{m}_Q^2\right)^3,
\end{aligned}
$$

$$
\begin{aligned}
\rho_5^0(s) =& \frac{\langle\bar{q}g_s\sigma Gq\rangle}{768\pi^6}\int dydz\, yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)^2\left(5s-2\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}g_s\sigma Gq\rangle}{3072\pi^6}\int dydz\, z\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^2\left(5s-2\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle}{128\pi^6}\int dydz\, (1-y-z)\left(s-\overline{m}_Q^2\right)^2 \\
&-\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle}{512\pi^6}\int dydz\, \frac{(1-y-z)^2}{y}\left(s-\overline{m}_Q^2\right)^2,
\end{aligned}
$$

$$
\begin{aligned}
\rho_7^0(s) =& \frac{m_Q^2\langle\bar{q}q\rangle}{576\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{z\,(1-y-z)^2}{y^2}\left(3s-2\overline{m}_Q^2\right) \\
&+\frac{m_Q^4\langle\bar{q}q\rangle}{288\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{(1-y-z)^2}{y^3} \\
&-\frac{m_Q^2\langle\bar{q}q\rangle}{96\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{(1-y-z)^2}{y^2}\left(s-\overline{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle}{768\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, (1-y-z)^2\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle}{288\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, yz\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle}{144\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \left(s-\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\rho_9^0(s) = -\frac{\langle\bar{q}q\rangle^3}{6\pi^2}\int dy\, y\,(1-y)\, s,
$$

$$
\begin{aligned}
\rho_{11}^0(s) =& \frac{\langle\bar{q}q\rangle^2\langle\bar{q}g_s\sigma Gq\rangle}{4\pi^2}\int dy\, y\,(1-y) \\
&\times\left[1+\left(s+\frac{s^2}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)\right] \\
&-\frac{\langle\bar{q}q\rangle^2\langle\bar{q}g_s\sigma Gq\rangle}{144\pi^2}\int dy\left[1+\frac{3s}{2}\delta\left(s-\widetilde{m}_Q^2\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\rho_{13}^0(s) =& -\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2}{16\pi^2}\int dy\, y\,(1-y) \\
&\times\left(1+\frac{s}{T^2}+\frac{s^2}{2T^4}+\frac{s^3}{2T^6}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}q\rangle^3}{162T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, \frac{1-y}{y^2}\, s\,\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{m_Q^4\langle\bar{q}q\rangle^3}{81T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, \frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle^3}{27T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, \frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle}{216}\left[\langle\bar{q}q\rangle^2\left\langle\frac{\alpha_s GG}{\pi}\right\rangle-\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{16\pi^2}\right] \\
&\times\int dy\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2}{288\pi^2}\int dy\left(1+\frac{s}{T^2}+\frac{3s^2}{2T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle^3}{36}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, y\,(1-y) \\
&\times\left(1+\frac{s}{T^2}+\frac{s^2}{2T^4}+\frac{s^3}{2T^6}\right)\delta\left(s-\widetilde{m}_Q^2\right).
\end{aligned}
$$

For the $QQcd\bar{q}$ states with $J^P=\frac{3}{2}^-$,

$$
\begin{aligned}
\rho_0^1(s) =& \frac{1}{245760\pi^8}\int dydz\, yz\,(1-y-z)^4 \\
&\times\left(s-\overline{m}_Q^2\right)^4\left(7s-2\overline{m}_Q^2\right) \\
&+\frac{m_Q^2}{49152\pi^8}\int dydz\, (1-y-z)^4\left(s-\overline{m}_Q^2\right)^4,
\end{aligned}
$$

$$
\begin{aligned}
\rho_4^1(s) =& -\frac{m_Q^2}{18432\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times\int dydz\, \frac{z\,(1-y-z)^4}{y^2}\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^4}{18432\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{(1-y-z)^4}{y^3}\left(s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2}{12288\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{(1-y-z)^4}{y^2}\left(s-\overline{m}_Q^2\right)^2 \\
&+\frac{1}{12288\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times\int dydz\, yz\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^2\left(5s-2\overline{m}_Q^2\right) \\
&+\frac{m_Q^2}{4096\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, (1-y-z)^2\left(s-\overline{m}_Q^2\right)^2 \\
&-\frac{1}{442368\pi^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times\int dydz\, (1-y-z)^4\left(s-\overline{m}_Q^2\right)^2\left(7s-4\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\rho_6^1(s) =& \frac{\langle\bar{q}q\rangle^2}{48\pi^4}\int dydz\, yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}q\rangle^2}{48\pi^4}\int dydz\, (1-y-z)\left(s-\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\rho_8^1(s) =& -\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{96\pi^4}\int dydz\, yz\left(3s-2\overline{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{96\pi^4}\int dydz,
\end{aligned}
$$

$$
\begin{aligned}
\rho_{10}^1(s) =& \left[\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{384\pi^4}+\frac{\langle\bar{q}q\rangle^2}{432\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\right] \\
&\times\int dy\, y\,(1-y)\left[1+s\,\delta\left(s-\widetilde{m}_Q^2\right)\right] \\
&-\frac{m_Q^2\langle\bar{q}q\rangle^2}{432\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{z\,(1-y-z)}{y^2}\left(1+\frac{s}{T^2}\right)\delta\left(s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^4\langle\bar{q}q\rangle^2}{432\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{1-y-z}{y^3}\delta\left(s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}q\rangle^2}{144\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{1-y-z}{y^2}\delta\left(s-\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle^2}{432\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, (1-y-z)\left[1+\frac{s}{4}\delta\left(s-\overline{m}_Q^2\right)\right] \\
&+\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{6912\pi^4}\int dydz\, (1-y-z)\left[1+\frac{s}{4}\delta\left(s-\overline{m}_Q^2\right)\right],
\end{aligned}
$$

$$
\rho_3^0(s) = -\frac{\langle\bar{q}q\rangle}{3072\pi^6}\int dydz\, yz\,(1-y-z)^2\left(s-\overline{m}_Q^2\right)^3\left(5s-\overline{m}_Q^2\right)
$$

$$-\frac{m_Q^2\langle\bar{q}q\rangle}{768\pi^6}\int dydz\ (1-y-z)^2\left(s-\overline{m}_Q^2\right)^3,$$

$$\rho_5^0(s)=\frac{\langle\bar{q}g_s\sigma Gq\rangle}{1536\pi^6}\int dydz\, yz\,(1-y-z)\times\left(s-\overline{m}_Q^2\right)^2\left(4s-\overline{m}_Q^2\right)+\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle}{512\pi^6}\times\int dydz\ (1-y-z)\left(s-\overline{m}_Q^2\right)^2,$$

$$\rho_7^0(s)=\frac{m_Q^2\langle\bar{q}q\rangle}{1152\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,\frac{z\,(1-y-z)^2}{y^2}\left(2s-\overline{m}_Q^2\right)+\frac{m_Q^4\langle\bar{q}q\rangle}{1152\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dydz\,\frac{(1-y-z)^2}{y^3}-\frac{m_Q^2\langle\bar{q}q\rangle}{384\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dydz\,\frac{(1-y-z)^2}{y^2}\left(s-\overline{m}_Q^2\right)+\frac{\langle\bar{q}q\rangle}{9216\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\ (1-y-z)^2\left(s-\overline{m}_Q^2\right)\left(5s-3\overline{m}_Q^2\right)-\frac{\langle\bar{q}q\rangle}{1152\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dydz\,yz\left(s-\overline{m}_Q^2\right)\left(3s-\overline{m}_Q^2\right)-\frac{m_Q^2\langle\bar{q}q\rangle}{576\pi^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dydz\ \left(s-\overline{m}_Q^2\right),$$

$$\rho_9^0(s)=-\frac{\langle\bar{q}q\rangle^3}{18\pi^2}\int dy\,y\,(1-y)\ s,$$

$$\rho_{11}^0(s)=\frac{\langle\bar{q}q\rangle^2\langle\bar{q}g_s\sigma Gq\rangle}{16\pi^2}\times\int dy\,y\,(1-y)\left[1+\left(\frac{4s}{3}+\frac{2s^2}{3T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)\right],$$

$$\rho_{13}^0(s)=-\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2}{96\pi^2T^2}\times\int dy\,y\,(1-y)\left(s+\frac{s^2}{T^2}+\frac{s^3}{T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right)-\frac{m_Q^2\langle\bar{q}q\rangle^3}{324T^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dy\,\frac{1-y}{y^2}\left(1-\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)+\frac{m_Q^4\langle\bar{q}q\rangle^3}{324T^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dy\,\frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right)-\frac{m_Q^2\langle\bar{q}q\rangle^3}{108T^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right)+\frac{\langle\bar{q}q\rangle}{648}\left[\langle\bar{q}q\rangle^2\left\langle\frac{\alpha_sGG}{\pi}\right\rangle-\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{16\pi^2}\right]\times\int dy\,\left(1+\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)-\frac{\langle\bar{q}q\rangle^3}{216T^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int dy\,y\,(1-y)\left(s+\frac{s^2}{T^2}+\frac{s^3}{T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right).$$

For the $QQcd\bar{q}$ states with $J^P=\frac{5}{2}^-$,

$$\rho_0^1(s)=\frac{1}{2457600\pi^8}\int dydz\,yz\,(1-y-z)^4\times(4+y+z)\left(s-\overline{m}_Q^2\right)^4\left(7s-2\overline{m}_Q^2\right)+\frac{m_Q^2}{491520\pi^8}\int dydz\ (1-y-z)^4\,(4+y+z)\left(s-\overline{m}_Q^2\right)^4,$$

$$\rho_4^1(s)=-\frac{m_Q^2}{184320\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,\frac{z\,(1-y-z)^4\,(4+y+z)}{y^2}\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right)-\frac{m_Q^4}{184320\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,\frac{(1-y-z)^4\,(4+y+z)}{y^3}\left(s-\overline{m}_Q^2\right)+\frac{m_Q^2}{122880\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,\frac{(1-y-z)^4\,(4+y+z)}{y^2}\left(s-\overline{m}_Q^2\right)^2-\frac{1}{221184\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,yz\,(1-y-z)^2\,(4-y-z)\left(s-\overline{m}_Q^2\right)^2\left(5s-2\overline{m}_Q^2\right)-\frac{m_Q^2}{73728\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\ (1-y-z)^2\,(4-y-z)\left(s-\overline{m}_Q^2\right)^2-\frac{1}{221184\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,z\,(1-y-z)^3\left(s-\overline{m}_Q^2\right)^2\left(5s-2\overline{m}_Q^2\right)$$

$$-\frac{m_Q^2}{294912\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\,\frac{(1-y-z)^3\,(3+y+z)}{y}\left(s-\overline{m}_Q^2\right)^2-\frac{1}{884736\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\ (1-y-z)^4\left(s-\overline{m}_Q^2\right)^2\left(7s-4\overline{m}_Q^2\right)+\frac{1}{1474560\pi^6}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\times\int dydz\ (1-y-z)^5\left(s-\overline{m}_Q^2\right)^2\left(3s-2\overline{m}_Q^2\right),$$

$$\rho_6^1(s)=\frac{\langle\bar{q}q\rangle^2}{96\pi^4}\times\int dydz\,yz\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(2s-\overline{m}_Q^2\right)+\frac{m_Q^2\langle\bar{q}q\rangle^2}{96\pi^4}\int dydz\ (1-y-z)\left(s-\overline{m}_Q^2\right),$$

$$\rho_8^1(s)=-\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{192\pi^4}\times\int dydz\,yz\ \left(3s-2\overline{m}_Q^2\right)-\frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{192\pi^4}\int dydz-\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{2304\pi^4}\int dydz\,z\,(1-y-z)\left(3s-2\overline{m}_Q^2\right)-\frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{2304\pi^4}\int dydz\,\frac{1-y-z}{y},$$

$$\rho_{10}^1(s)=\left[\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{768\pi^4}+\frac{\langle\bar{q}q\rangle^2}{864\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\right]$$

$$
\begin{aligned}
&\times \int dy\, y\,(1-y)\left[1+s\,\delta\left(s-\widetilde{m}_Q^2\right)\right] \\
&-\frac{m_Q^2\langle\bar{q}q\rangle^2}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, \frac{z\,(1-y-z)}{y^2}\left(1+\frac{s}{T^2}\right)\delta\left(s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^4\langle\bar{q}q\rangle^2}{864\pi^2T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{1-y-z}{y^3}\delta\left(s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}q\rangle^2}{288\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, \frac{1-y-z}{y^2}\delta\left(s-\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle^2}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, (1-y-z)\left[1+\frac{s}{4}\delta\left(s-\overline{m}_Q^2\right)\right] \\
&+\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle^2}{9216\pi^4}\int dydz\, \frac{1}{y}\delta\left(s-\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{27648\pi^4}\int dydz\, (1-4y-4z)\left[1+\frac{s}{2}\delta\left(s-\overline{m}_Q^2\right)\right] \\
&-\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle^2}{27648\pi^4}\int dydz\, \frac{1-y-z}{yz}\delta\left(s-\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\rho_3^0(s) = &-\frac{\langle\bar{q}q\rangle}{18432\pi^6} \\
&\times \int dydz\, yz\,(1-y-z)^2\,(2+y+z)\left(s-\overline{m}_Q^2\right)^3\left(5s-\overline{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle}{4608\pi^6}\int dydz\, (1-y-z)^2\,(2+y+z)\left(s-\overline{m}_Q^2\right)^3,
\end{aligned}
$$

$$
\begin{aligned}
\rho_5^0(s) = &\frac{\langle\bar{q}g_s\sigma Gq\rangle}{6144\pi^6} \\
&\times \int dydz\, yz\,(1-y-z)\,(1+y+z)\left(s-\overline{m}_Q^2\right)^2\left(4s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle}{2048\pi^6}\int dydz\, (1-y-z)\,(1+y+z)\left(s-\overline{m}_Q^2\right)^2,
\end{aligned}
$$

$$
\begin{aligned}
\rho_7^0(s) = &\frac{m_Q^2\langle\bar{q}q\rangle}{6912\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, \frac{z\,(1-y-z)^2\,(2+y+z)}{y^2}\left(2s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^4\langle\bar{q}q\rangle}{6912\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, \frac{(1-y-z)^2\,(2+y+z)}{y^3} \\
&-\frac{m_Q^2\langle\bar{q}q\rangle}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, \frac{(1-y-z)^2\,(2+y+z)}{y^2}\left(s-\overline{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle}{9216\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, (1+2y)\,z\,(1-y-z)\left(s-\overline{m}_Q^2\right)\left(3s-\overline{m}_Q^2\right) \\
&+\frac{m_Q^2\langle\bar{q}q\rangle}{9216\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, \frac{(1-y-z)\,(1+5y+z)}{y}\left(s-\overline{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle}{18432\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, (1-y-z)^2\left(s-\overline{m}_Q^2\right)\left(5s-3\overline{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle}{55296\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dydz\, (1-y-z)^3\left(s-\overline{m}_Q^2\right)\left(7s-5\overline{m}_Q^2\right),
\end{aligned}
$$

$$
\rho_9^0(s) = -\frac{\langle\bar{q}q\rangle^3}{36\pi^2}\int dy\, y\,(1-y)\, s,
$$

$$
\begin{aligned}
\rho_{11}^0(s) = &\frac{\langle\bar{q}q\rangle^2\langle\bar{q}g_s\sigma Gq\rangle}{32\pi^2} \\
&\times \int dy\, y\,(1-y)\left[1+\left(\frac{4s}{3}+\frac{2s^2}{3T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right)\right] \\
&+\frac{\langle\bar{q}q\rangle^2\langle\bar{q}g_s\sigma Gq\rangle}{3456\pi^2} \\
&\times \int dy\, \left[1+2s\,\delta\left(s-\widetilde{m}_Q^2\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\rho_{13}^0(s) = &-\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2}{192\pi^2T^2} \\
&\times \int dy\, y\,(1-y)\left(s+\frac{s^2}{T^2}+\frac{s^3}{T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle^3}{648T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dy\, \frac{1-y}{y^2}\left(1-\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{m_Q^4\langle\bar{q}q\rangle^3}{648T^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dy\, \frac{1}{y^3}\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{m_Q^2\langle\bar{q}q\rangle^3}{216T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dy\, \frac{1}{y^2}\delta\left(s-\widetilde{m}_Q^2\right) \\
&+\frac{\langle\bar{q}q\rangle^3}{1296}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dy\, \left(1+\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2}{13824\pi^2T^2} \\
&\times \int dy\, \left(s+\frac{4s^2}{T^2}\right)\delta\left(s-\widetilde{m}_Q^2\right) \\
&-\frac{\langle\bar{q}q\rangle^3}{432T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int dy\, y\,(1-y)\left(s+\frac{s^2}{T^2}+\frac{s^3}{T^4}\right)\delta\left(s-\widetilde{m}_Q^2\right),
\end{aligned}
$$

where $\int dydz = \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz$, $\int dy = \int_{y_i}^{y_f} dy$, $y_f = \frac{1+\sqrt{1-4m_Q^2/s}}{2}$, $y_i = \frac{1-\sqrt{1-4m_Q^2/s}}{2}$, $z_i = \frac{ym_Q^2}{ys-m_Q^2}$, $\overline{m}_Q^2 = \frac{(y+z)m_Q^2}{yz}$, $\widetilde{m}_Q^2 = \frac{m_Q^2}{y(1-y)}$, $\int_{y_i}^{y_f} dy \to \int_0^1$, $\int_{z_i}^{1-y} dz \to \int_0^{1-y} dz$, when the δ functions $\delta(s-\overline{m}_Q^2)$ and $\delta(s-\widetilde{m}_Q^2)$ appear. We can obtain the QCD spectral densities of the Ξ_{cc} and Ξ_{cc}^* with a simple replacement $m_s \to 0$, $\langle\bar{s}s\rangle \to \langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle \to \langle\bar{q}g_s\sigma Gq\rangle$ for the QCD spectral densities of the Ω_{cc} and Ω_{cc}^*, respectively.

References

1. R. Aaij et al., Phys. Rev. Lett. **119**, 112001 (2017)
2. A. De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D **12**, 147 (1975)
3. T. DeGrand, R.L. Jaffe, K. Johnson, J.E. Kiskis, Phys. Rev. D **12**, 2060 (1975)
4. H.G. Dosch, M. Jamin, B. Stech, Z. Phys. C **42**, 167 (1989)
5. M. Jamin, M. Neubert, Phys. Lett. B **238**, 387 (1990)
6. Z.G. Wang, Eur. Phys. J. C **71**, 1524 (2011)
7. L. Tang, X.Q. Li, Chin. Phys. C **36**, 578 (2012)
8. Z.G. Wang, Commun. Theor. Phys. **59**, 451 (2013)
9. R.T. Kleiv, T.G. Steele, A. Zhang, Phys. Rev. D **87**, 125018 (2013)
10. J.R. Zhang, M.Q. Huang, Phys. Rev. D **78**, 094007 (2008)
11. S. Narison, R. Albuquerque, Phys. Lett. B **694**, 217 (2011)
12. H.X. Chen, Q. Mao, W. Chen, X. Liu, S.L. Zhu, Phys. Rev. D **96**, 031501 (2017)
13. X.H. Hu, Y.L. Shen, W. Wang, Z.X. Zhao, arXiv:1711.10289
14. R.M. Albuquerque, S. Narison, Nucl. Phys. Proc. Suppl. **207–208**, 265 (2010)
15. T.M. Aliev, K. Azizi, M. Savci, Nucl. Phys. A **895**, 59 (2012)
16. T.M. Aliev, K. Azizi, M. Savci, J. Phys. **G40**, 065003 (2013)
17. K. Azizi, T.M. Aliev, M. Savci, J. Phys. Conf. Ser. **556**, 012016 (2014)
18. Z.G. Wang, Eur. Phys. J. A **45**, 267 (2010)
19. Z.G. Wang, Eur. Phys. J. C **68**, 459 (2010)
20. Z.G. Wang, Eur. Phys. J. A **47**, 81 (2011)
21. F.S. Navarra, M. Nielsen, S.H. Lee, Phys. Lett. B **649**, 166 (2007)
22. Z.G. Wang, Y.M. Xu, H.J. Wang, Commun. Theor. Phys. **55**, 1049 (2011)
23. M.L. Du, W. Chen, X.L. Chen, S.L. Zhu, Phys. Rev. D **87**, 014003 (2013)
24. Z.G. Wang, arXiv:1708.04545;
25. Z.G. Wang, Z.H. Yan, Eur. Phys. J. C **78**, 19 (2018)
26. Q.S. Zhou, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, arXiv:1801.04557
27. R. Aaij et al., Phys. Rev. Lett. **115**, 072001 (2015)
28. L. Maiani, A.D. Polosa, V. Riquer, Phys. Lett. B **749**, 289 (2015)
29. L. Maiani, A.D. Polosa, V. Riquer, Phys. Lett. B **750**, 37 (2015)
30. R.F. Lebed, Phys. Rev. D **92**, 114030 (2015)
31. R.F. Lebed, Phys. Lett. B **749**, 454 (2015)
32. R. Zhu, C.F. Qiao, Phys. Lett. B **756**, 259 (2016)
33. V.V. Anisovich, M.A. Matveev, J. Nyiri, A.V. Sarantsev, A.N. Semenova, arXiv:1507.07652
34. R. Ghosh, A. Bhattacharya, B. Chakrabarti, Phys. Part. Nucl. Lett. **14**, 550 (2017)
35. V.V. Anisovich, M.A. Matveev, J. Nyiri, A.V. Sarantsev, A.N. Semenova, Int. J. Mod. Phys. A **30**, 1550190 (2015)
36. H.Y. Cheng, C.K. Chua, Phys. Rev. D **92**, 096009 (2015)
37. V.V. Anisovich, M.A. Matveev, A.V. Sarantsev, A.N. Semenova, Mod. Phys. Lett. A **30**, 1550212 (2015)
38. G.N. Li, M. He, X.G. He, JHEP **1512**, 128 (2015)
39. Z.G. Wang, Eur. Phys. J. C **76**, 70 (2016)
40. Z.G. Wang, T. Huang, Eur. Phys. J. C **76**, 43 (2016)
41. Z.G. Wang, Eur. Phys. J. C **76**, 142 (2016)
42. Z.G. Wang, Nucl. Phys. B **913**, 163 (2016)
43. J.X. Zhang, Z.G. Wang, Z.Y. Di, Acta Phys. Polon. B **48**, 2013 (2017)
44. Z.G. Wang, Phys. Lett. B **685**, 59 (2010)
45. Z.G. Wang, Eur. Phys. J. C **68**, 479 (2010)
46. Z.G. Wang, Commun. Theor. Phys. **58**, 723 (2012)
47. Z.G. Wang, Eur. Phys. J. C **78**, 300 (2018)
48. Z.G. Wang, J.X. Zhang, Eur. Phys. J. C **78**, 503 (2018)
49. M. Karliner, J.L. Rosner, Phys. Rev. D **95**, 114012 (2017)
50. M. Padmanath, N. Mathur, Phys. Rev. Lett. **119**, 042001 (2017)
51. W. Wang, R.-L. Zhu, Phys. Rev. D **96**, 014024 (2017)
52. Z.G. Wang, Eur. Phys. J. C **77**, 325 (2017)
53. B. Chen, X. Liu, Phys. Rev. D **96**, 094015 (2017)
54. T.M. Aliev, S. Bilmis, M. Savci, arXiv:1704.03439
55. Z.G. Wang, arXiv:1806.10384
56. K. Azizi, Y. Sarac, H. Sundu, Phys. Rev. D **95**, 094016 (2017)
57. K. Azizi, Y. Sarac, H. Sundu, Phys. Rev. D **96**, 094030 (2017)
58. Y. Chung, H.G. Dosch, M. Kremer, D. Schall, Nucl. Phys. B **197**, 55 (1982)
59. E. Bagan, M. Chabab, H.G. Dosch, S. Narison, Phys. Lett. B **301**, 243 (1993)
60. D. Jido, N. Kodama, M. Oka, Phys. Rev. D **54**, 4532 (1996)
61. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**(385), 448 (1979)
62. L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. **127**, 1 (1985)
63. P. Pascual, R. Tarrach, *QCD: Renormalization for the Practitioner* (Springer, Berlin, 1984)
64. Z.G. Wang, T. Huang, Phys. Rev. D **89**, 054019 (2014)
65. Z.G. Wang, Eur. Phys. J. C **74**, 2874 (2014)
66. Z.G. Wang, T. Huang, Nucl. Phys. A **930**, 63 (2014)
67. Z.G. Wang, Commun. Theor. Phys. **63**, 466 (2015)
68. Z.G. Wang, Y.F. Tian, Int. J. Mod. Phys. A **30**, 1550004 (2015)
69. Z.G. Wang, T. Huang, Eur. Phys. J. C **74**, 2891 (2014)
70. Z.G. Wang, Eur. Phys. J. C **74**, 2963 (2014)
71. P. Colangelo, A. Khodjamirian, arXiv:hep-ph/0010175
72. C. Patrignani et al., Chin. Phys. C **40**, 100001 (2016)
73. S. Narison, R. Tarrach, Phys. Lett. **125 B**, 217 (1983)
74. S. Narison, QCD as a theory of hadrons from partons to confinement. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **17**, 1 (2007)
75. Z.G. Wang, Eur. Phys. J. C **76**, 387 (2016)
76. Z.G. Wang, Commun. Theor. Phys. **66**, 335 (2016)
77. M.J. Yan, X.H. Liu, S. Gonzalez-Solis, F.K. Guo, C. Hanhart, U.G. Meissner, B.S. Zou, arXiv:1805.10972