Eur. Phys. J. C (2018) 78:398
https://doi.org/10.1140/epjc/s10052-018-5890-1

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Scalar pair production in a magnetic field in de Sitter universe

Mihaela-Andreea Băloi[1], **Cosmin Crucean**[2,a], **Diana Popescu**[2]

[1] Politehnica University of Timişoara, V. Parvan Ave. 2, 300223 Timisoara, Romania
[2] West University of Timişoara, V. Parvan Ave. 4, 300223 Timisoara, Romania

Received: 29 March 2018 / Accepted: 10 May 2018 / Published online: 22 May 2018

Abstract The production of scalar particles by the dipole magnetic field in de Sitter expanding universe is analyzed. The amplitude and probability of transition are computed using perturbative methods. A graphical study of the transition probability is performed obtaining that the rate of pair production is important in the early universe. Our results prove that in the process of pair production by the external magnetic field the momentum conservation law is broken. We also found that the probabilities are maximum when the particles are emitted perpendicular to the direction of magnetic dipole momentum. The total probability is computed and is analysed in terms of the angle between particles momenta.

1 Introduction

The problem of particle generation in magnetic field in Minkowski space-time was first studied by Heisenberg and Euler [1] and this paper represents the basis for the today well known Schwinger effect. This paper proves for the first time how to solve the Dirac equation when a magnetic field is present, and these results represent the basis for the more recent studies that imply particle production in the presence of external fields [2–9] in curved backgrounds. An important result was obtained recently in [9], where the minimally coupled Klein–Gordon equation was solved in four dimensions in a de Sitter geometry. Further in [9] the rate of pair production was computed with the help of Bogoliubov method and the Minkowski limit was addressed. Another result which uses the nonperturbative treatment of scalar pair production in magnetic field on Robertson–Walker universe was discussed in [10], where the density number was computed using the Bogoliubov coefficients. In [10] the de Sitter case is not contained since the scale factor $a(t)$ was chosen to have a linear time dependence. The result of [10] has proved how to work out the density number when a magnetic field is present in Robertson–Walker spacetime. While the vast majority of nonperturbative computations are done in two dimensions, in [10] the computations were made in four dimensions. These results need to be completed with a perturbative treatment in the case of scalar field since the perturbative method allows a complete study of the case when the gravitational fields are strong in comparison with the magnetic fields.

In this paper we will study the problem of scalar pair production in dipolar magnetic field on de Sitter geometry. We will use the perturbative methods presented in [11–21], for computing the amplitude/probability of scalar pair production in magnetic field on de Sitter expanding universe. The problem of particle production when a magnetic field and a gravitational field are present seems to receive little attention in literature because of the technical difficulties in solving the field equations in the presence of magnetic fields. The perturbative approach to the problem of fermions production in magnetic field on de Sitter geometry was used in [13], and the main results prove that the probability of pair production is nonvanishing only in strong gravitational fields. However for a complete picture of the problem of pair production in gravitational fields one needs to combine the nonperturbative results [2–8], with the perturbative treatment of this problem. This will imply as we mentioned above that the nonperturbative computations to be extended in four dimensions, since only in these cases we will have a complete picture on how the field equations with external field will look like and how these equations can be solved. However some time ago is was argued [12], that the production due to the field interactions should also be taken into consideration. Despite of this observation, the perturbative approach only recently received attention and the results are based on the calculations of QED transition amplitudes that generates particle production in a curved background [13,14,20]. The problem of space expansion that generate particle production was first discussed in [22], and important results were also obtained in [4,5]. Since the de Sitter space-time could describe our universe, we believe that it is important to study the problem

[a] e-mail: crucean@quasar.physics.uvt.ro;cosmin.crucean@e-uvt.ro

of scalar particle production in magnetic fields in this geometry by using both perturbative and nonperturbative methods. The first step in our study will be the computation of the first order QED transition amplitude that generates scalar particles in the field of a magnetic dipole. Then we will present the main steps required for computing the total probability for the process of pair production in magnetic field.

The origin of magnetic fields in the Universe is a subject that has been approached by many authors [23–27,38], mostly motivated by the astronomical observations which show that, the galaxies and galaxy clusters have a proper magnetic field of weak intensity. The vast majority of the papers propose technical solutions of accurate measurements of the large scales magnetic fields, but an interesting idea that is analysed in the literature is that, these magnetic fields own their origin in the early stages of Universe evolution.

The paper begins with an introduction in the problem of pair production in magnetic field. In section two we compute the amplitude/probability of pair production in magnetic field. Section three is dedicated to the graphical study of our analytical results and limit cases, while in section four we compute the total probability. Our conclusions are presented in section five.

2 Probability calculation

We start this section by mentioning that we work in the chart with conformal time that covers only half of the whole de Sitter manifold. More precisely we consider the chart with conformal time $t_c \in (-\infty, 0)$, which covers the expanding portion of de Sitter manifold [29]:

$$ds^2 = dt^2 - e^{2\omega t} d\vec{x}^{\,2} = \frac{1}{(\omega t_c)^2}(dt_c^2 - d\vec{x}^{\,2}), \tag{1}$$

where ω is the expansion factor and $\omega > 0$, while the conformal time is defined as $t_c = -\frac{1}{\omega} e^{-\omega t}$.

In de Sitter expanding geometry the free Klein–Gordon equation has analytical solutions as it was shown in Refs. [22,30]. In this paper we will use the exact solutions of the Klein–Gordon equation with a defined momentum [22,30]:

$$f_{\vec{p}}\,(x) = \frac{1}{2}\sqrt{\frac{\pi}{\omega}}\,\frac{(-\omega t_c)^{3/2}}{(2\pi)^{3/2}}\, e^{-\pi\mu/2} H^{(1)}_{i\mu}\,(-p\,t_c)\, e^{i\,\vec{p}\cdot\vec{x}}\,, \tag{2}$$

where $H^1_\mu(z)$ is the Hankel function of first kind, $p = |\vec{p}|$ is the momentum modulus. We also use the notations:

$$\mu = \sqrt{k^2 - \tfrac{9}{4}}\,, \quad k = \frac{m}{\omega}\,, \tag{3}$$

with $m > 3\omega/2$. The fundamental solutions of negative frequencies are obtained by complex conjugation $f^*_{\vec{p}}(x)$.

The form of the potential that produces the dipole magnetic field is known from electrodynamics [31,32]:

$$\vec{A}_M = \frac{\vec{\mathcal{M}} \times \vec{x}}{|\vec{x}\,|^3}, \tag{4}$$

where $\vec{\mathcal{M}}$ is the magnetic dipole moment. The expression of the field of a magnetic dipole can be obtained as the curl of the vector potential [31,32]:

$$\vec{B}_M = \vec{\nabla} \times \vec{A}_M = \frac{3\vec{x}\,(\vec{\mathcal{M}}\cdot\vec{x}\,) - \vec{\mathcal{M}}(\vec{x}\cdot\vec{x}\,)}{|\vec{x}\,|^5}. \tag{5}$$

The expression of $\vec{A}$ in de Sitter geometry is established by using the conformal invariance of Maxwell equations. The metric given in Eq. (1) is conformal with the Minkowski metric, i.e. $g_{\mu\nu} = \Omega\eta_{\mu\nu}$. Then knowing that $\vec{A}_M$ is the vector potential in Minkowski space, then the vector potential in de Sitter geometry is [33]:

$$A^\mu = \Omega^{-1} A^\mu_M, \tag{6}$$

where $\Omega = (\omega t_c)^{-2}$ as can be seen from the line element given in Eq. (1), and is the conformal factor transformation. Taking now $A^0(x) = 0$, we obtain for the potential vector:

$$\vec{A}(x) = \frac{\vec{\mathcal{M}} \times \vec{x}}{|\vec{x}\,|^3}\,(\omega t_c)^2. \tag{7}$$

The transition amplitude of scalar pair production in external field, defined in the first order of perturbation theory in Coulomb gauge is [15,19]:

$$A_{i\to f} = -e \int \sqrt{-g(x)}\left[f^*_{\vec{p}\,'}(x)\ \overleftrightarrow{\partial_i}\ f^*_{\vec{p}}\,(x)\right] A^i(x) d^4x, \tag{8}$$

where $\sqrt{-g(x)} = (\omega t_c)^{-4}$.

We must specify that we work in Coulomb gauge i.e. $\nabla_i(\sqrt{-g}A^i) = 0$, and our amplitude is gauge invariant since the transformations $A^i \to A^i + \partial^i\Lambda$, leave the amplitude unchanged [36].

Amplitude calculation can be done by using the fundamental solutions given in (2) and (7). Then the four dimensional integral is split in a temporal integral and a spatial integral. By taking the bilateral derivative and then solving the spatial integral we obtain [13]:

$$\int d^3x\, \frac{\vec{x}}{|\vec{x}\,|^3}\, e^{-i(\vec{p}+\vec{p}\,')\vec{x}} = -\frac{4\pi i(\vec{p}+\vec{p}\,')}{|\vec{p}+\vec{p}\,'|^2}. \tag{9}$$

The transition amplitude can be expressed using the relation between Hankel functions and Bessel K functions [34,35]:

$$A_{i\to f} = -\frac{e}{2\pi^3|\vec{p}+\vec{p}\,'|^2}(\vec{\mathcal{M}}\times(\vec{p}+\vec{p}\,'))\cdot(\vec{p}-\vec{p}\,')\int_0^\infty dzzK_{-i\mu}(ipz)K_{-i\mu}(ip'z) \tag{10}$$

where we pass in the temporal integral to the new variable of integration $z = e^{-\omega t}/\omega = -t_c$. The computation of the temporal integral leads to the final expression for the transition amplitude corresponding to the process of pair production in dipolar magnetic field in terms of unit step functions θ, Euler gamma functions Γ and Gauss hypergeometric functions ${}_2F_1$:

$$A_{i\to f} = -\frac{e}{4\pi^3|\vec{p}+\vec{p}\,'|^2}(\vec{\mathcal{M}}\times(\vec{p}+\vec{p}\,'))\cdot(\vec{p}-\vec{p}\,')\times\left[\frac{\theta(p-p')}{p^2}f_k\left(\frac{p'}{p}\right)+\frac{\theta(p'-p)}{p'^2}f_k\left(\frac{p}{p'}\right)\right]. \tag{11}$$

The functions $f_k\left(\frac{p}{p'}\right)$ defined above are:

$$f_k\left(\frac{p'}{p}\right) = \left(\frac{p'}{p}\right)^{-i\mu}\Gamma(1-i\mu)\Gamma(1+i\mu)\,{}_2F_1\left(1,1-i\mu;2;1-\left(\frac{p'}{p}\right)^2\right)$$
$$= \frac{1}{\left(1-\left(\frac{p'}{p}\right)^2\right)}\left[\left(\frac{p'}{p}\right)^{i\mu}\Gamma(1+i\mu)\Gamma(-i\mu)+\left(\frac{p'}{p}\right)^{-i\mu}\Gamma(1-i\mu)\Gamma(i\mu)\right]. \tag{12}$$

The probability of pair production is obtained by taking the square modulus of the amplitude:

$$\mathcal{P} = |\mathcal{A}_{i\to f}|^2 = \frac{e^2}{16\pi^6|\vec{p}+\vec{p}\,'|^4}|(\vec{\mathcal{M}}\times(\vec{p}+\vec{p}\,'))\cdot(\vec{p}-\vec{p}\,')|^2\times\left[\frac{\theta(p-p')}{p^4}\left|f_k\left(\frac{p'}{p}\right)\right|^2+\frac{\theta(p'-p)}{p'^4}\left|f_k\left(\frac{p}{p'}\right)\right|^2\right]. \tag{13}$$

Now let us discuss the physical consequences of our calculation. Firstly, we observe that the momentum conservation law is broken in the process of pair production by the dipole field. This is a consequence of the presence of the external field in de Sitter background [13,20]. Secondly from the probability expression results that the fermions could not be emitted on the direction of magnetic field because the probability is vanishing due to the vectorial product $\vec{\mathcal{M}}\times(\vec{p}+\vec{p}\,')$. The most probable transitions will be those in which the scalar particles will be emitted perpendicular on the direction of the magnetic field, result obtained also in [13], for fermions.

Considering the above mentioned observations, let us fix the magnetic dipole moment on the $\vec{e}_3$ direction such that $\vec{\mathcal{M}} = \mathcal{M}\vec{e}_3$. Then because of the vectorial product we can consider the momenta vectors $\vec{p}$, $\vec{p}\,'$ are in the $(1,2)$ plane, such that $\vec{\mathcal{M}}\perp\vec{p}\,'$ and $\vec{\mathcal{M}}\perp\vec{p}$. Taking the polar coordinates for $\vec{p}$, $\vec{p}\,'$:

$$p_1 = p\cos\beta;\quad p_2 = p\sin\beta$$
$$p_1' = p'\cos\varphi;\quad p_2' = p'\sin\varphi, \tag{14}$$

we obtain that the angle between momenta vectors $\vec{p}$ and $\vec{p}\,'$ is just $\beta-\varphi$. Using the angular analysis above, the final expression for the probability of scalar pair production turns out to be:

$$\mathcal{P} = \frac{e^2\mathcal{M}^2}{16\pi^6}\frac{(p^2+p'^2-2pp'\cos(\beta-\varphi))}{(p^2+p'^2+2pp'\cos(\beta-\varphi))}\times\left[\frac{\theta(p-p')}{p^4}\left|f_k\left(\frac{p'}{p}\right)\right|^2+\frac{\theta(p'-p)}{p'^4}\left|f_k\left(\frac{p}{p'}\right)\right|^2\right]. \tag{15}$$

From the probability equation we observe that the momenta vectors $\vec{p}$, $\vec{p}\,'$ of the produced particles, could be on the same direction while their orientations could be opposite or the same.

3 Graphical results and limit cases

The probability equation (15) is analysed in terms of parameter $k = m/\omega$, by making the plots for given momenta ratios p/p' and given angle $\beta-\varphi$ between momenta vectors. These graphs are obtained by using MAPLE. We must specify that our computations for probability of pair production, were done in the case $m/\omega > 3/2$ and that is reflected in our graphs in terms of parameter k and we considered for modulus of magnetic moment the value $\mathcal{M} = 1$.

From Figs. 1, 2, 3 and 4 we observe that the probability drops to zero for large values of parameter k. The probability is nonvanishing only for $k \in (1.51, 2)$, where the gravity is still strong and we can conclude that this process is possible only in strong gravitational fields. Another observation is that the probability of pair production is sensibly larger when the angle between momenta vectors approaches π and the momenta ratio p/p' are close to unity as seen from Figs. 1 and 4.

The above stated conclusions can be confirmed if we plot the probability equation (15) as function of the angle $\beta-\varphi$ between momenta vectors. In these graphs k and p/p' are fixed (Figs. 5, 6).

Finally we conclude that there are higher probabilities for pair production processes that have as result particles with

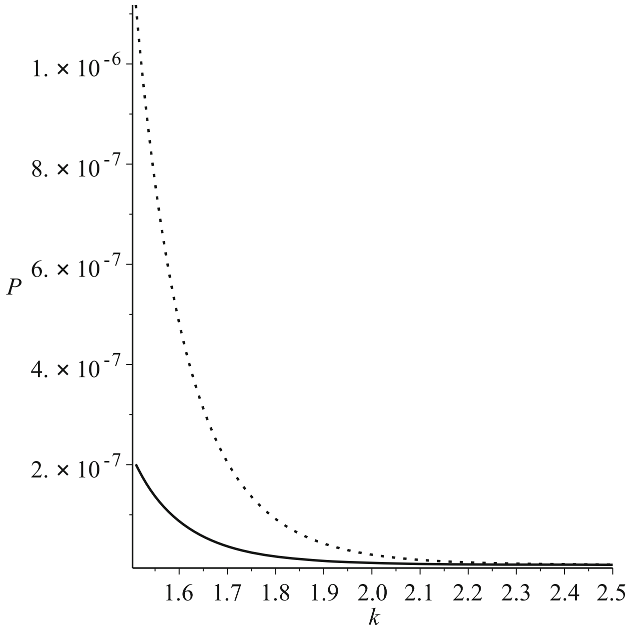

Fig. 1 $\mathcal{P}$ as a function of k for $p/p' = 0.9$ solid line and $p/p' = 0.8$ the point line. Angle $\beta - \varphi = 0$

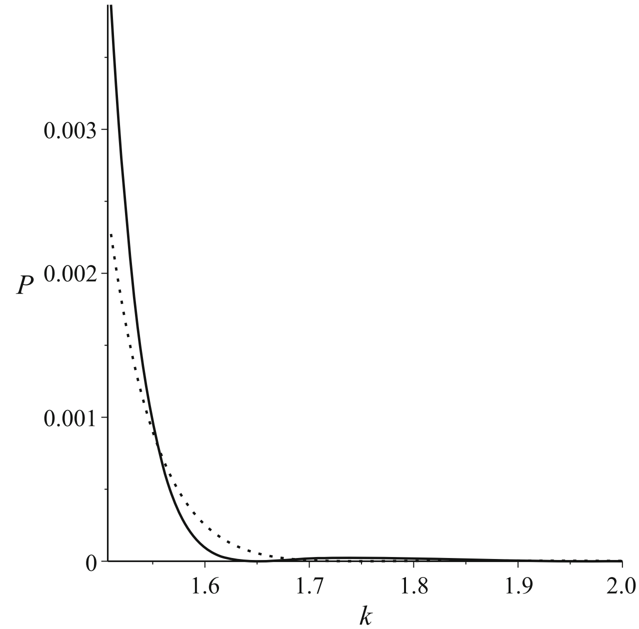

Fig. 3 $\mathcal{P}$ as a function of k for $p/p' = 0.01$ solid line and $p/p' = 0.03$ the point line. Angle $\beta - \varphi = 0$

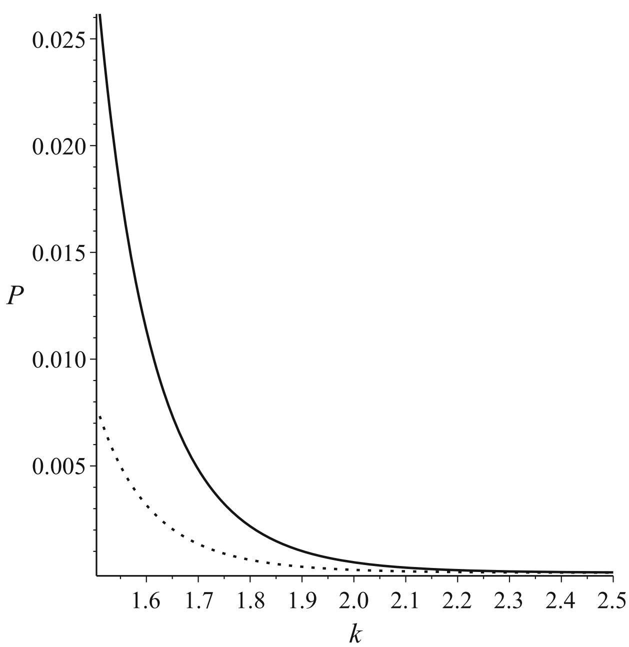

Fig. 2 $\mathcal{P}$ as a function of k for $p/p' = 0.9$ solid line and $p/p' = 0.8$ the point line. Angle $\beta - \varphi = \pi$

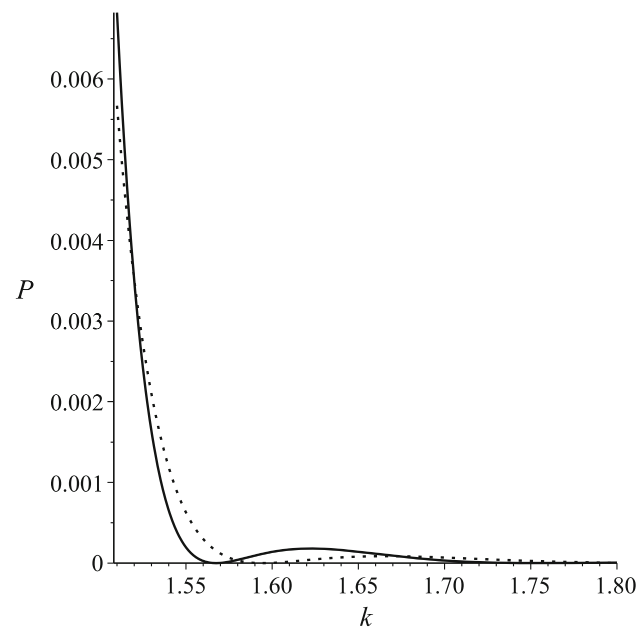

Fig. 4 $\mathcal{P}$ as a function of k for $p/p' = 0.001$ solid line and $p/p' = 0.003$ the point line. Angle $\beta - \varphi = \pi$

the momenta ratio close to unity and with the momenta on the same direction but opposite as orientation.

Let us study now the Minkowski limit for amplitude and probability. First we observe from our graphs that for $k \to \infty$ our probability is vanishing. This result is in fact the Minkowski limit which is obtained when $k = \infty$. Indeed in the Minkowski scalar QED [37] the first order processes of pair production in external fields are not allowed by the simultaneous energy–momentum conservation. In de Sitter case the translational invariance with respect to time is lost and as a consequence the amplitudes of particle production in the first order of perturbation theory are nonvanishing.

Further let us verify that our analytical results confirm the graphical predictions that the probability is vanishing in the Minkowski limit. For large k, the parameter $\mu \simeq k$. Then by

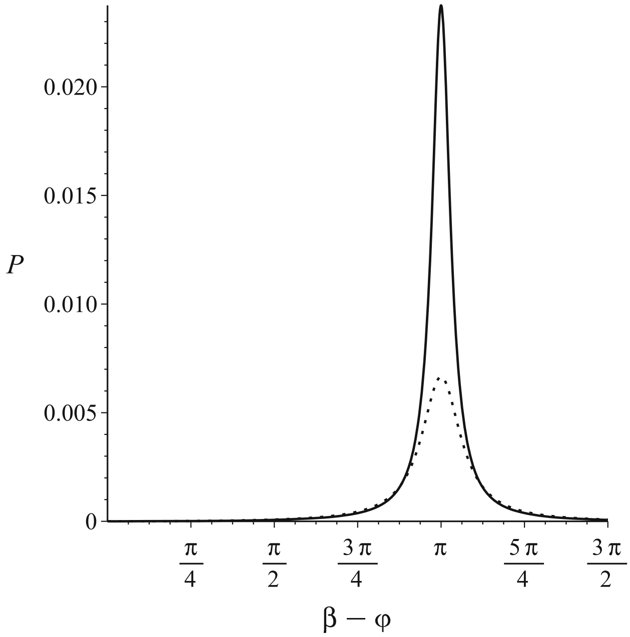

Fig. 5 $\mathcal{P}$ as a function of $\beta-\varphi$ for for $p/p'=0.9$ solid line and $p/p'=0.8$ the point line. Parameter $k=1.52$

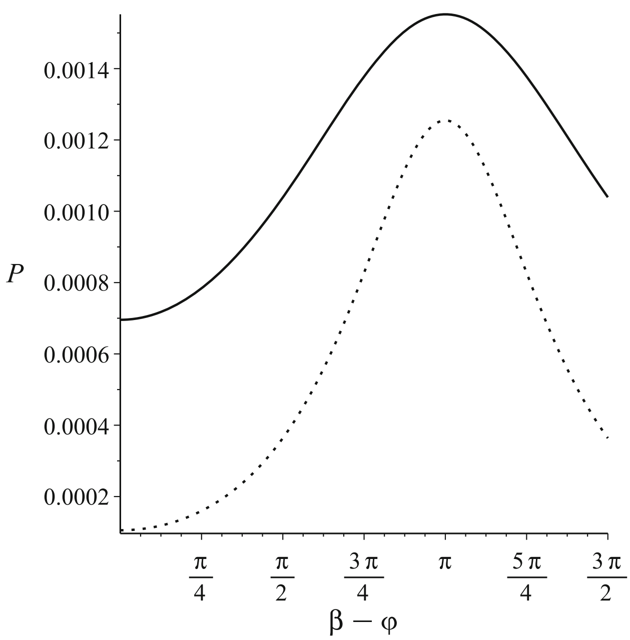

Fig. 6 $\mathcal{P}$ as a function of $\beta-\varphi$ for for $p/p'=0.3$ solid line and $p/p'=0.1$ the point line. Parameter $k=1.52$

using the Eqs. (27) and (28) from Appendix the function f_k that defines the amplitude can be brought for large k to the form:

$$f_{k>>1}\simeq\frac{2\pi i e^{-\pi k}}{\left(1-\left(\frac{p'}{p}\right)^2\right)}\left(\left(\frac{p'}{p}\right)^{-ik}-\left(\frac{p'}{p}\right)^{ik}\right). \tag{16}$$

The probability of pair production will then be proportional with:

$$\mathcal{P}\sim e^{-2\pi k}, \tag{17}$$

which clearly will be negligible for large k and will vanish for $k=\infty$. Here we remark that the probability is proportional with the factor $e^{-2\pi k}$, which is the same factor found in [6,7], where the problem of pair production in external fields on de Sitter geometry was studied by using a nonperturbative approach [6,7]. We must mention that a nonperturbative calculation of pair production in magnetic field on de Sitter geometry has been done in [9].

4 The total probability

In this section we will compute the total probability of scalar pair production in the field of the magnetic dipole on de Sitter geometry. If one use the nonperturbative approach to the problem of particle production in external field then the number of particles is evaluated by integrating after the final momenta and because of the approximations made for the out modes the momenta dependence in the density number of particles is lost. This has as a consequence the fact that the momenta integrals are solved using a cut-off method. Here we want to present the computation of the final momenta integrals in the case of perturbative approach of the subject, where the momenta dependence in the final amplitude and probability is preserved. The total probability is obtained integrating the probability equation (15) after the final momenta p, p':

$$\mathcal{P}_{tot}=\int\int\mathcal{P}\,d^3p\;d^3p'. \tag{18}$$

Since the graphical results suggest that the probability is significative in strong gravitational field we will rewrite the equation for function $f_k\left(\frac{p'}{p}\right)$ such that the parameter m/ω will be considered in the interval $m/\omega\in(1.51,2)$. We consider for the present calculation the case when $p>p'$ with the observation that the ratio of the momenta will be taken in the interval $p/p'\in(0.5,0.9)$. Then for parameters specified above, the f_k functions that define the probability, can be brought in the following form by using Eqs. (25) and (26) from Appendix:

$$f_\mu\left(\frac{p'}{p}\right)=\frac{2\pi\mu}{\sinh(\pi\mu)}\frac{p}{(p+p')}. \tag{19}$$

Further we present only the case $p>p'$ since for $p'>p$ the computations are similar. The integration after final momenta is taken such that $p\in(0,\infty)$ and $p'\in(0,p'_B)$, where

$\vec{\mathcal{P}}'_B = \vec{p}' + e\vec{A}$, is the particle momentum in the presence of the dipole magnetic field $\vec{B}$. Here the potential vector could be expressed in terms of magnetic dipole moment using Eq. (4), since we take $\vec{\mathcal{M}} = \mathcal{M}\vec{e}_3$, then the components of $\vec{A}$ are: $A_1 = -\frac{\mathcal{M}x_2}{|\vec{x}|^3}$, $A_2 = \frac{\mathcal{M}x_1}{|\vec{x}|^3}$, $A_3 = 0$. This method for cutting off the momentum was also used in [7], and in this way the number of particles was obtained as a finite quantity. An important remark is that one of the momenta integrals can be solved without any approximations and this is a consequence of the fact that in the perturbative calculation the probability is a function which depends on the particle momenta (see Eq. (15)). This also opens the way for using other regularization methods for obtaining the total probability.

As we point out in the previous section the pair of particles will be emitted perpendicular to the direction of dipole magnetic field. Let us consider the case when the momenta of the scalar particles have the same orientation $\beta - \varphi = 0$. By fixing the angle between the momenta vectors we obtain the total probability when the particles are emitted on given directions and it will be sufficiently to solve only the momenta modulus integrals in this case. Using now Eq. (19) and integrating after the final momenta p , p' in the above mentioned limits, we obtain the total probability in the case $\beta - \varphi = 0$ by using Eq. (29) from Appendix:

$$P_{tot} = \frac{e^2\mathcal{M}^2\mu^2}{24\pi^4\sinh^2(\pi\mu)}\,\mathcal{P}'^2_B. \tag{20}$$

The total probability can be computed using the same method for other values of the angle between momenta vectors $\beta - \varphi$ (see Eq. (30) from Apendix) and the results can be found bellow :

$$P_{tot} = \frac{e^2\mathcal{M}^2\mu^2}{24\pi^4\sinh^2(\pi\mu)} \begin{cases} \frac{(2\pi\sqrt{3}-3\sqrt{3}-6)}{\sqrt{3}-2}\,\mathcal{P}'^2_B & \text{for } \beta-\varphi=\pi/6 \\ \frac{(27-4\pi\sqrt{3})}{3}\,\mathcal{P}'^2_B & \text{for } \beta-\varphi=\pi/3 \\ \frac{(9+8\pi\sqrt{3})}{9}\,\mathcal{P}'^2_B & \text{for } \beta-\varphi=2\pi/3 \\ \frac{(10\pi\sqrt{3}-3\sqrt{3}+6)}{\sqrt{3}+2}\,\mathcal{P}'^2_B & \text{for } \beta-\varphi=5\pi/6 \end{cases} \tag{21}$$

A numerical calculation proves that the total probabilities increase when the angle between the momenta vectors $\beta - \varphi$ approaches π, result that confirms our graphical analysis from the previous section. A remarkable property of the total probability is that it becomes negligible as the factor $k = m/\omega$ takes larger values, due to the dependence on the factor $\sinh^{-2}(\pi\mu)$. This result proves that the process of pair production in dipolar magnetic field was important only in early universe when the gravitational fields were very strong. In the Minkowski limit the total probability is vanishing proving that the phenomenon of pair production in dipolar magnetic field is allowed as a perturbative process only in strong gravitational fields.

Another aspect concerning the problem of particle production in de Sitter expanding space-time is related to the spectator gauge fields which appear in magnetogenesis or in anisotropic inflation. These spectator fields [38] are present when one study the phenomenon of pair production in a time dependent electric field in de Sitter spacetime or the Schwinger effect [9]. The result obtained in [38] suggest that there are some effects that were not taken into consideration until now, such as the postulation of a time dependent current which could cause the violation of the weak energy condition and the second law of thermodynamics. This subject deserves further investigations since in the case of quantum field theory on de Sitter space-time, the energy operator is different from that of the flat case both in the theory of free Maxwell field and in the theory of fields interactions, as was shown in [14]. Using the Noether theorem, the energy operator written using the normal ordering, in terms of the electric and magnetic field strengths $\vec{E}$, $\vec{B}$, for the free Maxwell field turn out to be [14]:

$$\mathcal{H} = \int d^3x : \left[(-\omega t_c)\frac{1}{2}(\vec{E}^2 + \vec{B}^2) + \omega\vec{x}\cdot\vec{E}\wedge\vec{B} - \frac{\omega}{2}\partial_{t_c}(\vec{A}^2)\right] : . \tag{22}$$

The above expression is completely different from that of the flat space energy operator, because there appears terms that are specific to the de Sitter geometry. An extended discussion about this subject can be found in [14,33].

In the present paper we restrict to compute the amplitude of transition and probability for the process of pair production in the field of a magnetic dipole, using the QED formalism developed in [14,39]. Our calculations are valid only in early universe when the gravitational fields were strong in comparative with the magnetic fields and present the first derivation of the total probability for the process of pair production in magnetic field using a perturbative method. The result obtained in [9] study the problem of scalar pair production in strong magnetic fields using a nonperturbative method. The interesting idea is now to combine the two methods using the formalism developed in [3] for obtaining the contribution of both cosmological particle production and perturbative particle production. That will require some work to be done to translate the calculations in terms of density number of particles, as was shown in [12].

5 Concluding remarks

We present in this paper the first perturbative approach to the problem of scalar pair production in a magnetic field in a de Sitter geometry. The main result of our paper is related to the fact that the first order transition amplitude and probability are nonvanishing only in strong gravitational fields that corresponds in our case to the early universe conditions. Our study proves that the particles will be most probably emitted perpendicular to the magnetic field direction. This conclusion shows that there are differences comparatively with the problem of pair production in electric field where the particles are emitted parallel with the electric field direction [6,7,15]. In the Minkowski limit the amplitude and probability are vanishing since in Minkowski scalar QED [37] this process is forbidden by the energy–momentum conservation.

The problem of particle production when a gravitational field and a magnetic field are present seems to receive little attention in literature. This study is important since it is well known that black holes and neutron starts have also strong magnetic fields and it is fundamental to understand the problem of fields interactions in these geometries. For further study it will be important to combine the nonperturbative and perturbative results related to the problem of pair production in magnetic field on de Sitter spacetime for obtaining a complete picture of this phenomenon. This can be done by using the formalism proposed in [3], and we hope that in a future study to present these results.

Acknowledgements This work was supported by a grant of the Ministry of National Education and Scientific Research, RDI Programme for Space Technology and Advanced Research - STAR, project number 181/20.07.2017.

Appendix

Here we present the main steps for computing the amplitude of pair production in magnetic field. Using the relation that connects Hankel functions and Bessel K functions [34,35]:

$$H_\nu^{(1,2)}(z) = \mp\left(\frac{2i}{\pi}\right)e^{\mp i\pi\nu/2}K_\nu(\mp iz), \tag{23}$$

we arrive at integrals of the type [35]:

$$\int_0^\infty dz z^{-\lambda}K_\mu(az)K_\nu(bz) = \frac{2^{-2-\lambda}a^{-\nu+\lambda-1}b^{\nu}}{\Gamma(1-\lambda)}\Gamma\left(\frac{1-\lambda+\mu+\nu}{2}\right)\Gamma\left(\frac{1-\lambda-\mu+\nu}{2}\right)\times\Gamma\left(\frac{1-\lambda+\mu-\nu}{2}\right)\Gamma\left(\frac{1-\lambda-\mu-\nu}{2}\right)\,{}_2F_1\left(\frac{1-\lambda+\mu+\nu}{2},\frac{1-\lambda-\mu+\nu}{2};1-\lambda;1-\frac{b^2}{a^2}\right),$$
$$Re(a+b)>0\,,\ Re(\lambda)<1-|Re(\mu)|-|Re(\nu)|. \tag{24}$$

The function $f_k\left(\frac{p'}{p}\right)$ when calculating the total probability is obtained if we use:

$$a^{ix} = e^{ix\ln a},\ \ a\in R \tag{25}$$

and the next property of the natural logarithm:

$$\frac{y-1}{y}\leq \ln y\leq y-1\ \ ,y>0, \tag{26}$$

with the mention that in our case we can use that $\ln y\simeq y-1$.

The limit $k>>1$ in the functions f_k is obtained if we use the following identity between hypergeometric functions [34,35]:

$$ {}_2F_1(a,b;c;z) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\,{}_2F_1(a,b;a+b-c+1;1-z) +(1-z)^{c-a-b}\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}\,{}_2F_1(c-a,c-b;c-a-b+1;1-z), \tag{27}$$

and the well known relations between gamma Euler functions [35],

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)}. \tag{28}$$

The momenta integrals for computing the final formulas for the total probability are:

$$\int_0^\infty dp\,\frac{(p-p')^2}{(p+p')^4} = \frac{1}{3p'}. \tag{29}$$

$$\int_0^\infty dp\,\frac{(p^2+p'^2-\sqrt{3}pp')}{(p+p')^2(p^2+p'^2+\sqrt{3}pp')} = \frac{2\pi\sqrt{3}-3\sqrt{3}-6}{3p'(\sqrt{3}-2)},$$
$$\int_0^\infty dp\,\frac{(p^2+p'^2-pp')}{(p+p')^2(p^2+p'^2+pp')} = \frac{27-4\pi\sqrt{3}}{9p'},$$
$$\int_0^\infty dp\,\frac{(p^2+p'^2+pp')}{(p+p')^2(p^2+p'^2-pp')} = \frac{8\pi\sqrt{3}+9}{27p'},$$
$$\int_0^\infty dp\,\frac{(p^2+p'^2+\sqrt{3}pp')}{(p+p')^2(p^2+p'^2-\sqrt{3}pp')} = \frac{10\pi\sqrt{3}-3\sqrt{3}+6}{3p'(\sqrt{3}+2)}. \tag{30}$$

References

1. W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)
2. S.P. Gavrilov, D.M. Gitman, Phys. Rev. D **87**, 125025 (2013)
3. N.D. Birrel, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982)
4. L. Parker, Phys. Rev. Lett. **21**, 562 (1968)
5. L. Parker, Phys. Rev. **183**, 1057 (1969)
6. V.M. Villalba, Phys. Rev. D **52**, 3742 (1995)
7. J. Garriga, Phys. Rev. D **49**, 6343 (1994)
8. J. Haro, E. Elizalde, J. Phys. A **41**, 372003 (2008)
9. E. Bavarsad, S.P. Kim, C. Stahl, S.S. Xue, Phys. Rev. D **97**, 025017 (2018)
10. K. Sogut, A. Havare, Nucl. Phys. B **901**, 76 (2015)
11. D. Marolf, I.A. Morrison, M. Srednicki, Class. Quantum Gravity **30**, 155023 (2013)
12. N.D. Birrel, P.C.W. Davies, L.H. Ford, J. Phys. A **13**, 961 (1980)
13. C. Crucean, M.A. Băloi, Phys. Rev. D **93**, 044070 (2016)
14. I.I. Cotăescu, C. Crucean, Phys. Rev. D **87**, 044016 (2013)
15. M.A. Băloi, Mod. Phys. Lett. A **29**, 1450138 (2014)
16. K.H. Lotze, Nucl. Phys. B **312**, 673 (1989)
17. I.L. Buchbinder, E.S. Fradkin, D.M. Gitman, Fortschr. Phys. **29**, 187 (1981)
18. I.L. Buchbinder, L.I. Tsaregorodtsev, Int. J. Mod. Phys. A **7**, 2055 (1992)
19. C. Crucean, R. Racoceanu, A. Pop, Phys. Lett. B **665**, 409 (2008)
20. C. Crucean, Phys. Rev. D **85**, 084036 (2012)
21. K.H. Lotze, Class. Quantum Gravity **5**, 595 (1985)
22. E. Schrödinger, Physica **6**, 899 (1939)
23. A.H. Guth, Phys. Rev. D **23**, 347 (1981)
24. L.F. Abbott, S.Y. Pi, *Inflationary Cosmology* (World Scientific, Singapore, 1986)
25. M.S. Turner, L.M. Widrow, Phys. Rev. D **37**, 2743 (1988)
26. G.B. Field, S.M. Carroll, Phys. Rev. D **62**, 103008 (2000)
27. S. Kawati, A. Kokado, Phys. Rev. D **39**, 2959 (1989)
28. S. Kawati, A. Kokado, Phys. Rev. D **39**, 3612 (1989)
29. C.W. Misner, K.S. Thorne, J.A. Wheleer, *Gravitation* (W. H. Freeman and Company, New York, 1973)
30. I.I. Cotăescu, C. Crucean, A. Pop, Int. J. Mod. Phys. A **23**, 2563 (2008)
31. J.D. Jackson, *Classical Electrodynamics* (Wiley, Hoboken, 1962)
32. W. Greiner, *Classical Electrodynamics* (Springer, Berlin, 1998)
33. I.I. Cotăescu, C. Crucean, Prog. Theor. Phys. **124**, 1051 (2010)
34. G.N. Watson, *Theory of Bessel Functions* (Cambridge University Press, Cambridge, 1922)
35. I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, Cambridge, 2007)
36. C. Crucean, M.A. Băloi, Phys. Rev. D **95**, 048502 (2017)
37. S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995)
38. M. Giovannini, Phys. Rev. D **97**, 061301 (2018)
39. C. Crucean, M.A. Băloi, Int. J. Mod. Phys. A **30**, 1550088 (2015)