

Universal dual amplitudes and asymptotic expansions for $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$ in four dimensions

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Abstract Though the one-loop amplitudes of the Higgs boson to massless gauge bosons are finite because there is no direct interaction at tree level in the Standard Model, a well-defined regularization scheme is still required for their correct evaluation. We reanalyze these amplitudes in the framework of the four-dimensional unsubtraction and the loop-tree duality (FDU/LTD), and show how a local renormalization solves potential regularization ambiguities. The Higgs boson interactions are also used to illustrate new additional advantages of this formalism. We show that LTD naturally leads to very compact integrand expressions in four space-time dimensions of the one-loop amplitude with virtual electroweak gauge bosons. They exhibit the same functional form as the amplitudes with top quarks and charged scalars, thus opening further possibilities for simplifications in higher-order computations. Another outstanding application is the straightforward implementation of asymptotic expansions by using dual amplitudes. One of the main benefits of the LTD representation is that it is supported in a Euclidean space. This characteristic feature naturally leads to simpler asymptotic expansions.

1 Introduction

The $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$ are the golden channels for production and decay of the Higgs boson at the CERN's Large Hadron Collider (LHC). The one-loop contributions to the Hgg vertex are known since a long time ago [1–3], as well as the Higgs decay into a photon pair [4–8]. It is well known that these amplitudes are finite due to the absence of a direct interaction at tree level in the Standard Model. However,

contrary to what it is naively expected, dimensional regularization (DREG) – or another regularization technique [9] – and a well-defined renormalization scheme are still required for their correct evaluation. Indeed, the naive calculation in four space-time dimensions not only leads to incorrect results [10], but it also spoils gauge invariance and produces inconsistent physical effects such as the absence of decoupling in the limit $M_f^2 \gg M_H^2$, with M_f the mass of the virtual particle running in the loop.

We have recently proposed a new approach to dealing with perturbative computations avoiding DREG. The four-dimensional unsubtraction method (FDU) [11–13] is based on an integrand-level transformation that achieves a fully local cancellation of singularities. The key component of this approach is the loop-tree duality theorem (LTD) [14–18], which separates the loop contribution into sums of dual integrands obtained by applying a number of cuts equal to the number of loops.

A generic one-loop amplitude with N -internal propagators has the form

$$\mathcal{A}^{(1)} = \int_{\ell} \left(\prod_{i=1}^N G_F(q_i) \right) \mathcal{N}(\ell, \{p_k\}), \quad (1)$$

where

$$\int_{\ell} = -i \int \frac{d^d \ell}{(2\pi)^d}, \quad (2)$$

is the standard one-loop integration measure, $G_F(q_i) = (q_i^2 - m_i^2 + i0)^{-1}$ are Feynman propagators, and $\mathcal{N}(\ell, \{p_k\})$ is the numerator that has a polynomial dependence in the loop and external momenta. The corresponding LTD representation is obtained by setting sequentially the internal propagators on-shell,

$$\mathcal{A}^{(1)} = - \int_{\ell} \sum_{i=1}^N \tilde{\delta}(q_i) \left(\prod_{j \neq i} G_D(q_i; q_j) \right) \mathcal{N}(\ell, \{p_k\}), \quad (3)$$

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with $\tilde{\delta}(q_i) = i 2\pi \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$, and promoting the remaining propagators to dual propagators with a modified $i0$ -prescription: $G_D(q_i; q_j) = (q_j^2 - m_j^2 - i0 \eta \cdot (q_j - q_i))^{-1}$, where η is a future-like vector, i.e. $\eta^2 \geq 0$ and $\eta_0 > 0$. From now on, we set $\eta_\mu = (1, \mathbf{0})$.

In this paper, we reanalyze the $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$ scattering amplitudes at one-loop and their asymptotic expansion in the FDU/LTD formalism. The four-dimensional nature of the FDU/LTD approach allows one to get an alternative insight into the structure of these scattering amplitudes, unveiling the origin of local UV singularities that vanish in the integrated amplitude but lead to finite contributions. In the first place, we show how to apply the LTD theorem to obtain compact expressions for the amplitude integrand that exhibit the same functional form for virtual charged scalars, fermions (top quarks) or W gauge bosons. This is a highly non-trivial result as intermediate expressions with gauge bosons diverge faster in the UV than those with scalars and fermions. After that, we discuss the local renormalization of the one-loop amplitude by introducing a suitable counterterm that locally cancels the UV behavior of the one-loop integrand and allows a direct integration of the amplitude in $d = 4$ space-time dimensions. The second relevant result presented in this paper is related with asymptotic expansions. The simplicity and well-behaved convergence of the large-mass and small-mass asymptotic expansions of the Higgs boson amplitudes in the LTD formalism avoids considering complementary expansions in different regions of the loop momentum [19, 20].

2 Dual amplitudes for $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$

The one-loop scattering amplitudes of the Higgs boson to two massless gauge bosons have the form

$$\begin{aligned} |\mathcal{M}_{gg \rightarrow H}^{(1)}\rangle &= i g_S^2 \text{Tr}(\mathbf{T}^a \mathbf{T}^b) \varepsilon_a^\mu(p_1) \varepsilon_b^\nu(p_2) \mathcal{A}_{\mu\nu}^{(1,t)}, \\ |\mathcal{M}_{H \rightarrow \gamma\gamma}^{(1)}\rangle &= i e^2 \left(\sum_{f=\phi, t, W} e_f^2 N_C^f \mathcal{A}_{\mu\nu}^{(1,f)} \right) \\ &\quad \times (\varepsilon^\mu(p_1))^* (\varepsilon^\nu(p_2))^*, \end{aligned} \quad (4)$$

with ε the polarization vectors of the external gluons and photons, $\text{Tr}(\mathbf{T}^a \mathbf{T}^b) = T_R \delta_{ab}$ the color factor, N_C^f the number of colors, and e_f the electric charge. Eventually, the sum in Eq. (4) might include the other quarks, and the leptons with $N_C^l = 1$. By Lorentz invariance, the color and electric charge stripped tensor amplitude is given by

$$\mathcal{A}_{\mu\nu}^{(1,f)} = \sum_{i=1}^5 \mathcal{A}_i^{(1,f)} T_{\mu\nu}^i, \quad (5)$$

as a function of the tensor basis

$$T_i^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2 p_1^\nu p_2^\mu}{s_{12}}, g^{\mu\nu}, \frac{2 p_1^\mu p_2^\nu}{s_{12}}, \frac{2 p_1^\mu p_1^\nu}{s_{12}}, \frac{2 p_2^\mu p_2^\nu}{s_{12}} \right\}, \quad (6)$$

where $s_{12} = (p_1 + p_2)^2$, with $s_{12} = M_H^2$ if the Higgs boson is on-shell. We can extract the scalar coefficients $\mathcal{A}_i^{(1,f)}$ by using the projectors

$$P_1^{\mu\nu} = \frac{1}{d-2} \left(g^{\mu\nu} - \frac{2 p_1^\nu p_2^\mu}{s_{12}} - (d-1) \frac{2 p_1^\mu p_2^\nu}{s_{12}} \right), \quad (7)$$

$$P_2^{\mu\nu} = \frac{2 p_1^\mu p_2^\nu}{s_{12}}, \quad (8)$$

with $P_i^{\mu\nu} \mathcal{A}_{\mu\nu}^{(1,f)} = \mathcal{A}_i^{(1,f)}$. Because of gauge invariance, only the first coefficient $\mathcal{A}_1^{(1,f)}$ is relevant, while $\mathcal{A}_2^{(1,f)}$ should vanish upon integration. The other three coefficients do not contribute to the scattering amplitude after contracting with the polarization vectors.

The one-loop amplitude can be expressed in terms of the internal momenta $q_1 = \ell + p_1$, $q_2 = \ell + p_{12}$ with $p_{12} = p_1 + p_2$, $q_3 = \ell$; and $q_4 = \ell + p_2$ to account for the diagrams with the two photons/gluons exchanged. Explicitly, the one-loop amplitude with virtual top quarks is

$$\begin{aligned} \mathcal{A}_{\mu\nu}^{(1,t)} &= \frac{M_t}{\langle v \rangle} \int_\ell \left(\prod_{i=1}^3 G_F(q_i) \right) \\ &\quad \times \text{Tr} [\gamma_\nu (q_1 + M_t) \gamma_\mu (q_3 + M_t) (q_2 + M_t)] \\ &\quad + (p_1 \leftrightarrow p_2), \end{aligned} \quad (9)$$

with $\langle v \rangle$ the vacuum expectation value of the Higgs boson, and $G_F(q_i) = (q_i^2 - M_t^2 + i0)^{-1}$ the Feynman propagators.

For the W boson amplitude, we work in the unitary gauge because all of the propagating degrees of freedom are physical and the internal propagators

$$-i \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \frac{1}{q_i^2 - M_W^2 + i0}, \quad (10)$$

do not introduce additional poles in the loop momentum space, allowing a straightforward application of the LTD theorem [14]. We do not provide here the explicit expressions equivalent to Eq. (9) for the charged scalar and W boson loop amplitudes; they can be obtained straightforwardly from the standard Feynman rules.

Partial results for the W boson loop amplitude are more singular in the UV than the corresponding expressions for the charged scalar and top quark loops due to the presence of higher powers of the loop momentum. These additional powers are introduced through the $WW\gamma$ vertex, which is linear in the loop momentum, and the W propagator, as shown in Eq. (10). Also, the W loop amplitude receives contributions from bubble diagrams with $WW\gamma\gamma$ interaction vertices

that do not exist for the top quark loop. Those contributions are necessary to preserve gauge invariance. It is a remarkable feature of LTD that by setting the internal propagators on-shell the rank of the numerators of intermediate expressions is reduced automatically. Moreover, the gauge invariance cross-cancellations between the bubble and triangle diagrams explicitly arise in LTD without extra manipulations.

As a consequence, we obtain the following expressions for the scalar coefficients in Eq. (5) that exhibit the same functional form for charged scalars, fermions (top quarks) and W bosons:

$$\begin{aligned} \mathcal{A}_1^{(1,f)} = & g_f \int_{\ell} \tilde{\delta}(\ell) \\ & \times \left[\left(\frac{\ell_0^{(+)} }{q_{1,0}^{(+)} } + \frac{\ell_0^{(+)} }{q_{4,0}^{(+)} } + \frac{2(\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \right. \\ & \times \left(\frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} + c_2^{(f)} \right) \\ & \left. + \frac{2s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_3^{(f)} \right], \end{aligned} \quad (11)$$

$$\mathcal{A}_2^{(1,f)} = g_f \frac{c_3^{(f)}}{2} \int_{\ell} \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)} }{q_{1,0}^{(+)} } + \frac{\ell_0^{(+)} }{q_{4,0}^{(+)} } - 2 \right), \quad (12)$$

with $f = \phi, t, W$. The on-shell loop energies are given by

$$\begin{aligned} q_{1,0}^{(+)} &= \sqrt{(\ell + \mathbf{p}_1)^2 + M_f^2}, \quad q_{4,0}^{(+)} = \sqrt{(\ell + \mathbf{p}_2)^2 + M_f^2}, \\ \ell_0^{(+)} &= q_{2,0}^{(+)} = q_{3,0}^{(+)} = \sqrt{\ell^2 + M_f^2}. \end{aligned} \quad (13)$$

It is worth mentioning that Eqs. (11) and (12) were derived from the application of the LTD theorem to the projected amplitudes in Eq. (9) and the proper unification of the dual coordinate system, as carefully explained in Ref. [11, 12]. The coefficients $c_i^{(f)}$ have indeed the form $c_i^{(f)} = c_{i,0}^{(f)} + r_f c_{i,1}^{(f)}$ with $r_f = s_{12}/M_f^2$. For the three different flavors that we consider, $f = \phi, t, W$, these coefficients are given by

$$\begin{aligned} g_f &= \frac{2M_f^2}{\langle v \rangle s_{12}}, \quad c_{1,0}^{(f)} = \left(\frac{4}{d-2}, -\frac{8}{d-2}, \frac{4(d-1)}{d-2} \right), \\ c_{1,1}^{(f)} &= \left(0, 1, \frac{2(5-2d)}{d-2} \right), \\ c_{3,0}^{(f)} &= (d-2) c_{1,0}^{(f)}, \quad c_{3,1}^{(f)} = 0, \quad c_{23,0}^{(f)} = (d-4) \frac{c_{1,0}^{(f)}}{2}, \\ c_{23,1}^{(f)} &= \left(0, 0, \frac{d-4}{d-2} \right), \end{aligned} \quad (14)$$

with $c_2^{(f)} = c_{23}^{(f)} - c_3^{(f)}$. This result indicates that the calculation of the amplitude for other virtual states could be reduced to the determination of the scalar coefficients $c_i^{(f)}$. The universality of the expressions in Eqs. (11) and (12) could be supported by supersymmetric Ward identities at tree level similar to those relating amplitudes with heavy quarks and heavy

scalars [21, 22], because the dual representation is indeed a tree-level like object. It is also interesting to notice that the two-loop amplitudes for scalar and pseudoscalar Higgs bosons to two photons have been calculated in Ref. [23] based on the assumption that if two physical processes correspond to a similar set of Feynman diagrams, then their cross sections should be described by a common set of analytical functions. Their calculation is thus reduced to determine the coefficients of a linear combination of those functions by solving a large set of linear equations arising from comparing the asymptotic expansions of a given ansatz and a one-dimensional integral representation of the amplitude. Such motivation could also be argued in this case, since similar physical processes should be described by similar integrand representations although with different coefficients. The LTD approach appears to be suitable for this purpose. We leave that discussion and the possible extension to two loops as an open question for a future publication.

Although the coefficient $\mathcal{A}_2^{(1,f)}$ vanishes upon integration in d -dimensions, notice that the naive calculation with $d = 4$ leads to a finite contribution that violates gauge invariance. So, we can exploit that information to simplify the integrand-level expression for $\mathcal{A}_1^{(1,f)}$ by introducing non-trivial integrals which vanish in d -dimensions. Then we can rewrite $\mathcal{A}_1^{(1,f)}$ in the most compact form:

$$\begin{aligned} \mathcal{A}_1^{(1,f)} = & g_f s_{12} \int_{\ell} \tilde{\delta}(\ell) \\ & \times \left[\left(\frac{\ell_0^{(+)} }{q_{1,0}^{(+)} } + \frac{\ell_0^{(+)} }{q_{4,0}^{(+)} } + \frac{2(\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \right. \\ & \times \frac{M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} \\ & \left. + \frac{2s_{12}}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_{23}^{(f)} \right], \end{aligned} \quad (15)$$

which depends only on two independent coefficients. The integral proportional to the coefficient $c_1^{(f)}$ is indeed finite in the UV; thus, it will lead to the same result, up to $\mathcal{O}(\epsilon)$, if evaluated in four or d -dimensions. On the contrary, the remaining contribution must necessarily be calculated in d -dimensions because $c_{23}^{(f)} \propto d-4$ and the accompanying integral is logarithmically divergent. Since the coefficients $c_i^{(f)}$ depend on the nature of the particle circulating the loop through the associated Feynman rules, a consistent treatment of the dimensional extension of Dirac and Lorentz algebras is required to avoid potential mismatches in finite pieces. Explicitly, if we use the four-dimensional Dirac algebra by setting $d = 4$ in the very first steps of the calculation, we get $c_{23}^{(f)} = 0$ and the second line in Eq. (15) is absent. However, a fully d -dimensional calculation shows that this terms leads to a finite non-vanishing contribution, arising from the UV pole

of the bubble integral accompanying $c_{23}^{(f)}$. From the mathematical point of view, this behavior is due to the absence of a local regularization, i.e. the integrand-level functions appearing in Eq. (15) are not *integrable*. In the next section, we will discuss how to implement a completely local renormalization to achieve integrability in four dimensions.

3 Local renormalization and four-dimensional dual representation

Since there is no direct interaction of the Higgs boson to massless gauge bosons, the one-loop amplitude is UV finite and does not need to be renormalized. However, the integrand of the one-loop amplitude is locally singular in the UV. This explains the requirement of introducing a well-defined regularization scheme to treat it properly. Therefore, we define in this section a UV counter-term that exactly cancels locally the UV behavior of the one-loop amplitude, but integrates to zero and does not lead to any effective renormalization.

The approach that we follow in this paper differs slightly from the previously used in Refs. [11–13]. Instead of expanding around the UV propagator $G_F(q_{UV}) = (q_{UV}^2 - \mu_{UV}^2 + i0)^{-1}$ at Feynman integral level, we first switch to the LTD representation and then expand. The UV expansion of Eq. (15) is particularly simple because only the contribution proportional to $c_{23}^{(f)}$ presents a singular behavior. The UV counter-term is defined as

$$\mathcal{A}_{1,UV}^{(1,f)} = -g_f s_{12} \int_{\ell} \frac{1}{4(q_{UV,0}^{(+)})^3} \left(1 + \frac{1}{(q_{UV,0}^{(+)})^2} \frac{3\mu_{UV}^2}{d-4} \right) c_{23}^{(f)}, \quad (16)$$

with $q_{UV,0}^{(+)} = \sqrt{\ell^2 + \mu_{UV}^2}$. The measure of the integral in Eq. (16) is defined in the spatial components of the loop momentum, i.e. $\int_{\ell} = \int d^{d-1}\ell / (2\pi)^{d-1}$. The term proportional to μ_{UV}^2 is subleading in the UV limit and is used to fix the renormalization scheme. The factor $3/(d-4)$ has been adjusted to impose $\mathcal{A}_{1,UV}^{(1)} = 0$ in d -dimensions. This is the only place of the calculation where DREG is still necessary. Though, once the unintegrated UV counter-term has been computed, its four-dimensional limit can be used to regularize any other similar process. Notice that the renormalization scale μ_{UV} is arbitrary because the one-loop amplitude is indeed not renormalized. For the scalar and the top quark amplitudes, the Dyson prescription [24], which consists in subtracting the amplitude evaluated with vanishing external photon (or gluon) momenta, has a similar effect. It fails, however, for the W boson loop; although it correctly subtracts the leading non-decoupling term in the limit $M_W^2/s_{12} \rightarrow \infty$, it does not account properly for the relevant subleading contributions.

The difference of Eqs. (15) and (16) defines the locally renormalized amplitude $\mathcal{A}_{1,R}^{(1,f)}$. Remarkably, it has a smooth four-dimensional limit and can directly be calculated with $d = 4$,

$$\mathcal{A}_{1,R}^{(1,f)} \Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,UV}^{(1,f)} \right)_{d=4}. \quad (17)$$

Notice that $c_{23}^{(f)}$ vanishes in four dimensions and therefore the first term of the integrand in Eq. (16) vanishes, but $\hat{c}_{23}^{(f)} = c_{23}^{(f)}/(d-4)$ leads to a finite contribution because $c_{23}^{(f)} \propto d-4$ but it is multiplied by an integral whose leading UV divergence behaves as $1/(d-4)$. Explicitly, the final and most compact expression for the unintegrated loop amplitude of this paper is

$$\begin{aligned} \mathcal{A}_{1,R}^{(1,f)} \Big|_{d=4} = & g_f s_{12} \int_{\ell} \left[\frac{1}{2\ell_0^{(+)}} \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} \right) \right. \\ & \left. + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right] \\ & \times \frac{M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} + \frac{3\mu_{UV}^2}{4(q_{UV,0}^{(+)})^5} \hat{c}_{23}^{(f)}, \end{aligned} \quad (18)$$

with the coefficients $c_1^{(f)}$ and $\hat{c}_{23}^{(f)}$ evaluated at $d = 4$, i.e.

$$\begin{aligned} c_1^{(f)} &= (2, -4 + r_t, 6 - 3r_W), \\ \hat{c}_{23}^{(f)} &= (1, -2, 3 + r_W/2). \end{aligned} \quad (19)$$

The integrated amplitude reads

$$\mathcal{A}_{1,R}^{(1,f)} \Big|_{d=4} = \frac{g_f s_{12}}{16\pi^2} \left(\frac{M_f^2}{s_{12}} \log^2 \left(\frac{\beta_f - 1}{\beta_f + 1} \right) c_1^{(f)} + 2\hat{c}_{23}^{(f)} \right), \quad (20)$$

with $\beta_f = \sqrt{1 - 4M_f^2/(s_{12} + i0)}$, and it agrees with the expected well-known result [1–8]. For the explicit integration, we have used the following parametrization of the loop three-momentum:

$$\ell = M_f \xi (2\sqrt{v(1-v)} \mathbf{e}_{\perp}, 1 - 2v), \quad (21)$$

with ξ its modulus normalized to the internal mass M_f , and \mathbf{e}_{\perp} the unit vector in the transverse plane. The dual integration measure is

$$\int_{\ell} \tilde{\delta}(\ell) = \int_{\ell} \frac{1}{2\ell_0^{(+)}} = \frac{M_f^2}{4\pi^2} \int_0^{\infty} \xi_0^{-1} \xi^2 d\xi \int_0^1 dv, \quad (22)$$

with $\xi_0 = \sqrt{\xi^2 + 1}$. The square roots present in Eq. (18) can be transformed into rational functions of x by implementing the change of variables

$$\xi = \frac{1}{2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right), \quad (23)$$

with $x \in [1, \infty)$.

4 Asymptotic expansions in the Euclidean space of the loop three-momentum

LTD reduces the original d -dimensional integration domain with Minkowski metric, to a $(d-1)$ -dimensional space with Euclidean metric: the loop momentum spatial components. In the particular case $d=4$, this Euclidean space corresponds to the domain of the loop three-momentum. This is an interesting feature that allows one to circumvent potential difficulties that arise when performing asymptotic expansions of the integrand in a Minkowski space [19, 20]. We can use the production and decay of the Higgs boson to massless gauge bosons as benchmark example to illustrate the ease of performing asymptotic expansions in the LTD formalism. The method is also applicable to other more complex processes.

As starting example, we consider the large-mass limit $M_f^2 \gg s_{12}$ of the dual contribution with q_3 on-shell, i.e.

$$\tilde{\delta}(q_3) G_D(q_3; q_2) = \frac{\tilde{\delta}(q_3)}{s_{12} + 2q_3 \cdot p_{12} - i0}, \quad (24)$$

where $q_3 \cdot p_{12} = q_{3,0}^{(+)} \sqrt{s_{12}}$ in the center-of-mass frame. The on-shell energy $q_{3,0}^{(+)}$ is defined in Eq. (13). Since $q_{3,0}^{(+)} \geq M_f$, the asymptotic expansion for $M_f^2 \gg s_{12}$ is straightforwardly written as

$$\tilde{\delta}(q_3) G_D(q_3; q_2) = \frac{\tilde{\delta}(q_3)}{2q_3 \cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3 \cdot p_{12}} \right)^n. \quad (25)$$

Likewise, we shall expand the terms

$$\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} = \sum_{n=0}^{\infty} \frac{\Gamma(2n+1)}{\Gamma^2(n+1)} \left(-\frac{2\ell \cdot \mathbf{p}_1 + \mathbf{p}_1^2}{(2\ell_0^{(+)})^2} \right)^n, \quad (26)$$

which is a valid expansion because $\ell_0^{(+)} > M_f$. Notice that each term of the expansions in Eqs. (25) and (26) is less singular in the UV than the previous one, and is well behaved in the IR. There is no need to consider additional loop momentum regions to obtain the correct asymptotic expansion.

Taking into account the previous considerations, we obtain the following expansion in the center-of-mass frame, again with $d=4$:

$$\begin{aligned} \mathcal{A}_{1,R}^{(1,f)}(s_{12} < 4M_f^2) \Big|_{d=4} &= \frac{M_f^2}{2\langle v \rangle} \int_{\ell} \left[\frac{M_f^2}{(\ell_0^{(+)})^5} \left(\sum_{n=1}^{\infty} Q_n(z) \left(\frac{s_{12}}{(2\ell_0^{(+)})^2} \right)^{n-1} \right) c_1^{(f)} \right. \\ &\quad \left. + \frac{3\mu_{UV}^2}{(q_{UV,0}^{(+)})^5} \hat{c}_{23}^{(f)} \right], \end{aligned} \quad (27)$$

with $z = (2\ell \cdot \mathbf{p}_1)/(\ell_0^{(+)} \sqrt{s_{12}}) = \xi(1-2v)/\xi_0$, and

$$Q_n(z) = \frac{1}{1-z^2} (P_{2n}(z) - 1), \quad (28)$$

where $P_{2n}(z)$ is the Legendre polynomial. The asymptotic expansion of the amplitude in Eq. (27) can easily be integrated without using DREG. At the lowest orders, we find

$$\begin{aligned} \mathcal{A}_{1,R}^{(1,f)} \Big|_{d=4} &= \frac{s_{12}}{8\pi^2 \langle v \rangle} \left(\frac{2\hat{c}_{23,0}^{(f)} - c_{1,0}^{(f)}}{r_f} + 2\hat{c}_{23,1}^{(f)} - \frac{c_{1,0}^{(f)}}{12} - c_{1,1}^{(f)} \right. \\ &\quad - \left(\frac{c_{1,0}^{(f)}}{90} + \frac{c_{1,1}^{(f)}}{12} \right) r_f - \left(\frac{c_{1,0}^{(f)}}{560} + \frac{c_{1,1}^{(f)}}{90} \right) r_f^2 \\ &\quad - \left(\frac{c_{1,0}^{(f)}}{3150} + \frac{c_{1,1}^{(f)}}{560} \right) r_f^3 - \left(\frac{c_{1,0}^{(f)}}{16632} + \frac{c_{1,1}^{(f)}}{3150} \right) r_f^4 \\ &\quad \left. + \mathcal{O}(r_f^5) \right). \end{aligned} \quad (29)$$

The $1/r_f$ non-decoupling term vanishes because $\hat{c}_{23,0}^{(f)} = c_{1,0}^{(f)}/2$, leading to the following explicit results for the different internal particles:

$$\begin{aligned} \mathcal{A}_{1,R}^{(1,\phi)} \Big|_{d=4} &= \frac{s_{12}}{16\pi^2 \langle v \rangle} \left(-\frac{1}{3} - \frac{2}{45} r_\phi - \frac{1}{140} r_\phi^2 \right. \\ &\quad \left. - \frac{2}{1575} r_\phi^3 - \frac{1}{4158} r_\phi^4 + \mathcal{O}(r_\phi^5) \right), \\ \mathcal{A}_{1,R}^{(1,t)} \Big|_{d=4} &= \frac{s_{12}}{16\pi^2 \langle v \rangle} \left(-\frac{4}{3} - \frac{7}{90} r_t - \frac{1}{126} r_t^2 \right. \\ &\quad \left. - \frac{13}{12600} r_t^3 - \frac{8}{51975} r_t^4 + \mathcal{O}(r_t^5) \right), \\ \mathcal{A}_{1,R}^{(1,W)} \Big|_{d=4} &= \frac{s_{12}}{16\pi^2 \langle v \rangle} \left(7 + \frac{11}{30} r_W + \frac{19}{420} r_W^2 \right. \\ &\quad \left. + \frac{29}{4200} r_W^3 + \frac{41}{34650} r_W^4 + \mathcal{O}(r_W^5) \right). \end{aligned} \quad (30)$$

The asymptotic expansion for small M_f can also be obtained from Eq. (18)

$$\begin{aligned} \mathcal{A}_{1,R}^{(1,f)}(M_f^2 \ll s_{12}) \Big|_{d=4} &= \frac{M_f^2}{2\langle v \rangle} \int_{\ell} \left[\frac{-4m_f^2}{(\ell_0^{(+)})^3 (1-z^2)} \right. \\ &\quad \times \left(1 - \sum_{n=1}^{\infty} \frac{4(\ell_0^{(+)})^2 (s_{12} m_f^2 (2-m_f^2))^{n-1}}{(4\ell^2 - s_{12}(1+m_f^2))^n} \right) c_1^{(f)} \\ &\quad \left. + \frac{3\mu_{UV}^2}{(q_{UV,0}^{(+)})^5} \hat{c}_{23}^{(f)} \right], \end{aligned} \quad (31)$$

with $m_f^2 = -M_f^2/(s_{12} + i0)$. Once again, the terms of the expansion in Eq. (31) are less and less singular in the UV at higher orders, allowing a full calculation with $d=4$. Integration of Eq. (31) leads to the awaited logarithmic contributions

$$\begin{aligned}
& \mathcal{A}_{1,R}^{(1,f)}(M_f^2 \ll s_{12}) \Big|_{d=4} \\
&= \frac{s_{12}}{8\pi^2 \langle v \rangle} \left(2 \hat{c}_{23,1}^{(f)} - \left(2 \hat{c}_{23,0}^{(f)} + c_{1,1}^{(f)} L_f^2 \right) m_f^2 \right. \\
&\quad + \left(c_{1,0}^{(f)} L_f^2 + 4 c_{1,1}^{(f)} L_f \right) m_f^4 \\
&\quad - \left(4 c_{1,0}^{(f)} L_f + 2 c_{1,1}^{(f)} (2 + 3 L_f) \right) m_f^6 \\
&\quad + \left(2 c_{1,0}^{(f)} (2 + 3 L_f) + 4 c_{1,1}^{(f)} \left(3 + \frac{10}{3} L_f \right) \right) m_f^8 \\
&\quad \left. + \mathcal{O}(m_f^{10}) \right), \quad (32)
\end{aligned}$$

with $L_f = \log(m_f^2)$. As expected, the leading term in Eq. (32) vanishes for charged scalars and top quarks since $\hat{c}_{23,1}^{(f)} = 0$ for these particles, but it leads to a constant for the W boson loop with $\hat{c}_{23,1}^{(W)} = 1/2$. Explicitly,

$$\begin{aligned}
& \mathcal{A}_{1,R}^{(1,\phi)} \Big|_{d=4} = \frac{M_\phi^2}{8\pi^2 \langle v \rangle} \left(2 - 2 L_\phi^2 m_\phi^2 + 8 L_\phi m_\phi^4 \right. \\
&\quad \left. - 4 (2 + 3 L_\phi) m_\phi^6 \right) + \mathcal{O}(m_\phi^{10}), \\
& \mathcal{A}_{1,R}^{(1,t)} \Big|_{d=4} = \frac{M_t^2}{8\pi^2 \langle v \rangle} \left(-4 + L_t^2 - 4 (1 - L_t) L_t m_t^2 \right. \\
&\quad + 2 (2 - 5 L_t) m_t^4 \\
&\quad \left. + 4 \left(1 + \frac{8}{3} L_t \right) m_t^6 \right) + \mathcal{O}(m_t^{10}), \\
& \mathcal{A}_{1,R}^{(1,W)} \Big|_{d=4} = \frac{s_{12}}{8\pi^2 \langle v \rangle} (1 - 3 (2 - L_W^2) m_W^2 \\
&\quad - 6 (2 - L_W) L_W m_W^4 + 6 (2 - L_W) m_W^6 \\
&\quad - 4 (3 + L_W) m_W^8 + \mathcal{O}(m_W^{10})), \quad (33)
\end{aligned}$$

and these expressions are in agreement with the expansions shown in Ref. [25]. In both cases – the small- and the large-mass limits – all the asymptotic expansions have been calculated directly in four space-time dimensions. This is achievable thanks to the fact that in the Euclidean space of the loop three-momentum it was necessary to consider a single kinematical region to achieve the correct asymptotic expansion in either of the two limits. In fact, this is a direct consequence of dealing with integrable and locally regularized representations of the scattering amplitudes: there is a strict commutativity between integrals and parametric expansions at integrand level.

5 Conclusions

We have presented a very compact and universal integrand-level representation of the one-loop amplitude for the Higgs boson to two massless gauge bosons. The functional form of the amplitude is the same for internal scalars, fermions and

vector bosons, and could be supported by tree-level supersymmetric Ward identities or be motivated by the fact that similar physical processes should be described by similar integrand representations with different coefficients. Presumably, this universality could be exploited further at higher orders. The amplitude has been locally renormalized such that a pure four-dimensional expression free from potential scheme subtleties is obtained. All the previously known results were recovered within a pure four-dimensional representation of the loop amplitude.

Since the integration of the FDU/LTD amplitude effectively occurs in an Euclidean space, namely the loop three-momentum space, asymptotic expansions are easily implemented. In fact, the local regularization in an Euclidean space implies that the series expansion of the integrand commutes with the integral symbol. Thus, expanding the integrand in any parameter (for instance, the mass of the particle circulating the loop) an integrating order-by-order, will lead to the right result. The asymptotic expansion of the Higgs boson amplitudes leads to very simple expressions that can easily be integrated. The results obtained, although focused on the Higgs boson interactions, can be generalized to other processes. In particular, the methods presented in this article open new possibilities for more efficient implementations and further simplifications of higher-order computations and asymptotic expansions.

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