Eur. Phys. J. C (2017) 77:847
https://doi.org/10.1140/epjc/s10052-017-5408-2

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

The anomalous $U(1)_{\mathrm{anom}}$ symmetry and flavors from an SU(5) × SU(5)′ GUT in $\mathbf{Z}_{12-I}$ orbifold compactification

Jihn E. Kim[1,2], **Bumseok Kyae**[3], **Soonkeon Nam**[1]

[1] Department of Physics, Kyung Hee University, 26 Gyungheedaero, Dongdaemun–Gu, Seoul 02447, Republic of Korea
[2] Center for Axion and Precision Physics Research (IBS), 291 Daehakro, Yuseong-Gu, Daejeon 34141, Republic of Korea
[3] Department of Physics, Pusan National University, 2 Busandaehakro-63-Gil, Geumjeong–Gu, Busan 46241, Republic of Korea

Received: 10 August 2017 / Accepted: 22 November 2017 / Published online: 8 December 2017

Abstract In string compactifications, frequently the anomalous $U(1)$ gauge symmetry appears which belongs to $E_8 \times E_8'$ of the heterotic string. This anomalous $U(1)$ gauge boson obtains mass at the compactification scale ($\approx 10^{18}$ GeV) by absorbing one pseudoscalar (corresponding to the model-independent axion) from the second rank antisymmetric tensor field B_{MN}. Below the compactification scale a global symmetry $U(1)_{\mathrm{anom}}$ results whose charge Q_{anom} is the original gauge $U(1)$ charge. This is the most natural global symmetry, realizing the "invisible" axion. This global symmetry $U(1)_{\mathrm{anom}}$ is suitable for a flavor symmetry. In the simplest compactification model with the flipped SU(5) grand unification, all the low energy parameters are calculated in terms of the vacuum expectation values of the standard model singlets.

1 Introduction

In the effective theory low energy global symmetries are of fundamental importance for the strong CP solutions [1] and cosmology [2–4]. Using the bottom-up approach, the Kim–Shifman–Vainstein–Zhakarov (KSVZ) axion model [5,6] and the Dine–Fischer–Srednicki–Zhitnitsky (DFSZ) axion model [7,8] are of practical interest.[1] However, these global symmetries might be badly broken by gravitational effects [11–13].

On the other hand, a consistent top-down approach, a so-called string model, does not allow any global symmetry. In the compactifications of the heterotic string [14] there always exists the pseudoscalar from the second rank antisymmetric tensor field $B_{\mu\nu}$ $(\mu, \nu = 1, 2, 3, 4)$ [15], which is the so-called "model-independent axion (MI-axion)" [16]. If the MI axion is physical, its decay constant is of order 10^{15} GeV [17], which is shown in Fig. 1 of Ref. [18]. When this MI-axion degree is removed at the compactification scale a global $U(1)$ symmetry can survive in breaking down to realize the "invisible" axions at the intermediate scale M_{int} [5–8]. This happens in the compactifications with an anomalous $U(1)$ gauge symmetry [19,20]. The anomalous $U(1)$ gauge symmetry is a $U(1)$ subgroup of the $E_8 \times E_8'$ gauge group and the corresponding gauge boson obtains mass at the compactification scale ($\approx 10^{18}$ GeV) by absorbing the MI-axion degree. In this case a global symmetry called $U(1)_{\mathrm{anom}}$ is surviving down to a lower energy scale. Note that the so-called "model-dependent axions (MD-axions)" from B_{MN} $(M, N = 5, \ldots, 10)$ [21] do not match to any $U(1)$ subgroup of $E_8 \times E_8'$ because the heterotic string has only one anomalous $U(1)$ gauge symmetry. Because there is no global symmetry except the $U(1)_{\mathrm{anom}}$ global symmetry, the MD-axions must be removed at the compactification scale unless they become accidentally light [22]. Thus, those used in Refs. [23,24] must be accidentally realized. In the string compactification accidental symmetries are pointed out to be related to axions [25,26] and R symmetry [27]. Here we identify $U(1)_{\mathrm{anom}}$ as the needed Peccei–Quinn (PQ) symmetry [1] for the "invisible" axion [5–8]. Now a reasonable compactification model with an $U(1)_{\mathrm{anom}}$ symmetry can be considered in full detail. In this paper the model presented in Ref. [28] is chosen, based on $\mathbf{Z}_{12-I}$ orbifold compactification.[2] The model presented in Ref. [30] could also have been chosen. This model, however, contains many more singlets and hence is more complicated to be completely presented here. Even though the analyses are presented in the specific

[1] For the DFSZ model, only in the supersymmetric extension of the standard model the fine-tuning problem [9] is evaded by the μ term [10].

[a] e-mail: jihnekim@gmail.com

[2] Recently, a comparison of orbifold compactifications and free fermionic models has been studied [29].

model, the current method can be applied to any model in order to obtain complete knowledge on the "invisible" axion and flavor parameters.

The low energy gauge group obtained in [28] is $SU(5) \times U(1)_X \times U(1)^6 \times SU(5)' \times SU(2)'$ where the primed non-Abelian groups are from the hidden sector E_8'. The first two factors $SU(5) \times U(1)_X$ are the so-called rank-5 flipped-SU(5) [31–33]. Being a GUT, the flipped-SU(5) must resolve the doublet–triplet splitting problem in the Higgs quintets **5** and $\overline{\mathbf{5}}$: "Why are the color triplets superheavy while Higgs doublets remain light?" In this paper it is shown how the splitting is realized in terms of the complete spectrum in the model. It is in fact done beyond the dimensional analyses.

The global symmetry $U(1)_{\rm anom}$ is beyond the flipped-SU(5). Hence it can be used as a family symmetry. Since all the quantum numbers $Q_{\rm anom}$ of the global symmetry $U(1)_{\rm anom}$ are known, the order of magnitude of all the Yukawa couplings can be obtained, resolving the family parameters. That is, the mass matrices of the SM fermions can be obtained. Basically, it turns out that the flavor matrices are given by the multiples of the Yukawa coupling constants [27] instead of by the mass power suppressions via the Froggatt–Nielsen powers [34].

In Sect. 2 the definition of the quantum numbers and expression for $Q_{\rm anom}$ are presented in terms of six U(1) gauge charges of $E_8 \times E_8'$. It is derived which pair of **5** and $\overline{\mathbf{5}}$ is remaining toward the needed pair in the SUSY SM. Here we also discuss the 't Hooft mechanism which is working for the transfer of the global symmetry down to the axion window. In Sect. 3, the mass scales in the model where $U(1)_{\rm anom}$ is surviving as a PQ symmetry down to an intermediate scale, are discussed. In Sect. 4, the Yukawa mass matrices of $Q_{\rm em} = +\frac{2}{3}$ and $-\frac{1}{3}$ quarks, $Q_{\rm em} = -1$ charged leptons, and light SM neutrinos are presented. Section 5 is our conclusion. In the appendix the 't Hooft mechanism in the compactification process is discussed. For this occasion the correct entries for the previous tables of Refs. [35,36], taking into account its erratum, are presented.

2 Global charges and one pair of Higgs doublets in SUSY

In the open string theory with n Chan–Paton factors string amplitudes are U(n) invariant. This U(n), constructed with n(fundamental)–$\bar{n}$(anti-fundamental), is the world sheet global symmetry; viz. p. 374 of Ref. [37]. In the target space this is coordinate (x_M) dependent and hence U(n) is promoted to a gauge symmetry, which is the reason that the string theory does not allow any global symmetry. The basic reason might be the string movement in the world sheet. However, if the location of the string is fixed at a fixed point, variation of the string in the world sheet is not allowed. Hence in the target space global symmetry may not be promoted to a gauge symmetry. Fixed points are present in the symmetric orbifold compactifications and the existence of a global symmetry is not ruled out. However, in the smooth compactifications there will be no global symmetry. Thus an anomalous U(1) may not arise in smooth compactifications. To obtain a global symmetry producing an anomalous U(1) and hence the "invisible" axion, the orbifold compactification is being considered.

An $E_8 \times E_8'$ heterotic string model compactified on $\mathbf{Z}_{12-I}$ orbifold gives SU(5)×SU(5)′×SU(2)′ with seven $U(1)$s [28]. This GUT model has been studied for various aspects in Refs. [35,36,38,39]. Extra $U(1)$s are just a problem in orbifold compactification. In the Calabi–Yau compactifications for example, the rank due to extra $U(1)$s is easily reduced. But there is one important $U(1)$ factor which is a part of the flipped SU(5) GUT [31]. Because of the difficulty of obtaining an adjoint representation for a Higgs multiplet for breaking SU(5), the flipped SU(5) is probably the most favorable GUT in the orbifold compactification [37].[3] First we pay attention to the factor SU(5)$_{\rm flip}$ where our definition of SU(5)$_{\rm flip}$ is containing a gauge group $U(1)$: $SU(5) \times U(1)_X$. The second factor is the anomalous $U(1)$. Except these two $U(1)$ factors, $U(1)_X$ and $U(1)_{\rm anom}$, the non-Abelian gauge group is SU(5)×SU(5)′×SU(2)′ and the rest of the anomaly free factors are $\tilde{\rm U}(1)^5$. Since the rank of the original gauge group $E_8 \times E_8'$ is 16, the total number of $U(1)$ factors is 7. Their charges are named X and Q_i $(i = 1, \ldots, 6)$. $Q_{\rm anom}$ is a linear combination of Q_i $(i = 1, \ldots, 6)$. In Refs. [35,36] these charges are defined on the lattice as[4]

$$X = (-2, -2, -2, -2, -2;\, 0^3)(0^8)', \tag{1}$$

$$Q_{\rm anom} = 84Q_1 + 147Q_2 - 42Q_3 - 63Q_5 - 9Q_6, \tag{2}$$

where

$$Q_1 = (0^5; 12, 0, 0)(0^8)',$$
$$Q_2 = (0^5; 0, 12, 0)(0^8)',$$
$$Q_3 = (0^5; 0, 0, 12)(0^8)',$$
$$Q_4 = (0^8)(0^4, 0; 12, -12, 0)',$$
$$Q_5 = (0^8)(0^4, 0; -6, -6, 12)',$$
$$Q_6 = (0^8)(-6, -6, -6, -6, 18; 0, 0, 6)'.$$

In the orbifold compactification, there frequently appears an anomalous $U(1)_A$ gauge field A_μ from a subgroup of $E_8 \times E_8'$ [19,20]. The charge of this anomalous $U(1)_A$

[3] The Pati–Salam model [40] is also good but to break the part SU(2)$_L$×SU(2)$_R$, contained in the PS model, down to SU(2)×U(1) one needs a VEV of Δ ($\equiv$ **3** under SU(2)$_R$); hence it is like an adjoint of SU(2). Anyway, because of this complexity, it may be said that the flipped SU(5) is 'most favored' for the string compactification.

[4] For the definition, see Refs. [37,41].

is given in Eq. (2). In addition, the anomaly cancellation in 10 dimensions (10D) requires the so-called Green–Schwarz (GS) term in terms of the second rank antisymmetric tensor field B_{MN} $(M, N = 1, 2, \ldots, 10)$ [15]. This GS term always introduces the MI axion $a_{\rm MI}$, $\partial_\mu a_{\rm MI} \propto \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$ (μ, etc. $= 1, 2, 3, 4$) where $H^{\nu\rho\sigma}$ is the field strength of $B^{\rho\sigma}$ [16]. In this compactification with $U(1)_A$, A_μ absorbs $a_{\rm MI}$ to become massive at the compactification scale $m_A \approx 10^{18}$ GeV.[5] Below the scale m_A there remains a global symmetry which is called $U(1)_{\rm anom}$. Its charge is given by $Q_{\rm anom}$ presented in Eq. (2). In detail it works as follows. Suppose that five $\tilde{\rm U}(1)$ charges out of $Q_{1,\ldots,6}$ are broken and there is only one gauge symmetry remaining which we identify as $U(1)_{\rm anom}$. Then two continuous parameters can be considered, the MI-axion direction and the phase of the $U(1)_{\rm anom}$ transformation. Out of two continuous directions, only one phase or pseudoscalar is absorbed by the $U(1)_{\rm anom}$ gauge boson and one continuous direction survives. The remaining continuous degree corresponds to a global symmetry which is called the ’t Hooft mechanism [43,44]: “If both a gauge symmetry and a global symmetry are broken by one scalar vacuum expectation value (VEV), the gauge symmetry is broken and a global symmetry is surviving”. The resulting global charge is a linear combination of the original gauge and global charges. This will briefly be reviewed in the appendix. This counting of pseudoscalar degrees is not affected by changing the scales of the VEVs. Thus, when the anomalous $U(1)$ is arising at the compactification scale, the gauge symmetry $U(1)_{\rm anom}$ is broken and the MI-axion degree is removed and in addition the global $U(1)_{\rm anom}$ symmetry below the compactification scale results. The dilaton partner of the MI axion must remain heavy as in the usual Higgs mechanism since it does not find its partner below the compactification scale.

2.1 No gauged anomalous $U(1)$ below the compactification scale

There have been many discussions on the Fayet–Iliopoulos (FI) D-term for $U(1)_{\rm anom}$ gauge symmetry at the GUT scale, e.g. in Ref. [45]. However, there is no gauged $U(1)$ corresponding to the anomalous symmetry below the compactification scale. In models with a hierarchy between the compactification scale and the GUT scale there is no need to consider the $U(1)_{\rm anom}$ D-term, $\int d^4\theta\, \xi\, D$, below the compactification scale. The ξ term is in the D-term potential, $\frac{1}{2}D^2$, with $D = -\xi - e\phi^* Q_a \phi$ where e is the $U(1)_{\rm anom}$ gauge coupling. Our $U(1)_{\rm anom}$ is derived from the orbifold compactification of the $E_8 \times E_8'$ heterotic string. After compactification of the six internal space $M_4 \times K$ can be considered where M_4 is the Minkowski space and K is the internal space. In Fig. 1 some relevant fields living in K are shown. The effective symmetry group in the M_4 Lagrangian is gauge symmetries times some discrete groups without any global symmetry except that corresponding to $B_{\mu\nu}$. The 4D scalar B_{ij} are called the MD-axions, a_n $(n \geq 1)$, determined by the topology of the internal space. Reference [45] shows that the classical symmetries corresponding to $a_n \to a_n+$(constant) are broken by the world-sheet instanton effects. Figure 1 shows these fields living in K with some $U(1)$ gauge fields. In the compactification of $E_8 \times E_8'$ heterotic string there appears only one anomalous $U(1)$ if any of such terms are present. If so, the corresponding gauge boson obtains mass by absorbing $a_{\rm MI}$ as shown in the appendix and Fig. 1b. All the other $U(1)$s are anomaly free. a_n are not absorbed to gauge bosons at this stage. Now the massless states a_n and non-anomalous six $U(1)$s below the compactification scale can be considered as well as their Kähler potentials and FI D terms. However, for $U(1)_{\rm anom}$ we do not need to consider the corresponding D term.

For a consistency check, consider Fig. 1a again. If the throat is not cut, the space is still 10D. In this 10D the $U(1)_{\rm anom}$ subgroup of $E_8 \times E_8'$ can be considered. Let us consider the subgroup ${\rm SU(3)}_C\times {\rm SU(2)}_L\times {\rm U(1)}_Y\times {\rm U(1)_{anom}}\times \prod_{i=1}^5 \tilde{U}(1)_i$ of $E_8 \times E_8'$ where $\tilde{U}(1)_i$ are anomaly free. Before cutting the throat $E_8 \times E_8'$ is broken in order to separate $U(1)_{\rm anom}$ by the VEV of an appropriate adjoint representation (in 4D language) in the bulk. Of course, this adjoint representation is not present in our massless spectrum but is good enough to see the resulting effective low energy theory. If $U(1)_{\rm anom}$ is separated in this way it obtains mass by absorbing the MI axion by the VEVs of F_{ij} as shown in the appendix. Then superheavy masses are assigned to the adjoint representation introduced. So far, nothing has been introduced violating the effective symmetry

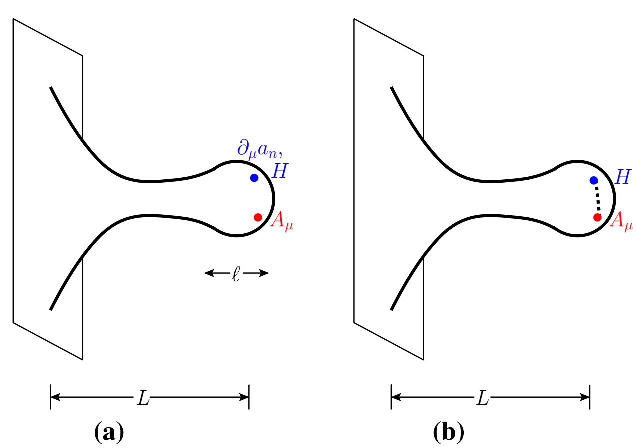

Fig. 1 Compactification, leading to $M_4 \times K$. The parallelogram depicts M_4, ℓ is a compactification size of K and L in one direction is shown pictorially as a neck

[5] If there is no anomalous $U(1)$ gauge symmetry, the MI axion is an axion at $m_{\rm MI} \approx 10^{16}$ GeV [17].

$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{\mathrm{anom}} \times \prod_{i=1}^{5} \tilde{U}(1)_i$. Next, the throat is cut to obtain the effective 4D theory which is $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \prod_{i=1}^{5} \tilde{U}(1)_i$. There cannot be a FI D term for $U(1)_{\mathrm{anom}}$ in this interpretation.

Even if we consider the FI term with a nonvanishing ξ, in case there is no hierarchy between the compactification scale and the GUT scale [46], it can be shown that a global symmetry can be derived below the scale of the anomalous gauge boson mass. Therefore, the following equation can be considered, including the D term with a nonvanishing ξ,

$$\begin{aligned}&\frac{1}{2}\partial^\mu a_{\mathrm{MI}}\partial_\mu a_{\mathrm{MI}} + M_{\mathrm{MI}} A_\mu \partial^\mu a_{\mathrm{MI}} + \left| -\xi + e\sum_a \phi_a^* Q_a \phi_a \right|^2 \\ &\quad + \Big[|(\partial_\mu - ieA_\mu)\phi_1|^2 + \cdots \Big] \\ &= (M_{\mathrm{MI}}\partial^\mu a_{\mathrm{MI}} - eV_1 \partial^\mu a_1)A_\mu + \cdots , \end{aligned} \tag{3}$$

where ϕ_a are assumed to carry only the anomalous charge for a moment, not carrying any non-anomalous charge Y and $\tilde{Q}_i$ $(i = 1, \ldots, 5)$. Let one ϕ_a, say ϕ_1 develops a VEV, V_1, by minimizing the FI term. Here, two phase fields, a_{MI} and a_1 [= the phase of ϕ_1 $(= (V_1 + \rho_1)e^{ia_1/V_1})/\sqrt{2}$] are considered and only one Goldstone boson is absorbed to A_μ,

$$\sqrt{M_{\mathrm{MI}}^2 + e^2 V_1^2}\,(\cos\theta_G\, a_{\mathrm{MI}} - \sin\theta_G\, a_1) \tag{4}$$

where $\tan\theta_G = eV_1/M_{\mathrm{MI}}$. The orthogonal direction

$$\theta' = \cos\theta_G \frac{a_{\mathrm{MI}}}{|M_{\mathrm{MI}}|} + \sin\theta_G \frac{a_1}{|eV_1|} \tag{5}$$

is surviving as a global direction below the scale $\sqrt{M_{\mathrm{MI}}^2 + e^2 V_1^2}$. With this global symmetry, breaking the five non-anomalous Ũ(1)'s around the GUT scale can be considered, leaving only one global symmetry to the axion window. If one gauge symmetry $U(1)_{\mathrm{anom}}$ were the whole story for the global symmetry, the next lower scale VEV of a scalar carrying nonzero VEV (probably at a GUT scale) will break the global $U(1)_{\mathrm{anom}}$. In the orbifold compactification however, there appear many gauge $U(1)$s (six in our example) which are anomaly-free except the above $U(1)_{\mathrm{anom}}$, say $U(1)_4$ in our example. The D term of $U(1)_4$ is $|\phi_2^* Q_4 \phi_2|^2$. Generally, ϕ_2 carries the $U(1)_{\mathrm{anom}}$ charge, for example in Table 2 any ϕ_2 carrying nonzero Q_4 also carries Q_{anom}. So, the VEV of ϕ_2 will break both $U(1)_4$ and the global $U(1)_{\mathrm{anom}}$ obtained above. Below $\langle\phi_2\rangle$ appears the global symmetry $U(1)_{\mathrm{anom}}$. Applying the 't Hooft mechanism repeatedly until all anomaly-free gauge $U(1)$s are broken except $U(1)_Y$, the global $U(1)_{\mathrm{anom}}$ interpretable as $U(1)_{\mathrm{PQ}}$ from string compactification is obtained.[6] Then, at the intermediate scale 10^{9-11} GeV, one $U(1)_{\mathrm{anom}}$ breaking VEV f_a of a SM singlet scalar Φ breaks the global symmetry $U(1)_{\mathrm{anom}}$ spontaneously and the needed "invisible" axion at the intermediate scale results. The global symmetry whose shift angle θ' is broken at an intermediate scale to create the "invisible" axion by some scalar field carrying the anomalous charge. To observe this only one scalar field ϕ_1 develops a VEV V_1 as before. Now a global charge surviving below the scale $\sqrt{M_{\mathrm{MI}}^2 + e^2 V_1^2}$ is defined as[7]

$$\hat{Q}' = Q_{\mathrm{anom}} + x\, Q_a \tag{6}$$

where Q_a is an anomaly-free gauge $U(1)$ charge. For the condition $\hat{Q}'|\phi_1\rangle = 0$ so that $\hat{Q}'$ is a good generator of the global symmetry, $x = -Q_{\mathrm{anom}}(\phi_1)/Q_a(\phi_1)$ is fixed. Since Q_a is an anomaly-free generator, a constant multiple of Q_a in Eq. (6) can be added to give the same anomaly coefficient for $U(1)'$–$\mathrm{SU}(3)_c$–$\mathrm{SU}(3)_c$. A new global charge can be taken

$$\hat{Q}' = Q_{\mathrm{anom}}. \tag{7}$$

Actually, a more general proof was given in Ref. [43]. Consider $\hat{Q}' = Q_{\mathrm{anom}} + x_1 Q_a + x_2 Q_b$ where Q_a and Q_b are anomaly-free gauge charges. Then $\mathrm{Tr}\,\hat{Q}' = Q_{\mathrm{anom}}$ and $\mathrm{Tr}\,\hat{Q}' Q_p Q_q = Q_{\mathrm{anom}} Q_p Q_q$ where $\{p, q\} = \{a, b\}$. In fact, in Ref. [43] instead of Q_{anom} of Eq. (2), it was shown to have exactly the same traces with

$$\hat{Q}' = 63(0^5; 16, 28, -8)(0^5; 6, 6, -12)'. \tag{8}$$

In the tables of the present paper Q_{anom} is used, and in the Tables of [43] $\hat{Q}'$ is used but the anomaly-related quantities such as $\mathrm{Tr}\,\hat{Q}' Q^\alpha_{\mathrm{color}} Q^\beta_{\mathrm{color}}$ and $\mathrm{Tr}\,\hat{Q}' Q_{\mathrm{em}} Q_{\mathrm{em}}$ are exactly the same in both cases.

2.2 "Invisible" axion in the axion window

For simplicity's sake, a hierarchy is assumed between the compactification and the GUT scales. ϕ is chosen, not carrying any gauge charge. The VEV of ϕ is assumed to be at the axion window and breaks the $U(1)_{\mathrm{anom}}$ global symmetry. In this case, the actual global symmetry breaking scale is a mixture of two effects: the MI-axion direction and the hypothetical intermediate scale axion direction (the phase of ϕ). When the original anomalous gauge charge of ϕ is Q_a, ϕ $(\sim ve^{ia_\phi/v})$, the QCD axion a created at the intermediate scale (determined by the VEV of ϕ) is a combination of a_ϕ and a_{MI},

[6] It is easy to see this by counting the number of continuous degrees. Two phases are introduced, a_{MI} and a_1. One combination is absorbed to A^μ_{anom}. One may worry that the other combination, (5), might become massive because two terms for a_{MI} and a_1 are considered. However, it

Footnote 6 continued
does not work that way because both generators, $U(1)_{\mathrm{anom}}$ and Q_a in the FI D-term in Eq. (3), are proportional. Equation (4) explicitly shows that only one combination is removed to A^μ_{anom}.

[7] The sign in front of Q_{anom} belongs to the sign convention of the $\overline{\theta}$ term and we choose + sign here.

Table 1 The $SU(5)\times U(1)_X$ states. Here, + represents helicity $+\frac{1}{2}$ and – represents helicity $-\frac{1}{2}$. Sum of $Q_{\rm anom}$ is multiplied by the index of the fundamental representation of $\mathrm{SU}(3)_c$, $\frac{1}{2}$. The PQ symmetry, being chiral, counts quark and antiquark in the same way. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively. The bold entries in the column $Q_{\rm anom}$ are $Q_{\rm anom}/126$

Sect.	Colored states	$\mathrm{SU}(5)_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
U	$(\underline{+++--};--+)(0^8)'$	$\overline{\mathbf{10}}_{-1}$		-6	-6	$+6$	0	0	0	$-1638(\mathbf{-13})$	C_2	-3276
U	$(\underline{+----};+--)(0^8)'$	$\mathbf{5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(\underline{+----};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(\underline{+++--};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(\underline{1\,0\,0\,0\,0};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
T_4^0	$(\underline{-1\,0\,0\,0\,0};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_6$	$+1008$
T_6^0	$(\underline{1\,0\,0\,0\,0};0\,0\,0)(0^5;\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(\underline{-1\,0\,0\,0\,0};0\,0\,0)(0^5;\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(\underline{-1\,0\,0\,0\,0};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(\mathbf{-\frac{54}{7}})$	C_9	-1296
T_7^0	$(\underline{+1\,0\,0\,0\,0};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(\mathbf{-\frac{54}{7}})$	C_{10}	-1296
T_3^0	$(\underline{+++--};0\,0\,0)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(\mathbf{-\frac{33}{7}})$	C_{11}	-1188
T_9^0	$(\underline{++---};0\,0\,0)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(\mathbf{+\frac{33}{7}})$	C_{12}	$+1188$
				-16	-28	$+8$	0	$+18$	$+6$	-3492		-5406

$$
a = \cos\theta\, a_\phi + \sin\theta\, a_{\rm MI}, \quad \text{with } \sin\theta = \frac{g Q_a v}{\sqrt{M_{\rm MI}^2 + g^2 Q_a^2 v^2}} \tag{9}
$$

where the antisymmetric tensor field strength is the MI-axion, $H_{\mu\nu\rho} = M_{MI}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma a_{MI}$ [16]. Thus, for $v \ll M_{MI}$ the desired "invisible" axion at the intermediate scale is obtained.

Here we stress again that the exact global symmetries from the string compactification require anomalous gauge symmetries after compactification. From the $E_8 \times E_8'$ heterotic string there is only one such anomalous gauge symmetry as discussed above.[8] Any other global symmetries must be accidental as discussed for QCD axions in [25,26] and for axion-like particles in [48].

2.3 Three families in the flipped SU(5)

The three families of the minimal supersymmetric standard model (MSSM) and one pair of Higgs doublets are required as a result. To have the "invisible" axion, it is further required that $U(1)_{\rm anom}$ is broken at the intermediate scale, $M_{\rm int} \approx 10^{11}$ GeV. At the $\mathrm{SU}(5)_{\rm flip}$ GUT level, three copies of $\overline{\mathbf{10}}_{-1} \oplus \mathbf{5}_{+3} \oplus \mathbf{1}_{-5}$ are necessary. In Table 1 SU(5) non-singlet fields with the global quantum numbers where the axion–photon–photon couplings are presented in the last column are presented. One family appears in the untwisted sector U and two families appear in the twisted sector T_4^0.

In the $\mathrm{SU}(5)_{\rm flip}$ GUT, a pair of $\overline{\mathbf{10}}_{-1} \oplus \mathbf{10}_{+1}$ is needed to break the rank 5 group $\mathrm{SU}(5)_{\rm flip}$ down to the rank 4 group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. They appear in T_3^0 and T_9^0 in Table 1. The vacuum expectation values (VEVs) of these pairs achieve the doublet–triplet splitting discussed in Sect. 4.1.

For the Higgs quintets T_6 has pairs with multiplicity 3. More importantly, two pairs appear in T_4^0 and one pair appears in T_7^0. The two pairs appearing in T_4^0 are not distinguished and the Higgsino mass matrix elements are democratic. Their Yukawa couplings take the form of $C_5C_6\sigma_1$ that conserves the $U(1)_{\rm anom}$ symmetry,

$$
M^{2\times 2}_{\rm demo} = \begin{pmatrix} M & M \\ M & M \end{pmatrix} \tag{10}
$$

where $M \sim \langle\sigma_1\rangle$. The σ_1 multiplicity is 3, as shown in Table 2. The twisted sector T_4^0 satisfies the $\mathbf{Z}_3$ orbifold selection rules and the multiplicity 3 belonging to the permutation symmetry S_3 splits into S_3 representations $\mathbf{2}\oplus\mathbf{1}$. Three σ_1's under S_3 in the $\mathbf{Z}_3$ compactification can be combined to [49,50]

$$
\begin{aligned}
\Phi_0 &= \frac{1}{\sqrt{3}}\left(\sigma_1^a + \sigma_1^b + \sigma_1^c\right), \\
\Phi_+ &= \frac{1}{\sqrt{3}}\left(\sigma_1^a + \omega\,\sigma_1^b + \bar\omega\,\sigma_1^c\right), \\
\Phi_- &= \frac{1}{\sqrt{3}}\left(\sigma_1^a + \bar\omega\,\sigma_1^b + \omega\,\sigma_1^c\right),
\end{aligned} \tag{11}
$$

where $\omega = e^{2\pi i/3}$ and $\bar\omega = e^{4\pi i/3}$ are the cube roots of unity. Φ_0 is a singlet $\mathbf{1}$ and Φ_+ and Φ_- form a doublet $\mathbf{2}$. Suppose that $\langle\Phi_0\rangle \neq 0$ and $\langle\Phi_+\rangle = \langle\Phi_-\rangle = 0$. C_5 and C_6 belong to $\mathbf{2}$ of S_3, and their multiplication gives $\mathbf{2}\times\mathbf{2} = \mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{2}$,

[8] There can be more anomalous gauge symmetries from Type-I and Type II-B [47].

Table 2 Left-handed $SU(5)\times U(1)_X \times \mathrm{SU}(5)' \times \mathrm{SU}(2)'$ singlet states. $(N^L)_j$ is the notation for the oscillator mode of the oscillating string with j denoting the coordinate in the internal space. $\mathcal{P}(f_{0,+,-})$ is the multiplicity of the corresponding spectrum in the twisted sector $T^{0,+,-}$. In this table there is only $\mathcal{P}(f_0)$. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively

Sectors	Neutral singlet states	$\mathrm{SU}(5)_X$	$(N^L)_j$	$\mathcal{P}(f_0)$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	La.
T_4^0	$(0^5\,;\frac{-2}{3}\,\frac{-2}{3}\,\frac{-2}{3})\,(0^8)'$	$\mathbf{1}_0$	0	3	−8	−8	−8	0	0	0	$-1512(\mathbf{-12})$	σ_1
T_4^0	$(0^5\,;\frac{-2}{3}\,\frac{1}{3}\,\frac{1}{3})\,(0^8)'$	$\mathbf{1}_0$	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	−8	+4	+4	0	0	0	$-252(\mathbf{-2})$	σ_2
T_4^0	$(0^5\,;\frac{1}{3}\,\frac{-2}{3}\,\frac{1}{3})\,(0^8)'$	$\mathbf{1}_0$	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	+4	−8	+4	0	0	0	$-1008(\mathbf{-8})$	σ_3
T_4^0	$(0^5\,;\frac{1}{3}\,\frac{1}{3}\,\frac{-2}{3})\,(0^8)'$	$\mathbf{1}_0$	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	+4	+4	−8	0	0	0	$+1260(\mathbf{+10})$	σ_4
T_6	$(0^5\,;0\,1\,0)\,(0^5\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	$\mathbf{1}_0$	0	2	0	+12	0	+12	0	0	$+1764(\mathbf{+14})$	σ_5
T_6	$(0^5\,;0\,0\,1)\,(0^5\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	$\mathbf{1}_0$	0	2	0	0	+12	−12	0	0	$-504(\mathbf{-4})$	σ_6
T_6	$(0^5\,;0\,-\,1\,0)\,(0^5\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	$\mathbf{1}_0$	0	2	0	−12	0	−12	0	0	$-1764(\mathbf{-14})$	σ_7
T_6	$(0^5\,;0\,0\,-\,1)\,(0^5\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	$\mathbf{1}_0$	0	2	0	0	−12	+12	0	0	$+504(\mathbf{+4})$	σ_8
T_2^0	$(0^5\,;\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3})\,(0^5\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	$\mathbf{1}_0$	$2_{\bar 1}, 2_3$	1, 1	−4	−4	−4	−12	0	0	$-756(\mathbf{-6})$	σ_9
T_2^0	$(0^5\,;\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3})\,(0^5\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	$\mathbf{1}_0$	$2_{\bar 1}, 2_3$	1, 1	−4	−4	−4	+12	0	0	$-756(\mathbf{-6})$	σ_{10}
T_3	$(0^5\,;\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})\,(0^5\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\mathbf{1}_0$	0	1	−6	−6	−6	+12	−9	−3	$-540(\mathbf{-\frac{30}{7}})$	σ_{11}
T_3	$(0^5\,;\frac{-1}{2}\,\frac{1}{2}\,\frac{1}{2})\,(0^5\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\mathbf{1}_0$	0	1	−6	+6	+6	+12	−9	−3	$+720(\mathbf{+\frac{40}{7}})$	σ_{12}
T_3	$(0^5\,;\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})\,(0^5\,\frac{-1}{4}\,\frac{3}{4}\,\frac{-1}{2})'$	$\mathbf{1}_0$	0	1	+6	+6	−6	−12	−9	−3	$+2232(\mathbf{+\frac{124}{7}})$	σ_{13}
T_3	$(0^5\,;\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})\,(0^5\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	$1_1, 1_3$	2,1	+6	+6	−6	0	+9	+3	$+1044(\mathbf{+\frac{58}{7}})$	σ_{14}
T_9	$(0^5\,;\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})\,(0^5\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	0	1	+6	+6	+6	−12	+9	+3	$+540(\mathbf{+\frac{30}{7}})$	σ_{15}
T_9	$(0^5\,;\frac{1}{2}\,\frac{-1}{2}\,\frac{-1}{2})\,(0^5\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	0	2	+6	−6	−6	−12	+9	+3	$-720(\mathbf{-\frac{40}{7}})$	σ_{16}
T_9	$(0^5\,;\frac{-1}{2}\,\frac{-1}{2}\,\frac{1}{2})\,(0^5\,\frac{1}{4}\,\frac{-3}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	0	2	−6	−6	+6	+12	+9	+3	$-2232(\mathbf{-\frac{124}{7}})$	σ_{17}
T_9	$(0^5\,;\frac{-1}{2}\,\frac{-1}{2}\,\frac{1}{2})\,(0^5\,\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	$\mathbf{1}_0$	$1_{\bar 1}, 1_{\bar 3}$	1,1	−6	−6	+6	0	−9	−3	$-1044(\mathbf{-\frac{58}{7}})$	σ_{18}
T_1^0	$(0^5\,;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0^5\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	3_3	1	−2	−2	−2	−12	+9	+3	$-972(\mathbf{-\frac{54}{7}})$	σ_{19}
T_1^0	$(0^5\,;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0^5\,\frac{1}{4}\,\frac{-3}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	3_3	1	−2	−2	−2	+12	+9	+3	$-972(\mathbf{-\frac{54}{7}})$	σ_{20}
T_1^0	$(0^5\,;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0^5\,\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	$\mathbf{1}_0$	$\{1_1, 1_3\}$	1	−2	−2	−2	0	−9	−3	$+216(\mathbf{+\frac{12}{7}})$	σ_{21}
			$\{2_3, 1_2\}$	1								
			6_3	1								
T_7^0	$\left(0^5\,;\frac{5}{6}\,\frac{-1}{6}\,\frac{-1}{6}\right)(0^5\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	$2_{\bar 1}$	1	+10	−2	−2	0	+9	+3	$+36(\mathbf{+\frac{2}{7}})$	σ_{22}
T_7^0	$\left(0^5\,;\frac{-1}{6}\,\frac{5}{6}\,\frac{-1}{6}\right)(0^5\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	$2_{\bar 1}$	1	−2	+10	−2	0	+9	+3	$+792(\mathbf{+\frac{44}{7}})$	σ_{23}
T_7^0	$\left(0^5\,;\frac{-1}{6}\,\frac{-1}{6}\,\frac{5}{6}\right)(0^5\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	$\mathbf{1}_0$	$2_{\bar 1}$	1	−2	−2	+10	0	+9	+3	$-1476(\mathbf{-\frac{82}{7}})$	σ_{24}

$$
\begin{aligned}
\Psi_0 &= \frac{1}{\sqrt{3}}\left(C_5^{(1)}C_6^{(1)} + C_5^{(2)}C_6^{(2)}\right),\\
\Psi_0' &= \frac{1}{\sqrt{3}}\left(C_5^{(1)}C_6^{(1)} - C_5^{(2)}C_6^{(2)}\right),\\
\Psi_+ &= \frac{1}{\sqrt{3}}\left(C_5^{(1)}C_6^{(2)} + \omega\, C_5^{(2)}C_6^{(1)}\right),\\
\Psi_- &= \frac{1}{\sqrt{3}}\left(C_5^{(1)}C_6^{(2)} + \bar\omega\, C_5^{(2)}C_6^{(1)}\right),
\end{aligned} \tag{12}
$$

where Ψ_+ and Ψ_- form a doublet under interchange (1) $\leftrightarrow$ (2). Thus, the singlet VEV $\langle\Phi_0\rangle$ can couple with either Ψ_0 or Ψ_0'. In this way one pair becomes superheavy. This result is equivalent to the democratic mass matrix (10). Namely, the determinant of $M^{2\times 2}_{\rm demo}$ is 0 and only one pair obtains mass $2M$. The remaining pair is massless at this stage. For the three pairs in T_6 the Higgsino mass matrix can be studied in the same way. Since it belongs to T_6 we consider the $\mathbf{Z}_2$ and S_2 permutation which allows only the following singlet combinations:

$$
\begin{aligned}
\Psi^{(a)} &= \frac{1}{\sqrt{3}}\left(C_7^{(1)}C_8^{(1)} + C_7^{(2)}C_8^{(2)} + C_7^{(3)}C_8^{(3)}\right),\\
\Psi^{(b)} &= \frac{1}{\sqrt{3}}\left(C_7^{(1)}C_8^{(1)} - C_7^{(2)}C_8^{(2)} + C_7^{(3)}C_8^{(3)}\right),\\
\Psi^{(c)} &= \frac{1}{\sqrt{3}}\left(C_7^{(1)}C_8^{(1)} + C_7^{(2)}C_8^{(2)} - C_7^{(3)}C_8^{(3)}\right),\\
\Psi^{(d)} &= \frac{1}{\sqrt{3}}\left(C_7^{(1)}C_8^{(1)} - C_7^{(2)}C_8^{(2)} - C_7^{(3)}C_8^{(3)}\right).
\end{aligned} \tag{13}
$$

If we require the invariance of the mass matrix under S_2, i.e. under the interchange of any two pairs out of (1), (2) and (3), only the term $\Psi^{(a)}$ is allowed. Then only one pair obtains a superheavy mass and two pairs remain light. Again it is like taking a democratic mass matrix. Even if the two pairs from T_6 remain light, their contribution to the unification point of couplings is null because they are the $\mathrm{SU(5)_{flip}}$ complete multiplets. However, the absolute magnitude of the gauge coupling constant at the unification point is affected. Nevertheless we will not discuss these complete multiplets anymore in this paper since the massless pairs do not affect our discussion on the flavor problems.

There are the Yukawa couplings $C_5C_{11}\sigma_{21}$ and $C_6C_{12}\sigma_{21}$ which conserve the $U(1)_{\rm anom}$ symmetry. Among the remaining two light pairs (one from T_4^0 and the other from T_7^0) one obtains mass and finally there will be only one light pair left. We have the 3×3 Higgsino mass matrix[9]

$$M^{3\times3}_{\rm Higgsino}=\begin{array}{cc} \begin{matrix} H_u^{(T4)_1} & H_u^{(T4)_2} & H_u^{(T7)} \end{matrix} & \\ \begin{pmatrix} M & M & m\\ M & M & m\\ m & m & 0\end{pmatrix} & \begin{matrix} H_d^{(T4)_1}\\ H_d^{(T4)_2}\\ H_d^{(T7)}\end{matrix}\end{array} \tag{14}$$

where $m\sim\langle\sigma_{21}\rangle$. As expected, the determinant of $M^{3\times3}_{\rm Higgsino}$ is 0, and there remain two light pairs as far as the (33) element is 0. The heaviest eigenstate of (14) is

$$\Psi^{M_c}=\frac{1}{\sqrt{2}}\left(\Psi_1^{T_4^0}+\Psi_2^{T_4^0}\right),\ \text{mass}=2M, \tag{15}$$

where Ψ is $H_{u,d}$. The Higgsino pair of the MSSM contains

$$\Psi^{0}=\frac{1}{\sqrt{2}}\left(\Psi_1^{T_4^0}-\Psi_2^{T_4^0}\right),\ \text{mass}=0. \tag{16}$$

The other state with a nonzero (33) element will be presented later.

3 Mass scales

Below the Planck scale $M_{\rm P}$ four scales are being considered: the compactification scale allowing large masses $M_{\rm vec}$ to vector-like pairs, the GUT scale $M_{\rm GUT}$, the intermediate $M_{\rm int}$ and the electroweak scale $v_{\rm ew}$. The principle of removing vector-like pairs is just the gauge principle as emphasized in [43]. If extra symmetries are introduced one must include another assumption(s) how those extra symmetries are broken. The hierarchy of scales that we consider is

$$\mathrm{E_8\times E_8'}\Big|_{M_{\rm vec}}\longrightarrow\ \mathrm{GUT}\ \longrightarrow\ \Big|_{M_{\rm GUT}}$$
$$\text{SM and “invisible” axion}\Big|_{M_{\rm int}}\longrightarrow\ \mathrm{SU(3)_c\times U(1)_{em}}\Big|_{v_{\rm ew}}$$

[9] In the (33) position, ε will be introduced later.

where ${M_{\rm vec}}^2$ is the order of the string tension, α'^{-1}. The particles removed at the compactification scale are the vector-like sets. In Table 3 charged singlets are listed. The vector-like sets, including the charges $Q_{\rm anom}$ and $\mathbf{Z}_{12}$ orbifolds, must be removed at $M_{\rm vec}$. The $U(1)$s which can be broken at the GUT scales are five anomaly free $\tilde{\rm U}(1)$'s. $U(1)_{\rm anom}$ global symmetry is required to be broken at the axion window and works as a global symmetry at the GUT scale. Any singlet of Table 3 can have a GUT scale VEV, leaving a global symmetry below $M_{\rm GUT}$ via the 't Hooft mechanism [44]. This process can be repeated in order to break all five anomaly free $\tilde{\rm U}(1)$'s, leaving only $U(1)_{\rm anom}$ global symmetry below $M_{\rm GUT}$. The doublet–triplet splitting can be of the form "colored particles=GUT scale and $H_{u,d}$=light". Even if the masses of colored particles are a bit smaller than M_{GUT}, proton stability can be achieved. Proton decay by dimension 6 operators of quarks and leptons (by the exchange of colored scalars) is helped by the Yukawa couplings for the first family members by order $10^{-5.5}$. Then it is not problematic. With the SUSY assumption dimension 5 operators of quark and lepton superfields, $W_4\propto\overline{\mathbf{10}}_{-1}\cdot\overline{\mathbf{10}}_{-1}\cdot\overline{\mathbf{10}}_{-1}\cdot\mathbf{5}_{+3}$ are the leading contribution. This is disastrously dangerous if the colored scalar masses are somewhat smaller than $M_{\rm GUT}$. There can arise a $\mathbf{Z}_4$ from a subgroup of an anomaly free $\tilde{\rm U}(1)$ gauge group, eliminating W_4 as shown in [38]. There is no fast proton decay problem [51].

There are two GUT scale sets,

$$C_7^{T_6}+C_8^{T_6},\ C_{11}^{T_3}+C_{12}^{T_9}. \tag{17}$$

These are $SU(5)\times U(1)_X$ non-singlets and these vector-like sets cannot be external light fields. In addition the superpotential terms cannot be generated through the intermediate states of these vector-like fields. Diagrams with these internal $M_{\rm GUT}$ mass states must contain loops which cannot generate superpotential terms because of the non-renomalization theorem. Thus high dimension superpotential terms composed of light fields cannot be generated with a suppression factor $M_{\rm GUT}$ or $M_{\rm vec}$ in our model.

If a high dimension superpotential term of light fields is generated, the relevant mass suppression factor must be the intermediate scale $M_{\rm int}$. This conclusion, depending on our detailed model, is very different from the general strategy in the MSSM where the μ term for example has the suppression factor $M_{\rm GUT}$ or $M_{\rm P}$ [10].

4 Yukawa couplings

Now, we search for a possibility for a nonzero (33) element of $M^{3\times3}_{\rm Higgsino}$ in Eq. (14) which respects the $U(1)_{\rm anom}$ symmetry and the $\mathbf{Z}_{12-I}$ selection rules. At the level of dimension 10 there appears one

Table 3 Electromagnetically charged singlets

Sect.	Charged singlet states	$SU(5)_X$	Mu.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	La.	$Q_a^{\gamma\gamma}$
U	$(+\,+\,+\,+\,+;-\,+\,-)\,(0^8)'$	$\mathbf{1}_{-5}$		-6	$+6$	-6	0	0	0	$+630(\mathbf{+5})$	S_1	$+630$
T_4^0	$(+\,+\,+\,+\,+;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0^8)'$	$\mathbf{1}_{-5}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	S_{24}	-378
T_4^+	$((\frac{1}{6})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{1}{2})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{2})'$	$\mathbf{1}_{-5/3}$	2	-2	$+2$	$+6$	$+4$	$+10$	-10	$-666(\mathbf{-\frac{37}{7}})$	S_2	-74
	$((\frac{1}{6})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{1}{2})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-1}{6}\,\frac{1}{2}\,\frac{-1}{2})'$	$\mathbf{1}_{-5/3}$	2	-2	$+2$	$+6$	-8	-8	-16	$+522(\mathbf{+\frac{29}{7}})$	S_3	$+58$
T_4^-	$((\frac{-1}{6})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{6})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{1}{6}\,\frac{+1}{2}\,\frac{-1}{2})'$	$\mathbf{1}_{5/3}$	2	-2	-6	$+2$	-4	-10	$+10$	$-594(\mathbf{-\frac{33}{7}})$	S_4	-66
	$((\frac{-1}{6})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{6})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{1}{6}\,\frac{-1}{2}\,\frac{+1}{2})'$	$\mathbf{1}_{5/3}$	2	-2	-6	$+2$	$+8$	$+8$	$+16$	$-1782(\mathbf{-\frac{99}{7}})$	S_5	-198
T_2^+	$((\frac{1}{3})^5;\frac{-1}{3}\,\frac{1}{3}\,0)\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{-1}{3}\,0\,\frac{1}{2})'$	$\mathbf{1}_{-10/3}$	1	-4	$+4$	0	-4	$+8$	$+16$	$-396(\mathbf{-\frac{22}{7}})$	S_6	-176
	$((\frac{-1}{6})^5;\frac{1}{6}\,\frac{-1}{6}\,\frac{1}{2})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{2}{3}\,0\,\frac{-1}{2})'$	$\mathbf{1}_{5/3}$	1	$+2$	-2	$+6$	$+8$	-10	$+10$	$+162(\mathbf{+\frac{9}{7}})$	S_7	$+18$
	$((\frac{-1}{6})^5;\frac{1}{6}\,\frac{-1}{6}\,\frac{1}{2})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{-1}{3}\,0\,\frac{1}{2})'$	$\mathbf{1}_{5/3}$	1	$+2$	-2	$+6$	-4	$+8$	$+16$	$-1026(\mathbf{-\frac{57}{7}})$	S_8	-114
T_2^-	$((\frac{-1}{3})^5;\frac{-1}{3}\,0\,\frac{1}{3})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{1}{3}\,0\,\frac{-1}{2})'$	$\mathbf{1}_{10/3}$	1	-4	0	$+4$	$+4$	-8	-16	$+144(\mathbf{+\frac{8}{7}})$	S_9	$+64$
	$((\frac{1}{6})^5;\frac{1}{6}\,\frac{-1}{2}\,\frac{-1}{6})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-2}{3}\,0\,\frac{1}{2})'$	$\mathbf{1}_{-5/3}$	1	$+2$	-6	-2	-8	$+10$	-10	$-1170(\mathbf{-\frac{65}{7}})$	S_{10}	-130
	$((\frac{1}{6})^5;\frac{1}{6}\,\frac{-1}{2}\,\frac{-1}{6})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{1}{3}\,0\,\frac{-1}{2})'$	$\mathbf{1}_{-5/3}$	1	$+2$	-6	-2	$+4$	-8	-16	$+18(\mathbf{+\frac{1}{7}})$	S_{11}	$+2$
T_1^+	$((\frac{-1}{3})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{1}{2})\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{1}{12}\,\frac{-1}{4}\,0)'$	$\mathbf{1}_{10/3}$	1	-2	$+2$	$+6$	$+4$	$+1$	-13	$-72(\mathbf{-\frac{4}{7}})$	S_{12}	-32
	$((\frac{1}{6})^5;\frac{-2}{3}\,\frac{2}{3}\,0)\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{1}{12}\,\frac{-1}{4}\,0)'$	$\mathbf{1}_{-5/3}$	1	-8	$+8$	0	$+4$	$+1$	-13	$+558(\mathbf{+\frac{31}{7}})$	S_{13}	$+62$
	$((\frac{1}{6})^5;\frac{1}{3}\,\frac{-1}{3}\,0)\,(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{1}{12}\,\frac{-1}{4}\,0)'$	$\mathbf{1}_{-5/3}$	2	$+4$	-4	0	$+4$	$+1$	-13	$-198(\mathbf{-\frac{11}{7}})$	$2S_{14}$	-22
T_1^-	$((\frac{1}{3})^5;\frac{-1}{6}\,\frac{1}{2}\,\frac{1}{6})\left((\frac{-1}{6})^4\,\frac{1}{2};\frac{5}{12}\,\frac{-1}{4}\,0\right)'$	$\mathbf{1}_{-10/3}$	1	-2	$+6$	$+2$	$+8$	-1	$+13$	$+576(\mathbf{+\frac{32}{7}})$	S_{15}	$+256$
	$((\frac{-1}{6})^5;\frac{-2}{3}\,0\,\frac{-1}{3})\left((\frac{-1}{6})^4\,\frac{1}{2};\frac{5}{12}\,\frac{-1}{4}\,0\right)'$	$\mathbf{1}_{5/3}$	1	-8	0	-4	$+8$	-1	$+13$	$-558(\mathbf{-\frac{31}{7}})$	S_{16}	-62
	$((\frac{-1}{6})^5;\frac{1}{3}\,0\,\frac{2}{3})\left((\frac{-1}{6})^4\,\frac{1}{2};\frac{5}{12}\,\frac{-1}{4}\,0\right)'$	$\mathbf{1}_{5/3}$	1	$+4$	0	$+8$	$+8$	-1	$+13$	$-54(\mathbf{-\frac{3}{7}})$	S_{17}	-6
T_7^+	$((\frac{-1}{3})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{-1}{2})\left(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-5}{12}\,\frac{1}{4}\,0\right)'$	$\mathbf{1}_{-10/3}$	1	-2	$+2$	-6	-8	$+1$	-13	$+432(\mathbf{+\frac{24}{7}})$	S_{18}	$+192$
	$((\frac{1}{6})^5;\frac{1}{3}\,\frac{2}{3}\,0)\left(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-5}{12}\,\frac{1}{4}\,0\right)'$	$\mathbf{1}_{5/3}$	1	$+4$	$+8$	0	-8	$+1$	-13	$+1566(\mathbf{+\frac{87}{7}})$	S_{19}	$+174$
	$((\frac{1}{6})^5;\frac{-2}{3}\,\frac{-1}{3}\,0)\left(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{-1}{2};\frac{-5}{12}\,\frac{1}{4}\,0\right)'$	$\mathbf{1}_{5/3}$	1	-8	-4	0	-8	$+1$	-13	$-1206(\mathbf{-\frac{67}{7}})$	S_{20}	-134
T_7^-	$((\frac{1}{3})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{6})\,((\frac{-1}{6})^4,\frac{1}{2};\frac{-1}{12}\,\frac{1}{4}\,0)'$	$\mathbf{1}_{10/3}$	1	-2	-6	$+2$	-4	-1	$+13$	$-1188(\mathbf{-\frac{66}{7}})$	S_{21}	-528
	$((\frac{-1}{6})^5;\frac{-2}{3}\,0\,\frac{2}{3})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{-1}{12}\,\frac{1}{4}\,0)'$	$\mathbf{1}_{-5/3}$	1	-8	0	$+8$	-4	-1	$+13$	$-1062(\mathbf{-\frac{59}{7}})$	S_{22}	-118
	$((\frac{-1}{6})^5;\frac{1}{3}\,0\,\frac{-1}{3})\,((\frac{-1}{6})^4\,\frac{1}{2};\frac{-1}{12}\,\frac{1}{4}\,0)'$	$\mathbf{1}_{-5/3}$	2	$+4$	0	-4	-4	-1	$+13$	$+450(\mathbf{+\frac{25}{7}})$	S_{23}	$+100$
				-16	-28	$+8$	0	$+18$	$+42$	-7632		-1162

$$W \propto \frac{1}{M_{\rm int}^7} H_u^{(T_7^0)} H_d^{(T_7^0)} \sigma_{12}^{(T3)} \sigma_{23}^{(T_7^0)} \sigma_{21}^{(T_7^0)} \sigma_{21}^{(T_7^0)} \sigma_8^{(T6)} \times \sigma_8^{(T6)} \sigma_8^{(T6)} \sigma_1^{(T_4^0)} \tag{18}$$

where we used the $M_{\rm int}$ as the suppression factor. This term introduces a nonzero entry ε in the (33) element which cannot be very small because the VEVs of the singlets are also at the scale $M_{\rm int}$. In this case, the remaining two eigenstates of (14) are

$$\frac{1}{N}\begin{pmatrix} 1 \\ 1 \\ -\frac{M}{m} + \frac{\varepsilon}{2m} - \sqrt{2 + \left(\frac{M}{m} - \frac{\varepsilon}{2m}\right)^2} \end{pmatrix},$$
$$\text{mass} = M + \frac{\varepsilon}{2} - \sqrt{2m^2 + \left(M - \frac{\varepsilon}{2}\right)^2} \tag{19}$$

and the massless one is still Eq. (16),

$$\mathbf{5}^{T_4^0}, \overline{\mathbf{5}}^{T_4^0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \ \text{mass} = 0. \tag{20}$$

At this level, the Higgs doublets are those appearing in T_4^0. However, the doublets from T_4^0 obtain mass at the electroweak scale when soft masses of order $m_{3/2}^2$ are introduced, which is a well-known fact in the supergravity phenomenology. This will not be discussed here.

4.1 Doublet–triplet splitting

For the successful MSSM, the multiplet (20) must split into heavy colored ones and light Higgs doublets. This doublet–triplet splitting problem is achieved by the VEVs of $\overline{\mathbf{10}}_{-1}$

Table 4 Three left-handed states belonging to the vector-like spectrum appearing in Fig. 2

Sect.	$P+kV$	SU(5)$_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label
T_8^0	$(\underline{+ + - - -}; \frac{+1}{6}\,\frac{+1}{6}\,\frac{+1}{6})\,(0^8)'$	$\mathbf{10}_{+1}$	2	$+2$	$+2$	$+2$	0	0	0	$+378(\frac{+\mathbf{21}}{\mathbf{7}})$	C_{v1}
T_4^0	$(\underline{+ + + - -}; \frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\frac{-\mathbf{21}}{\mathbf{7}})$	C_{v2}
T_5^0	$(\underline{-1\,0\,0\,0\,0}; \frac{+1}{6}\,\frac{+1}{6}\,\frac{+1}{6})\,(0^5; \frac{+1}{4}, \frac{+1}{4}, \frac{-2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	$+2$	$+2$	$+2$	0	-9	-3	$+972(\frac{+\mathbf{54}}{\mathbf{7}})$	C_{v3}

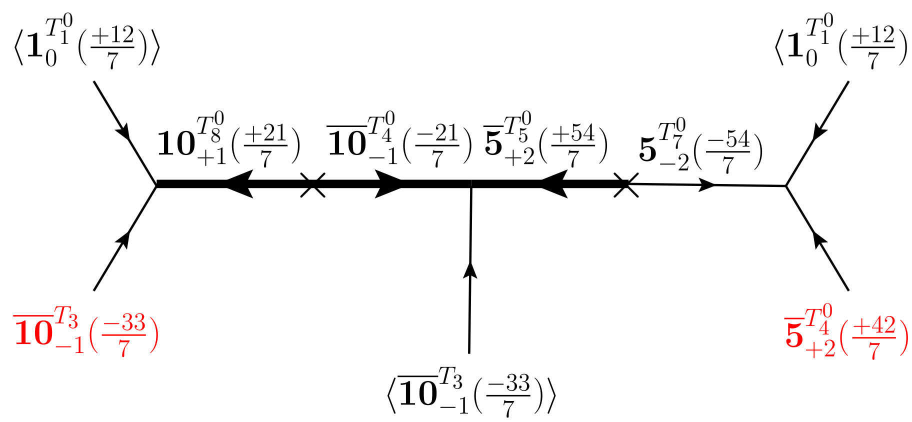

Fig. 2 A high dimensional term. The fractional numbers in the brackets are $Q_{\rm anom}/126$

and $\mathbf{10}_{+1}$ appearing in T_3 and T_9. Here it is explicitly shown from the detailed string compactification model discussed above. It is shown that the suppression factor is the mass of the vector-like pair $M_{\rm vec}$. The VEVs of $\overline{\mathbf{10}}_{-1}$ and $\mathbf{10}_{+1}$ give mass to the colored triplets of the Higgs quintets, $\mathbf{5}_{-2}^{T_4^0}$ and $\overline{\mathbf{5}}_{+2}^{T_4^0}$. But the problem is the scale for the effective operator. We find the following operators:

$$\text{VEV of }\overline{\mathbf{10}}\text{: } \frac{1}{M_{\rm vec}M_{\rm int}}\,\langle C_{11}^{T3}\rangle\langle\sigma_{21}^{T_1^0}\rangle\langle\sigma_{21}^{T_1^0}\rangle\, C_{11}^{T3}\, C_6^{T_4^0}$$
$$\to d_{{\rm from}\,\overline{\mathbf{10}}_{-1}}\, d^c_{{\rm from}\,\overline{\mathbf{5}}_{+2}}, \tag{21}$$
$$\text{VEV of }\mathbf{10}\text{: } \frac{1}{M_{\rm vec}M_{\rm int}}\,\langle C_{12}^{T9}\rangle\langle\sigma_{19}^{T_1^0}\rangle\langle\sigma_{20}^{T_1^0}\rangle\, C_{12}^{T9}\, C_5^{T_4^0}$$
$$\to d^c_{{\rm from}\,\mathbf{10}_{+1}}\, d_{{\rm from}\,\mathbf{5}_{-2}}, \tag{22}$$

where the $1/M_{\rm vec}$ suppression exists because there appear heavy vector-like states in the tree diagram.[10] Two thick lines of Fig. 2 are vector-like states of Ref. [28] because there is no massless (left-handed) states in T_8^0 and T_5^0. The cross in the RHS is of the order $M_{\rm int}$ because $\mathbf{1}_0^{T_7^0}\,(\frac{-\mathbf{54}}{\mathbf{7}})$ is present in Table 2. On the other hand the cross in the LHS is expected to be of the order $M_{\rm vec}$ because $\overline{\mathbf{10}}^{T_4^0}\,(\frac{-\mathbf{21}}{\mathbf{7}})$ does not appear in Table 2. Even though the left-handed states of Table 2 do not contain a state with $Q_{\rm anom} = \frac{-21}{7}$, the vector-like pairs ($\mathbf{10}$ and $\overline{\mathbf{10}}$ shown as two thick arrows) can fulfill the quantum numbers because we will not require the masslessness conditions in the orbifold selection rules for the vector-like states. Also, the thick arrow line for $\overline{\mathbf{5}}_{+2}$ does not appear in Table 2. These three states are denoted by the 'vector-like states' which are shown in Table 4. The left-mover and right-mover masses in the heterotic string with $\mathbf{Z}_{12-I}$ compactification in the $k^{\rm th}$ twisted sector are given

$$M_L^2 = \frac{(P+kV)^2}{2} - \frac{2\tilde{c}_k}{2} + \frac{2\tilde{N}_L}{2}, \tag{23}$$
$$M_R^2 = \frac{(\tilde{s}+k\phi)^2}{2} - \frac{2c_k}{2} + \frac{2\tilde{N}_R}{2}, \tag{24}$$

where $\tilde{N}_{L,R}$ are the oscillator numbers and $2\tilde{c}_{4,8} = \frac{3}{2}$ and $2c_{4,8} = \frac{1}{2}$ are given in Ref. [41]. In the model of [41], the L and R vectors are

$$V = \left(0^5; \frac{-1}{6}, \frac{-1}{6}, \frac{-1}{6}\right)\left(0^5; \frac{+1}{4}, \frac{+1}{4}, \frac{-2}{4}\right)', \tag{25}$$
$$\phi = \left(\frac{5}{12}, \frac{4}{12}, \frac{1}{12}\right). \tag{26}$$

The QCD-color field in $\overline{\mathbf{5}}_{+2}$ finds the partner in $\overline{\mathbf{10}}_{-1}$, both of which are shown in red in Fig. 2. Similarly, $\mathbf{5}_{-2}$ finds the partner in $\mathbf{10}_{+1}$. Thus there remain only massless Higgs doublets from Eq. (20), H_u and H_d. The doublet–triplet splitting is realized. One pair of $\mathbf{5}_{-2}$ and $\overline{\mathbf{5}}_{+2}$ needs one pair of $\mathbf{10}_{+1}$ and $\overline{\mathbf{10}}_{-1}$ for the splitting.

In view of the longevity of proton, note that the dangerous $\overline{\mathbf{10}}_H \cdot \overline{\mathbf{10}} \cdot \overline{\mathbf{10}} \cdot \mathbf{5}$ and $\overline{\mathbf{10}}_H \cdot \mathbf{5} \cdot \mathbf{5} \cdot \mathbf{1}$ couplings are allowed in $SU(5) \times U(1)_X$, judging from the gauge quantum numbers alone. In the ordinary SO(10), which is the covering group

[10] Loop diagrams are not considered because of the non-renormalization theorem.

Table 5 The SU(5)′ representations. Notations are the same as in Table 1

	$P+kV$	SU(5)′	Mu.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	La.	$Q_a^{\gamma\gamma}$
T_1^0	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\underline{-1\,0^3}\,0;\frac{1}{4}\,\frac{1}{4}\,\frac{1}{2})'$	$\overline{\mathbf{10}}'_0$	1	−2	−2	−2	0	+3	+9	$-648(-\mathbf{\frac{36}{7}})$	T_1'	0
	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\underline{\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2}\,\frac{-1}{2}}\,\frac{1}{2};\frac{-1}{4}\,\frac{-1}{4}\,0)'$											
T_1^0	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\underline{1\,0\,0\,0}\,0;\frac{1}{4}\,\frac{1}{4}\,\frac{1}{2})'$	$(\mathbf{5}',\mathbf{2}')_0$	1	−2	−2	−2	0	+3	−3	$-540(-\mathbf{\frac{30}{7}})$	F_1'	0
	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0;\frac{-3}{4}\,\frac{-3}{4}\,\frac{-1}{2})'$											
	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\underline{\frac{1}{2}\,\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2}}\,\frac{-1}{2};\frac{-1}{4}\,\frac{-1}{4}\,0)'$											
	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2};\frac{-1}{4}\,\frac{-1}{4}\,0)'$											
T_1^0	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(\underline{\frac{-1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}}\,\frac{-1}{2};\frac{-1}{4}\,\frac{-1}{4}\,0)'$	$\overline{\mathbf{5}}'_0$	1	−2	−2	−2	0	+3	−15	$-432(-\mathbf{\frac{24}{7}})$	F_2'	0
	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,-1;\frac{1}{4}\,\frac{1}{4}\,\frac{1}{2})'$											
T_1^+	$((\frac{1}{6})^5;\frac{1}{3}\,\frac{-1}{3}\,0)\left(\underline{\frac{-5}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}}\,\frac{1}{2};\frac{1}{12}\,\frac{-1}{4}\,0\right)'$	$\overline{\mathbf{5}}'_{-5/3}$	1	+4	−4	0	+4	+1	+11	$-414(-\mathbf{\frac{23}{7}})$	F_3'	−230
	$((\frac{1}{6})^5;\frac{1}{3}\,\frac{-1}{3}\,0)(\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3}\,0;\frac{7}{12}\,\frac{1}{4}\,\frac{1}{2})'$											
T_4^+	$((\frac{1}{6})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{1}{2})\left(\underline{\frac{2}{3}\,\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3}}\,0;\frac{1}{3}\,0\,0\right)'$	$\mathbf{5}'_{-5/3}$	3	−2	+2	+6	+4	−2	+2	$-18(-\mathbf{\frac{1}{7}})$	F_4'	−10
	$((\frac{1}{6})^5;\frac{-1}{6}\,\frac{1}{6}\,\frac{1}{2})(\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{6}\,\frac{1}{2};\frac{-1}{6}\,\frac{-1}{2}\,\frac{-1}{2})'$											
T_4^-	$((\frac{-1}{6})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{6})\left(\underline{\frac{-2}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}}\,0;\frac{-1}{3}\,0\,0\right)'$	$\overline{\mathbf{5}}'_{5/3}$	3	−2	−6	+2	−4	+2	−2	$-1242(-\mathbf{\frac{69}{7}})$	F_5'	−690
	$((\frac{-1}{6})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac{1}{6})((\frac{-1}{6})^4\frac{-1}{2};\frac{1}{6}\,\frac{1}{2}\,\frac{1}{2})'$											
T_7^-	$((\frac{-1}{6})^5;\frac{1}{3}\,0\,\frac{-1}{3})\left(\underline{\frac{5}{6}\,\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6}}\,\frac{-1}{2};\frac{-1}{12}\,\frac{1}{4}\,0\right)'$	$\mathbf{5}'_{5/3}$	1	+4	0	−4	−4	−1	−11	$+666(+\mathbf{\frac{37}{7}})$	F_6'	+370
	$((\frac{-1}{6})^5;\frac{1}{3}\,0\,\frac{-1}{3})(\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}\,0;\frac{-7}{12}\,\frac{-1}{4}\,\frac{-1}{2})'$											
				−16	−28	+8	0	+18	+6	−3492		−1960

of $SU(5)\times U(1)_X$, these terms are forbidden by imposing the R-parity quantum numbers -1 and $+1$, respectively, for the matter and Higgs fields. In our case the GUT scale Higgs boson $\overline{\mathbf{10}}_H$ and $\mathbf{10}_H$ are C_{11} and C_{12} of Table 1. These are from T_3^0 and T_9^0. The orbifold selection rules allow for the superpotential term $C_{11}C_{12}$ but do not allow for $C_{11}C_4C_4C_3$ and $C_{11}C_3C_3\cdot\mathbf{1}$ from the fields in Table 1. In the latter operator containing the L-violating LLE^c, $\mathbf{1}$ may be chosen from T_1^0 such as $\sigma_{19,20,21}$ of Table 2. However, LLE^c alone does not trigger proton decay. Basically, the R-parity interpretation for the proton longevity in SO(10) GUT, up to dimension 5 operators, is automatic from our orbifold selection rules [37] in case of the $SU(5)\times U(1)_X$. However, a complete study of the proton longevity is outside the scope of this paper.

4.2 The CKM and PMNS matrices

Earlier an attempt was made to obtain a CKM matrix from standard-like models implied by a $\mathbf{Z}_2\times\mathbf{Z}_2$ fermionic construction [42]. Obtaining CKM and PMNS matrices based on the model of Ref. [28] was attempted. As commented on before, if a high dimension superpotential term of light fields is generated, the relevant mass suppression factor must be the intermediate scale $M_{\rm int}$. Suppose we have an effective operator for the $Q_{\rm em}=\frac{2}{3}$ quarks,

$$\frac{1}{M^n}(\text{SM singlets of Table 2})\cdot\overline{\mathbf{10}}_{-1}\mathbf{5}_3\mathbf{5}_{-2}. \tag{27}$$

In Tables 5 and 6, all the particles that transform non-trivially under SU(5)′×SU(2)′ are listed. There is no vector-like pair including $Q_{\rm anom}$ charges. Thus there is no tree diagram of an intermediate state with mass $M_{\rm vec}$ and any operator with sub-GUT scale fields must have the mass suppression parameter $M_{\rm int}$. Thus the suppression mass in Eq. (27) must be $M_{\rm int}$.

With this in mind the Yukawa matrices and the fermion masses are discussed. The $Q_{\rm em}=+\frac{2}{3}$ quark mass matrix, consistent with the $U(1)_{\rm anom}$ symmetry, the orbifold selection rules and the multiplicity 2 conditions of the Higgs doublets and matter fermions in T_4^0, is

$$M_u\propto\begin{array}{c}\\ \overline{\mathbf{10}}^U_{-1}\\ \overline{\mathbf{10}}^{T_4^A}_{-1}\\ \overline{\mathbf{10}}^{T_4^S}_{-1}\end{array}\begin{array}{c}\begin{array}{ccc}\mathbf{5}^U_3 & \mathbf{5}^{T_4^A}_3 & \mathbf{5}^{T_4^S}_3\end{array}\\ \begin{pmatrix}\frac{\sigma_2\sigma_4}{M_{\rm int}^2}, & 0, & \frac{\alpha_1\sigma_4}{M_{\rm int}}\\ 0, & 0, & 1\\ \frac{\alpha_2\sigma_2}{M_{\rm int}}, & 1, & 0\end{pmatrix}\end{array}v_u\ ,\quad\text{with }\langle H_u\rangle=\frac{v_u}{\sqrt{2}}, \tag{28}$$

where superscripts S and A in T_4^0 denote the symmetric and antisymmetric combinations of the multiplicity 2 fields and α_1 and α_2 are coupling parameters. Similarly, the $Q_{\rm em}=-\frac{1}{3}$ quark mass matrix is given by

Table 6 The SU(2)$'$ representations. Notations are the same as in Table 1. We listed only the upper component of SU(2)$'$ from which the lower component can be obtained by applying T^- of SU(2)$'$

Sect.	$P+kV$	SU(2)$'$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
T_1^0	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(\underline{1\,0\,0\,0}\,0;\frac14\,\frac14\,\frac12)'$ $(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0\,0\,0\,0\,0;\frac{-3}{4}\,\frac{-3}{4}\,\frac{-1}{2})'$	$(\mathbf{5}',\mathbf{2}')_0$	1	−2	−2	−2	0	+3	−3	$-540(\mathbf{-\frac{30}{7}})$	D_1'	In Table 5
T_1^0	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})\,(0\,0\,0\,0\,1;\frac14\,\frac14\,\frac12)'$	$\mathbf{2}'_0$	1	−2	−2	−2	0	+3	+21	$-756(\mathbf{-6})$	D_2	0
T_1^+	$((\frac16)^5;\frac13\,\frac{-1}{3}\,0)\,(\frac16\,\frac16\,\frac16\,\frac16\,\frac12;\frac{1}{12}\,\frac34\,0)'$	$\mathbf{2}'_{-5/3}$	1	+4	−4	0	−8	−5	+5	$+18(\mathbf{+\frac{1}{7}})$	D_3	+4
T_1^-	$((\frac{-1}{6})^5;\frac{-2}{3}\,0\,\frac{-1}{3})\,(\frac13\,\frac13\,\frac13\,\frac13\,0;\frac{-1}{12}\,\frac14\,\frac12)'$	$\mathbf{2}'_{5/3}$	1	−8	0	−4	−4	+5	−5	$-774(\mathbf{-\frac{43}{7}})$	D_4	−172
T_1^-	$((\frac{-1}{6})^5;\frac13\,0\,\frac23)\,(\frac13\,\frac13\,\frac13\,\frac13\,0;\frac{-1}{12}\,\frac14\,\frac12)'$	$\mathbf{2}'_{5/3}$	1	+4	0	+8	−4	+5	−5	$-270(\mathbf{-\frac{15}{7}})$	D_5	−60
T_2^+	$((\frac{-1}{6})^5;\frac16\,\frac{-1}{6}\,\frac12)\,(\frac13\,\frac13\,\frac13\,\frac13\,0;\frac16\,\frac12\,0)'$	$\mathbf{2}'_{5/3}$	1	+2	−2	+6	−4	−4	−8	$-54(\mathbf{-\frac{3}{7}})$	D_6	−12
T_2^-	$((\frac16)^5;\frac16\,\frac{-1}{2}\,\frac{-1}{6})\,(\frac16\,\frac16\,\frac16\,\frac16\,\frac12;\frac13\,0\,\frac12)'$	$\mathbf{2}'_{-5/3}$	1	+2	−6	−2	+4	+4	+8	$-954(\mathbf{-\frac{53}{7}})$	D_7	−212
T_4^+	$((\frac16)^5;\frac{-1}{6}\,\frac16\,\frac12)\,(\frac16\,\frac16\,\frac16\,\frac16\,\frac12;\frac{-1}{6}\,\frac12\,\frac12)'$	$\mathbf{2}'_{-5/3}$	2	−2	+2	+6	−8	+4	+8	$-450(\mathbf{-\frac{25}{7}})$	D_8	−100
T_4^-	$((\frac{-1}{6})^5;\frac{-1}{6}\,\frac{-1}{2}\,\frac16)\,(\frac13\,\frac13\,\frac13\,\frac13\,0;\frac23\,0\,0)'$	$\mathbf{2}'_{5/3}$	2	−2	−6	+2	+8	−4	−8	$-810(\mathbf{-\frac{45}{7}})$	D_9	−180
T_7^+	$((\frac16)^5;\frac13\,\frac23\,0)\,(\frac16\,\frac16\,\frac16\,\frac16\,\frac12;\frac{7}{12}\,\frac14\,0)'$	$\mathbf{2}'_{5/3}$	1	+4	+8	0	+4	−5	+5	$+1782(\mathbf{+\frac{99}{7}})$	D_{10}	+396
T_7^+	$((\frac16)^5;\frac{-2}{3}\,\frac{-1}{3}\,0)\,(\frac16\,\frac16\,\frac16\,\frac16\,\frac12;\frac{7}{12}\,\frac14\,0)'$	$\mathbf{2}'_{5/3}$	1	−8	−4	0	+4	−5	+5	$-990(\mathbf{-\frac{55}{7}})$	D_{11}	−220
T_7^-	$((\frac{-1}{6})^5;\frac13\,0\,\frac{-1}{3})\,\left(\frac13\,\frac13\,\frac13\,\frac13\,0;\frac{5}{12}\,\frac{-1}{4}\,\frac12\right)'$	$\mathbf{2}'_{-5/3}$	1	+4	0	−4	+8	+5	−5	$+234(\mathbf{+\frac{13}{7}})$	D_{12}	+52
				−16	−28	+8	0	+18	+6	−3492	$\sum_i =$	−784

$$
\begin{array}{cc}
 & \begin{array}{ccc} \overline{\mathbf{10}}_{-1}^{U} & \overline{\mathbf{10}}_{-1}^{T_4^A} & \overline{\mathbf{10}}_{-1}^{T_4^S} \end{array} \\
M_d \propto \begin{array}{c} \overline{\mathbf{10}}_{-1}^{U} \\ \overline{\mathbf{10}}_{-1}^{T_4^A} \\ \overline{\mathbf{10}}_{-1}^{T_4^S} \end{array} & \begin{pmatrix} \frac{\sigma_4^2}{M_{\rm int}^2}, & 0\,, & \frac{\beta_1\sigma_4}{M_{\rm int}} \\ 0\,, & 0\,, & 1 \\ \frac{\beta_1\sigma_4}{M_{\rm int}}, & 1\,, & 0 \end{pmatrix} v_d\ , \quad \text{with } \langle H_d\rangle = \frac{v_d}{\sqrt{2}},
\end{array}
\tag{29}
$$

where β_1 is a coupling parameter.

In Table 3 charged singlets are listed that will be needed for the charged lepton Yukawa couplings. Here $\mathbf{1}_{-5}$ in U and T_4^0 are $Q_{\rm em} = +1$ charged leptons. Thus the $Q_{\rm em} = -1$ charged lepton mass matrix is

$$
\begin{array}{cc}
 & \begin{array}{ccc} \mathbf{5}_3^{U} & \mathbf{5}_3^{T_4^A} & \mathbf{5}_3^{T_4^S} \end{array} \\
M_e \propto \begin{array}{c} \mathbf{1}_{-5}^{U} \\ \mathbf{1}_{-5}^{T_4^A} \\ \mathbf{1}_{-5}^{T_4^S} \end{array} & \begin{pmatrix} \frac{\sigma_2\sigma_3}{M_{\rm int}^2}, & 0, & \frac{\gamma_1\sigma_3}{M_{\rm int}} \\ 0, & 0, & 1 \\ \frac{\gamma_2\sigma_2}{M_{\rm int}}, & 1, & 0 \end{pmatrix} v_d\ .
\end{array}
\tag{30}
$$

Similarly, the neutrino mass matrix can be written

$$
\begin{array}{cc}
 & \begin{array}{ccc} \mathbf{5}_{+3}^{U} & \mathbf{5}_{+3}^{T_4^A} & \mathbf{5}_{+3}^{T_4^S} \end{array} \\
M_\nu \propto \begin{array}{c} \mathbf{5}_{+3}^{U} \\ \mathbf{5}_{+3}^{T_4^A} \\ \mathbf{5}_{+3}^{T_4^S} \end{array} & \begin{pmatrix} \frac{\sigma_4\sigma_6^2\sigma_{13}\sigma_{16}}{M_{\rm int}^5}, & 0, & \frac{\{\sigma_2^2\sigma_4,\,\sigma_2\sigma_5\sigma_9\}}{M_{\rm int}^3} \\ 0, & 0, & \frac{\sigma_1^3\sigma_4^2}{M_{\rm int}^5} \\ \frac{\{\sigma_2^2\sigma_4,\,\sigma_2\sigma_5\sigma_9\}}{M_{\rm int}^3}, & \frac{\sigma_1^3\sigma_4^2}{M_{\rm int}^5}, & 0 \end{pmatrix} \frac{v_u^2}{M_{\rm int}}.
\end{array}
\tag{31}
$$

The effective interaction for M_ν based on $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{\rm anom}$ introduces two H_u insertions. Note that u-type quarks obtain mass from the coupling $\overline{\mathbf{10}}_{-1}\cdot\mathbf{5}_{+3}\cdot\mathbf{5}_{H_u}$ where $\mathbf{5}_{H_u}$ is $\mathbf{5}_{-2}$ in the flipped SU(5). Hence the neutrino mass matrix comes from the square of this which introduces v_u^2. The coefficient of $\mathbf{5}_{+3}\mathbf{5}_{+3}v_u^2$ can be $\langle\overline{\mathbf{10}}_{-1}\rangle^2/(\text{suppression mass})^3 = \mathrm{M_{GUT}}^2/(\text{suppression mass})^3$ which we take as $1/M_{\rm int}$. In addition there are nonzero factors of $\sigma^3/M_{\rm int}^3$ or $\sigma^5/M_{\rm int}^5$. As illustration, $\sigma/M_{\rm int} = 0.1 - 0.5$ and the largest neutrino mass $m_{\nu_{\rm max}} = 0.5$ eV are taken. Then $M_{\rm int} \approx 10^8 - 10^{11}$ GeV. Obtaining $M_{\rm int}$ from first principles is beyond the scope of this paper.

Inspecting the mass matrices it is concluded that a physical phase in σ_4 leads to the CKM and PMNS phases.[11] For example, there can be a phase generated by the following superpotential:

$$
W = m\sigma_6\sigma_8 + \frac{1}{M^2}\sigma_6\sigma_7\sigma_2\sigma_4^2 \tag{32}
$$

where m and M are real parameters, and all fields develop nonvanishing VEVs. Then

$$
\sigma_4 = \pm i\left| mM^2\,\frac{\sigma_8}{\sigma_2\sigma_7}\right|^{1/2} e^{i(\delta_8-\delta_2-\delta_7)/2} \tag{33}
$$

where δ_i are the phases of σ_i. If $\delta_8 = \delta_2 = \delta_7 = 0$, the CKM and PMNS phases are determined as $\pm\frac{\pi}{2}$.

[11] The phases of other singlets can be rotated away by redefining the phases of some fermions.

The form of mass matrices in Eqs. (28), (29), (30), and (31) can describe the quark and lepton masses successfully by various ratios of the singlet VEVs. Here, however, we will not try to find the relevant ratios.

4.3 The axion–photon–photon coupling

Because all the $Q_{\rm anom}$ charges of the electromagnetically charged fermions are known, the axion–photon–photon coupling $c_{a\gamma\gamma}$ can be calculated. In Tables 5 and 6 the typos in the previous tables [35,36] are corrected. Summing the $Q_a^{\gamma\gamma}$ columns of Tables 1, 3, 5, and 6, we obtain

$$c_{a\gamma\gamma} \simeq \frac{-9312}{-3492} - 2 = \frac{2}{3}, \tag{34}$$

which must be the case if $U(1)_{\rm anom}$ is the PQ symmetry [39]. The subtraction of ≈ 2 for $m_u/m_d \simeq 0.5$ is due to the contribution from the condensation of light quarks.

5 Conclusion

The only allowed global symmetry $U(1)_{\rm anom}$ from the heterotic string is used to find the flavor structure of the SM. This global symmetry is the most natural choice for the "invisible" axion from string theory. In addition, $U(1)_{\rm anom}$ is describing a flavor symmetry. In the flipped SU(5) grand unification of Ref. [28] the mass matrices of quarks and leptons are calculated. It turns out that the fermion mass hierarchy in the SM results from the number of powers of Yukawa couplings, which is a common case in string compactification. Also it is shown how the doublet–triplet splitting in the flipped SU(5) GUT is realized in the model.

Acknowledgements J.E.K. thanks J. Ashfaque and P.K.S. Vaudrevange for pointing out typos in Ref. [35,36]. J.E.K. is supported in part by the National Research Foundation (NRF) grant funded by the Korean Government (MEST) (NRF-2015R1D1A1A01058449) and the IBS (IBS-R017-D1-2016-a00), and B.K. is supported in part by the NRF-2016R1D1A1B03931151.

Appendix: On the 't Hooft mechanism

Superstring axions arising from the mother gauge symmetry $E_8 \times E_8'$ have been considered very early [52]. However, the case has been realized in [53–55] with the anomalous $U(1)$ [19,20] after the orbifold compactification is known which is possible even from the mother gauge symmetry SO(32) because the anomalous $U(1)$ may arise from the spectrum in the twisted sectors. Basically, Barr excluded SO(32) from the untwisted sector spectrum [52]. In all these cases the working principle is the 't Hooft mechanism with the anomalous gauge $U(1)$ from compactification where the $U(1)$ is a subgroup of $E_8 \times E_8'$ or SO(32). In this string compactification the Green–Schwarz (GS) counter term [15] is essential.

The conventional 't Hooft mechanism in gauge theories [44] is the following. Out of two continuous transformation parameters $\alpha(x)$ and β, acting on the field ϕ,

$$\phi \to e^{i\alpha(x)Q_{\rm gauge}} e^{i\beta Q_{\rm global}}\phi, \tag{35}$$

the α direction becomes the longitudinal mode of heavy gauge boson. The above transformation can be rewritten

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\rm gauge}} e^{i\beta(Q_{\rm global}-Q_{\rm gauge})}\phi. \tag{36}$$

Redefining the local direction as $\alpha'(x) = \alpha(x)+\beta$ the transformation is obtained

$$\phi \to e^{i\alpha'(x)Q_{\rm gauge}} e^{i\beta(Q_{\rm global}-Q_{\rm gauge})}\phi. \tag{37}$$

Thus the charge $Q_{\rm global} - Q_{\rm gauge}$ is reinterpreted as the new global charge and is not broken by the VEV, $\langle\phi\rangle$. Basically, the direction β remains as the unbroken continuous direction.

When considering the MI-axion kinetic energy term in 10D [15],

$$-\frac{3\kappa^2}{2g^4\varphi^2} H_{MNP}H^{MNP}, \quad \text{with} \quad M, N, P = \{1, 2, \ldots, 10\}, \tag{38}$$

which is parametrized after compactification to 4D

$$\frac{1}{2\cdot 3!M_{MI}^2} H_{\mu\nu\rho}H^{\mu\nu\rho}, \quad \text{with} \quad \mu, \nu, \rho = \{1, 2, 3, 4\}. \tag{39}$$

The GS action in the differential form is [15]

$$S_1' = \frac{c}{108\,000}\int \Big\{30B\left[(\mathrm{tr}_1 F^2)^2 + (\mathrm{tr}_2 F^2)^2 - \mathrm{tr}_1 F^2\, \mathrm{tr}_2 F^2\right] + \cdots\Big\} \tag{40}$$

where tr_1 and tr_2 are for the E_8 and E_8' representations, which is relevant for our model discussed in the paper. In 4D, it leads to an interaction

$$S_1' \propto -\frac{c}{10800}\Big\{H_{\mu\nu\rho}A_\sigma\,\epsilon^{\mu\nu\rho\sigma}\epsilon^{ijklmn}\langle F_{ij}\rangle\langle F_{kl}\rangle\langle F_{mn}\rangle + \cdots\Big\} \to \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}A^\sigma \tag{41}$$

where we have taken VEVs of internal gauge field strengths, F_{ij} etc. The superstring carries the electric field A_μ, whose group space value is simply denoted A, a kind of matrix.

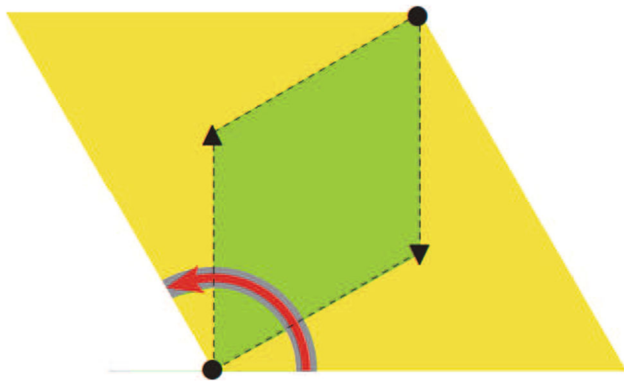

Fig. 3 The flux $\langle F_{ij}\rangle$ at a fixed point

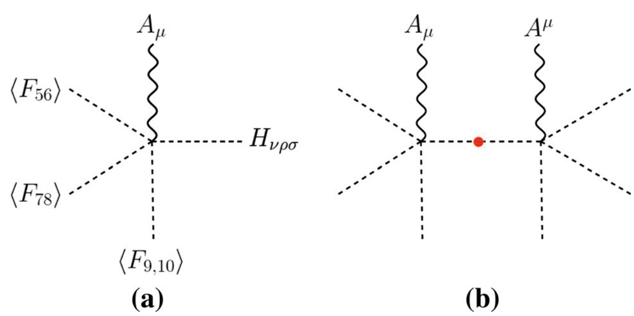

Fig. 4 The GS terms

First, it is noted how the VEVs are assigned to the field strengths in Eq. (41). Then 't Hooft mechanism is discussed. The line integral around a fixed point gives an integer. However, in the orbifold compactification we need to line-integrate only a part of 2π. In Fig. 3 the (limegreen) fundamental domain of Z_3 is shown. The line integral is for the matrix value gauge field A_μ which in our case is denoted by the shift vector and Wilson lines, V, $V \pm a_i$ in the (ij) internal plane. By Stokes' theorem, it is $\frac{1}{3}$ of the surface integral of F_{ij} which is assumed to be located at the fixed point $F_{ij}\delta^2(\sigma-\sigma_0)$ where σ_0 is the location of the fixed point in the two-torus. This delta function limit of a zero-length string must be neglected for the string tension contribution. In this way, VEVs are assigned to field strengths in Eq. (41).

It is noted that one scalar degree $H_{\mu\nu\rho}$ is the MI axion [16]

$$H_{\mu\nu\rho} = M_{MI}\epsilon_{\mu\nu\rho\sigma}\,\partial^\sigma a_{MI}. \tag{42}$$

In Fig. 4a, the first term of Eq. (41) is shown, transferring one derivative of $F_{\mu\nu}$ to $B_{\rho\sigma}$. Equations (39) and (42) result in the following:

$$\frac{1}{2}\partial^\mu a_{MI}\partial_\mu a_{MI} + M_{MI}A_\mu\partial^\mu a_{MI}. \tag{43}$$

The global direction with respect to the 't Hooft mechanism is the a_{MI} direction. Equation (43) can be expressed by adding the contribution of Fig. 4b [which is obtained from Fig. 4a],

$$\frac{1}{2}(\partial_\mu a_{MI})^2 + M_{MI}A_\mu\partial^\mu a_{MI} + \frac{1}{2}M_{MI}^2 A_\mu A^\mu \to \frac{1}{2}M_{MI}^2\left(A_\mu + \frac{1}{M_{MI}}\partial_\mu a_{MI}\right)^2. \tag{44}$$

Thus the MI-axion degree is completely removed below the gauge boson mass scale where the heavy anomalous gauge boson including the longitudinal degree is defined to be $\tilde{A}_\mu = A_\mu + (1/M_{MI})\partial_\mu a_{MI}$. The MI axion is dynamicaly removed from the theory except that it couples to the anomaly [56]. This coupling is defining a θ parameter at low energy depending on some cosmological determination of $\langle a_{\rm MI}\rangle$. The resulting global symmetry is broken by the usual Higgs mechanism and it is expected that it is achieved at the intermediate scale.

Now, the charge of the $U(1)_a$ subgroup of $E_8\times E_8'$ is denoted Q_a that was anomalous in the beginning. Supposing that ϕ carries nonzero gauge charge Q_a. All terms involving ϕ should respect the original $U(1)_a$ gauge symmetry. Thus the potential V is a function of $\phi^*\phi$ only.

Below M_{MI} we do not consider the FI term because there is no $U(1)_a$ gauge symmetry. However, to discuss the scales of the $U(1)_{\rm anom}$ breaking, we consider the ϕ couplings together with the a_{MI} degree. The ϕ coupling to A_μ is given for $\phi = (\frac{v+\rho}{\sqrt{2}})e^{ia_\phi/v}$,

$$\begin{aligned}|D_\mu\phi|^2 &= |(\partial_\mu - igQ_aA_\mu)\phi|^2_{\rho=0}\\ &= \frac{1}{2}(\partial_\mu a_\phi)^2 - gQ_aA_\mu\partial^\mu a_\phi + \frac{g^2}{2}Q_a^2v^2A_\mu^2\\ &= \frac{g^2}{2}Q_a^2v^2\left(A_\mu - \frac{1}{gQ_av}\partial^\mu a_\phi\right)^2.\end{aligned} \tag{45}$$

The gauge boson A_μ obtains its mass from two contributions, Eqs. (44) and (45); this can be written as

$$\begin{aligned}&\frac{1}{2}\left(M_{MI}^2 + g^2Q_a^2v^2\right)(A_\mu)^2\\ &+A_\mu(M_{MI}\partial^\mu a_{MI} - gQ_av\partial^\mu a_\phi)\\ &+\frac{1}{2}\left[(\partial_\mu a_{MI})^2 + (\partial^\mu a_\phi)^2\right]. \qquad (9)\end{aligned} \tag{46}$$

A new global degree is defined, a, interpretable as the "invisible" axion

$$a = \cos\theta\, a_\phi + \sin\theta\, a_{MI} \tag{47}$$

where

$$\sin\theta = \frac{gQ_av}{\sqrt{M_{MI}^2 + g^2Q_a^2v^2}}. \tag{48}$$

Thus, if $v \ll M_{MI}$, the global symmetry breaking scale can be at the intermediate scale. The axion has the dominant component from the phase of ϕ. The determination of v in the effective field theory framework is such that the coefficient of $\phi^*\phi$ in the potential is given by $-$(intermediate scale)^2.

References

1. R.D. Peccei, H.R. Quinn, CP conservation in the presence of instantons. Phys. Rev. Lett. **38**, 1440 (1977). https://doi.org/10.1103/PhysRevLett.38.1440
2. J. Preskill, M.B. Wise, F. Wilczek, Cosmology of the invisible axion. Phys. Lett. B **120**, 127 (1983). https://doi.org/10.1016/0370-2693(83)90637-8
3. L.F. Abbott, P. Sikivie, A cosmological bound on the invisible axion. Phys. Lett. B **120**, 133 (1983). https://doi.org/10.1016/0370-2693(83)90638-X
4. M. Dine, W. Fischler, The not so harmless axion. Phys. Lett. B **120**, 137 (1983). https://doi.org/10.1016/0370-2693(83)90639-1
5. J.E. Kim, Weak interaction singlet and strong CP invariance. Phys. Rev. Lett. **43**, 103 (1979). https://doi.org/10.1103/PhysRevLett.43.103
6. M.A. Shifman, V.I. Vainstein, V.I. Zakharov, Can confinement ensure natural CP invariance of strong interactions? Nucl. Phys. B **166**, 4933 (1980). https://doi.org/10.1016/0550-3213(80)90209-6
7. M. Dine, W. Fischler, M. Srednicki, A simple solution to the strong CP problem with a harmless axion. Phys. Lett. B **104**, 199 (1981). https://doi.org/10.1016/0370-2693(81)90590-6
8. A.P. Zhitnitsky, On possible suppression of the axion hadron interactions. Sov. J. Nucl. Phys. **31**, 260 (1980)
9. H.K. Dreiner, F. Staub, L. Ubaldi, From the unification scale to the weak scale: a self consistent supersymmetric Dine–Fischler–Srednicki–Zhitnitsky axion model. Phys. Rev. D **90**, 055016 (2014). arXiv:1402.5977 [hep-ph]
10. J.E. Kim, H.P. Nilles, The μ problem and the strong CP problem. Phys. Lett. B **138**, 150 (1984). https://doi.org/10.1016/0370-2693(84)91890-2
11. S.M. Barr, D. Seckel, Planck-scale corrections to axion models. Phys. Rev. D **46**, 539 (1992). https://doi.org/10.1103/PhysRevD.46.539
12. M. Kamionkowski, J. March-Russel, Planck-scale physics and the Peccei-Quinn mechanism. Phys. Lett. B **282**, 137 (1992). arXiv:hep-th/9202003
13. R. Holman, S.D.H. Hsu, T.W. Kephart, R.W. Kolb, R. Watkins, L.M. Widrow, Solutions to the strong CP problem in a world with gravity. Phys. Lett. B **282**, 132 (1992). arXiv:hep-ph/9203206
14. D.J. Gross, J.A. Harvey, E.J. Martinec, R. Rohm, The heterotic string. Phys. Rev. Lett. **54**, 502 (1984). https://doi.org/10.1103/PhysRevLett.54.502
15. M.B. Green, J.H. Schwarz, Anomaly cancellation in supersymmetric D = 10 gauge theory and superstring theory. Phys. Lett. B **149**, 117 (1984). https://doi.org/10.1016/0370-2693(84)91565-X
16. E. Witten, Some properties of O(32) superstrings. Phys. Lett. B **149**, 351 (1984). https://doi.org/10.1016/0370-2693(84)90422-2
17. K. Choi, J.E. Kim, Harmful axions in superstring models. Phys. Lett. B **154**, 393 (1985). https://doi.org/10.1016/0370-2693(85)90416-2
18. J.E. Kim, Axionic domain wall number related to $U(1)_{anom}$ global symmetry. Phys. Lett. B **759**, 58 (2016). arXiv:1604.00716 [hep-ph]
19. J.J. Atick, L. Dixon, A. Sen, String calculation of Fayet–Iliopoulos D-terms in arbitrary supersymmetric compactifications. Nucl. Phys. B **292**, 109 (1987). https://doi.org/10.1016/0550-3213(87)90639-0
20. M. Dine, I. Ichinose, N. Seiberg, F-terms and D-terms in string theory. Nucl. Phys. B **293**, 253 (1987). https://doi.org/10.1016/0550-3213(87)90072-1
21. E. Witten, Cosmic superstrings. Phys. Lett. B **153**, 243 (1985). https://doi.org/10.1016/0370-2693(85)90540-4
22. X.G. Wen, E. Witten, World sheet instantons and the Peccei-Quinn symmetry. Phys. Lett. B **166**, 397 (1986). https://doi.org/10.1016/0370-2693(86)91587-X
23. J.E. Kim, Model dependent axion as quintessence with almost massless hidden sector quarks. JHEP **0006**, 016 (2000). arXiv:hep-ph/9907528
24. K. Choi, Axions and the strong CP problem in M theory. Phys. Rev. D **56**, 6588 (1997). arXiv:hep-th/9706171
25. K.-S. Choi, I.-W. Kim, J.E. Kim, String compactification. QCD axion and axion–photon–photon coupling. JHEP **0703**, 116 (2007). arXiv:hep-ph/0612107
26. K.-S. Choi, H.P. Nilles, S. Ramos-Sanchez, P.K.S. Vaudrevange, Accions. Phys. Lett. B **675**, 381 (2009). arXiv:0902.3070 [hep-th]
27. R. Kappl, H.P. Nilles, S. Ramos-Sanchez, M. Ratz, Ki Schmidt-Hoberg, P.K.S. Vaudrevange, Large hierarchies from approximate R symmetries. Phys. Rev. Lett. **102**, 121602 (2009). arXiv:0812.2120 [hep-th]
28. J.H. Huh, J.E. Kim, B. Kyae, $SU(5)_{flip} \times SU(5)'$ from $\mathbf{Z}_{12-I}$. Phys. Rev. D **80**, 115012 (2009). arXiv:0904.1108 [hep-ph]
29. P. Athanasopoulos, A.E. Faraggi, S. Groot Nibbelink, V.M. Mehta, Heterotic free fermionic and symmetric toroidal orbifold models. JHEP **1604**, 038 (2016). arXiv:1602.03082 [hep-th]
30. J.-H. Kim, J.E. Kim, B. Kyae, Superstring standard model from $\mathbf{Z}_{12-I}$ orbifold compactification with and without exotics, and effective R-parity. JHEP **0706**, 034 (2007). arXiv:hep-ph/0702278
31. S.M. Barr, A new symmetry breaking pattern for SO(10) and proton decay. Phys. Lett. B **112**, 219 (1982). https://doi.org/10.1016/0370-2693(82)90966-2
32. J.-P. Derendinger, J.E. Kim, D.V. Nanopoulos, Anti-SU(5). Phys. Lett. B **139**, 170 (1984). https://doi.org/10.1016/0370-2693(84)91238-3
33. I. Antoniadis, J.R. Ellis, J.S. Hagelin, D.V. Nanopoulos, The flipped SU(5)×U(1) string model revamped. Phys. Lett. B **231**, 65 (1989). https://doi.org/10.1016/0370-2693(89)90115-9
34. C.D. Froggatt, H.B. Nielsen, Hierarchy of quark masses, Cabibbo angles and CP violation. Nucl. Phys. B **147**, 277 (1979). https://doi.org/10.1016/0550-3213(79)90316-X
35. J.E. Kim, Calculation of axion–photon–photon coupling in string theory. Phys. Lett. B **735**, 95 (2014)
36. J.E. Kim, Calculation of axion–photon–photon coupling in string theory. Phys. Lett. B **741**, 327 (2014). arXiv:1405.6175 [hep-ph]
37. K.S. Choi, J.E. Kim, *Quarks and Leptons from Orbifolded Superstring*. Lect. Notes. Phys. 696 (Springer, Heidelberg, 2006)
38. J.E. Kim, Abelian discrete symmetries $\mathbf{Z}_N$ and $\mathbf{Z}_{nR}$ from string orbifolds. Phys. Lett. B **726**, 450 (2013). arXiv:1308.0344 [hep-th]
39. J.E. Kim, S. Nam, Couplings between QCD axion and photon from string compactification. Phys. Lett. B **759**, 149 (2016). arXiv:1603.02145 [hep-ph]
40. J. Pati, Abdus Salam, Unified lepton–hadron symmetry and a gauge theory of the basic interactions. Phys. Rev. D **8**, 1240 (1973). https://doi.org/10.1103/PhysRevD.8.1240
41. J.E. Kim, B. Kyae, Flipped SU(5) from $\mathbf{Z}_{12-I}$ orbifold with Wilson line. Phys. Lett. B **770**, 47 (2007). arXiv:hep-th/0608086
42. A. Faraggi, Cabibbo–Kobayashi–Maskawa mixing in superstring derived standard-like models. Nucl. Phys. B **416**, 63 (1994). https://doi.org/10.1016/0550-3213(94)90578-9
43. J.E. Kim, Naturally realized two dark Z's near the electroweak scale. Phys. Rev. D **96**, 055033 (2017). arXiv:1703.10925 [hep-ph]
44. G. 't Hooft, *Renormalizable Lagrangians for massive Yang–Mills fields*. Nucl. Phys. B **35** 167,(1971).https://doi.org/10.1016/0550-3213(71)90139-8

45. M. Dine, N. Seiberg, E. Witten, Fayet-Iliopoulos terms in supergravity. Nucl. Phys. B **289**, 589 (1987). https://doi.org/10.1016/0550-3213(87)90395-6
46. J.J. Atick, A. Sen, Two-loop dilaton tadpole induced by Fayet–Iliopoulos D-terms in compactified heterotic string theories. Nucl. Phys. B **296**, 157 (1988). https://doi.org/10.1016/0550-3213(88)90385-9
47. L.E. Ibanez, R. Rabadan, A.M. Uranga, Anomalous U(1)'s in type I and type IIB D = 4, N=1 string vacua. Nucl. Phys. B **542**, 112 (1999). arXiv:hep-th/9808139
48. A.G. Dias, A.C.B. Machado, C.C. Nishi, A. Ringwald, P. Vaudrevange, The quest for an intermediate-scale accidental axion and further ALPs. JHEP **1406**, 037 (2014). arXiv:1403.5760 [hep-ph]
49. K.-S. Choi, J.E. Kim, Three family $\mathbf{Z}_3$ orbifold trinification, MSSM and doublet triplet splitting problem. Phys. Lett. B **567**, 87 (2003). arXiv:hep-ph/0305002
50. G. Segrè, H.A. Weldon, Natural suppression of strong P and T violations and calculable mixing angles in SU(2)×U(1). Phys. Rev. Lett. **42**, 1191 (1979). https://doi.org/10.1103/PhysRevLett.42.1191
51. H.M. Lee, S. Raby, M. Ratz, G.G. Ross, R. Schieren, K. Schmidt-Hoberg, P.K.S. Vaudrevange, Discrete R symmetries for the MSSM and its singlet extensions. Nucl. Phys. B **850**, 1 (2012). arXiv:1102.3595 [hep-ph]
52. S.M. Barr, Harmless axions in superstring theories. Phys. Lett. B **158**, 397 (1985). https://doi.org/10.1016/0370-2693(85)90440-X
53. J.E. Kim, The strong CP problem in orbifold compactifications and an SU(3)×SU(2)×U(1)^n model. Phys. Lett. B **207**, 434 (1988). https://doi.org/10.1016/0370-2693(88)90678-8
54. J.L. Lopez, D.V. Nanopoulos, Can an anomalous U(1) solve the strong CP problem? Phys. Lett. B **245**, 111 (1990). https://doi.org/10.1016/0370-2693(90)90173-4
55. E. Halyo, Can the axions of standard-like superstring models solve the strong CP problem? Phys. Lett. B **318**, 597 (1993). https://doi.org/10.1016/0370-2693(93)90460-Y
56. P. Svrcek, E. Witten, Axions in string theory. JHEP **0606**, 051 (2006). arXiv:hep-th/0605206