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Testing violation of the Leggett–Garg-type inequality in neutrino oscillations of the Daya Bay experiment

Qiang Fu[1,2,3,a], Xurong Chen[1,b]

[1] Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
[2] Lanzhou University, Lanzhou 730000, China
[3] University of Chinese Academy of Sciences, Beijing 100049, China

Abstract The Leggett–Garg inequality (LGI), derived under the assumption of realism, acts as the temporal Bell inequality. It is studied in electromagnetic and strong interaction like photonics, superconducting qubits and nuclear spin. The weak interaction two-state oscillations of neutrinos affirmed the violation of Leggett–Garg-type inequalities (LGtI). We make an empirical test for the deviation of experimental results with the classical limits by analyzing the survival probability data of reactor neutrinos at a distinct range of baseline dividing energies, as an analog to a single neutrino detected at different times. A study of the updated data of the Daya Bay experiment unambiguously depicts an obvious cluster of data over the classical bound of LGtI and shows a 6.1σ significance of the violation of them.

1 Introduction

Nonclassical features of the quantum system have experienced extensive study since the inception of quantum mechanics. After a long debate between the believers of local realism and quantum mechanics, a breakthrough study, Bell's inequality (BI), was provided by Bell [1]. The unique feature of BI is its testable formula from the consequence of the famous hypothesis called local realism (LR). The LR believers assume that any observable value of an object, even if not detected, must have a definite value and that results of any individual measurement of the observables remain unaffected if they have a space-like separation. Extensive experimental investigations [2–4] over the past several decades

[a] e-mail: fuqiang@impcas.ac.cn
[b] e-mail: xchen@impcas.ac.cn

tested the violation of BI. These studies conclude that any local realism view of a microscopic object needs to be nonlocal. Based on these studies of BI, Leggett and Garg further derived a new series of inequalities [5] on the assumption of macrorealism (MR), now known as the Leggett–Garg inequalities (LGIs), that any system behaving as a macroscopic realism must obey. From the structure of the LGIs, we can see them as an analog of Bell's inequalities in temporal interpretation, which also makes it possible to implement a rigorous test of quantum mechanics on a macroscopic level, which is usually very difficult in designing experiments in space-like separation condition. By testing the LGtI, we can also perform a rigorously loophole-free test of quantum mechanics [2,6–8].

Besides the nonlocal behavior and quantum correlation between different particles, for single particle states there can also exist entanglement by the flavor transition [9]. Due to the oscillation of neutrino flavors, it offers an ideal source to test quantum mechanics in the case of the weak interaction and from a macroscopic point of view. For two-flavor neutrino oscillation, a two-level state's matrix can be expressed in the form

$$\rho = \frac{1}{4}\left[I \otimes I + (\mathbf{r} \cdot \sigma) \otimes I + I \otimes (\mathbf{s} \cdot \sigma) + \sum_{n,m=1}^{3} T_{mn}(\sigma_m \otimes \sigma_n) \right]. \tag{1}$$

Here the elements of the matrix T are $T_{mn} = Tr\left[\rho(\delta_m \otimes \delta_n)\right]$. For a two-qubit situation, many kinds of quantum correlations [10–12] like entanglement, fidelity, quantum discord and geometric discord have been derived to get their general expressions. Mermin and Svetlichny [13,14] inequalities were derived for three or even many body system that have two macroscopic distinct states. Using these results of

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two-level states, a series of quantum-information theory calculations [15–18] have been applied on neutrino flavor oscillation in the last several years. These quantum correlations can be directly linked to the probabilities of flavor oscillation, which leads to violation of the classical boundary limits when assuming the neutrino oscillation mixing angle is not vanishing.

Neutrino flavor oscillation is such a special process in that it is merely affected by the neutrino's own properties like mass square differences, mixing angles and the energies. Neutrinos just interact with matter by weak interaction with a quite low cross section. The influence of the environment on the neutrinos' propagation is much more negligible comparing with an optical or an electrical system, which makes neutrinos an ideal particle on testing the LGIs. As the mass eigenstate of a neutrino is not the same as its flavor eigenstate, during propagation, neutrinos undergo flavor mixing as regards the three flavored eigenstates. The MINOS experiment has been studied in Ref. [19], which observed the violation of Leggett–Garg-type inequalities, K3 and K4 terms, with a significance greater than 6σ [19]. The MINOS experiment is an accelerator neutrino experiment using decay in flight neutrinos with a fixed baseline distance of 735 km and a large range of ν_μ energy from 0.5 to 50 GeV, which happens to cover the largest violation of LGIs K3 and K4. The Daya Bay Collaboration reported an updated data analysis of the electron anti-neutrino disappearance channel [20], which gives a best fit of $sin^2 2\theta_{13} = 0.084 \pm 0.005$. We will investigate whether the Daya Bay reactor neutrino experiment can observe the violation of LGIs.

2 The Leggett–Garg-type inequalities

We focus on the simplest L–G-type inequality, which is constructed as follows. Consider a system with two absolutely distinguishable states corresponding to an observable quantity $Q(t)$ which can have two different values, $+1$ or -1. Assume that whenever the system was being measured, the observable quantity occupies a value of either $+1$ or -1 for being in state 1 or 2, respectively. Then we can define a macroscopic observable $Q(t)$ for the macroscopic system. We have $C_{t_i,t_j} = \langle Q(t_i)Q(t_j)\rangle$ as its two-time correlation function, where $Q(t_i)$ and $Q(t_j)$ are the observable quantity's values when being measured at time t_i and time t_j. In this work we consider the two states as the survival of electron anti-neutrino and the disappearance of the electron anti-neutrino. As the neutrinos are being created in the reactor by beta decay process, they are totally in the state of the flavor eigenstate. Since the PMNS matrix does not change with time, the two-flavor neutrino oscillation obeys the same survival probability. We shall introduce this stationarity assumption [21], which requires that

the evolution of the neutrino for different ordered time intervals is the same. Then $C(t_i, t_j) = C(t_i - t_j)$ (if $t_i < t_j$). Next, consider a sequence of times t_1, t_2, t_3 and t_4 (here, $t_1 < t_2 < t_3 < t_4$). If we take a series of measurements for $Q(t)$ in these four times, it is straightforward to determine four time correlations (C_{12}, C_{23}, C_{34} and C_{14}). Then it is possible to adopt the stationary condition on the standard LGI procedure leading to K4 LG-type inequality involving four correlation functions. For any sequence of measurements, any $Q(t_i)$ has a definite observable value, regardless of the choice of the pair $Q(t_i)Q(t_j)$ it belongs to. So, the combination $Q(t_1)Q(t_2) + Q(t_2)Q(t_3) + Q(t_3)Q(t_4) - Q(t_1)Q(t_4)$ lies always between -2 and +2. Similarly, the K_3 inequality lies between -1 and +1. If all the terms in the above formula are replaced by time correlations (average), the Leggett–Garg-type inequalities are in the form

$$K_3 \equiv C_{12} + C_{23} - C_{13} \leq 1,$$
$$K_4 \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2. \tag{2}$$

The above inequalities impose a constraint on macroscopic realism as regards the temporal separated joint probabilities in any two-state system.

3 Three-flavoured neutrino oscillations

It has extensively been verified that the flavor component of a neutrino oscillates during its propagation. The oscillation properties of different neutrino flavors are determined by their mixing angles (θ_{12}, θ_{23}, and θ_{13}), a CP phase of the Pontecorvo–Maki–Nakagawa–Sakata matrix and their mass-squared differences ($\Delta m_{32}^2, \Delta m_{21}^2$) [22,23]. Here, we will treat the Leggett–Garg-type inequalities using the updated measurement of $\overline{\nu_e}$ survival channel in the results of the Daya Bay experiment, where give the latest best fit of the mixing angle θ_{13} with large significance [20]. The measurement was updated later with a full detector configuration [20]. For the θ_{13} measurement, one used the baseline length dividing the $\overline{\nu_e}$ energy as the variable to depict the survival probability of $\overline{\nu_e}$ as

$$P_{\overline{\nu_e} \to \overline{\nu_e}} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\frac{1.267\Delta m_{21}^2 L}{E}$$
$$- \sin^2 2\theta_{13}\sin^2\frac{1.267\Delta m_{ee}^2 L}{E}, \tag{3}$$

where E is the energy of $\overline{\nu_e}$ in MeV, L is the propagation distance between the near and far point detector, θ_{12} is the solar neutrino mixing angle and Δm_{21}^2 is their mass-squared difference in eV^2. Notice that Δm_{ee}^2 is an effective mass-squared difference [24] in electron anti-neutrino disappearance with the form

$$\Delta m_{ee}^2 = \cos^2\theta_{12}\Delta m_{31}^2 + \sin^2\theta_{12}\Delta m_{32}^2. \tag{4}$$

Since $m_{12}^2 = (7.50 \pm 0.20) \times 10^{-5}$ eV^2, while $m_{ee}^2 = (2.42 \pm 0.11) \times 10^{-3}$ eV^2 according to Ref. [20], we can choose an appropriate value of the ratio L/E to make one of the terms of $\sin^2 \frac{1.267 \Delta m^2 L}{E}$ vanishing. For the Daya Bay experiment, the effect of the parameter θ_{12} becomes far less, sufficiently so that, compared with θ_{13}, θ_{12} can be regarded as negligibly small. Given that there is an initial pure electron anti-neutrino source, after propagation for a time t, the survival probability of $\bar{\nu}_e$ will be

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{1.267 \Delta m_{ee}^2 ct}{E}. \tag{5}$$

However, there is the MSW (Mikheyev–Smirnov–Wolfenstein) effect (usually called the matter effect) during the propagation of neutrino in matter. The effect is only significant for high energy neutrinos and a long range of matter, like the solar neutrino experiment. The KamLAND and Super-K P_{ee} day–night discrepancies are only obvious for larger than 6 MeV neutrinos [25,26]. Furthermore, the solar neutrino experiments involve the matter effect caused by the electron in the solar medium, which electron density $\epsilon_\odot$ is much larger than that in the Earth. Generally speaking, a neutrino vector of state in flavor basis $|\nu(t)\rangle = (\nu_e(t)\ \nu_\mu(t)\ \nu_\tau(t))^T$ obeys the Schrödinger equation:

$$i \frac{d}{dt} |\nu(t)\rangle = \mathcal{H} |\nu(t)\rangle \tag{6}$$

where the Hamiltonian can be replaced by an effective one as

$$\mathcal{H} \simeq \frac{1}{2E} U \, \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) U^\dagger + \text{diag}(V, 0, 0), \tag{7}$$

where V is the effective charged potential contribution to ν_e [27], given in the form

$$V(x) \simeq 7.56 \times 10^{-14} \left(\frac{\rho(x)}{\text{g/cm}^3} \right) Y_e(x) \text{ eV}, \tag{8}$$

where $\rho(x)$ is the matter density along the track path of the neutrino, $Y_e(x)$ (for the Earth $\simeq 0.5$) is the number of electrons normalized to the number of nucleons. For the matter of constant density, the series expansion for three-flavor neutrino oscillation probabilities can be derived from the Hamiltonian Eq. (7) [28]. For ν_e survival, the survival probability expansion to second order is

$$P_{ee} = 1 - \alpha \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2} - 4\sin^2 \theta_{13} \frac{\sin^2 (A-1)\Delta}{(A-1)^2}, \tag{9}$$

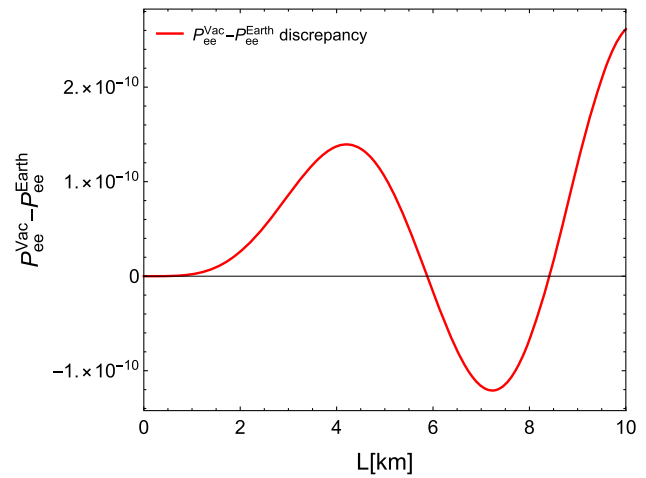where $\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \simeq 0.0297$, and the abbreviation for A and Δ is

Fig. 1 Illustration of the matter effect of the Earth in the interval of 0–10 km. The plot is the discrepancy of vacuum P_{ee}^{Vac} and the Earth's P_{ee}^{Earth}. The neutrino is 6 MeV monoenergetic, and $\rho(x) = 2.65$ g/cm^3 for standard rock

$$\Delta \equiv \frac{1.27 \Delta m_{31}^2 L}{E} \frac{[\text{ev}^2][\text{km}]}{[\text{GeV}]},$$
$$A \equiv \frac{2EV}{\Delta m_{31}^2 \times 10^{-3}} \frac{[\text{GeV}][\text{eV}]}{[\text{eV}^2]}. \tag{10}$$

For this Daya Bay analysis, we calculate the discrepancy of the P_{ee} probability of a 6 MeV neutrino in the range of 0 to 10 km covering the range of the experiment, about 2 km. From Fig. 1, we can draw the conclusion that the matter effect is too small to be included in the "short" baseline.

According to the expansion of the P_{ee} with matter effect mentioned above, we will use the oscillation probability just in the vacuum. Using the stationary assumption, one can derive the four joint probabilities $P_{\nu_\alpha, \nu_\beta}(t_i, t_j)$, here α and β stands for $\bar{\nu}_e$ and another neutrino flavor, i and j are from 1 to 4 defined above. The two-time correlation function C_{12} in this $P_{\bar{\nu}_e \to \bar{\nu}_e}$ is given by

$$C_{12} = 1 - \left[\sin 2\theta_{13} \sin \left(\frac{1.267 \Delta m_{ee}^2}{E} c(t_2 - t_1) \right) \right]^2$$
$$= 2 P_{\bar{\nu}_e \to \bar{\nu}_e}(t_2 - t_1) - 1. \tag{11}$$

Similarly, the correlation functions C_{23}, C_{34}, and C_{14} can be calculated. Using the Eq. (5), the quantity K_n^Q can be evaluated as defined in Eq. (2). By choosing the time intervals in a particular way, we can achieve a maximum value of K_4 when $t_4 - t_3 = t_3 - t_2 = t_2 - t_1 = \delta t$. Under this condition, the correlation functions depend on the baseline length L and the neutrino energy E_ν. We select the neutrinos' measured L_{eff}/E to make the oscillation phase $\psi_a = \frac{1.267 \Delta m^2}{E} c(\delta t)$ obey the sum rule: $\psi_{12} + \psi_{23} + \psi_{34} = \psi_{14}$. We have an experimental arrangement in which measurements occur at some fixed distance from the neutrino sources. Assuming the

neutrino begins in the pure $|\bar{\nu}_e\rangle$ state,

$$K_n^Q = -2 + 2\sum_{a=1}^{n-1} P_{ee}(\psi_a) - 2P_{ee}\left(\sum_{a=1}^{n-1}\psi_a\right). \quad (12)$$

Here n can be 3 or 4 in this paper, which corresponds to K_3 or K_4 LGI. In quantum mechanics, the commutators of operators can be nonvanishing. However, in a classical system, operators with observable values must commute; then the macrorealism derived K_n will become

$$K_n^C = \sum_{a=1}^{n-1} C_{i,i+1} - \prod_{a=1}^{n-1} C_{i,i+1}. \quad (13)$$

The Daya Bay Collaboration released updated oscillation results as a function of the effective baseline distance L_{eff} over the average energy $\langle E_\nu \rangle$ in bins [20]. For their six anti-neutrino detectors (ADs) placed in three separate experimental halls (EHs) and three nuclear reactors neutrino sources, the effective baseline varies for each detected anti-neutrino. The Daya Bay experiment covers an energy between 1 and 8 MeV. The ranges of effective baseline and energy correspond to a phase range of $(0, 3/4\pi)$, within which the violations of LGI will be observed near the minimum point of the anti-neutrino survival probability.

To test the violations of the K_3 and K_4 inequalities, we address the data from the Daya Bay neutrino experiment. The Daya Bay experiment extracted the survival probabilities of neutrinos using Daya Bay and Ling-Ao nuclear power stations' reactors. We use all the measurement positions including EH1, EH2 and EH3 of the Daya Bay. The reactors provide different sources of neutrinos with several fixed baselines and an energy spectrum with peaks. We make a θ_{13} fit over the Daya Bay updated data and get the fit error band and center value of P_{ee} shown in Fig. 2. With the best fit of $\sin^2 2\theta_{13}$ and the 1σ error band of it, we generate a large set of pseudodata. Then we select all sets of data points in Fig. 3 which obey the sum rule of phase with the precision of 0.5% ($\psi_1 + \psi_2 \in \psi_3$) and 0.1% ($\psi_1 + \psi_2 + \psi_3 \in \psi_4$) for the K_3 and K_4 respectively. For the K_3 (K_4) situation, 48 (56) correlation triples (quadruples) satisfy the sum rule. Meanwhile the updated measurement only includes the static errors and we simply assumed that the errors at small phase of the oscillation probability are the fitting error.

The violation of Leggett–Garg-type inequalities has been tested and confirmed by the MINOS experiment, with the K_3 and K_4 being inconsistent with the realism prediction over 5σ [19]. Since the violation of Leggett–Garg-type inequalities happens when the mixing angle of two flavors is not zero, we suppose that the violation could be observed in the $\bar{\nu}_e$ survival channel at Daya Bay. In order to estimate the significance from events as regards the number of violations, we simulated the statistical quantity by creating a large sample of pseudodata based on the fitting result of the observed P_{ee}
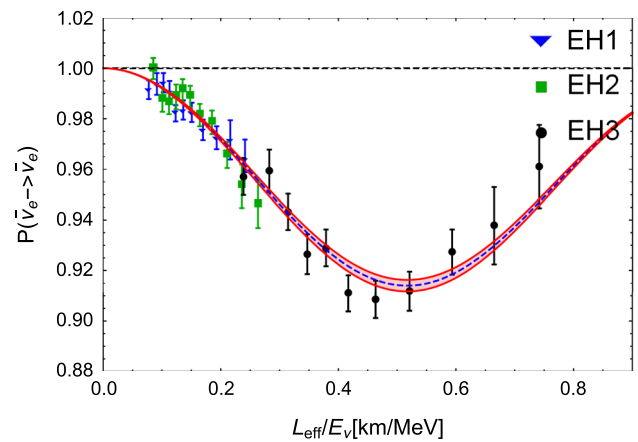

Fig. 2 The electron anti-neutrino survival probability versus effective propagation distance L_{eff} over anti-neutrino energy E_ν. The dashed blue curve indicates the prediction fitting directly to the measured Daya Bay values of P_{ee}. The red band indicates a 1σ confidence interval around the fitted prediction. The blue triangles, green rectangles and black dots are the binned data of the Daya Bay EH1, EH2 and EH3 from Ref. [20], respectively

values. The pseudodata is generated by a Gaussian distribution model with the means and variances matched to the center values and deviations of the best fit. Each set of simulated data gives an artificial number of LGI violations for K_3 and K_4, from which we can calculate the level of inconsistency of the predictions between quantum and classical K_n.

To estimate the confidence level of these results being inconsistent with expectation and prediction from realism, we make a fit of the histogram filled by predicted LGI violations number under the realism model of Eq. (7) to a beta-binomial distribution, thus to estimate the deviation of classical predictions from the actually observed number of LGI violations. For the actual number of LGI violations (41 in 48 data points), there exists a 6.1σ deviation from the expected distribution of the classical prediction.

A similar statistical test is made for LGI K_4. Using the filter of the phase sum rule described above, we get a number of 30 (in total of 56 data points) exceeding the classical limits. As Fig. 4 shows, there are obvious clusters of points over the classical bound of K_3 and K_4. The discrepancy between the observed events number and the classical predicted events originating from the fluctuation is very clear. Our K_4 data also possesses a 6σ deviation from the classical prediction.

4 Discussions

The results mentioned above clearly constrain the validity of quantum mechanics in such a macroscopic area. Values of LGtI K_3 and K_4 are violated with the QM prediction at a confidence level of over 6σ compared with the classical bound
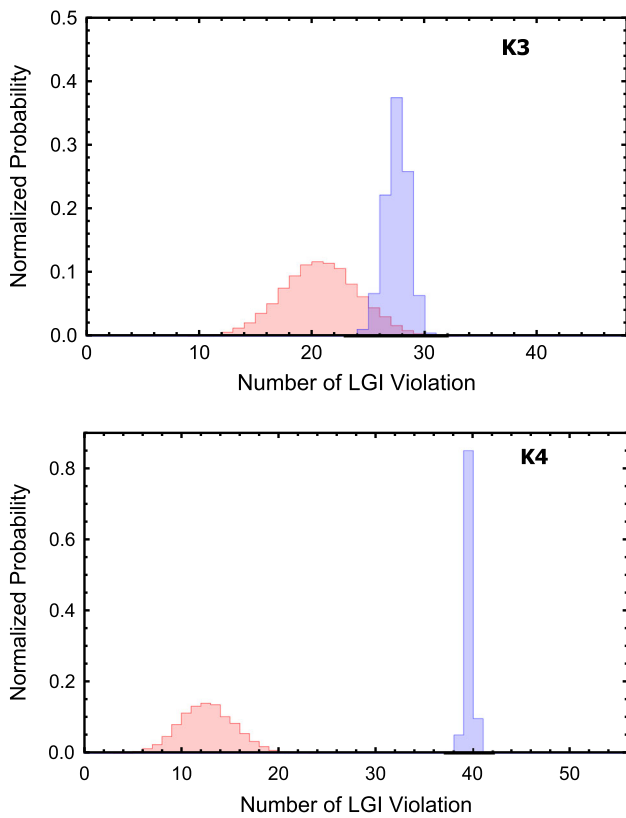
Fig. 3 The histograms of the number of K3 (upper) and K4 (lower) values that violate the LGI bound. The left curves with red filling indicate the expected classical distributions, while the right curves with blue filling indicate the corresponding quantum quantity

Fig. 4 The distribution of K3 (upper) and K4 (lower) versus the effective propagation length divided by neutrino energy reconstructed from P_{ee}. The black dot data show a cluster over the LGI bound. We also show the expected distributions of classical (red circles) and quantum (blue circle) predictions. Note that the K_3 and K_4 can be multiple values, since there are many triples and quadruples satisfying the phase sum rule

for the neutrino θ_{13} mixing in our estimation. Anti-electron neutrino oscillations also violate the limits of Leggett–Garg inequality. The detected violations act as a new affirmation of quantum nonlocality existing in the neutrino system during its long-range propagation. These violations were observed over the near and far detectors placed at three experimental halls (EHs) with the baseline long enough to make the test not being a Bell-like inequality test. Besides, it should be worthwhile to make a detailed data analysis on the Daya Bay experiment involving three-flavor neutrino oscillation, in order to achieve more data points of LGI K_3 and K_4. It could be worth to test the quantum mechanics in such a weak interaction context. Although tests of incompatible of LGI and QM have been achieved in photonics and electronic experiments [29,30], nuclear spin qubits [31] and even condensed states [32], there are few reports of LGI violation in particle physics. Even though the MINOS and Daya Bay experimental setup show the LGtI violations, these two experiments are all in the context of two-flavor neutrino oscillation, which can not reveal the CP violation. Since entanglement exists between a pair of neutral meson and anti-meson, which will violate the Bell inequality [33], three-flavor oscillation analysis involving neutrinos and anti-neutrinos may shed light on the study of CP-violating phase. [34]
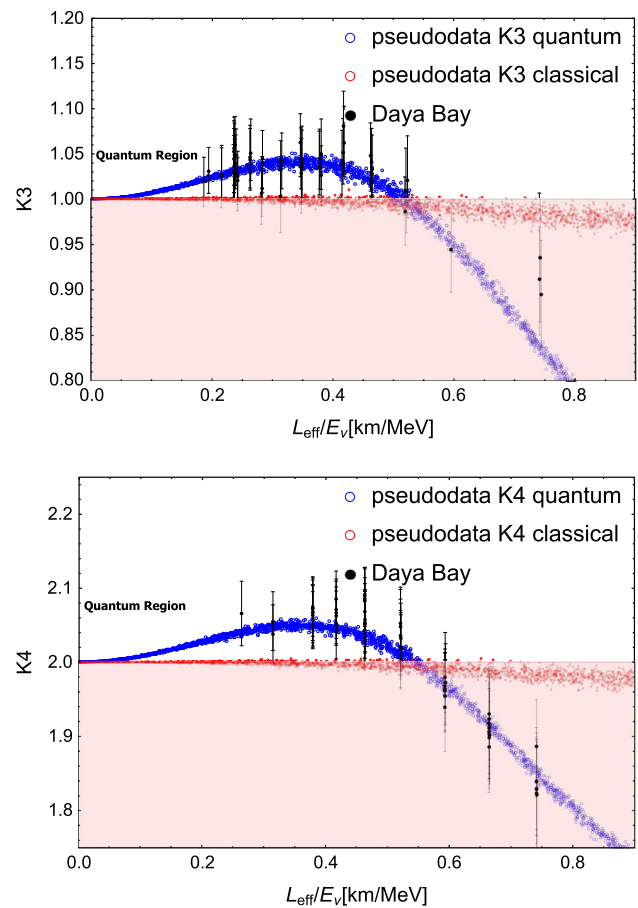
References

1. J.S. Bell, On the Einstein–Podolsky–Rosen paradox. Physics **1**, 195 (1964)
2. B. Hensen et al., Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. Nature **526**, 682 (2015). arXiv:1508.05949

3. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Violation of Bell's inequality under strict Einstein locality conditions. Phys. Rev. Lett. **81**, 5039 (1998). arXiv:quant-ph/9810080

4. A. Aspect, P. Grangier, G. Roger, Experimental realization of Einstein–Podolsky–Rosen–Bohm Gedankenexperiment: a new violation of Bell's inequalities. Phys. Rev. Lett. **49**, 91 (1982)

5. A.J. Leggett, A. Garg, Quantum mechanics versus macroscopic realism: Is the flux there when nobody looks? Phys. Rev. Lett. **54**, 857 (1985)

6. L.K. Shalm, E. Meyer-Scott, B.G. Christensen, P. Bierhorst, M.A. Wayne, M.J. Stevens, T. Gerrits, S. Glancy, D.R. Hamel, M.S. Allman, K.J. Coakley, S.D. Dyer, C. Hodge, A.E. Lita, V.B. Verma, C. Lambrocco, E. Tortorici, A.L. Migdall, Y. Zhang, D.R. Kumor, W.H. Farr, F. Marsili, M.D. Shaw, J.A. Stern, C. Abellán, W. Amaya, V. Pruneri, T. Jennewein, M.W. Mitchell, P.G. Kwiat, J.C. Bienfang, R.P. Mirin, E. Knill, S.W. Nam, Strong loophole-free test of local realism. Phys. Rev. Lett. **115**, 250402 (2015)

7. M. Giustina, M.A.M. Versteegh, S. Wengerowsky, J. Handsteiner, A. Hochrainer, K. Phelan, F. Steinlechner, J. Kofler, J.-A. Larsson, C. Abellán, W. Amaya, V. Pruneri, M.W. Mitchell, J. Beyer, T. Gerrits, A.E. Lita, L.K. Shalm, S.W. Nam, T. Scheidl, R. Ursin, B. Wittmann, A. Zeilinger, Significant-loophole-free test of Bell's theorem with entangled photons. Phys. Rev. Lett. **115**, 250401 (2015)

8. J. Gallicchio, A.S. Friedman, D.I. Kaiser, Testing Bells inequality with cosmic photons: closing the setting-independence loophole. Phys. Rev. Lett. **112**, 110405 (2014). arXiv:1310.3288

9. M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, Entropy, entanglement, and transition probabilities in neutrino oscillations. EPL **85**, 0002 (2009). arXiv:0707.4476

10. S. Hill, W.K. Wootters, Entanglement of a pair of quantum bits. Phys. Rev. Lett. **78**, 5022 (1997)

11. W.K. Wootters, Entanglement of formation of an arbitrary state of two qubits. Phys. Rev. Lett. **80**, 2245 (1998)

12. B. Dakić, V. Vedral, i c v Brukner, Necessary and sufficient condition for nonzero quantum discord. Phys. Rev. Lett. **105**, 190502 (2010)

13. N.D. Mermin, Extreme quantum entanglement in a superposition of macroscopically distinct states. Phys. Rev. Lett. **65**, 1838 (1990)

14. G. Svetlichny, Distinguishing three-body from two-body nonseparability by a Bell-type inequality. Phys. Rev. D **35**, 3066 (1987)

15. D. Gangopadhyay, D. Home, A.S. Roy, Probing the Leggett-Garg inequality for oscillating neutral kaons and neutrinos. Phys. Rev. A **88**, 022115 (2013)

16. S. Banerjee, A.K. Alok, R. Srikanth, B.C. Hiesmayr, A quantum-information theoretic analysis of three-flavor neutrino oscillations. Eur. Phys. J. C **75**, 487 (2015)

17. A.K. Alok, S. Banerjee, S.U. Sankar, Quantum correlations in terms of neutrino oscillation probabilities. Nucl. Phys. B **909**, 65 (2016). arXiv:1411.5536

18. D. Gangopadhyay, A.S. Roy, Three-flavoured neutrino oscillations and the Leggett-Garg inequality. Eur. Phys. J. C **77**, 260 (2017)

19. J.A. Formaggio, D.I. Kaiser, M.M. Murskyj, T.E. Weiss, Violation of the Leggett-Garg inequality in neutrino oscillations. Phys. Rev. Lett. **117**, 050402 (2016)

20. F.P. An, A.B. Balantekin, HRe Band, New measurement of antineutrino oscillation with the full detector configuration at daya bay. Phys. Rev. Lett. **115**, 111802 (2015)

21. S.F. Huelga, T.W. Marshall, E. Santos, Proposed test for realist theories using rydberg atoms coupled to a high-*q* resonator. Phys. Rev. A **52**, R2497 (1995)

22. B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge. Sov. Phys. JETP **26**, 984 (1968). [Zh. Eksp. Teor. Fiz. **53**, 1717 (1967)]

23. Z. Maki, M. Nakagawa, S. Sakata, Remarks on the unified model of elementary particles. Prog. Theor. Phys. **28**, 870 (1962)

24. H. Nunokawa, S. Parke, R.Z. Funchal, Another possible way to determine the neutrino mass hierarchy. Phys. Rev. D **72**, 013009 (2005)

25. E. Lisi, D. Montanino, Earth regeneration effect in solar neutrino oscillations: an analytic approach. Phys. Rev. D **56**, 1792 (1997)

26. F. Vissani, Solar neutrino physics on the beginning of 2017 (2017). arXiv:1706.05435

27. L. Wolfenstein, Neutrino oscillations in matter. Phys. Rev. D **17**, 2369 (1978)

28. E.K. Akhmedov, R. Johansson, M. Lindner, T. Ohlsson, T. Schwetz, Series expansions for three flavor neutrino oscillation probabilities in matter. JHEP **04**, 078 (2004). arXiv:hep-ph/0402175

29. C. Emary, Leggett-Garg inequalities for the statistics of electron transport. Phys. Rev. B **86**, 085418 (2012)

30. J. Kofler, i c v Brukner, Condition for macroscopic realism beyond the Leggett-Garg inequalities. Phys. Rev. A **87**, 052115 (2013)

31. G. Waldherr, P. Neumann, S.F. Huelga, F. Jelezko, J. Wrachtrup, Violation of a temporal Bell inequality for single spins in a diamond defect center. Phys. Rev. Lett. **107**, 090401 (2011)

32. Z.-Q. Zhou, S.F. Huelga, C.-F. Li, G.-C. Guo, Experimental detection of quantum coherent evolution through the violation of Leggett-Garg-type inequalities. Phys. Rev. Lett. **115**, 113002 (2015)

33. B.C. Hiesmayr, A. Di Domenico, C. Curceanu, A. Gabriel, M. Huber, J.-Å. Larsson, P. Moskal, Revealing Bell's nonlocality for unstable systems in high energy physics. Eur. Phys. J. C **72**, 1856 (2012)

34. J. Naikoo, A.K. Alok, S. Banerjee, S.U. Sankar, G. Guarnieri, B.C. Hiesmayr, Legget–Garg-type inequalities and the neutrino mass-degeneracy problem (2017). arXiv:1710.05562