

A search for sterile neutrinos with the latest cosmological observations

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Abstract We report the result of a search for sterile neutrinos with the latest cosmological observations. Both cases of massless and massive sterile neutrinos are considered in the Λ CDM cosmology. The cosmological observations used in this work include the Planck 2015 temperature and polarization data, the baryon acoustic oscillation data, the Hubble constant direct measurement data, the Planck Sunyaev–Zeldovich cluster counts data, the Planck lensing data, and the cosmic shear data. We find that the current observational data give a hint of the existence of massless sterile neutrino (as dark radiation) at the 1.44σ level, and the consideration of an extra massless sterile neutrino can indeed relieve the tension between observations and improve the cosmological fit. For the case of massive sterile neutrino, the observations give a rather tight upper limit on the mass, which implies that actually a massless sterile neutrino is more favored. Our result is consistent with the recent result of neutrino oscillation experiment done by the Daya Bay and MINOS collaborations, as well as the recent result of cosmic ray experiment done by the IceCube collaboration.

1 Introduction

Since the discovery of the acceleration of expansion of the universe, dark energy has been proposed to explain the cause of the acceleration phenomenon [1–7]. The cosmological constant Λ [8] is the preferred candidate for dark energy. In practice, the Λ cold dark matter (Λ CDM) model has been achieving successes in fitting various observational data (it is still the best model so far in fitting data; see, e.g., Refs. [9, 10]). However, it has been found that several important astrophysical observations are inconsistent with the Planck observation (based on the Λ CDM cosmology), to some extent. For example, the fit results of the six-parameter

base Λ CDM model based on the Planck+WP+highL data [11] are in tension with the direct measurement of the Hubble constant [12], the Planck Sunyaev–Zeldovich clusters counts [13], and the cosmic shear measurement of CFHTLenS survey [14].

In our previous work [15], we have shown that involving a light (sub-eV) sterile neutrino species in the Λ CDM model can help to reconcile the tensions between Planck and above-mentioned astrophysical observations (see also Refs. [16–19]). It was shown that [15], under the CMB+BAO constraint, when taking the sterile neutrino into account, the inconsistencies with Planck are improved from 2.4σ to 1.0σ for H_0 observation, from 4.3σ to 2.0σ for SZ cluster counts observation, and from 2.3σ to 1.7σ for cosmic shear observation. For other relevant studies of sterile neutrinos, see, e.g., Refs. [20–30]. Thus, to relieve the tensions in cosmological observations, a natural choice is to consider the model with light sterile neutrinos.

The possibility of the existence of light sterile neutrinos has been motivated to explain the anomalies of short-baseline neutrino experiments [31–40]. It seems that the fully thermalized ($\Delta N_{\text{eff}} \approx 1$) sterile neutrinos with eV-scale mass are needed to explain these results [41–43]. The cosmological observations play an important role in constraining the total mass of the active neutrinos (see, e.g., Refs. [44–66]). In fact, since the sterile neutrinos have some effects on the evolution of the universe, the cosmological observations can also provide independent evidence in searching for sterile neutrinos [15].

Recently, some important updated observational data were released. For example, Riess et al. [67] reported the new result of local measurement of the Hubble constant, $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is 3.3σ higher than the result of $66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ derived by the Planck mission based on the Λ CDM model with $\sum m_\nu = 0.06 \text{ eV}$ using the latest Planck CMB data. Moreover, the BAO measurements were also updated for the CMASS and LOWZ

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galaxy samples, done by the Data Release 12 (DR12) [68] of SDSS-III BOSS (Baryon Oscillation Spectroscopic Survey).

The aim of this work is to search for the sterile neutrinos by using the latest cosmological observations. We shall investigate how the sterile neutrinos can relieve the tensions in the current observations and whether the current cosmological data can provide evidence for the existence of sterile neutrino.

This paper is organized as follows. In Sect. 2, we introduce the analysis method and the observational data we use in this paper. The results are given and discussed in Sect. 3. Conclusion is given in Sect. 4.

2 Analysis method and observational data

In this section, we introduce the analysis method and the observational data that will be used in this paper.

2.1 Analysis method

We will consider the two cases of massless and massive sterile neutrinos in the framework of Λ CDM cosmology. The massless sterile neutrino serves as the dark radiation, and thus when this case is considered, an additional parameter N_{eff} needs to be added in the model; this case is called Λ CDM+ N_{eff} model in this paper. When the sterile neutrino is considered to be massive, then one needs to add another extra parameter, $m_{\nu, \text{sterile}}^{\text{eff}}$, in the model; this case is thus called Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model in this paper.

In the case of massive sterile neutrino, the true mass of a thermally distributed sterile neutrino reads $m_{\text{sterile}}^{\text{thermal}} = (N_{\text{eff}} - 3.046)^{-3/4} m_{\nu, \text{sterile}}^{\text{eff}}$. In order to avoid a negative $m_{\text{sterile}}^{\text{thermal}}$, N_{eff} must be larger than 3.046 in a universe with sterile neutrinos. In this work, the active neutrino mass is kept at 0.06 eV (i.e., the minimal-mass normal hierarchy is assumed).

We place constraints on the Λ CDM cosmology with sterile neutrinos by using the current observational data. The conventions used in this paper are consistent with those adopted by the Planck collaboration [69], i.e., those used in the `cambridge` Boltzmann code [70]. There are six independent cosmological parameters in the base Λ CDM model,

$$\mathbf{P} = \{\omega_b, \omega_c, 100\theta_{\text{MC}}, \tau, \ln(10^{10} A_s), n_s\},$$

where $\omega_b \equiv \Omega_b h^2$ and $\omega_c \equiv \Omega_c h^2$ are the present-day baryon and cold dark matter densities, respectively, θ_{MC} is the ratio between the sound horizon and the angular diameter distance at the decoupling epoch, τ is the Thomson scattering optical depth due to reionization, A_s is the amplitude of initial curvature perturbation power spectrum at $k = 0.05 \text{ Mpc}^{-1}$, and n_s is its spectral index. In addition,

there are two additional free parameters, N_{eff} and $m_{\nu, \text{sterile}}^{\text{eff}}$, for describing the sterile neutrino. Thus, there are seven independent parameters in total for the Λ CDM+ N_{eff} model, and there are eight independent parameters in total for the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model. Other parameters, such as Ω_m , σ_8 , H_0 , and so on, are the derived parameters.

We use the `cosmoMC` package [71] to infer the posterior probability distributions of parameters. Flat priors for the base parameters are used. The prior ranges for the base parameters are chosen to be much wider than the posterior ranges in order not to affect the results of parameter estimation.

2.2 Observational data

In this paper, the data sets we use include the cosmic microwave background (CMB), the baryon acoustic oscillations (BAO), the Hubble constant (H_0), the Planck Sunyaev–Zeldovich (SZ), the Planck lensing, and the weak lensing (WL) observations.

The CMB data: We use the Planck 2015 CMB temperature and polarization data [72] in our calculations. We consider the combination of the likelihood at $30 \leq \ell \leq 2500$ in the temperature (TT), the cross-correlation of temperature and polarization (TE), and the polarization (EE) power spectra and the Planck low- ℓ likelihood in the range of $2 \leq \ell \leq 29$, which is denoted “Planck TT,TE,EE+lowP”, following the nomenclature of the Planck collaboration [69].

The BAO data: In order to break the geometric degeneracy, it is necessary to consider the BAO data. We use the LOWZ ($z_{\text{eff}} = 0.32$) and CMASS ($z_{\text{eff}} = 0.57$) samples of BOSS DR12 [68], as well as the 6dFGS (six-degree-field galaxy survey) ($z_{\text{eff}} = 0.106$) sample [73] and the SDSS MGS (main galaxy sample) ($z_{\text{eff}} = 0.15$) sample [74].

The H_0 measurement: We use the recently measured new local value of the Hubble constant, $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, reported in Ref. [67].

The SZ data: The counts of rich clusters of galaxies are from the sample of Planck thermal Sunyaev–Zeldovich (SZ) cluster observation [75].

The Lensing data: We use the Planck lensing data [76], which provide additional information at low redshift.

The WL data: We use the cosmic shear data of weak lensing (WL) from the CFHTLenS survey [77].

In what follows, we will use these observational data to place constraints on the Λ CDM cosmology with sterile neutrinos. We will compare the Λ CDM, Λ CDM+ N_{eff} , and Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ models under the uniform data sets. The basic data combination adopted in this paper is the Planck TT,TE,EE+lowP+BAO combination. In order to show the impacts from the other astrophysical observations on measuring the properties of the sterile neutrino, we also further combine the H_0 +SZ+Lensing+WL data in the anal-

ysis. Thus, in our analysis, we use the two data combinations: (i) Planck TT,TE,EE+lowP+BAO, and (ii) Planck TT,TE,EE+lowP+BAO+ H_0 +SZ+Lensing+WL. For convenience, we occasionally use the abbreviations “CMB+BAO” and “CMB+BAO+other” for them in the paper. In the next section, we will report and discuss the fitting results of the cosmological models in the light of these data sets.

3 Results and discussion

In this section, we report the fitting results of the cosmological models (the Λ CDM model, the Λ CDM+ N_{eff} model, and the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model) and discuss the implications of these results in the search for sterile neutrinos. We will discuss the cases of massless and massive sterile neutrinos, respectively, in the two subsections.

Detailed fit values for the three models for cosmological parameters are given in Table 1. In the table, we quote the $\pm 1\sigma$ errors, but for the parameters that cannot be well constrained, we quote the 95.4% CL upper limits.

When we make comparison for the three models from the statistical point of view, we must be aware of the fact that they have different numbers of parameters. In general, a model with more parameters tends to give a better fit to the same data, i.e., it tends to have a smaller χ_{min}^2 . Thus, when the comparison is made for models with different parameter numbers, the simple comparison of χ_{min}^2 is not appropriate because it is unfair. A punishment mechanism must be considered for those models with more parameters. The simplest way, for our purpose in this work, is to consider the Akaike information criterion (AIC) [78], with the definition $\text{AIC} = \chi_{\text{min}}^2 + 2k$, where k is the number of parameters of a model. A model with a lower AIC is more favored by data. So, when we make model selection for two models, we will calculate the difference of AIC for them, i.e., $\Delta\text{AIC} = \Delta\chi_{\text{min}}^2 + 2\Delta k$. In the case of this paper, we take the Λ CDM model as a reference model, and thus the Λ CDM+ N_{eff} model has $\Delta k = 1$ and the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model has $\Delta k = 2$. They are considered to be more favored over the Λ CDM model provided that the Λ CDM+ N_{eff} model has $\Delta\chi^2 < -2$ and the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model has $\Delta\chi^2 < -4$.

3.1 The case of massless sterile neutrino

The massless sterile neutrinos serve as the dark radiation, and thus in this case the effective number of relativistic species N_{eff} is treated as a free parameter. The total energy density of radiation in the universe is given by

$$\rho_r = \left[1 + N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} \right] \rho_\gamma,$$

Table 1 Fitting results for the Λ CDM model, the Λ CDM+ N_{eff} model, and the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model. We quote $\pm 1\sigma$ errors, but for the parameters that cannot be well constrained, we quote the 95.4% CL upper limits

Data Model	Planck TT,TE,EE+lowP+BAO			Planck TT,TE,EE+lowP+BAO+ H_0 +SZ+lensing+WL		
	Λ CDM	Λ CDM+ N_{eff}	Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$	Λ CDM	Λ CDM+ N_{eff}	Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$
$\Omega_b h^2$	0.0223 ± 0.0001	0.0225 ± 0.0002	0.0224 ± 0.0002	0.0224 ± 0.0001	0.0226 ± 0.0002	0.0226 ± 0.0002
$\Omega_c h^2$	0.1186 ± 0.0010	$0.1212^{+0.0017}_{-0.0027}$	$0.1187^{+0.0044}_{-0.0027}$	0.1175 ± 0.0010	$0.1214^{+0.0021}_{-0.0028}$	$0.1210^{+0.0026}_{-0.0031}$
$100\theta_{\text{MC}}$	1.0409 ± 0.0003	1.0406 ± 0.0004	$1.0407^{+0.0004}_{-0.0003}$	1.0411 ± 0.0003	1.0406 ± 0.0004	1.0407 ± 0.0004
τ	$0.085^{+0.017}_{-0.016}$	$0.088^{+0.018}_{-0.017}$	0.090 ± 0.017	0.072 ± 0.012	0.071 ± 0.012	0.079 ± 0.014
$\ln(10^{10} A_s)$	$3.103^{+0.033}_{-0.032}$	$3.114^{+0.036}_{-0.033}$	$3.116^{+0.034}_{-0.035}$	3.072 ± 0.022	3.081 ± 0.023	3.096 ± 0.028
n_s	$0.9676^{+0.0039}_{-0.0040}$	$0.9732^{+0.0055}_{-0.0064}$	$0.9719^{+0.0050}_{-0.0078}$	$0.9699^{+0.0040}_{-0.0039}$	$0.9774^{+0.0056}_{-0.0058}$	$0.9789^{+0.0064}_{-0.0075}$
$m_{\nu,\text{sterile}}^{\text{eff}}$	<0.7279	<0.2417
N_{eff}	...	<3.4384	<3.4273	...	$3.29^{+0.11}_{-0.17}$	$3.30^{+0.12}_{-0.20}$
Ω_m	$0.3080^{+0.0061}_{-0.0062}$	0.3048 ± 0.0067	$0.3085^{+0.0067}_{-0.0068}$	$0.3011^{+0.0057}_{-0.0058}$	$0.2975^{+0.0059}_{-0.0064}$	$0.3010^{+0.0065}_{-0.0070}$
σ_8	0.832 ± 0.013	0.841 ± 0.015	$0.817^{+0.030}_{-0.021}$	$0.8154^{+0.0085}_{-0.0084}$	$0.826^{+0.010}_{-0.011}$	$0.814^{+0.018}_{-0.013}$
H_0	$67.81^{+0.47}_{-0.46}$	$68.81^{+0.71}_{-1.02}$	$68.30^{+0.53}_{-1.00}$	68.33 ± 0.45	$69.71^{+0.84}_{-1.01}$	$69.44^{+0.84}_{-1.14}$
χ_{min}^2	12952.922	12952.976	12953.554	13140.214	13138.042	13141.042

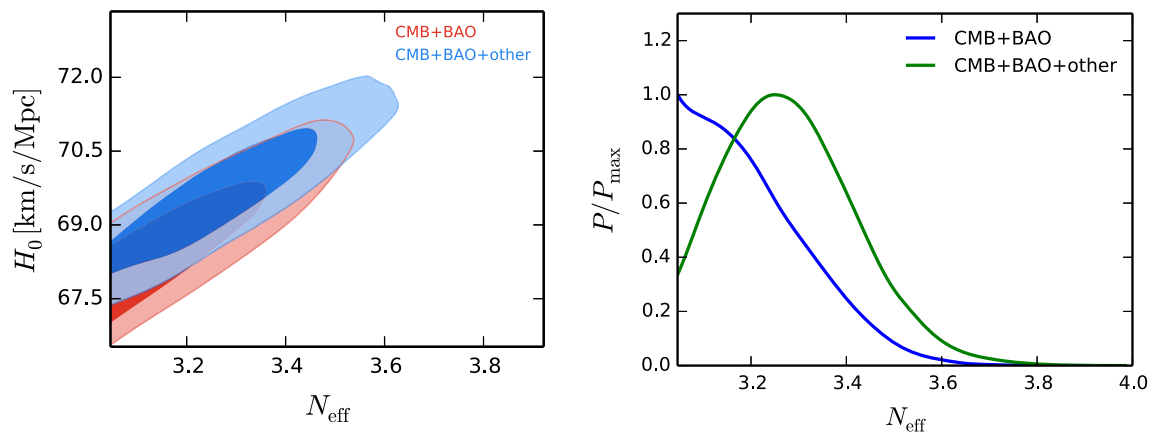


Fig. 1 Constraint results for the $\Lambda\text{CDM}+N_{\text{eff}}$ model from the CMB+BAO data combination and the CMB+BAO+other data combination. *Left panel* Two-dimensional marginalized posterior contours (68.3

and 95.4% CL) in the $N_{\text{eff}}-H_0$ plane. *Right panel* One-dimensional marginalized posterior distributions for N_{eff}

where ρ_γ is the energy density of photons. The standard case of three-generation neutrinos leads to $N_{\text{eff}} = 3.046$ [79, 80]. The detection of $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046 > 0$ indicates the presence of extra relativistic particle species in the universe, and in this paper we take the fitting result of $\Delta N_{\text{eff}} > 0$ as evidence of the existence of massless sterile neutrinos.

In this subsection, we constrain the $\Lambda\text{CDM}+N_{\text{eff}}$ model by using two combinations of data sets, namely, the CMB+BAO and CMB+BAO+other combinations. As mentioned above, hereafter, we shall use “CMB” to denote Planck TT,TE,EE+lowP and use “other” to denote H_0 +SZ+lensing+WL. The free parameters include six base parameters and N_{eff} , and we fix the active neutrino mass $\sum m_\nu = 0.06$ eV (two massless and one massive active neutrinos). The results of constraints on N_{eff} can be found in Table 1, and in this table the constraint results of other free parameters are also listed.

To directly show how the effective number of relativistic species N_{eff} affects the constraints on H_0 , we plot the two-dimensional posterior distribution contours (68.3% and 95.4% CL) in the $N_{\text{eff}}-H_0$ plane for the $\Lambda\text{CDM}+N_{\text{eff}}$ model, under the constraints from the two data combinations in the left panel of Fig. 1. We find that, in the $\Lambda\text{CDM}+N_{\text{eff}}$ model, H_0 is positively correlated with N_{eff} , for the two data combinations. Moreover, under the constraints of CMB+BAO+other data, the ΛCDM model gives $H_0 = 68.33 \pm 0.45$ $\text{km s}^{-1} \text{Mpc}^{-1}$, and the $\Lambda\text{CDM}+N_{\text{eff}}$ model gives $H_0 = 69.71^{+0.84}_{-1.01}$ $\text{km s}^{-1} \text{Mpc}^{-1}$, indicating that the tension with the direct measurement of Hubble constant, $H_0 = 73.00 \pm 1.75$ $\text{km s}^{-1} \text{Mpc}^{-1}$, is relieved to be at 2.58σ level and 1.69σ level, respectively. Therefore, the addition of the parameter N_{eff} can indeed relieve the tension between Planck data and H_0 direct measurement.

In the right panel of Fig. 1, we show the one-dimensional marginalized posterior distributions of N_{eff} for the $\Lambda\text{CDM}+$

N_{eff} model using the two data combinations. We find that the CMB+BAO data can only give an upper limit, $N_{\text{eff}} < 3.44$ (95.4% CL), but the CMB+BAO+other data can well constrain N_{eff} , giving the result of $N_{\text{eff}} = 3.29^{+0.11}_{-0.17}$, which indicates a detection of $\Delta N_{\text{eff}} > 0$ at the 1.44σ level.

Fitting to the CMB+BAO data, the ΛCDM and $\Lambda\text{CDM}+N_{\text{eff}}$ models yield a similar χ^2_{min} , implying that with this data combination adding the parameter N_{eff} cannot improve the fit. But, when fitting to the CMB+BAO+other data, the $\Lambda\text{CDM}+N_{\text{eff}}$ model leads to an increase of $\Delta\chi^2 = -2.172$, compared to the ΛCDM model, indicating that in this case the addition of the parameter N_{eff} can evidently improve the fit. Actually, when we use the information criterion to make a model selection, we have $\Delta\text{AIC} = -0.172$ for the $\Lambda\text{CDM}+N_{\text{eff}}$ model in this case, which shows that the $\Lambda\text{CDM}+N_{\text{eff}}$ model is only slightly better than the ΛCDM model from the statistical point of view.

Therefore, we find that the current CMB+BAO+other data can give a hint of the existence of massless sterile neutrino (as dark radiation) at the 1.44σ level, and the consideration of an extra massless sterile neutrino can indeed relieve the tension between observations and improve the cosmological fit.

3.2 The case of massive sterile neutrino

In this subsection, we further consider the case of massive sterile neutrino, i.e., we consider two extra parameters, N_{eff} and $m_{\nu, \text{sterile}}^{\text{eff}}$, compared to the ΛCDM model.

We make a comparison for the ΛCDM model and $\Lambda\text{CDM}+N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model using the two data combinations. In Fig. 2, we show the posterior distribution contours in the Ω_m-H_0 plane, for the two models. In the left panel, we show the case of fitting to the CMB+BAO data,

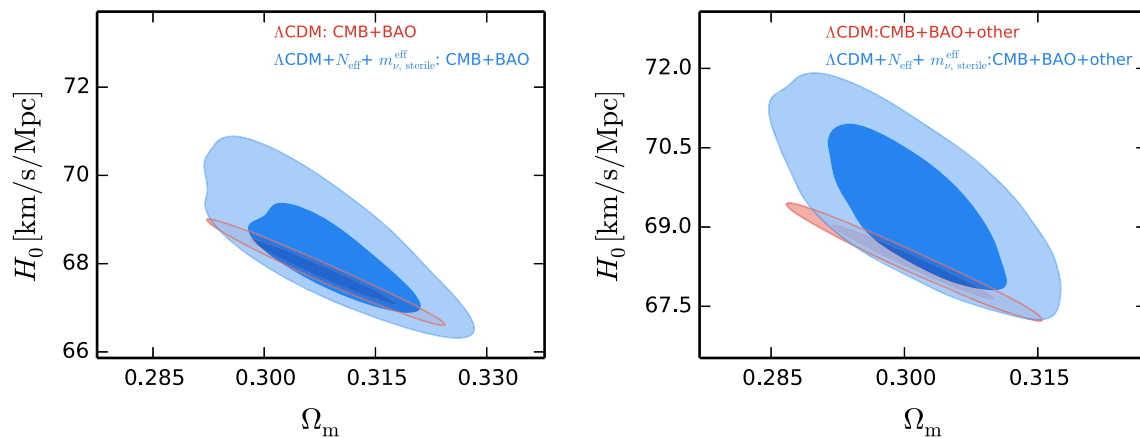


Fig. 2 The two-dimensional marginalized contours (68.3 and 95.4% CL) in the Ω_m – H_0 plane, for the Λ CDM and Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ models, from the constraints of CMB+BAO (left) and CMB+BAO+other (right)

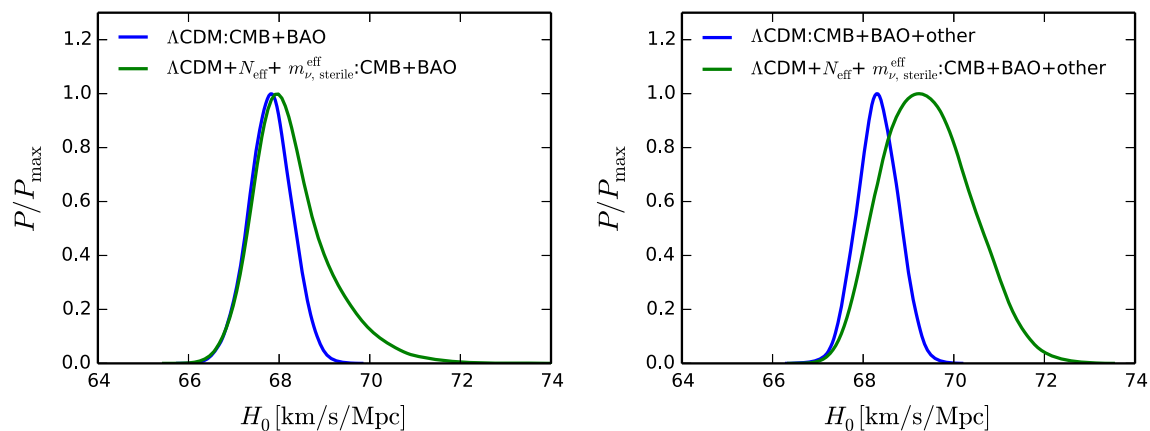


Fig. 3 The one-dimensional posterior distributions of H_0 in the Λ CDM and Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ models, from the constraints of CMB+BAO (left) and CMB+BAO+other (right)

and in the right panel, we show the case of fitting to the CMB+BAO+other data. We find that H_0 is anti-correlated with Ω_m in the two models, and the consideration of massive sterile neutrino amplifies the parameter space, in particular, in the direction of H_0 . Actually, the involvement of massive sterile neutrino can indeed give a much higher value of H_0 .

We show the one-dimensional posterior distributions of H_0 for the two models in Fig. 3 (left panel for CMB+BAO and right panel for CMB+BAO+other). From this figure, we can clearly see that once the massive sterile neutrino is considered in cosmology, then under the constraint of CMB+BAO+other data the fit value of H_0 can become much larger. Under the constraints of CMB+BAO data, we obtain $H_0 = 67.81^{+0.47}_{-0.46}$ km s^{−1} Mpc^{−1} for the Λ CDM model and $H_0 = 68.30^{+0.53}_{-1.00}$ km s^{−1} Mpc^{−1} for the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model, which indicates that the consideration of massive sterile neutrino improves the tension with the Hubble constant direct measurement ($H_0 = 73.00 \pm 1.75$ km s^{−1} Mpc^{−1}) from 2.86σ to 2.57σ . So, in this case, the tension can only be slightly relieved. Under the constraints

of CMB+BAO+other data, we obtain $H_0 = 68.33 \pm 0.45$ km s^{−1} Mpc^{−1} for the Λ CDM model and $H_0 = 69.44^{+0.84}_{-1.14}$ km s^{−1} Mpc^{−1} for the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model, which indicates that the involvement of massive sterile neutrino improves the tension from 2.58σ to 1.83σ . Thus, in this case, the tension can be largely relieved. But we must admit that the improvement for the tension (from 2.58σ to 1.83σ) is mainly owing to the fact that the introduction of two extra parameters leads to the posterior distribution of H_0 becoming much broader (see the right panel of Fig. 3).

Then we show the constraint results for the parameters N_{eff} and $m_{\nu,\text{sterile}}^{\text{eff}}$ in the Λ CDM+ $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model (see Table 1):

$$\left. \begin{array}{l} N_{\text{eff}} < 3.4273 \\ m_{\nu,\text{sterile}}^{\text{eff}} < 0.7279 \text{ eV} \end{array} \right\} \text{CMB+BAO,}$$

$$\left. \begin{array}{l} N_{\text{eff}} = 3.30^{+0.12}_{-0.20} \\ m_{\nu,\text{sterile}}^{\text{eff}} < 0.2417 \text{ eV} \end{array} \right\} \text{CMB+BAO+other.}$$

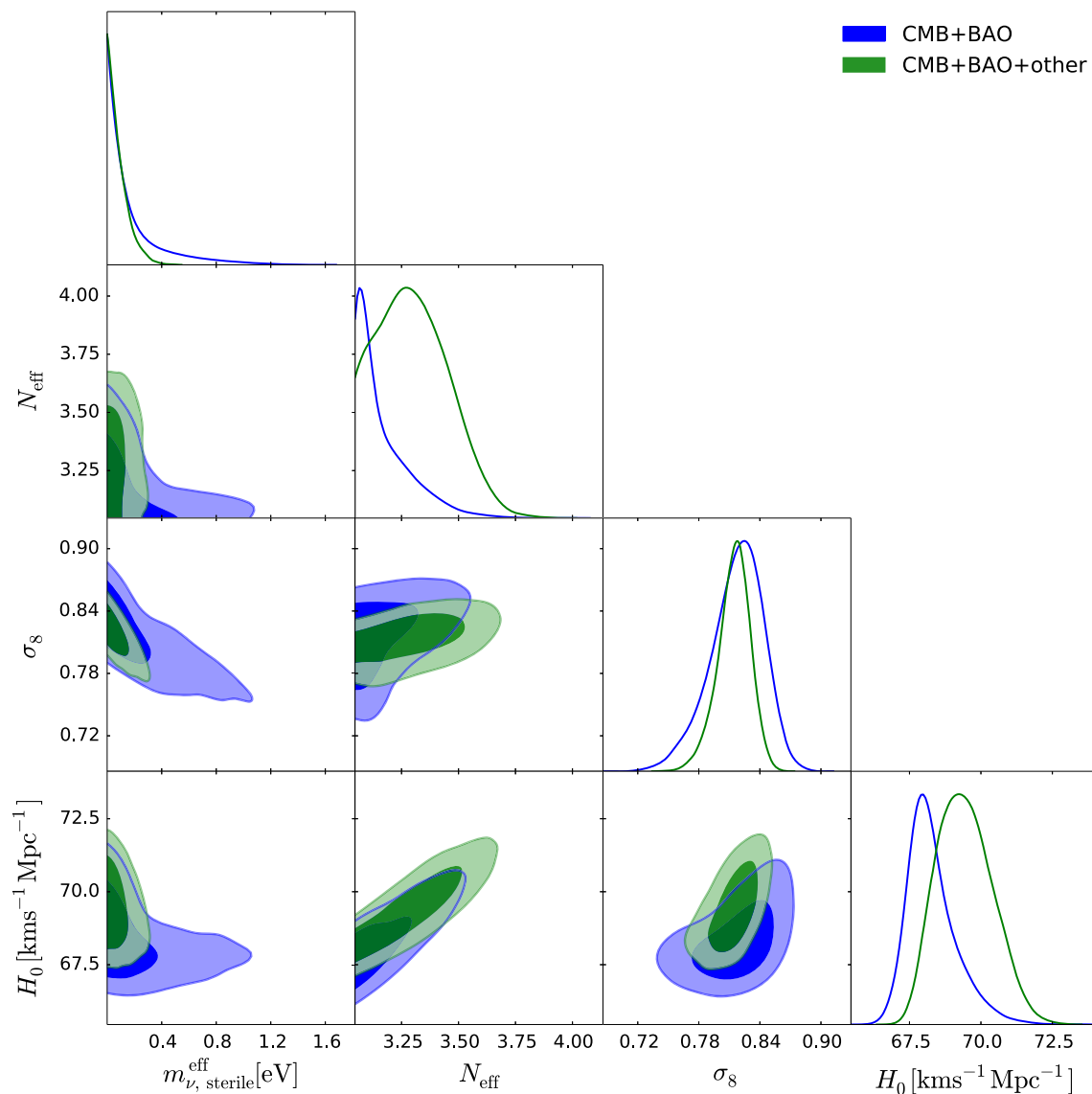


Fig. 4 The one-dimensional posterior distributions and two-dimensional marginalized contours (68.3 and 95.4% CL) for the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model, from the constraints of the CMB+BAO and CMB+BAO+other data combinations

We find that N_{eff} cannot be well constrained using only the CMB+BAO data, but the addition of H_0 , SZ, Lensing, and WL data can significantly improve the constraint on N_{eff} , favoring $\Delta N_{\text{eff}} > 0$ at the 1.27σ statistical significance. For the mass of sterile neutrino, the CMB+BAO data give $m_{\nu, \text{sterile}}^{\text{eff}} < 0.7279$ eV (95.4% CL), and further including the H_0 +SZ+Lensing+WL data leads to the result of $m_{\nu, \text{sterile}}^{\text{eff}} < 0.2417$ eV (95.4% CL). Evidently, adding low-redshift data tightens the constraint on $m_{\nu, \text{sterile}}^{\text{eff}}$ significantly. This indicates that the SZ cluster data (as well as the H_0 , Lensing, and WL data) play an important role in constraining the mass of sterile neutrino. In Fig. 4, we show the one- and two-dimensional marginalized posterior distributions of the parameters N_{eff} , $m_{\nu, \text{sterile}}^{\text{eff}}$, σ_8 , and

H_0 , for the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model. We find that, in the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model, σ_8 is anti-correlated with $m_{\nu, \text{sterile}}^{\text{eff}}$, and H_0 is positively correlated with N_{eff} , which leads to the fact that considering massive sterile neutrinos in cosmology can effectively relieve the tensions among the current observations.

Compared to the Λ CDM model, the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model does not provide an improved fit to the current observational data. Considering the massive sterile neutrino in cosmology leads to an increase of $\Delta\chi^2 = 0.632$ under the CMB+BAO constraint and an increase of $\Delta\chi^2 = 0.828$ under the CMB+BAO+other constraint. That is to say, the Λ CDM+ $N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model has $\Delta\text{AIC} = 4.632$ and 4.828 for the two data-combination cases, which shows that the

massive sterile neutrino is not favored by the current cosmological observations.

Therefore, we find that the current CMB+BAO+other data give a rather tight upper limit on $m_{\nu, \text{sterile}}^{\text{eff}}$ and favor $\Delta N_{\text{eff}} > 0$ at the 1.27σ level. Together with the constraint results for the massless sterile neutrino, we can conclude that the current observations do not seem to favor a massive sterile neutrino, but favor a massless sterile neutrino in some sense (only at the more than 1σ statistical significance). Our result is consistent with the recent result of neutrino oscillation experiment by the Daya Bay and MINOS collaborations [81], as well as the recent result of cosmic ray experiment by the IceCube collaboration [82].

4 Conclusion

The aim of this work is to search for sterile neutrinos using the latest cosmological observations. We consider the two cases of massless and massive sterile neutrinos, corresponding to the $\Lambda\text{CDM}+N_{\text{eff}}$ model and the $\Lambda\text{CDM}+N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model, respectively. The observational data used in this paper include the Planck TT,TE,EE+lowP data, the BAO data, the H_0 direct measurement, the Planck SZ cluster counts data, the Planck CMB lensing data, and the cosmic shear data.

For the $\Lambda\text{CDM}+N_{\text{eff}}$ model, the CMB+BAO+other data give $N_{\text{eff}} = 3.29^{+0.11}_{-0.17}$ (68.3% CL), favoring $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046 > 0$ at the 1.44σ level. Therefore, there is a hint of the existence of massless sterile neutrinos, from the current cosmological observations. We also find that the addition of the parameter N_{eff} can indeed relieve the tension between the Planck observation and the recent H_0 direct measurement (the tension is reduced to be at the 1.69σ level).

For the $\Lambda\text{CDM}+N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model, using the CMB+BAO data, we obtain $m_{\nu, \text{sterile}}^{\text{eff}} < 0.7279$ eV (95.4% CL) and $N_{\text{eff}} < 3.4273$ (95.4% CL). Thus, in this case, only upper limits on N_{eff} and $m_{\nu, \text{sterile}}^{\text{eff}}$ (for the massive sterile neutrino) can be derived. Further including the other (H_0 +SZ+Lensing+WL) data significantly improves the constraints, and in this case we obtain $m_{\nu, \text{sterile}}^{\text{eff}} < 0.2417$ eV (95.4% CL) and $N_{\text{eff}} = 3.30^{+0.12}_{-0.20}$ (68.3% CL). Thus, the current observations give a rather tight upper limit on $m_{\nu, \text{sterile}}^{\text{eff}}$ and favor $\Delta N_{\text{eff}} > 0$ at the 1.27σ level. This result seems to favor a massless sterile neutrino, in tension with the previous short-baseline neutrino oscillation experiments that prefer the mass of sterile neutrino at around 1 eV. But our result is consistent with the recent result of neutrino oscillation experiment done by the Daya Bay and MINOS collaborations [81], as well as the recent result of cosmic ray experiment done by the IceCube collaboration [82].

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