# Charged vector particle tunneling from a pair of accelerating and rotating and 5D gauged super-gravity black holes 

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Received: 22 March 2017 / Accepted: 21 April 2017 / Published online: 9 May 2017
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#### Abstract

The aim of this paper is to study the quantum tunneling process for charged vector particles through the horizons of more generalized black holes by using the Proca equation. For this purpose, we consider a pair of charged accelerating and rotating black holes with Newman-UntiTamburino parameter and a black hole in 5D gauged supergravity theory, respectively. Further, we study the tunneling probability and corresponding Hawking temperature for both black holes by using the WKB approximation. We find that our analysis is independent of the particles species whether or not the background black hole geometries are more generalized.


## 1 Introduction

A black hole ( BH ) is considered as an object which absorbs all the matter/energy from the environing area into it due to its intense gravitational field. General relativity (GR) depicts that a BH swallows all particles that collides the horizon of the BH. In 1974, Hawking predicted that a BH behaves like a black body having a specific temperature, known as the Hawking temperature, which allows a BH to emit radiation (called Hawking radiation) from its horizon by assuming quantum field hypotheses in the background of the curved spacetime.

A particle's action of a quantum mechanical nature is used in order to calculate the Hawking radiation spectrum from different BHs [1,2]. The analysis of Hawking radiation as a quantum tunneling phenomenon and accretion onto some particular BHs has attracted the attention of many researchers [3-8]. Various efforts have been made to examine this radiation spectrum from BH sy considering the quantum mechan-

[^0]ics of scalar, Dirac, fermion and photon particles etc. Many researchers [9-12] have studied vector particle tunneling to obtain more information as regards the Hawking temperature and the radiation spectrum from different BHs . The charged vector particle tunneling from Kerr-Newman BH [13] and charged black string [14] are important contributions toward BH physics.

The charged fermions tunneling from Reissner-Nord-ström-de Sitter BH with a global monopole [15] is studied by using the WKB approximation and Dirac equation to evaluate the tunneling process for charged particles as well as the Hawking temperature. In this paper the authors have evaluated the tunneling probability and the Hawking temperature for charged fermion tunneling from event horizon. The tunneling process for Plebanski-Demianski BHs is determined by the graphical behavior of the Hawking temperature of an ingoing and outgoing charged fermion from the event horizon [16]. The Hawking temperature for charged NUT (Newman-Unti-Tamburino) BH solutions to the field equations is considered with rotation and acceleration. A BH can be studied by a small measurement through quantum field theory on a curved background [17]. The tunneling probability for an outgoing particle is ruled by the imaginary part of the particle's action. A large number of attempts [18-26] have been made to calculate the tunneling of charged and uncharged scalar and Dirac particles with different BH configurations. The tunneling of spin- $\frac{1}{2}$ particles by the event horizon of the Rindler spacetime was explained and the Unruh temperature has been calculated [27]. Kraus and Wilczek [28,29] took a semi-classical process to analyze Hawking radiation as a tunneling event. This process contains the calculation for the phenomenon of s-wave emission across the event horizon. In [30], it has been shown that the Hawking radiation from a rotating wormhole may emit all types of particles.

This paper deals with the study of the Hawking radiation process of charged vector particles from the horizons of a
pair of accelerating and rotating BH s and a BH in 5D gauged super-gravity.

Vector particles (spin-1 bosons) such as $Z$ (uncharged) and $W^{ \pm}$(charged) bosons are of great importance in the Standard Model. In the background of BH geometries, the behavior of the bosons can be determined by using the Proca equation. First, we formulate the field equations of charged $W^{ \pm}$bosons by using the Lagrangian of the Glashow-WeinbergSalam model [31]. Then we shall investigate particle emission process by using the Hamilton-Jacobi definition and WKB approximation to the derived equation for the charged case in the considered BH geometries. By putting the determinant (of the coefficient matrix) equal to zero, we can solve for the radial function. Consequently, we compute the tunneling rate of the charged vector particles from the horizons of BHs and find the corresponding Hawking temperature values in both cases.

The paper is planned as follows: we discuss in Sect. 2 the tunneling rate and Hawking temperature for charged accelerating and rotating BH solutions with NUT parameter. Section 3 is devoted to an investigation of charged vector particle tunneling and the Hawking temperature for BH in 5D gauged super-gravity spacetime, by investigating the $W^{ \pm}$ bosons observation. Section 4 provides a summary of the results for both cases.

## 2 Accelerating and rotating black holes with NUT parameter

In general, the NUT parameter is affiliated with the gravitomagnetic monopole, related to the bending properties of the environing spacetime due to the fundamental mass, its accurate physical significance could not be determined. The generalization for multi dimensional Kerr-NUT-de Sitter spacetime [32,33] and its physical implication [34] are also investigated. As a BH , the dominance on the NUT parameter, the revolution parameter sets the spacetime free of bending singularities and the result can be thought of as a NUTlike result. If the revolution parameter commands the NUT parameter, the result is Kerr-like and a closed chain of bending singularity forms. The behavior of this form of the singularity structure is independent of the existence on the cosmology constant.

There are lots of BHs which are associated with the NUT parameter and lots of investigations have been made to examine their physical effects in the space of colliding waves. The significance of the NUT parameter makes itself accurately felt when a motionless Schwarzschild mass is absorbed in a stationary source and allows for electromagnetism [35]. The NUT parameter refers to the bend of the electromagnetism leaving out the fundamental Schwarzschild mass. In the absence of an electromagnetic field, it reduces to the bend-
ing of the vacuum spacetime [36]. The bend of the surrounding space pair with the mass of reference yields the NUT parameter.

The line element for accelerating and rotating BHs with NUT parameter is defined as [37]

$$
\begin{align*}
\mathrm{d} s^{2}= & -\frac{1}{\Omega^{2}}\left[\frac{Q}{\rho^{2}}\left(\mathrm{~d} t-\left(a \sin ^{2} \theta+4 l \sin ^{2} \frac{\theta}{2}\right) \mathrm{d} \phi\right)^{2}\right. \\
& -\frac{\rho^{2}}{Q} \mathrm{~d} r^{2} \frac{\tilde{P}}{\rho^{2}}\left(a \mathrm{~d} t-\left(r^{2}+(a+l)^{2}\right) \mathrm{d} \phi\right)^{2} \\
& \left.-\frac{\rho^{2}}{\tilde{P}} \sin ^{2} \theta \mathrm{~d} \theta^{2}\right], \tag{2.1}
\end{align*}
$$

where

$$
\begin{aligned}
\Omega= & 1-\frac{\alpha}{\omega}(l+a \cos \theta) r, \rho^{2}=r^{2}+(l+a \cos \theta)^{2} \\
Q= & {\left[\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right)\left(1+2 \alpha l \frac{r}{\omega}\right)-2 M r+\frac{\omega^{2} \tilde{k} r^{2}}{a^{2}-l^{2}}\right] } \\
& \times\left[1+\alpha \frac{a-l}{\omega} r\right]\left[1-\alpha \frac{a+l}{\omega} r\right], \\
\tilde{P}= & \sin ^{2} \theta\left(1-a_{3} \cos \theta-a_{4} \cos ^{2} \theta\right)=P \sin ^{2} \theta, \\
a_{3}= & 2 M \frac{\alpha a}{\omega}-4 \frac{a l \alpha^{2}}{\omega^{2}}\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \\
a_{4}= & -\frac{\alpha^{2} a^{2}}{\omega^{2}}\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) .
\end{aligned}
$$

Here, $M$ denotes the mass of pairs of BHs, $e$ and $g$ indicate the electric and magnetic charges, respectively, while $l$ is the NUT parameter of $\mathrm{BH}, \alpha$ and $\omega$ indicate acceleration and rotation of the sources, respectively. Also, $a$ is the Kerr-like rotation parameter and $\tilde{k}$ is given by

$$
\left(\frac{\omega^{2}}{a^{2}-l^{2}}+3 \alpha^{2} l^{2}\right) \tilde{k}=1+2 \frac{\alpha l}{\omega} M-3 \frac{\alpha^{2} l^{2}}{\omega^{2}}\left(\tilde{e}^{2}+\tilde{g}^{2}\right)
$$

Here, $\alpha, \omega, M, \tilde{e}, \tilde{g}$ and $\tilde{k}$ are arbitrary real parameters. We would like to mention that $\omega$ depends on the NUT parameter $l$ and the Kerr-like rotation parameter $a$. The $\alpha$ twisting property of BHs is proportional to the rotation $\omega$. Also, $\omega$ depends on rotation parameters $l$ and $a$. The parameters $\alpha$, $\omega, M, \tilde{e}, \tilde{g}$ and $\tilde{k}$ vary independently. If $\alpha$ is equal to zero, then the metric in Eq. (2.1) leads to the Kerr-Newman-NUT solution. If $l=0$, then the metric in Eq. (2.1) gives the couple of charged and rotating BHs. In this case, if $\tilde{e}$ and $\tilde{g}$ are equal to zero, we have a Schwarzschild BH and if $l$ and $a$ are equal to zero it leads to a C-metric.

The metric (2.1) can be rewritten as
$\mathrm{d} s^{2}=-f(r, \theta) \mathrm{d} t^{2}+\frac{\mathrm{d} r^{2}}{g(r, \theta)}+\Sigma(r, \theta) \mathrm{d} \theta^{2}+k(r, \theta) \mathrm{d} \phi^{2}$

$$
\begin{equation*}
-2 H(r, \theta) \mathrm{d} t \mathrm{~d} \phi, \tag{2.2}
\end{equation*}
$$

where $f(r, \theta), g(r, \theta), \Sigma(r, \theta), K(r, \theta)$ and $H(r, \theta)$ are given by the following equations:

$$
\begin{aligned}
f(r, \theta)= & \frac{Q-P a^{2} \sin ^{2} \theta}{\rho^{2} \Omega^{2}}, \quad g(r, \theta)=\frac{Q \Omega^{2}}{\rho^{2}} \\
\Sigma= & (r, \theta)=\frac{\rho^{2}}{\Omega^{2} P}, \\
k(r, \theta)= & \frac{1}{\Omega^{2} \rho^{2}}\left(\sin ^{2} \theta P\left(r^{2}+(a+l)^{2}\right)^{2}\right. \\
& \left.-Q\left(a \sin ^{2} \theta+4 l \sin ^{2} \frac{\theta}{2}\right)^{2}\right) \\
H(r, \theta)= & \frac{1}{\Omega^{2} \rho^{2}}\left(\sin ^{2} \theta P a\left(r^{2}+(a+l)^{2}\right)\right. \\
& \left.-Q\left(a \sin ^{2} \theta+4 l \sin ^{2} \frac{\theta}{2}\right)\right)
\end{aligned}
$$

The electromagnetic potential for these BHs is given by

$$
\begin{align*}
A= & \frac{1}{a\left(r^{2}+(l+a \cos \theta)^{2}\right)}[-\tilde{e} r(a \mathrm{~d} t-\mathrm{d} \phi((l+a) \\
& \left.\left.-\left(l^{2}+a^{2} \cos ^{2} \theta+2 a l \cos \theta\right)\right)\right) \\
& -\tilde{g}\left(l+a \cos \theta\left(a \mathrm{~d} t-\mathrm{d} \phi\left(r^{2}+(l+a)^{2}\right)\right)\right] \tag{2.3}
\end{align*}
$$

The event horizons are obtained for $g(r, \theta)=\frac{Q \Omega^{2}}{\rho^{2}}=0$, which implies that $\Omega \neq 0$, so $Q=0$, which yields the following real roots of $r$ :
$r_{\alpha 1}=\frac{\omega}{\alpha(a+l)}, \quad r_{\alpha 2}=\frac{-\omega}{\alpha(a-l)} \quad r_{ \pm}=\frac{a^{2}-l^{2}}{\omega^{2} \tilde{k}}$
$\left[-\left(\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \frac{\alpha l}{\omega}-M\right)\right.$
$\pm \sqrt{\left.\left(\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \frac{\alpha l}{\omega}-M\right)^{2}-\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \frac{\omega^{2} \tilde{k}}{\alpha^{2}-l^{2}}\right]}$,
where $r_{\alpha 1}$ and $r_{\alpha 2}$ are acceleration horizons and $r_{ \pm}$represent the outer and inner horizons, respectively, such that

$$
\begin{aligned}
& \left(\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \frac{\alpha l}{\omega}-M\right)^{2} \\
& \quad-\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right) \frac{\omega^{2} \tilde{k}}{a^{2}-l^{2}}>0
\end{aligned}
$$

The angular velocity at BH outer (event) horizon is defined by
$\check{\Omega}=\frac{a}{r_{+}^{2}+(a+l)^{2}}$.
In order to investigate the tunneling spectrum for charged vector particles through the BH horizon, we will consider Proca equation with electromagnetic effects. In a curved spacetime with electromagnetic field, the motion of massive spin-1 charged vector fields is depicted by the given

Proca equation by using the Lagrangian of the W-bosons of Glashow-Weinberg-Salam model [10]
$\frac{1}{\sqrt{-\mathbf{g}}} \partial_{\mu}\left(\sqrt{-g} \psi^{\nu \mu}\right)+\frac{m^{2}}{h^{2}} \psi^{\nu}+\frac{i}{h} e A_{\mu} \psi^{\nu \mu}+\frac{i}{h} e F^{\nu \mu} \psi_{\mu}=0$,
where $\mathbf{g}$ is the determinant of the coefficient matrix, $m$ is particles mass and $\psi^{\mu \nu}$ is an anti-symmetric tensor, i.e.,

$$
\begin{aligned}
\psi_{\nu \mu} & =\partial_{\nu} \psi_{\mu}-\partial_{\mu} \psi_{\nu}+\frac{i}{h} e A_{\nu} \psi_{\mu}-\frac{i}{h} e A_{\mu} \psi_{\nu} \text { and } F^{\mu \nu} \\
& =\nabla^{\mu} A^{\nu}-\nabla^{v} A^{\mu}
\end{aligned}
$$

Here, $A_{\mu}$ is considered as the electromagnetic potential of the BH , $e$ denotes the charge of the W-bosons and $\nabla_{\mu}$ is the geometrically covariant derivative. Since the equation of motion for the $W^{+}$and $W^{-}$bosons is similar, the tunneling processes should be similar too. For simplification, here we will consider the $W^{+}$boson case; the results of this case can be extended to $W^{-}$bosons due to the digitalization of the line element. For $W^{+}$field, the values of the components of $\psi^{\mu}$ and $\psi^{\nu \mu}$ are obtained as follows:
$\psi^{0}=\frac{-k \psi_{0}-H \psi_{3}}{f k+H^{2}}, \quad \psi^{1}=g \psi_{1}, \quad \psi^{2}=\Sigma^{-1} \psi_{2}$,
$\psi^{3}=\frac{-H \psi_{0}+f \psi_{3}}{f k+H^{2}}$,
$\psi^{01}=\frac{-k g \psi_{01}-H g \psi_{13}}{f k+H^{2}}, \quad \psi^{02}=\frac{-k \psi_{02}-H \psi_{23}}{\Sigma\left(f k+-H^{2}\right)}$,
$\psi^{03}=\frac{-\psi_{03}}{f k+H^{2}}$,
$\psi^{12}=g \Sigma^{-1} \psi_{12}, \quad \psi^{13}=\frac{g\left(f \psi_{13}-H \psi_{01}\right)}{f k+H^{2}}$,
$\psi^{23}=\frac{g \psi_{23}-H \psi_{02}}{\Sigma\left(f k+H^{2}\right)}$.
The electromagnetic vector potential for this BH is given by [38]

$$
\begin{align*}
A= & \frac{1}{a\left[r^{2}+(l+a \cos \theta)^{2}\right]}[-\tilde{e} r[a \mathrm{~d} t \\
& \left.-\mathrm{d} \phi(l+a)^{2}-\left(l^{2}+a^{2} \cos ^{2} \theta+2 l a \cos \theta\right)\right] \\
& \left.-\tilde{g}(l+a \cos \theta)\left[a \mathrm{~d} t-\mathrm{d} \phi r^{2}+(l+a)^{2}\right]\right] \tag{2.7}
\end{align*}
$$

Applying the WKB approximation [39], i.e.,

$$
\begin{equation*}
\psi_{\nu}=c_{\nu} \exp \left[\frac{i}{\hbar} S_{0}(t, r, \theta, \phi)+\Sigma \hbar^{n} S_{n}(t, r, \theta, \phi)\right], \tag{2.8}
\end{equation*}
$$

to the Proca equation (2.6) and neglecting the terms for $n=$ $1,2,3,4, \ldots$, we obtain the following set of equations:
$k g\left[c_{1}\left(\partial_{1} S_{0}\right)\left(\left(\partial_{1} S_{0}\right)+e A_{0}\right)\right.$

$$
\begin{align*}
& \left.-c_{0}\left(\partial_{1} S_{0}\right)^{2}\right]-H g\left[c_{3}\left(\partial_{1} S_{0}\right)^{2}\right. \\
& \left.-c_{1}\left(\partial_{1} S_{0}\right)\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right]+\frac{k}{\Sigma}\left[c _ { 2 } ( \partial _ { 2 } S _ { 0 } ) \left(\left(\partial_{0} S_{0}\right)\right.\right. \\
& \left.\left.+e A_{3}\right)-c_{0}\left(\partial_{2} S_{0}\right)^{2}\right] \\
& -\frac{H}{\Sigma}\left[c_{3}\left(\partial_{2} S_{0}\right)^{2}-c_{2}\left(\partial_{2} S_{0}\right)\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right] \\
& +\left[c_{3}\left(\partial_{3} S_{0}\right)\left(\left(\partial_{0} S_{0}\right)+e A_{0}\right)\right. \\
& \left.-c_{0}\left(\partial_{3} S_{0}\right)\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right]+e A_{3} k g\left[c_{1}\left(\left(\partial_{0} S_{0}\right)+e A_{0}\right)\right. \\
& \left.-c_{0}\left(\partial_{1} S_{0}\right)\right] \\
& -m^{2} k c_{0}-m^{2} H c_{3}-e A_{3} H g\left[c_{3}\left(\partial_{1} S_{0}\right)\right. \\
& \left.-c_{1}\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right]=0 \tag{2.9}
\end{align*}
$$

$$
\begin{align*}
& k g {\left[c_{1}\left(\partial_{0} S_{0}\right)\left(\left(\partial_{0} S_{0}\right)+e A_{0}-c_{0}\left(\partial_{1} S_{0}\right)\left(\partial_{0} S_{0}\right)\right]\right.} \\
& \quad-H g\left[c_{3}\left(\partial_{1} S_{0}\right)\left(\partial_{0} S_{0}\right)\right. \\
&\left.\quad-c_{1}\left(\partial_{0} S_{0}\right)\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right]+\frac{g\left(f k+H^{2}\right)}{\Sigma} \\
& \quad \times\left[c_{2}\left(\partial_{1} S_{0}\right)\left(\partial_{2} S_{0}\right)-c_{1}\left(\partial_{2} S_{0}\right)^{2}\right] \\
& \quad+g f\left[c_{3}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)-c_{1}\left(\partial_{3} S_{0}\right)\left(\left(\partial_{3} S_{0}\right)+e A_{3}\right)\right] \\
& \quad+g H\left[c _ { 1 } ( \partial _ { 3 } S _ { 0 } ) \left(\left(\partial_{0} S_{0}\right)\right.\right. \\
&\left.\left.\quad-e A_{0}\right)-c_{0}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)\right]+e A_{0} k g\left[c _ { 1 } \left(\left(\partial_{0} S_{0}\right)\right.\right. \\
&\left.\left.\quad-e A_{0}\right)-c_{0}\left(\partial_{1} S_{0}\right)\right] \\
& \quad-m^{2} g c_{1}(f k-H)-e A_{0} H g\left[c_{3}\left(\partial_{1} S_{0}\right)\right. \\
&\left.\quad-c_{1}\left(\left(\partial_{3} S_{0}\right)-e A_{3}\right)\right] \\
& \quad+e A_{3} g f\left[c_{3}\left(\partial_{1} S_{0}\right)-c_{1}\left(\left(\partial_{3} S_{0}\right)\right.\right. \\
&\left.\left.\quad+e A_{3}\right)\right]+e A_{3} H\left[c_{1}\left(\left(\partial_{0} S_{0}\right)+e A_{0}\right)\right. \\
&\left.\quad-c_{0}\left(\partial_{1} S_{0}\right)\right]=0, \tag{2.10}
\end{align*}
$$

$$
\frac{k}{\Sigma}\left[c_{2}\left(\partial_{2} S_{0}\right)^{2}-c_{0}\left(\partial_{2} S_{0}\right)\left(\partial_{0} S_{0}\right)+e A_{0}\left(\partial_{0} S_{0}\right) c_{2}\right]
$$

$$
-\frac{H}{\Sigma}\left[c_{3}\left(\partial_{1} S_{0}\right)\left(\partial_{2} S_{0}\right)\right.
$$

$$
\left.-c_{2}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)\right]-\frac{g}{\Sigma}\left[c_{2}\left(\partial_{1} S_{0}\right)^{2}\right.
$$

$$
\left.-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{2} S_{0}\right)\right](f k-H)
$$

$$
+\frac{f}{\Sigma}\left[c_{3}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)-c_{2}\left(\partial_{3} S_{0}\right)^{2}-e A_{3} c_{2\left(\partial_{3} S_{0}\right)}\right]
$$

$$
+\frac{H}{\Sigma}\left[\left(\partial_{3} S_{0}\right)\left(\partial_{0} S_{0}\right) c_{2}\right.
$$

$$
\left.-c_{0}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)+c_{2} e A_{0}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)\right]
$$

$$
-m^{2} c_{2} \Sigma^{-1}(f k-H)
$$

$$
+e A_{0} \frac{k}{\Sigma}\left[c_{2}\left(\partial_{0} S_{0}\right)-c_{0}\left(\partial_{2} S_{0}\right)\right.
$$

$$
\left.+e A_{0} c_{2}\right]-e A_{0} \frac{H}{\Sigma}\left[c_{3}\left(\partial_{2} S_{0}\right)-c_{2}\left(\partial_{3} S_{0}\right)\right.
$$

$$
\left.-c_{2} e A_{3}\right]+e A_{3} \frac{f}{\Sigma}\left[c_{3}\left(\partial_{2} S_{0}\right)-c_{2}\left(\partial_{3} S_{0}\right)\right.
$$

$$
\left.-e A_{3} c_{2}\right]+e A_{3} \frac{H}{\Sigma}\left[c_{2}\left(\partial_{0} S_{0}\right)\right.
$$

$$
\begin{equation*}
\left.-c_{0}\left(\partial_{2} S_{0}\right)+e A_{0} c_{2}\right]=0 \tag{2.11}
\end{equation*}
$$

$\left[c_{3}\left(\partial_{0} S_{0}\right)^{2}-c_{0}\left(\partial_{3} S_{0}\right)\left(\partial_{0} S_{0}\right)+e A_{0} c_{3}\left(\partial_{0} S_{0}\right)\right.$

$$
\begin{align*}
& \left.-e A_{3} c_{0}\left(\partial_{0} S_{0}\right)\right] \\
& +g-H\left[c_{1}\left(\partial_{0} S_{0}\right)\left(\partial_{1} S_{0}\right)-c_{0}\left(\partial_{1} S_{0}\right)^{2}+e A_{0} c_{1}\left(\partial_{1} S_{0}\right)\right] \\
& -f g\left[c_{3}\left(\partial_{2} S_{0}\right)^{2}\right. \\
& \left.-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)-e A_{3} c_{1}\left(\partial_{0} S_{0}\right)\right]-\frac{H}{\Sigma}\left[c_{2}\left(\partial_{0} S_{0}\right)\left(\partial_{2} S_{0}\right)\right. \\
& -c_{0}\left(\partial_{2} S_{0}\right)^{2} \\
& \left.+e A_{0} c_{2}\left(\partial_{2} S_{0}\right)\right]-f g\left[c_{3}\left(\partial_{1} S_{0}\right)^{2}-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)\right. \\
& \left.-e A_{3} c_{1}\left(\partial_{0} S_{0}\right)\right] \\
& -\frac{H}{\Sigma}\left[c_{2}\left(\partial_{0} S_{0}\right)\left(\partial_{2} S_{0}\right)-c_{0}\left(\partial_{2} S_{0}\right)^{2}+e A_{0} c_{2}\left(\partial_{2} S_{0}\right)\right] \\
& -\frac{f}{\Sigma}\left[c_{3}\left(\partial_{2} S_{0}\right)^{2}\right. \\
& \left.-c_{2}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)-e A_{3} c_{2}\left(\partial_{2} S_{0}\right)\right] \\
& +e A_{0}\left[c_{3}\left(\partial_{0} S_{0}\right)-c_{0}\left(\partial_{3} S_{0}\right)\right. \\
& +e A_{0} c_{3}-e A_{3} c_{0}+m^{2}\left[H c_{0}+c_{3} f\right]=0 . \tag{2.12}
\end{align*}
$$

Using the technique of separation of variables, we can choose
$S_{0}=-(E-j \check{\Omega}) t+W(r)+N \phi+\Theta(\theta)$,
where $E$ and $j$ represent the particle's energy and angular momentum, respectively. From Eqs. (2.9)-(2.12), we can obtain a matrix equation,
$G\left(c_{0}, c_{1}, c_{2}, c_{3}\right)^{T}=0$,
which implies a $4 \times 4$ matrix labeled " $G$ ", whose components are given as follows:
$G_{11}=-\dot{W}^{2} k g-\frac{k N^{2}}{\Sigma}-\dot{\Theta}^{2}-\dot{\Theta} e A_{3}-m^{2} k-e A_{3} k g \dot{W}$,
$G_{12}=-\dot{W} k g(E-j \check{\Omega})+k g \dot{W} e A_{0}+H g \dot{W} \dot{\Theta}$ $+H g \dot{W} e A_{3}-e A_{3} k g(E-j \check{\Omega})+k g e^{2} A_{3} A_{0}$ $+e A_{3} H g \dot{\Theta}+H g e^{2} A_{3}$,
$G_{13}=-\frac{k}{\Sigma}(E-j \check{\Omega}) N+\frac{k}{\Sigma} N e A_{3}+\frac{H}{\Sigma} \dot{\Theta} N+\frac{H}{\Sigma} N e A_{3}$,
$G_{14}=-\dot{W}^{2} H g-\frac{H}{\Sigma} N^{2}-\dot{\Theta}^{2}(E-j \check{\Omega})+\dot{\Theta} e A_{0}$ $-m^{2} H-e A_{3} g H \dot{W}$,
$G_{21}=k g(E-j \check{\Omega}) \dot{W}-g H \dot{W} \dot{\Theta}-e A_{0} k g \dot{W}-e A_{3} H \dot{W}$,
$G_{22}=-k g(E-j \check{\Omega})\left(-(E-j \check{\Omega})+e A_{0}\right.$ $-g H(E-j \check{\Omega})\left(\dot{\Theta}+e A_{3}\right)$ $\left.+\frac{g}{\Sigma} N^{2}(f k-H)-g f \dot{\Theta}\left(\dot{\Theta}+e A_{3}\right)\right)$ $+g H \dot{\Theta}\left(-(E-j \check{\Omega})-e A_{0}\right)+e A_{0} k g(-(E-j \check{\Omega})$ $+e A_{0} g H\left(\dot{\Theta}-e A_{3}\right)-e A_{3} g f\left(\dot{\Theta}+e A_{3}\right)$,

$$
e A_{3} H\left(-(E-j \check{\Omega})-m^{2} g(f k-H)\right.
$$

$G_{23}=\frac{g}{\Sigma} \dot{W} N\left(f k-H^{2}\right)$,
$G_{24}=H g(E-j \check{\Omega}) \dot{W}+g f \dot{W} \dot{\Theta}-e A_{0} g H \dot{W}+e A_{3} g f \dot{W}$,
$G_{31}=\frac{k}{\Sigma}(E-j \check{\Omega}) N-\frac{H}{\Sigma} N \dot{\Theta}-e A_{3} H N$,
$G_{32}=\frac{g}{\Sigma} \dot{W} N(E-j \check{\Omega})$,
$G_{33}=-\frac{k}{\Sigma}\left[(E-j \check{\Omega})^{2}-e A_{0}(E-j \check{\Omega})\right]+\frac{H}{\Sigma} \dot{W} \dot{\Theta}$
$-\frac{g}{\Sigma}\left[\dot{W}^{2}\left(f k+H^{2}\right)\right]-\frac{f}{\Sigma}\left[(\dot{\Theta})^{2}+e A_{3}(\dot{\Theta})\right]$
$+\frac{H}{\Sigma}\left[\dot{\Theta}\left((E-j \check{\Omega})+e A_{0} N\right)\right]$
$-m^{2} \Sigma^{-1}\left(f k+H^{2}\right)-e A_{0} \frac{k}{\Sigma}[(E-j \check{\Omega})$
$\left.-e A_{0}\right]+e A_{0} \frac{H}{\Sigma}\left[\dot{\Theta}+e A_{3}\right]$
$-e A_{3} \frac{f}{\Sigma}\left[\dot{\Theta}+e A_{3}\right]$,
$G_{34}=-\frac{H}{\Sigma} N \dot{W}+\frac{f}{\Sigma} N \dot{\Theta}-e A_{0} N \frac{H}{\Sigma}+e A_{3}+N \frac{f}{\Sigma}$,
$G_{41}=(E-j \check{\Omega}) \dot{\Theta}+(E-j \check{\Omega}) e A_{3}+g H \dot{W}^{2}+N^{2} \frac{H}{\Sigma}$
$+m^{2} H+(E-j \check{\Omega}) e A_{0}$
$-e^{2} A_{0} A_{3}, \quad G_{42}=g H \dot{W}(E-j \check{\Omega})-g H e A_{0} \dot{W}$
$+f g \dot{W} \dot{\Theta}-\operatorname{fge}_{3}(E-j \check{\Omega})$,
$G_{43}=\frac{H}{\Sigma}(E-j \check{\Omega}) N-\frac{H}{\Sigma} N e A_{0}+\frac{f}{\Sigma} N \dot{\Theta}+N e A_{3}$,
$G_{44}=(E-j \check{\Omega})^{2}-(E-j \check{\Omega}) e A_{0}-f g \dot{W}^{2}$
$-\frac{f}{\Sigma} N-m^{2} f-e A_{0}\left[(E-j \check{\Omega})-e A_{0}\right]$,
where $\dot{W}=\partial_{r} S_{0}, \dot{\Theta}=\partial_{\theta} S_{0}$ and $N=\partial_{\phi} S_{0}$. For the nontrivial solution, the absolute value $\mathbf{G}$ equals zero, and we solve the resultant equation for the radial part so that we get the following integral:
$\operatorname{Im} W^{ \pm}= \pm \int \sqrt{\frac{\left(E-e A_{0}-j \check{\Omega}\right)^{2}+X}{f(r) g(r)}} \mathrm{d} r$
where + and - represent the radial functions of outgoing and incoming particles, respectively, while the function $X$

Expanding the functions $f(r)$ and $g(r)$ in Taylor's series near the horizon, we get
$f\left(r_{+}\right) \approx f^{\prime}\left(r_{+}\right)\left(r-r_{+}\right), \quad g\left(r_{+}\right) \approx g^{\prime}\left(r_{+}\right)\left(r-r_{+}\right)$.

Using the above expressions in Eq. (2.14), one can see that the resulting equation has poles at $r=r_{+}$. For the calculation of the Hawking temperature by using the tunneling method, it is required that we regularize the singularity by a specific complex contour to bypass the pole. For our standard co-ordinates of the BH metric, the tunneling of outgoing particles can be obtained by taking an infinitesimal half circle below the pole $r=r_{+}$, while for the ingoing particle such a contour is taken above the pole. Further, in order to calculate the semi-classical tunneling probability, it is required that the resulting wave equation must be multiplied by its complex conjugate. In this way, the part of the trajectory that starts from outside of the BH and continues to the observer will not contribute to the calculation of the final tunneling probability and can be ignored because it will be completely real. Therefore, the only part of the trajectory that contributes to the tunneling probability is the contour around the BH horizon.

Hence using Eqs. (2.14) and (2.15), and integrating the resulting equation around the pole, we get
$\operatorname{Im} W^{ \pm}= \pm i \pi \frac{E-e A_{0}-j \check{\Omega}}{2 \kappa\left(r_{+}\right)}$,
and the surface gravity is [36]

$$
\begin{aligned}
\kappa\left(r_{+}\right)= & {\left[\frac{\left[\frac{\alpha l}{\omega}\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right)-M+\frac{\omega^{2} \tilde{k}}{a^{2}-l^{2}} r_{+}\right]}{\left[r_{+}^{2}+(a+l)^{2}\right]}\right.} \\
& \left.\times\left[1+\frac{\alpha(a-l)}{\omega} r_{+}\right] \times\left[1-\frac{\alpha(a+l)}{\omega} r_{+}\right]\right] .
\end{aligned}
$$

The tunneling probability for charged vector particles is given by

$$
\begin{aligned}
\Gamma & =\frac{\operatorname{Prob}[\operatorname{emission}]}{\operatorname{Prob}[\text { absorption }]}=\frac{\exp \left[-2\left(\operatorname{Im} W^{+}+\operatorname{Im} \Theta\right)\right]}{\exp \left[-2\left(\operatorname{Im} W^{-}-\operatorname{Im} \Theta\right)\right]}=\exp \left[-4 \operatorname{Im} W^{+}\right] \\
& =\exp \left[-2 \pi \frac{E-e A_{0}-j \check{\Omega}}{\left[\frac{\left[\frac{\alpha l}{\omega}\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right)-M+\frac{\omega^{2} \tilde{k}}{a^{2}-l^{2}} r_{+}\right]}{\left[r_{+}^{2}+(a+l)^{2}\right]} \times\left[1+\frac{\alpha(a-l)}{\omega} r_{+}\right] \times\left[1-\frac{\alpha(a+l)}{\omega} r_{+}\right]\right]}\right]
\end{aligned}
$$

can be defined as $X=-\Sigma^{-1} f N-m^{2} f-H g(E-j \check{\Omega})-$ $g f \dot{\Theta}+e A_{0} g H-e A_{3} g f ; \Omega$ is the angular velocity on the event horizon.

Now, finally we can calculate the Hawking temperature by comparing the above result with the Boltzmann formula $\Gamma_{B}=e^{-\left(E-e A_{0}-j \check{\Omega}\right) / T_{H}}$, to get
$T_{H}=\left[\frac{\left[\left[\frac{\alpha l}{\omega}\left(\omega^{2} \tilde{k}+\tilde{e}^{2}+\tilde{g}^{2}\right)-M+\frac{\omega^{2} \tilde{k}}{a^{2}-l^{2}} r_{+}\right] \times\left[1+\frac{\alpha(a-l)}{\omega} r_{+}\right]\left[1-\frac{\alpha(a+l)}{\omega} r_{+}\right]\right]}{2 \pi\left[r_{+}^{2}+(a+l)^{2}\right]}\right]$.

The Hawking temperature depends on $A_{0}$, the vector potential, $E$, the energy, $\check{\Omega}$, the angular momentum; $M$ is the mass of the pair of BHs, $e$ and $g$ are electric and magnetic charges, respectively, $a$ is the rotation of a $\mathrm{BH}, l$ is the NUT parameter, $\alpha$ represents the acceleration of the sources and $\omega$ the rotation of the sources.

We would like to mention that the Hawking temperature of charged vector particles given in Eq. (2.17) is the same as the Hawking temperature of fermion particles in Eq. (4.20) of [36]. Thus the Hawking temperature is independent of the particle species.

## 3 Black holes in 5D gauged super-gravity

The gauged theory is stated as a super-gravity theory in which the gravitino, the superpartner of the graviton is charged under some internal gauge group. However, the gauged super-gravity is more significant as compared to the ungauged case, because this theory has a negative cosmological constant, so it is defined on an anti-de Sitter space. Here, for the discussion of charged vector particles tunneling spectrum from a BH in 5D gauged super-gravity, we evaluate the tunneling probability of particles and the corresponding Hawking temperature at the BH horizon. Such BH solutions occur in $D=5 N=8$ gauged super-gravity (symmetry) [40]. Firstly, this solution was formulated in [41] as a particular case (STU model) of solutions of the $D=5 N=2$ gauged super-gravity equations of motion. The metric for the BH in 5D gauged super-gravity is [40]

$$
\begin{align*}
\mathrm{d} s^{2}= & -\left(H_{1} H_{2} H_{3}\right)^{-\frac{2}{3}} f \mathrm{~d} t^{2} \\
& +\left(H_{1} H_{2} H_{3}\right)^{\frac{1}{3}}\left(f^{-1} \mathrm{~d} r^{2}+r^{2} \mathrm{~d} \Omega_{3, k}^{2}\right) \tag{3.1}
\end{align*}
$$

where
$f=k-\frac{\mu}{r^{2}}+g^{2} r^{2} H_{1} H_{2} H_{3}, \quad H_{i}=1+\frac{q_{i}}{r^{2}}($ for $i=1,2,3)$,
and $\mathrm{d} \Omega_{3, k}^{2}$ is the metric on $S^{3}$ with unit radius if $k=1$, or the metric on $\mathbf{R}^{3}$ if $k=0$; here $\mu$ is the non-extremality parameter [41], which is related to the ADM mass, $g=1 / L$ is the inverse radius of $A d S_{5}$ related to the cosmological constant $\Lambda=-6 g^{2}=-6 / L^{2}$, and $q_{i}$ are charges entering the metric. The three gauge field potentials $A_{\mu}^{i}$ from the solution of equation of motion are of the form
$A_{0}^{i}=\frac{\tilde{q_{i}}}{r^{2}+q_{i}} \quad($ for $i=1,2,3)$,
where the $\tilde{q}_{i}$ are the physical charges, which are conserved and the Gauss law is applicable to such charges.

The line element can be rewritten as

$$
\begin{align*}
\mathrm{d} s^{2}= & -\tilde{A}(r) \mathrm{d} t^{2}+\tilde{B}^{-1}(r) \mathrm{d} r^{2}+\tilde{C}(r) \mathrm{d} \theta^{2} \\
& +\tilde{D}(r) \mathrm{d} \phi^{2}+\tilde{E}(r) \mathrm{d} \zeta^{2} \tag{3.2}
\end{align*}
$$

where

$$
\begin{aligned}
& \tilde{A}(r)=f\left(H_{1} H_{2} H_{3}\right)^{-\frac{2}{3}} \quad \tilde{B}^{-1}(r)=f^{-1}\left(H_{1} H_{2} H_{3}\right)^{\frac{1}{3}} \\
& \tilde{C}(r)=r^{2}\left(H_{1} H_{2} H_{3}\right)^{\frac{1}{3}} \quad \tilde{D}(r)=r^{2} \sin ^{2} \theta\left(H_{1} H_{2} H_{3}\right)^{\frac{1}{3}} \\
& \tilde{E}(r)=r^{2} \sin ^{2} \theta \sin ^{2} \phi\left(H_{1} H_{2} H_{3}\right)^{\frac{1}{3}}
\end{aligned}
$$

The horizons of the metric (3.2) can be determined when $f(r)=0$. For this purpose we follow [40] and assume that $g^{2}=1$ (by the choice of units as in [40]). Hence, in this case the outer horizon is located at
$r_{+}=\sqrt{\frac{\sqrt{\left(1+q_{i}\right)^{2}+4 \mu}-\left(1+q_{i}\right)}{2}}$,
for $\sqrt{\left(1+q_{i}\right)^{2}+4 \mu}>\left(1+q_{i}\right)$ and $i=1,2,3$.
In the Proca equation (2.6) the components of $\psi^{\nu}$ and $\psi^{\mu \nu}$ are given by

$$
\begin{aligned}
\psi^{0} & =-\tilde{A}^{-1} \psi_{0}, \quad \psi^{1}=\tilde{B} \psi_{1}, \quad \psi^{2}=\tilde{C}^{-1} \psi_{2} \\
\psi^{3} & =\tilde{D}^{-1} \psi_{3}, \quad \psi^{4}=\tilde{E}^{-1} \psi_{4}, \\
\psi^{o 1} & =-\tilde{B} \tilde{A}^{-1} \psi_{01}, \quad \psi^{02}=-(\tilde{A} \tilde{C})^{-1} \psi_{02}, \\
\psi^{03} & =-(\tilde{A} \tilde{D})^{-1} \psi_{03} \\
\psi^{04} & =-(\tilde{A} \tilde{E})^{-1} \psi_{04}, \quad \psi^{12}=\tilde{B} \tilde{C}^{-1} \psi_{12}, \\
\psi^{13} & =\tilde{B} \tilde{D}^{-1} \psi_{13}, \quad \psi^{14}=\tilde{B} \tilde{E}^{-1} \psi_{14} \\
\psi^{23} & =(\tilde{C} \tilde{D})^{-1} \psi_{23}, \quad \psi^{24}=(\tilde{C} \tilde{E})^{-1} \psi_{24}, \\
\psi^{34} & =(\tilde{D} \tilde{E})^{-1} \psi_{34} .
\end{aligned}
$$

By using Eq. (2.6), we obtain the following set of equations (for simplicity, we assume $A_{0} \equiv A_{0}^{i}$ for all $i$ ):

$$
\begin{aligned}
& \tilde{B}\left[c_{0}\left(\partial_{1} S_{0}\right)^{2}-c_{1}\left(\partial_{0} S_{0}\right)\left(\partial_{1} S_{0}\right)-e A_{0} c_{1}\left(\partial_{1} S_{0}\right)\right] \\
& \quad+\tilde{C}^{-1}\left[c_{0}\left(\partial_{2} S_{0}\right)^{2}-c_{2}\left(\partial_{0} S_{0}\right) c_{0}\left(\partial_{2} S_{0}\right)-e A_{0} c_{2}\left(\partial_{2} S_{0}\right)\right] \\
& \quad+\tilde{D}^{-1}\left[C_{0}\left(\partial_{3} S_{0}\right)^{2}-c_{3}\left(\partial_{3} S_{0}\right)\left(\partial_{0} S_{0}\right)\right. \\
& \left.\quad-e A_{0} c_{3}\left(\partial_{3} S_{0}\right)\right]+\tilde{E}^{-1}\left[c_{0}\left(\partial_{4} S_{0}\right)^{2}-c_{4}\left(\partial_{4} S_{0}\right)\left(\partial_{0} S_{0}\right)\right. \\
& \left.\quad-e A_{0} c_{4}\left(\partial_{4} S_{0}\right)\right]+m^{2} c_{0}=0, \\
& \tilde{A}^{-1}\left[c_{0}\left(\partial_{1} S_{0}\right)\left(\partial_{0} S_{0}\right)-c_{1}\left(\partial_{0} S_{0}\right)^{2}-e A_{0} c_{1}\left(\partial_{0} S_{0}\right)\right] \\
& \quad+\tilde{C}^{-1}\left[c_{1}\left(\partial_{2} S_{0}\right)^{2}-c_{2}\left(\partial_{1} S_{0}\right)\left(\partial_{2} S_{0}\right)\right]+\tilde{D}^{-1}\left[c_{1}\left(\partial_{3} S_{0}\right)^{2}\right.
\end{aligned}
$$

$$
\begin{align*}
& \left.\quad-c_{3}\left(\partial_{3} S_{0}\right)\left(\partial_{1} S_{0}\right)\right]+\tilde{E}^{-1}\left[c_{1}\left(\partial_{4} S_{0}\right)^{2}\right. \\
& \left.\quad-c_{4}\left(\partial_{1} S_{0}\right) c_{0}\left(\partial_{4} S_{0}\right)\right]+e A_{0} \tilde{A}^{-1}\left[c_{0}\left(\partial_{1} S_{0}\right)\right. \\
& \left.\quad-c_{1}\left(\partial_{0} S_{0}\right)\right]+m^{2} c_{1}=0,  \tag{3.4}\\
& \tilde{A}^{-1}\left[c_{2}\left(\partial_{0} S_{0}\right)^{2}-c_{0}\left(\partial_{0} S_{0}\right)\left(\partial_{2} S_{0}\right)+e A_{0} c_{2}\left(\partial_{0} S_{0}\right)\right] \\
& \quad-\tilde{B}\left[c_{2}\left(\partial_{1} S_{0}\right)^{2}-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{2} S_{0}\right)\right] \\
& \quad+\tilde{D}^{-1}\left[c_{3}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)\right. \\
& \left.-c_{2}\left(\partial_{3} S_{0}\right)^{2}\right]+\tilde{E}^{-1}\left[c_{4}\left(\partial_{2} S_{0}\right)\left(\partial_{4} S_{0}\right)\right. \\
& \left.\quad-c_{2}\left(\partial_{4} S_{0}\right)^{2}\right]+e A_{0} \tilde{A}^{-1}\left[c_{2}\left(\partial_{0} S_{0}\right)-c_{0}\left(\partial_{2} S_{0}\right)\right. \\
& \left.\quad+e A_{0} c_{2}\right]-m^{2} c_{2}=0,  \tag{3.5}\\
& \tilde{A}^{-1}\left[c_{3}\left(\partial_{0} S_{0}\right)^{2}-c_{0}\left(\partial_{0} S_{0}\right)\left(\partial_{3} S_{0}\right)+e A_{0} c_{3}\left(\partial_{0} S_{0}\right)\right] \\
& \quad-\tilde{B}\left[c_{3}\left(\partial_{1} S_{0}\right)^{2}-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{3} S_{0}\right)\right]-\tilde{C}^{-1}\left[c_{3}\left(\partial_{2} S_{0}\right)^{2}\right. \\
& \left.\quad-c_{2}\left(\partial_{2} S_{0}\right)\left(\partial_{3} S_{0}\right)\right]+\tilde{E}^{-1}\left[c_{4}\left(\partial_{4} S_{0}\right)\left(\partial_{3} S_{0}\right)\right. \\
& \left.\quad-c_{3}\left(\partial_{4} S_{0}\right)^{2}\right]+e A_{0} \tilde{A}^{-1}\left[c_{3}\left(\partial_{0} S_{0}\right)-c_{0}\left(\partial_{3} S_{0}\right)\right. \\
& \left.\quad+e A_{0} c_{3}\right]-m^{2} c_{3}=0,  \tag{3.6}\\
& \tilde{A}^{-1} \\
& \quad\left[c_{4}\left(\partial_{0} S_{0}\right)^{2}-c_{0}\left(\partial_{0} S_{0}\right)\left(\partial_{4} S_{0}\right)+e A_{0} c_{4}\left(\partial_{0} S_{0}\right)\right] \\
& \quad-\tilde{B}\left[c_{4}\left(\partial_{1} S_{0}\right)^{2}-c_{1}\left(\partial_{1} S_{0}\right)\left(\partial_{4} S_{0}\right)\right]-\tilde{C}^{-1}\left[c_{4}\left(\partial_{2} S_{0}\right)^{2}\right. \\
& \left.\quad-c_{2}\left(\partial_{2} S_{0}\right)\left(\partial_{4} S_{0}\right)\right] \\
& \quad-\tilde{D}^{-1}\left[c_{4}\left(\partial_{3} S_{0}\right)^{2}-c_{3}\left(\partial_{3} S_{0}\right)\left(\partial_{4} S_{0}\right)\right]  \tag{3.7}\\
& \quad+e A_{0} \tilde{A}^{-1}\left[c_{4}\left(\partial_{0} S_{0}\right)-c_{0}\left(\partial_{4} S_{0}\right)+e A_{0} c_{4}\right]-m^{2} c_{4}=0 .
\end{align*}
$$

We carry out the separation of variables:

$$
\begin{equation*}
S_{0}=-\left(E-j \check{\Omega}_{1}\right) t+W(r)+\Theta(\zeta, \vartheta)+N \phi \tag{3.8}
\end{equation*}
$$

where $\check{\Omega}_{1}$ is the angular velocity for BH given by Eq. (3.2).
For the above $S_{0}$ the preceding set of equations (3.3)-(3.7) can be written in terms of a matrix equation, $\Lambda\left(c_{0}, c_{1}, c_{2}, c_{3}\right.$, $\left.c_{4}\right)^{T}=0$, and the elements of the required matrix have the following form:

$$
\begin{aligned}
\Lambda_{00}= & \tilde{B} \dot{W}^{2}+\tilde{C}^{-1}\left(\partial_{2} \Theta\right)^{2}+\tilde{D}^{-1}\left(\partial_{3} \Theta\right)^{2}+\tilde{E}^{-1} N+m^{2} \\
\Lambda_{01}= & \tilde{B}\left[\left(E-j \check{\Omega}_{1}\right) \dot{W}-e A_{0} \dot{W}\right] \\
\Lambda_{02}= & \tilde{C}^{-1}\left(E-j \check{\Omega}_{1}\right)\left(\partial_{2} \Theta\right) \\
\Lambda_{03}= & \tilde{D}^{-1}\left(E-j \check{\Omega}_{1}\right)\left(\partial_{3} \Theta\right)-\tilde{D}^{-1} e A_{0}\left(\partial_{3} \Theta\right), \\
\Lambda_{04}= & \tilde{E}^{-1}\left(E-j \check{\Omega}_{1}\right) N-\tilde{E}^{-1} j e A_{0}, \\
\Lambda_{10}= & -\tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right) \dot{W}+e A_{0} \tilde{A}^{-1} \dot{W} \\
\Lambda_{11}= & -\tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)^{2}+e A_{0}\left(E-j \check{\Omega}_{1}\right) \tilde{A}^{-1} \\
& +\tilde{C}^{-1}\left(\partial_{2} \Theta\right)^{2}+\tilde{D}^{-1}\left(\partial_{3} \Theta\right)^{2}+\tilde{E}^{-1} N^{2} \\
& +e A_{0} \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)+m^{2} \\
\Lambda_{12}= & -\tilde{C}^{-1} \dot{W}\left(\partial_{2} \Theta\right), \quad \Lambda_{13}=-\tilde{D}^{-1} \dot{W}\left(\partial_{3} \Theta\right), \\
\Lambda_{14}= & -\tilde{E}^{-1} \dot{W} N, \quad \Lambda_{20}=\tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)\left(\partial_{2} \Theta\right) \\
& -\tilde{A}^{-1} e A_{0}\left(\partial_{2} \Theta\right), \\
\Lambda_{21}= & \tilde{B} \dot{W}\left(\partial_{2} \Theta\right),
\end{aligned}
$$

$$
\Lambda_{22}=\tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)^{2}-e A_{0}\left(E-j \check{\Omega}_{1}\right) \tilde{A}^{-1}-\tilde{B} \dot{W}^{2}
$$

$$
\begin{aligned}
& -\tilde{D}^{-1}\left(\partial_{3} \Theta\right)^{2}-\tilde{E}^{-1} N^{2} \\
& +e A_{0} \tilde{A}^{-1}\left[e A_{0}-\left(E-j \check{\Omega}_{1}\right)\right]-m^{2}, \\
\Lambda_{23}= & \tilde{D}^{-1}\left(\partial_{2} \Theta\right)\left(\partial_{3} \Theta\right), \quad \Lambda_{24}=\tilde{E}^{-1}\left(\partial_{2} \Theta\right) N, \\
\Lambda_{30}= & \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)\left(\partial_{3} \Theta\right)-e A_{0} \tilde{A}^{-1}\left(\partial_{3} \Theta\right), \\
\Lambda_{31}= & \tilde{B} \dot{W}\left(\partial_{3} \Theta\right), \quad \Lambda_{32}=\tilde{C}^{-1}\left(\partial_{2} \Theta\right)\left(\partial_{3} \Theta\right), \\
\Lambda_{33}= & \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)^{2}-e A_{0} \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)-\tilde{B} \dot{W}^{2} \\
& -\tilde{E}^{-1} N^{2}-\tilde{C}^{-1}\left(\partial_{2} \Theta\right)^{2}-m^{2} \\
& -e A_{0} \tilde{A}^{-1}\left[\left(E-j \check{\Omega}_{1}\right)-e A_{0}\right], \\
\Lambda_{34}= & \tilde{E}^{-1} j\left(\partial_{3} \Theta\right), \quad \Lambda_{40}=\tilde{A}^{-1}\left(\left(E-j \check{\Omega}_{1}\right) N\right. \\
& -e A_{0} \tilde{A}^{-1} j, \quad \Lambda_{41}=\tilde{B} \dot{W} N, \\
\Lambda_{42}= & \tilde{C}^{-1}\left(\partial_{2} \Theta\right) N, \quad \Lambda_{43}=\tilde{D}^{-1}\left(\partial_{3} \Theta\right) N, \\
\Lambda_{44}= & \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right)^{2}-e A_{0} \tilde{A}^{-1}\left(E-j \check{\Omega}_{1}\right) \\
& -\tilde{B} \dot{W}^{2}-\tilde{C}^{-1}\left(\partial_{2} \Theta\right)^{2} \\
& -\tilde{D}^{-1}\left(\partial_{3} \Theta\right)^{2}-m^{2}-e A_{0} \tilde{A}^{-1}\left[\left(E-j \check{\Omega}_{1}\right)-e A_{0}\right] .
\end{aligned}
$$

For the non-trivial solution, the determinant $\Lambda$ is equal to zero and using the same technique as discussed in the previous section, we get

$$
\begin{align*}
\operatorname{Im} W^{ \pm} & = \pm \int \sqrt{\frac{\left(E-e A_{0}-j \check{\Omega}_{1}\right)^{2}+\tilde{X}}{\tilde{A} \tilde{B}}} \\
& = \pm \iota \pi \frac{\left(E-e A_{0}-j \check{\Omega}_{1}\right)}{2 \kappa\left(r_{+}\right)} \tag{3.9}
\end{align*}
$$

where

$$
\begin{align*}
\tilde{X}= & -\tilde{A} \tilde{C}^{-1}\left(\partial_{2} \Theta\right)^{2}-\tilde{A} \tilde{D}^{-1}\left(\partial_{3} \Theta\right)^{2}-\tilde{A} m^{2} \\
& -\tilde{E}^{-1}\left(\partial_{2} \Theta\right) N . \tag{3.10}
\end{align*}
$$

Since the BH given by Eq. (3.2) is non-rotating, $\check{\Omega}_{1}=0$. The surface gravity for this BH is given by [40]
$\kappa\left(r_{+}\right)=\frac{2 r_{+}^{6}+r_{+}^{4}\left(1+\sum_{i=1}^{3} q_{i}\right)-\prod_{i=1}^{3} q_{i}}{r_{+}^{2} \sqrt{\prod_{i=1}^{3}\left(r_{+}^{2}+q_{i}\right)}}$.
The required tunneling probability as discussed in the previous section is
$\tilde{\Gamma}=\frac{\tilde{\Gamma}_{\text {emission }}}{\tilde{\Gamma}_{\text {absorption }}}=e^{-4 \operatorname{Im} W^{+}}=e^{-2 \pi \frac{\left(E-e A_{0}\right)\left(r_{+}^{2} \sqrt{\prod_{i=1}^{3}\left(r_{+}^{2}+q_{i}\right)}\right.}{2 r_{+}^{6}+r_{+}^{4}\left(1+\sum_{i=1}^{3} q_{i}\right)-\prod_{i=1}^{3} q_{i}}}$.
The Hawking temperature in this case is given by
$\tilde{T}_{H}=\frac{\left[2 r_{+}^{6}+r_{+}^{4}\left(1+\sum_{i=1}^{3} q_{i}\right)-\prod_{i=1}^{3} q_{i}\right]}{2 \pi r_{+}^{2} \sqrt{\prod_{i=1}^{3}\left(r_{+}^{2}+q_{i}\right)}}$.
The Hawking temperature is related to the energy $E$, the potential $A_{0}$, the angular momentum $j$, the radial coordinate at the outer horizon $r_{+}$and the charge $q_{i}$. We would like
to mention that the Hawking temperature of charged vector particles given by Eq. (3.12) is the same as the Hawking temperature of 5D gauged super-gravity BH in Eq. (9) in Ref. [40].

## 4 Outlook

During the tunneling process when a particle with electropositive energy crosses the horizon, it appears as Hawking radiation. Likewise, a particle with electronegative energy burrows in weave, it is assimilated by the BH, so its mass falls and finally disappears. Thus, the movement of the particles may be in the configuration of outgoing and incoming, performing the particle's action turns out to be complex and real, respectively. The emission rate of a tunneling particles from the BH is associated with the imaginary component of the particles' action, which in fact is related to the Boltzmann factor based on the Hawking temperature.

In this paper, we have extended the work of vector particle tunneling for more generalized BHs in 4D and 5D spaces and recovered their corresponding Hawking temperatures at which particles tunnel through horizons. For this purpose, we have used the Proca equation with the background of electromagnetism to investigate the tunneling of charged vector particles from accelerating and rotating BHs in 4D and 5D BHs having electric and magnetic charges with a NUT parameter. We have implemented the WKB approximation to the Proca equation, which leads to the set of field equations; then we use separation of variables to solve these equations. We solve for the radial part by using the determinant of the coefficient matrix being equal to zero. Using the surface gravity, we have formulated the tunneling probability and the Hawking temperature for both BHs at the outer horizon. All these quantities depend on the defining parameters of the BHs . It is worthwhile to mention here that the back-reaction effects of the emitted particle on the BH geometry and self-gravitating effects have been neglected, the derived Hawking temperature is only a leading term. Thus one does not need to calculate the appropriate solution of the semi-classical Einstein field equations for the geometry of the background BH in equilibrium with its Hawking radiation [42].

From our analysis we have concluded that the Hawking temperature at which particles tunnel through the horizon is independent of the species of particles. In particular the nature of the background BH geometries, for the particles having different spins (either spin up or down) or zero spin, the tunneling probabilities will be seen to be the same by considering semi-classical effects. Thus, their corresponding Hawking temperatures must be the same for all kinds of particles. For both cases, we have carried out the calculations for more general BHs, i.e., a pair of charged accelerating and rotating BHs with NUT parameter (which is a more general
case of BHs as compared to the BH taken in [43]) and a BH in 5D gauged super-gravity. Our findings are similar to the statement that the temperature of tunneling particles is independent of the species of the particles, and this result is also valid for different coordinate frames by using specific coordinate transformations. The authors of Ref. [43] have proved it for the Kerr BH (which only is rotating), while we have proved it for more generalized BHs. Hence, the conclusion still holds if background BH geometries are more general.

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