Eur. Phys. J. C (2017) 77:48
DOI 10.1140/epjc/s10052-017-4615-1

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Inönü–Wigner contraction and $D = 2 + 1$ supergravity

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Received: 25 November 2016 / Accepted: 5 January 2017 / Published online: 25 January 2017

Abstract We present a generalization of the standard Inönü–Wigner contraction by rescaling not only the generators of a Lie superalgebra but also the arbitrary constants appearing in the components of the invariant tensor. The procedure presented here allows one to obtain explicitly the Chern–Simons supergravity action of a contracted superalgebra. In particular we show that the Poincaré limit can be performed to a $D = 2 + 1$ (p, q) AdS Chern–Simons supergravity in presence of the exotic form. We also construct a new three-dimensional (2, 0) Maxwell Chern–Simons supergravity theory as a particular limit of (2, 0) AdS–Lorentz supergravity theory. The generalization for $\mathcal{N} = p + q$ gravitinos is also considered.

1 Introduction

The three-dimensional (super)gravity theory represents an interesting toy model in order to approach higher-dimensional (super)gravity theories, which are not only more difficult but also leads to tedious calculations. Additionally, the $D = 2+1$ model has the remarkable property to be written as a gauge theory using the Chern–Simons (CS) formalism [1,2]. In particular, the three-dimensional supersymmetric extension of General Relativity [3,4] can be obtained as a CS gravity theory using $(A)dS$ or Poincaré supergroup. A wide class of $\mathcal{N}$-extended Supergravities and further extensions have been studied in diverse contexts in, e.g., [5–25].

The derivation of a supergravity action for a given superalgebra is not, in general, a trivial task and its construction is not always ensured. On the other hand, several (super)algebras can be obtained as an Inönü–Wigner (IW) contraction of a given (super)algebra [38,39]. Nevertheless, the Chern–Simons action based on the IW contracted (super)algebra cannot always be obtained by rescaling the gauge fields and considering some limit as in the (anti)commutation relations. In particular it is well known that, in the presence of the exotic Lagrangian, the Poincaré limit cannot be applied to a (p, q) AdS CS supergravity [7]. This difficulty can be overcome extending the $\mathfrak{osp}(2, p) \otimes \mathfrak{osp}(2, q)$ superalgebra by introducing the automorphism generators $\mathfrak{so}(p)$ and $\mathfrak{so}(q)$ [9]. In such a case, the IW contraction can be applied and reproduces the Poincaré limit leading to a new (p, q) Poincaré supergravity which includes additional $\mathfrak{so}(p) \oplus \mathfrak{so}(q)$ gauge fields.

Here, we present a generalization of the IW contraction by considering not only the rescaling of the generators but also the constants of the non-vanishing components of an invariant tensor. The method introduced here ensures the construction of any CS action based on a contracted (super)algebra. In particular, we show that the Poincaré limit can be applied to a (p, q) AdS supergravity in the presence of the exotic Lagrangian without introducing extra fields as in Ref. [9]. Subsequently, we apply the method to different (p, q) AdS–Lorentz supergravities whose IW contraction leads to diverse (p, q) Maxwell supergravities. The possibility to turn the IW contraction into an algebraic operation is not new and has already been presented in the context of asymptotic symmetries and higher spin theories in Ref. [26]. Other interesting results using diverse flat limit contractions in supergravity can be found in Ref. [27].

At the bosonic level, the Maxwell symmetries have lead to interesting gravity theories allowing to recover General Relativity from Chern–Simons and Born–Infeld (BI) theories [28–31]. On the other hand, the AdS–Lorentz and its generalizations allow one to recover the Pure Lovelock [32–34] Lagrangian in a matter-free configuration from CS and BI theories [35,36]. At the supersymmetric level, the Maxwell superalgebra provides a pure supergravity action in the MacDowell–Mansouri formalism [37]. More recently, a three-dimensional CS action based on the minimal Maxwell

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superalgebra has been presented in Ref. [22] using the expansion procedure. Here, we show that the same result can be obtained using our alternative approach. Besides, we show that the Maxwell limit can also be applied in a (p, q) enlarged supergravity leading to a (p, q) Maxwell supergravity with an exotic Lagrangian.

The organization of the present work is as follows: in Sect. 2, we apply our approach to $\mathcal{N} = 1$ and $\mathcal{N} = p+q$ *AdS* CS supergravities. In particular, we show that the Poincaré limit can be applied to (p, q) *AdS* CS supergravity theories in the presence of the exotic Lagrangian. In Sect. 3, we discuss the Inönü–Wigner contraction of an expanded supergravity. In particular, we describe the general scheme. In Sect. 4, we apply our procedure to a $\mathcal{N} = 1$ expanded CS supergravity. In Sect. 5, we present the CS formulation of the $(2, 0)$ and (p, q) Maxwell supergravities and discuss their relations to $(2, 0)$ and (p, q) *AdS*–Lorentz supergravities, respectively. Section 6 concludes our work with some comments and possible developments.

2 Inönü–Wigner contraction and the invariant tensor

The standard Inönü–Wigner contraction [38,39] of a Lie (super)algebra $\mathfrak{g}$ consists basically in properly rescaling the generators by a parameter σ and applying the limit $\sigma \to \infty$ corresponding to a contracted (super)algebra.

Despite having the proper contracted (super)algebra following the IW scheme, the contracted invariant tensor cannot be trivially obtained. This is particularly regrettable since the invariant tensor is an essential ingredient in the construction of a Chern–Simons action.

In this paper, we present a generalization of the standard Inönü–Wigner contraction considering the rescaling not only of the generators but also of the constants appearing in the invariant tensor. The method introduced here allows one to obtain the non-vanishing components of the invariant tensor of an IW contracted (super)algebra. Thus, the construction of any CS action based on an IW contracted (super)algebra is ensured. In particular, we apply the method to different (p, q) *AdS*–Lorentz superalgebras whose IW contraction leads to diverse (p, q) Maxwell superalgebras.

Let us first apply the approach to the *AdS* supergravity in order to derive the Poincaré supergravity.

2.1 Poincaré and $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ supergravity

As in the bosonic level, the IW contraction of the *AdS* superalgebra leads to the Poincaré one. Besides, the Poincaré CS supergravity action can be obtained considering a particular limit after an appropriate rescaling of the fields of the super *AdS* CS action. Nevertheless, in the presence of torsion the exotic Lagrangian, which has no Poincaré limit [7], is added to the *AdS* CS supergravity.

The three-dimensional Chern–Simons action is given by

$$I_{\mathrm{CS}}^{(2+1)} = k \int \left\langle A dA + \frac{2}{3} A^3 \right\rangle, \tag{1}$$

where A corresponds to the gauge connection one-form and $\langle \ldots \rangle$ denotes the invariant tensor. In the case of the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra, the connection one-form is given by

$$A = \frac{1}{2}\omega^{ab} \tilde{J}_{ab} + \frac{1}{l} e^a \tilde{P}_a + \frac{1}{\sqrt{l}} \psi^\alpha \tilde{Q}_\alpha, \tag{2}$$

where $\tilde{J}_{ab}$, $\tilde{P}_a$ and $\tilde{Q}_\alpha$ are the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ generators. The gauge fields e^a, ω^{ab} and ψ are the dreibein, the spin connection and the gravitino, respectively. Here, the length scale l is introduced purposely in order to have dimensionless generators $T_A = \{\tilde{J}_{ab}, \tilde{P}_a, \tilde{Q}_\alpha\}$ such that the connection one-form $A = A_\mu^A T_A dx^\mu$ must also be dimensionless. Since the dreibein $e^a = e^a_\mu dx^\mu$ is related to the spacetime metric $g_{\mu\nu}$ through $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$, it must have dimensions of length. Then the "true" gauge field should be considered as e^a/l. In the same way, we consider $\psi/\sqrt{l}$ as the supersymmetry gauge field since the gravitino $\psi = \psi_\mu dx^\mu$ has dimensions of $(\text{length})^{1/2}$.

The (anti)-commutation relations for the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra are given by

$$\left[\tilde{J}_{ab}, \tilde{J}_{cd}\right] = \eta_{bc}\tilde{J}_{ad} - \eta_{ac}\tilde{J}_{bd} - \eta_{bd}\tilde{J}_{ac} + \eta_{ad}\tilde{J}_{bc}, \tag{3}$$

$$\left[\tilde{J}_{ab}, \tilde{P}_c\right] = \eta_{bc}\tilde{P}_a - \eta_{ac}\tilde{P}_b, \tag{4}$$

$$\left[\tilde{P}_a, \tilde{P}_b\right] = \tilde{J}_{ab}, \tag{5}$$

$$\left[\tilde{J}_{ab}, \tilde{Q}_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab}\tilde{Q}\right)_\alpha, \left[\tilde{P}_a, \tilde{Q}_\alpha\right] = -\frac{1}{2}\left(\Gamma_a\tilde{Q}\right)_\alpha, \tag{6}$$

$$\left\{\tilde{Q}_\alpha, \tilde{Q}_\beta\right\} = -\frac{1}{2}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta}\tilde{P}_a\right], \tag{7}$$

where C denotes the charge conjugation matrix, Γ_α represents the Dirac matrices and $\Gamma_{ab} = \frac{1}{2}[\Gamma_a, \Gamma_b]$.

The non-vanishing components of an invariant tensor for the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra are given by

$$\left\langle \tilde{J}_{ab}\tilde{P}_c \right\rangle = \mu_1 \epsilon_{abc}, \tag{8}$$

$$\left\langle \tilde{J}_{ab}\tilde{J}_{cd} \right\rangle = \mu_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{9}$$

$$\left\langle \tilde{P}_a\tilde{P}_b \right\rangle = \mu_0 \eta_{ab}, \tag{10}$$

$$\left\langle \tilde{Q}_\alpha\tilde{Q}_\beta \right\rangle = 2\left(\mu_1 - \mu_0\right) C_{\alpha\beta}, \tag{11}$$

where μ_0 and μ_1 are arbitrary constants. Then, considering the invariant tensor (8)–(11) and the connection one-form in the general expression for the $D=3$ CS action, we have

$$\begin{aligned}I_{\mathrm{CS}}^{(2+1)} = k\int\Big[&\frac{\mu_0}{2}\left(\omega^a{}_b d\omega^b{}_a+\frac{2}{3}\omega^a{}_c\omega^c{}_b\omega^b{}_a+\frac{2}{l^2}e^aT_a-\frac{2}{l}\bar{\psi}\Psi\right)\\&+\frac{\mu_1}{l}\left(\epsilon_{abc}R^{ab}e^c+\frac{1}{3l^2}\epsilon_{abc}e^ae^be^c+2\bar{\psi}\Psi\right)\\&-d\left(\frac{\mu_1}{2l}\epsilon_{abc}\omega^{ab}e^c\right)\Big],\end{aligned} \tag{12}$$

where

$$R^{ab}=d\omega^{ab}+\omega^a{}_c\omega^{cb},\ T^a=de^a+\omega^a{}_be^b,$$
$$\Psi=d\psi+\frac{1}{4}\omega_{ab}\Gamma^{ab}\psi+\frac{1}{2l}e_a\Gamma^a\psi.$$

It is well known that the following rescaling of the generators:

$$\tilde{J}_{ab}\to\bar{J}_{ab},\quad \tilde{P}_a\to\sigma^2\bar{P}_a,\quad \tilde{Q}_\alpha\to\sigma\bar{Q}_\alpha$$

leads to the Poincaré superalgebra in the limit $\sigma\to\infty$. It seems natural to construct a Poincaré CS supergravity action combining the corresponding rescaling of the generators with the *AdS* invariant tensor given by Eqs. (8)–(11). However, such rescaling of the generators leads to a trivial invariant tensor and then to a trivial CS action. In order to obtain the right Poincaré limit at the level of the action, a rescaling of the arbitrary constants appearing in the invariant tensor should also be considered. Indeed, a rescaling which preserves the curvatures structure is given by

$$\mu_0\to\mu_0,\quad \mu_1\to\sigma^2\mu_1.$$

Then, considering the rescaling of both the generators and the constants, one can see that the limit $\sigma\to\infty$ leads to the non-vanishing components of the invariant tensor for the Poincaré superalgebra,

$$\langle\bar{J}_{ab}\bar{P}_c\rangle_{\mathcal{P}}=\mu_1\epsilon_{abc}, \tag{13}$$
$$\langle\bar{J}_{ab}\bar{J}_{cd}\rangle_{\mathcal{P}}=\mu_0\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{14}$$
$$\langle\bar{Q}_\alpha\bar{Q}_\beta\rangle_{\mathcal{P}}=2\mu_1C_{\alpha\beta}, \tag{15}$$

where $\bar{J}_{ab}$, $\bar{P}_a$ and $\bar{Q}_\alpha$ are the Poincar é generators. Considering the Poincaré gauge connection one-form and the non-vanishing components of the Poincaré invariant tensor, the CS action reduces to

$$\begin{aligned}I_{\mathrm{CS}}^{(2+1)}=k\int&\frac{\mu_0}{2}\left(\omega^a{}_bd\omega^b{}_a+\frac{2}{3}\omega^a{}_c\omega^c{}_b\omega^b{}_a\right)\\&+\frac{\mu_1}{l}\left(\epsilon_{abc}R^{ab}e^c+2\bar{\psi}\Psi\right)-d\left(\frac{\mu_1}{2l}\epsilon_{abc}\omega^{ab}e^c\right),\end{aligned} \tag{16}$$

where the fermionic curvature is now given by

$$\Psi=d\psi+\frac{1}{4}\omega_{ab}\Gamma^{ab}\psi.$$

Let us note that the present approach allows one to trivially obtain the Poincar é limit from the $\mathfrak{osp}\,(2|1)\otimes\mathfrak{sp}\,(2)$ CS action. One could suggest that the same result can be obtained considering $l\to\infty$,, nevertheless the presence of the exotic Lagrangian forbids such limit. Additionally, one can notice that the gravitino does not contribute anymore to the exotic form.

2.2 (p,q) Poincaré and $\mathfrak{osp}\,(2|p)\otimes\mathfrak{osp}\,(2|q)$ supergravity

Let us now consider the (p,q) *AdS* supergravity theories, which can be viewed as a direct sum of *AdS* superalgebras. It is well known that the (p,q) Poincaré superalgebra can be derived as a Inönü–Wigner contraction of the (p,q) *AdS* superalgebra. However, as was mentioned in Refs. [9,11,40], the Poincaré limit at the level of the action requires enlargement of the *AdS* superalgebra, considering a direct sum of the $\mathfrak{so}\,(p)\oplus\mathfrak{so}\,(q)$ algebra and the (p,q) *AdS* superalgebra. Here, we show that our approach allows one to obtain the Poincaré limit without introducing additional gauge fields. In particular, the non-vanishing components of the invariant tensor of the (p,q) Poincaré superalgebra are obtained from the *AdS* ones.

The supersymmetric extension of the *AdS* algebra contains $\mathcal{N}=p+q$ gravitinos, and it is spanned by the set of generators $\left\{\tilde{J}_{ab},\tilde{P}_a,\tilde{T}^{ij},\tilde{T}^{IJ},\tilde{Q}^i_\alpha,Q^I_\alpha\right\}$ which satisfy [9]

$$\left[\tilde{J}_{ab},\tilde{J}_{cd}\right]=\eta_{bc}\tilde{J}_{ad}-\eta_{ac}\tilde{J}_{bd}-\eta_{bd}\tilde{J}_{ac}+\eta_{ad}\tilde{J}_{bc}, \tag{17}$$
$$\left[\tilde{T}^{ij},\tilde{T}^{kl}\right]=\delta^{jk}\tilde{T}^{il}-\delta^{ik}\tilde{T}^{jl}-\delta^{jl}\tilde{T}^{ik}+\delta^{il}\tilde{T}^{jk}, \tag{18}$$
$$\left[\tilde{T}^{IJ},\tilde{T}^{KL}\right]=\delta^{JK}\tilde{T}^{IL}-\delta^{IK}\tilde{T}^{JL}-\delta^{JL}\tilde{T}^{IK}+\delta^{IL}\tilde{T}^{JK}, \tag{19}$$
$$\left[\tilde{J}_{ab},\tilde{P}_c\right]=\eta_{bc}\tilde{P}_a-\eta_{ac}\tilde{P}_b,\left[\tilde{P}_a,\tilde{P}_b\right]=\tilde{J}_{ab}, \tag{20}$$
$$\left[\tilde{T}^{ij},\tilde{Q}^k_\alpha\right]=\left(\delta^{jk}\tilde{Q}^i_\alpha-\delta^{ik}\tilde{Q}^j_\alpha\right),\left[\tilde{T}^{IJ},\tilde{Q}^K_\alpha\right]=\left(\delta^{JK}\tilde{Q}^I_\alpha-\delta^{IK}\tilde{Q}^J_\alpha\right), \tag{21}$$
$$\left[\tilde{J}_{ab},\tilde{Q}^i_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}\tilde{Q}^i\right)_\alpha,\left[\tilde{P}_a,\tilde{Q}^i_\alpha\right]=-\frac{1}{2}\left(\Gamma_a\tilde{Q}^i\right)_\alpha, \tag{22}$$
$$\left[\tilde{J}_{ab},\tilde{Q}^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}\tilde{Q}^I\right)_\alpha,\left[\tilde{P}_a,\tilde{Q}^I_\alpha\right]=\frac{1}{2}\left(\Gamma_a\tilde{Q}^I\right)_\alpha, \tag{23}$$
$$\left\{\tilde{Q}^i_\alpha,\tilde{Q}^j_\beta\right\}=-\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab}-2\left(\Gamma^aC\right)_{\alpha\beta}\tilde{P}_a\right]+C_{\alpha\beta}\tilde{T}^{ij}, \tag{24}$$
$$\left\{\tilde{Q}^I_\alpha,\tilde{Q}^J_\beta\right\}=\frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab}+2\left(\Gamma^aC\right)_{\alpha\beta}\tilde{P}_a\right]-C_{\alpha\beta}\tilde{T}^{IJ}, \tag{25}$$

where $i,j=1,\ldots,p$ and $I,J=1,\ldots,q$. Here, the $\tilde{T}^{ij}$ and $\tilde{T}^{IJ}$ generators correspond to internal symmetry generators and satisfy a $\mathfrak{so}\,(p)$ and $\mathfrak{so}\,(q)$ algebra, respectively.

One can introduce the $\mathfrak{osp}\,(2|p)\times\mathfrak{osp}\,(2|q)$ connection one-form A given by

$$A = \frac{1}{2}\omega^{ab}\tilde{J}_{ab} + \frac{1}{l}e^a\tilde{P}_a + \frac{1}{2}A^{ij}\tilde{T}_{ij} + \frac{1}{2}A^{IJ}\tilde{T}_{IJ} + \frac{1}{\sqrt{l}}\bar{\psi}_i\tilde{Q}^i + \frac{1}{\sqrt{l}}\bar{\psi}_I\tilde{Q}^I. \tag{26}$$

The non-vanishing components of the invariant tensor for the (p,q) AdS superalgebra are given by

$$\left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \mu_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{27}$$
$$\left\langle \tilde{J}_{ab}\tilde{P}_c\right\rangle = \mu_1\epsilon_{abc}, \tag{28}$$
$$\left\langle \tilde{P}_a\tilde{P}_b\right\rangle = \mu_0\eta_{ab}, \tag{29}$$
$$\left\langle \tilde{Q}^i_\alpha\tilde{Q}^j_\beta\right\rangle = 2\left(\mu_1 - \mu_0\right)C_{\alpha\beta}\delta^{ij}, \tag{30}$$
$$\left\langle \tilde{Q}^I_\alpha\tilde{Q}^J_\beta\right\rangle = 2\left(\mu_1 + \mu_0\right)C_{\alpha\beta}\delta^{IJ}, \tag{31}$$
$$\left\langle \tilde{T}^{ij}\tilde{T}^{kl}\right\rangle = 2\left(\mu_0 - \mu_1\right)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right), \tag{32}$$
$$\left\langle \tilde{T}^{IJ}\tilde{T}^{KL}\right\rangle = 2\left(\mu_0 + \mu_1\right)\left(\delta^{IL}\delta^{KJ} - \delta^{IK}\delta^{LJ}\right), \tag{33}$$

where μ_0 and μ_1 are arbitrary constants. Considering the connection one-form A and the non-vanishing components of the invariant tensor in the three-dimensional CS general expression (1), we obtain the $\mathfrak{osp}(2|p) \otimes \mathfrak{osp}(2|q)$ supergravity action in three dimensions:

$$\begin{aligned} I^{(2+1)}_{\text{CS}} = k\int \frac{\mu_0}{2}\left(\omega^a{}_b d\omega^b{}_a + \frac{2}{3}\omega^a{}_c\omega^c{}_b\omega^b{}_a + \frac{2}{l^2}e^aT_a\right) \\ +\frac{\mu_1}{l}\epsilon_{abc}\left(R^{ab}e^c + \frac{1}{3l^2}e^ae^be^c\right) \\ +\left(\mu_0-\mu_1\right)\left[A^{ij}dA^{ji} + \frac{2}{3}A^{ik}A^{kj}A^{ji}\right] \\ +\left(\mu_0+\mu_1\right)\left[A^{IJ}dA^{JI} + \frac{2}{3}A^{IK}A^{KJ}A^{JI}\right] \\ +2\left(\mu_1-\mu_0\right)\left[\frac{1}{l}\bar{\psi}^i\left(\Psi^i + A^{ij}\psi^j\right)\right] \\ +2\left(\mu_1+\mu_0\right)\left[\frac{1}{l}\bar{\psi}^I\left(\Psi^I + A^{IJ}\psi^J\right)\right], \end{aligned} \tag{34}$$

where

$$R^{ab} = d\omega^{ab} + \omega^a{}_c\omega^{cb}, T^a = de^a + \omega^a{}_ce^c,$$
$$\Psi^i = d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i + \frac{1}{2l}e_a\Gamma^a\psi^i,$$
$$\Psi^I = d\psi^I + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^I - \frac{1}{2l}e_a\Gamma^a\psi^I.$$

Let us note that an off-shell formulation for $\mathfrak{osp}(2|p) \times \mathfrak{osp}(2|q)$ supergravity is not ensured when $p+q > 1$. Interestingly, diverse off-shell formulations for (p,q) AdS supergravity when $p+q \le 3$ can be found in Refs. [41,42].

One can note that the (p,q) Poincaré superalgebra can be obtained from the (p,q) AdS one considering the following rescaling of the generators:

$$\tilde{J}_{ab} \to \bar{J}_{ab}, \quad \tilde{T}^{ij} \to \sigma^2\bar{T}^{ij}, \quad \tilde{P}_a \to \sigma^2\bar{P}_a, \quad \tilde{Q}_\alpha \to \sigma\bar{Q}_\alpha$$

and the limit $\sigma \to \infty$:

$$\left[\bar{J}_{ab},\bar{J}_{cd}\right] = \eta_{bc}\bar{J}_{ad} - \eta_{ac}\bar{J}_{bd} - \eta_{bd}\bar{J}_{ac} + \eta_{ad}\bar{J}_{bc}, \tag{35}$$
$$\left[\bar{J}_{ab},\bar{P}_c\right] = \eta_{bc}\bar{P}_a - \eta_{ac}\bar{P}_b, \tag{36}$$
$$\left[\bar{J}_{ab},\bar{Q}^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab}\bar{Q}^i\right)_\alpha, \tag{37}$$
$$\left[\bar{J}_{ab},\bar{Q}^I_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab}\bar{Q}^I\right)_\alpha, \tag{38}$$
$$\left\{\bar{Q}^i_\alpha,\bar{Q}^j_\beta\right\} = \delta^{ij}\left(\Gamma^aC\right)_{\alpha\beta}\bar{P}_a + C_{\alpha\beta}\bar{T}^{ij}, \tag{39}$$
$$\left\{\bar{Q}^I_\alpha,\bar{Q}^J_\beta\right\} = \delta^{IJ}\left(\Gamma^aC\right)_{\alpha\beta}\bar{P}_a - C_{\alpha\beta}\bar{T}^{IJ}, \tag{40}$$

Here, T^{ij} and T^{IJ} behave as central charges and no longer satisfy a $\mathfrak{so}(p)$ and a $\mathfrak{so}(q)$ algebra, respectively. Indeed, when p or q is greater than 1, the (p,q) Poincaré superalgebra corresponds to a central extension of the $\mathcal{N}$-extended Poincaré superalgebra.

At the level of the action, we have to consider a rescaling of the constants appearing in the invariant tensor. Indeed, a rescaling which preserves the curvature structure is given by

$$\mu_0 \to \mu_0, \quad \mu_1 \to \sigma^2\mu_1.$$

Then the limit $\sigma \to \infty$ leads to the non-vanishing components of the invariant tensor for the Poincaré superalgebra,

$$\left\langle \bar{J}_{ab}\bar{J}_{cd}\right\rangle_{\mathcal{P}} = \mu_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{41}$$
$$\left\langle \bar{J}_{ab}\bar{P}_c\right\rangle_{\mathcal{P}} = \mu_1\epsilon_{abc}, \tag{42}$$
$$\left\langle \bar{Q}^i_\alpha\bar{Q}^j_\beta\right\rangle_{\mathcal{P}} = 2\mu_1C_{\alpha\beta}\delta^{ij}, \tag{43}$$
$$\left\langle \bar{Q}^I_\alpha\bar{Q}^J_\beta\right\rangle_{\mathcal{P}} = 2\mu_1C_{\alpha\beta}\delta^{IJ}, \tag{44}$$

where $\bar{J}_{ab}$, $\bar{P}_a$, $\bar{Q}^i_\alpha$ and $\bar{Q}^I_\alpha$ correspond now to the Poincaré generators. As was noticed in Ref. [9], there are no components of the (p,q) Poincaré invariant tensor including the $\bar{T}^{ij}$ and $\bar{T}^{IJ}$ generators. Indeed, considering the (p,q) Poincaré connection one-form and the invariant tensor (41)–(44) in the general expression for the CS action we find

$$\begin{aligned} I^{(2+1)}_{\text{CS}} = k\int \frac{\mu_0}{2}\left(\omega^a{}_b d\omega^b{}_a + \frac{2}{3}\omega^a{}_c\omega^c{}_b\omega^b{}_a\right) \\ +\frac{\mu_1}{l}\left(\epsilon_{abc}R^{ab}e^c + 2\bar{\psi}^i\Psi^i + 2\bar{\psi}^I\Psi^I\right), \end{aligned} \tag{45}$$

where

$$\Psi^i = d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i,$$
$$\Psi^I = d\psi^I + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^I.$$

The Poincaré CS action (45) can be directly obtained from the (p, q) AdS one considering the rescaling of the constants ($\mu_0 \to \mu_0$, $\mu_1 \to \sigma^2\mu_1$), the rescaling of the fields:

$$\omega_{ab} \to \omega_{ab}, A^{ij} \to \sigma^{-2}A^{ij}, e_a \to \sigma^{-2}e_a,$$
$$\psi^i \to \sigma^{-1}\psi^i, \psi^I \to \sigma^{-1}\psi^I,$$

and the limit $\sigma \to \infty$. Thus, the Poincaré limit can be applied without introducing extra fields and/or change of basis. Nevertheless, as in the $\mathcal{N} = 1$ case, the gravitino does not contribute to the exotic form. Besides, no $\mathfrak{so}(p)$ or $\mathfrak{so}(q)$ gauge fields appear in the Lagrangian. In order to obtain more interesting supergravity actions whose gravitino appears in the exotic term, it is necessary to consider our approach to enlarged supersymmetries.

On the other hand, let us note that the term proportional to μ_1 reproduces the action of Ref. [9] when $F(A) = 0$. Naturally, the same procedure can be applied to the direct sum of (p, q) AdS superalgebra and $\mathfrak{so}(p) \oplus \mathfrak{so}(q)$ algebra, which would lead to the most general action for (p, q) Poincaré superalgebra [9].

3 Inönü–Wigner contraction of an S-expanded supergravity

The development of the Lie (super)algebra expansion method has played an important role in deriving new (super)gravity theories [43–55]. In particular, the semigroup expansion method (S-expansion) allows to find explicitly the non-vanishing components of an invariant tensor for an expanded (super)algebra in terms of the original one [47]. This feature is particularly useful since the invariant tensor is a crucial ingredient in the construction of a Chern–Simons action.

Nevertheless, the CS action for a contracted superalgebra cannot be naively obtained by rescaling the generators appearing in the non-vanishing components of an invariant tensor and considering some limit.

As in the previous section, we show that by applying the rescaling to both generators and constants of the invariant tensor, the CS action for a contracted superalgebra can be obtained. In particular, we present the general scheme in order to derive an IW contracted supergravity from an S-expanded one.

First, we shall consider the contraction of the following subspace decomposition of the Lie S-expanded superalgebra $\mathfrak{G} = S \times \mathfrak{g}$:

$$\mathfrak{G} = W_0 \oplus W_1 \oplus W_2, \tag{46}$$

where W_0 corresponds to a subalgebra, W_1 corresponds to the fermionic subspace, and W_2 is generated by boost generators. Such a decomposition satisfies

$$[W_0, W_0] \subset W_0, [W_0, W_2] \subset W_2, \tag{47}$$
$$[W_0, W_1] \subset W_1, [W_1, W_2] \subset W_1, \tag{48}$$
$$[W_1, W_1] \subset W_0 \oplus W_2, [W_2, W_2] \subset W_0, \tag{49}$$

where each subspace is generated by sets of generators

$$W_p = \Big\{ X_p^{(i)} = \lambda_{i+p} X_p \text{ with}$$
$$i = 0, 2, 4 \ldots, n-p \text{ and } p = 0, 1, 2 \Big\}.$$

Here X_p are the generators of the original superalgebra $\mathfrak{g}$ and λ_{i+p} is an element of a semigroup S satisfying some explicit multiplication law of the $S_{\mathcal{M}}$ family [56].

The IW contraction of $\mathfrak{G}$ is obtained considering the rescaling of the expanded generators

$$X_p^{(i)} = \sigma^{i+p} X_p^{(i)}$$

and applying the limit $\sigma \to \infty$.

On the other hand, according to Theorem VII.2 of Ref. [47], the invariant tensor for an S-expanded superalgebra $\mathfrak{G}$ can be obtained from the original ones through

$$\left\langle T_{(A,j)} T_{(B,k)} \right\rangle_{\mathfrak{G}} = \tilde{\alpha}_i K_{jk}^{\ \ i} \left\langle T_A T_B \right\rangle_{\mathfrak{g}}, \tag{50}$$

with $T_{(A,j)} = \lambda_j T_A$. Here $\tilde{\alpha}_i$ are arbitrary constants and $K_{jk}^{\ \ i}$ is the 2-selector for the semigroup S defined as

$$K_{jk}^{\ \ i} = \begin{cases} 1, & \text{when } i = i(j,k), \\ 0, & \text{otherwise,} \end{cases}$$

with $\lambda_{i(j,k)} = \lambda_j \lambda_k$.

The IW contraction of the invariant tensor is obtained, considering the rescaling of the generators

$$T_{(A,j)} = \sigma^j T_{(A,j)},$$

the rescaling of the constant $\tilde{\alpha}_i$,

$$\tilde{\alpha}_i \to \sigma^i \tilde{\alpha}_i$$

and applying the limit $\sigma \to \infty$.

The approach considered here offers a close relation between expansion and contraction. Interestingly, as we shall see, our procedure allows us to obtain the contracted supergravity in presence of expanded Pontryagin–Chern–Simons form.

In the following sections, we shall present known and new Maxwell supergravities considering the present IW approach to diverse S-expanded supergravities.

4 Inönü–Wigner contraction and $\mathcal{N} = 1$ supergravity

Here, we present a generalization of the Inönü–Wigner contraction combining the S-expansion method and the rescaling of the invariant tensor and generators. In particular, we show the $D = 3$ CS action based on a $\mathcal{N} = 1$ Maxwell superalgebra.

A non-standard supersymmetrization of the Maxwell algebra was introduced in Refs. [57,58] which can be obtained as an IW contraction of the standard AdS–Lorentz superalgebra[1] [59,60]. Nevertheless, the non-standard Maxwell supersymmetric action and its physical relevance remains poorly explored due to its unusual anticommutation relations. Indeed, the P_a generators of the non-standard Maxwell superalgebra are not expressed as bilinear expressions of the fermionic generators Q,

$$\left\{Q_\alpha, Q_\beta\right\} = -\frac{1}{2}\left(\Gamma^{ab}C\right)_{\alpha\beta} Z_{ab}.$$

This feature prevents construction of a supergravity action based on this peculiar supersymmetry. Despite this particularity, there is an alternative in order to construct a $\mathcal{N} = 1$ supergravity action based on the Maxwell supersymmetries.

A particular Maxwell superalgebra, also called the minimal supersymmetrization of the Maxwell algebra [61–64] differs from the non-standard Maxwell one since it possesses an additional fermionic generator. Interestingly, a minimal Maxwell superalgebra can be derived as an Inönü–Wigner contraction of a new minimal AdS–Lorentz superalgebra introduced in Ref. [65].

Before studying the explicit IW contraction at the level of the invariant tensor, we first present the explicit construction of a CS supergravity invariant under the minimal AdS–Lorentz superalgebra. To this purpose, we will apply the S-expansion procedure analogously to the four-dimensional case [65].

4.1 Minimal AdS–Lorentz exotic supergravity

Following the procedure of Ref. [65], a minimal AdS–Lorentz superalgebra can be derived as an S-expansion of the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra. Indeed, considering $S_{\mathcal{M}}^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ as the abelian semigroup whose elements satisfy

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha+\beta \le 4, \\ \lambda_{\alpha+\beta-4}, & \text{if } \alpha+\beta > 4, \end{cases} \tag{51}$$

and after extracting a resonant subalgebra of $S_{\mathcal{M}}^{(4)} \times (\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2))$, the minimal AdS–Lorentz superalgebra is obtained [65]. This algebra corresponds to a supersymmetric extension of the $\mathfrak{so}(2,2) \oplus \mathfrak{so}(2,1)$ algebra $= \{J_{ab}, P_a, Z_{ab}\}$ and is generated by $\{J_{ab}, P_a, \tilde{Z}_{ab}, \tilde{Z}_a, Z_{ab}, Q_\alpha, \Sigma_\alpha\}$. This superalgebra, as in the Maxwell case, is quite different from the standard AdS–Lorentz superalgebra discussed in Refs. [19,59]. In fact, besides extra bosonic generators $\{\tilde{Z}_{ab}, \tilde{Z}_a\}$, it also has more than one spinor generator. The explicit (anti)commutation relations can be found in Appendix A for $\mathcal{N} = 1$.

The construction of a Chern–Simons action for the minimal AdS–Lorentz superalgebra requires the gauge connection one-form A:

$$\begin{aligned} A = {} & \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{2}\tilde{k}^{ab}\tilde{Z}_{ab} + \frac{1}{2}k^{ab} Z_{ab} + \frac{1}{l}e^a P_a \\ & + \frac{1}{l}\tilde{h}^a \tilde{Z}_a + \frac{1}{\sqrt{l}}\psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\xi^\alpha \Sigma_\alpha. \end{aligned} \tag{52}$$

Since we have considered a dimensionless connection one-form, a factor l has to be introduced for the dreibein field and the dreibein like field $\tilde{h}^a$. The same argument applies for the spinor fields.

Another crucial ingredient necessary to write down a CS supergravity action is the invariant tensor. Following Theorem VII.2 of Ref. [47], the non-vanishing components of an invariant tensor for the super minimal AdS–Lorentz are given by

$$\langle J_{ab} J_{cd}\rangle_S = \tilde{\alpha}_0 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{53}$$

$$\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_S = \tilde{\alpha}_2 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_2\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{54}$$

$$\left\langle \tilde{Z}_{ab} Z_{cd}\right\rangle_S = \tilde{\alpha}_2 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_2\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{55}$$

$$\langle J_{ab} Z_{cd}\rangle_S = \tilde{\alpha}_4 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{56}$$

$$\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_S = \tilde{\alpha}_4 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{57}$$

$$\langle Z_{ab} Z_{cd}\rangle_S = \tilde{\alpha}_4 \left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{58}$$

$$\langle J_{ab} P_c\rangle_S = \langle Z_{ab} P_c\rangle_S = \left\langle \tilde{Z}_{ab}\tilde{Z}_c\right\rangle_S = \tilde{\alpha}_2\left\langle \tilde{J}_{ab}\tilde{P}_c\right\rangle = \beta_2\epsilon_{abc}, \tag{59}$$

$$\left\langle J_{ab}\tilde{Z}_c\right\rangle_S = \left\langle Z_{ab}\tilde{Z}_c\right\rangle_S = \left\langle \tilde{Z}_{ab} P_c\right\rangle_S = \tilde{\alpha}_4\left\langle \tilde{J}_{ab}\tilde{P}_c\right\rangle = \beta_4\epsilon_{abc}, \tag{60}$$

$$\langle P_a P_b\rangle_S = \left\langle \tilde{Z}_a\tilde{Z}_b\right\rangle_S = \tilde{\alpha}_4\left\langle \tilde{P}_a\tilde{P}_b\right\rangle = \alpha_4\eta_{ab}, \tag{61}$$

$$\left\langle P_a\tilde{Z}_b\right\rangle_S = \tilde{\alpha}_2\left\langle \tilde{P}_a\tilde{P}_b\right\rangle = \alpha_2\eta_{ab}, \tag{62}$$

$$\langle Q_\alpha Q_\beta\rangle_S = \langle \Sigma_\alpha\Sigma_\beta\rangle_S = \tilde{\alpha}_2\left\langle \tilde{Q}_\alpha\tilde{Q}_\beta\right\rangle = 2\left(\beta_2 - \alpha_2\right)C_{\alpha\beta}, \tag{63}$$

$$\langle Q_\alpha \Sigma_\beta\rangle_S = \tilde{\alpha}_4\left\langle \tilde{Q}_\alpha\tilde{Q}_\beta\right\rangle = 2\left(\beta_4 - \alpha_4\right)C_{\alpha\beta}, \tag{64}$$

where $\{\tilde{J}_{ab}, \tilde{P}_a, \tilde{Q}_\alpha\}$ generate the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra (see Eqs. (8)–(11)) and where we have defined

$$\alpha_0 \equiv \tilde{\alpha}_0\mu_0, \quad \alpha_2 \equiv \tilde{\alpha}_2\mu_0, \quad \alpha_4 \equiv \tilde{\alpha}_4\mu_0,$$
$$\beta_2 \equiv \tilde{\alpha}_2\mu_1, \quad \beta_4 \equiv \tilde{\alpha}_4\mu_1.$$

Here $\tilde{\alpha}_0, \tilde{\alpha}_2, \tilde{\alpha}_4$ are arbitrary constants as μ_0 and μ_1. The CS supergravity action can be written considering the connection one-form (52) and the non-vanishing component of

[1] Also known as Poincaré semi-simple extended superalgebra.

the invariant (53)–(64) in the general three-dimensional CS expression

$$I_{\rm CS}^{(2+1)} = k\int\left\langle AdA + \frac{2}{3}A^3\right\rangle.$$

Thus, we have modulo boundary terms

$$\begin{aligned}
I_{\rm CS}^{(2+1)} = k\int & \frac{\alpha_0}{2}\left[\omega^a_{\ b}d\omega^b_{\ a} + \frac{2}{3}\omega^a_{\ c}\omega^c_{\ b}\omega^b_{\ a}\right]\\
&+\frac{\beta_2}{l}\left[\epsilon_{abc}\left(R^{ab}e^c + \frac{1}{3l^2}e^ae^be^c\right.\right.\\
&\left.\left.+K^{ab}e^c\tilde{K}^{ab}\tilde{h}^c + \frac{1}{l^2}\tilde{h}^a\tilde{h}^be\right) + 2\bar{\psi}\Psi + 2\bar{\xi}\,\Xi\right]\\
&+\alpha_2\left[R^a_{\ b}\tilde{k}^b_{\ a} + K^a_{\ b}\tilde{k}^b_{\ a} + \tilde{K}^a_{\ b}k^b_{\ a} + \frac{1}{l^2}e^aH_a\right.\\
&\left.+\frac{1}{l^2}\tilde{h}^aK_a - \frac{2}{l}\bar{\psi}\Psi - \frac{2}{l}\bar{\xi}\,\Xi\right]\\
&+\frac{\beta_4}{l}\left[\epsilon_{abc}\left(R^{ab}\tilde{h}^c + \frac{1}{3l^2}\tilde{h}^a\tilde{h}^b\tilde{h}^c + \tilde{K}^{ab}e^c + K^{ab}\tilde{h}^c\right.\right.\\
&\left.\left.+\frac{1}{l^2}e^ae^b\tilde{h}^c\right) + 2\bar{\xi}\Psi + 2\bar{\psi}\,\Xi\right]\\
&+\alpha_4\left[R^a_{\ b}k^b_{\ a} + K^a_{\ b}k^b_{\ a} + \tilde{K}^a_{\ b}\tilde{k}^b_{\ a} + \frac{1}{l^2}e^aK_a\right.\\
&\left.+\frac{1}{l^2}\tilde{h}^aH_a - \frac{2}{l}\bar{\xi}\Psi - \frac{2}{l}\bar{\psi}\,\Xi\right],
\end{aligned} \tag{65}$$

where

$$\begin{aligned}
K^{ab} &= Dk^{ab} + k^a_{\ d}k^d_{\ b} + \tilde{k}^a_{\ d}\tilde{k}^d_{\ b},\ \tilde{K}^{ab} = D\tilde{k}^{ab} + k^a_{\ d}\tilde{k}^d_{\ b} + k_b^{\ d}\tilde{k}^a_{\ d},\\
H^a &= D\tilde{h}^a + k^a_{\ b}\tilde{h}^b + \tilde{k}^a_{\ b}e^b,\ K^a = T^a + k^a_{\ b}e^b + \tilde{k}^a_{\ b}\tilde{h}^b,\\
\Psi &= d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i + \frac{1}{4}k_{ab}\Gamma^{ab}\psi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\xi^i\\
&\quad+\frac{1}{2l}e_a\Gamma^a\xi^i + \frac{1}{2l}\tilde{h}_a\Gamma^a\psi^i,\\
\Xi^i &= d\xi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}k_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi^i\\
&\quad+\frac{1}{2l}e_a\Gamma^a\psi^i + \frac{1}{2l}\tilde{h}_a\Gamma^a\xi^i\,.
\end{aligned}$$

The CS action (65) is locally gauge invariant under the minimal AdS–Lorentz superalgebra and is split into five independent pieces proportional to $\alpha_0, \alpha_2, \alpha_4, \beta_2$ and β_4.. In particular, the term proportional to α_0 corresponds to the exotic form, while the α_2 and α_4 terms contain exotic like Lagrangians plus fermionic terms.

Let us note that the CS action (65) reproduces the three-dimensional generalized cosmological constant term introduced in Refs. [56,66] when $\tilde{k}^{ab} = \tilde{h}^a = 0$. A generalized supersymmetric cosmological term has also been introduced in a four-dimensional MacDowell–Mansouri like action constructed out of the curvature two-form, based on an AdS–Lorentz superalgebra [65,67]. Besides, the bosonic part corresponds to the AdS–Lorentz CS action presented in Refs. [68,69].

4.2 The Maxwell limit

One is tempted to consider an appropriate rescaling of the fields at the level of the action (65) and apply some limit in order to derive the Maxwell supergravity action. However, although a minimal Maxwell superalgebra can be obtained as an IW contraction of the minimal AdS–Lorentz superalgebra, the IW contraction of the action (65) would reproduce a trivial CS action. To reproduce a non-trivial CS supergravity action based on the $\mathcal{N}=1$ Maxwell symmetries, it is necessary to extend the rescaling of the generators to the α and β constants appearing in the non-vanishing components of the invariant tensor. A rescaling which preserves the curvature structure is given by

$$\begin{aligned}
&\alpha_4 \to \sigma^4\alpha_4,\ \beta_4 \to \sigma^4\beta_4,\ \alpha_2 \to \sigma^2\alpha_2,\\
&\beta_2 \to \sigma^2\beta_2,\ \alpha_0 \to \alpha_0.
\end{aligned}$$

Then, considering the rescaling of the constants and the generators

$$\begin{aligned}
&\tilde{Z}_{ab} \to \sigma^2\tilde{Z}_{ab},\ Z_{ab} \to \sigma^4 Z_{ab},\ P_a \to \sigma^2 P_a,\ J_{ab} \to J_{ab},\\
&\tilde{Z}_a \to \sigma^4\tilde{Z}_a,\ Q_\alpha \to \sigma Q_\alpha,\ \Sigma \to \sigma^3\Sigma,
\end{aligned}$$

and applying the limit $\sigma \to \infty$, we recover the $\mathcal{N}=1$ Maxwell non-vanishing components of the invariant tensor,

$$\langle J_{ab}J_{cd}\rangle_{\mathcal{M}} = \alpha_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{66}$$
$$\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} = \alpha_2\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{67}$$
$$\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{68}$$
$$\langle J_{ab}Z_{cd}\rangle_{\mathcal{M}} = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), \tag{69}$$

$$\langle J_{ab}P_c\rangle_{\mathcal{M}} = \beta_2\epsilon_{abc}, \tag{70}$$
$$\left\langle \tilde{Z}_{ab}P_c\right\rangle_{\mathcal{M}} = \left\langle J_{ab}\tilde{Z}_c\right\rangle_{\mathcal{M}} = \beta_4\epsilon_{abc}, \tag{71}$$
$$\langle P_aP_b\rangle_{\mathcal{M}} = \alpha_4\eta_{ab}, \tag{72}$$
$$\left\langle Q_\alpha Q_\beta\right\rangle_{\mathcal{M}} = 2\left(\beta_2 - \alpha_2\right)C_{\alpha\beta}, \tag{73}$$
$$\left\langle Q_\alpha\Sigma_\beta\right\rangle_{\mathcal{M}} = 2\left(\beta_4 - \alpha_4\right)C_{\alpha\beta}. \tag{74}$$

Here, the generators now satisfy the (anti)commutation relations of a minimal Maxwell superalgebra (see Appendix B for $p=1, q=0$). Then using the Maxwell connection one-form

$$A = \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{2}\tilde{k}^{ab}\tilde{Z}_{ab} + \frac{1}{2}k^{ab}Z_{ab} + \frac{1}{l}e^aP_a + \frac{1}{l}\tilde{h}^a\tilde{Z}_a + \frac{1}{\sqrt{l}}\psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\xi^\alpha\Sigma_\alpha$$

and the non-vanishing components of the invariant tensor, we derive the three-dimensional CS supergravity action for the $\mathcal{N}=1$ Maxwell superalgebra [22]:

$$
\begin{aligned}
I^{(2+1)}_{\mathcal{M}-\mathrm{CS}} = k\int_M \Big[& \frac{\alpha_0}{2}\left(\omega^a_{\ b} d\omega^b_{\ a} + \frac{2}{3}\omega^a_{\ c}\omega^c_{\ b}\omega^b_{\ a}\right) \\
& + \frac{\beta_2}{l}\left(\epsilon_{abc} R^{ab} e^c + 2\bar{\psi}\Psi\right) \\
& + \alpha_2\left(R^a_{\ b}\tilde{k}^b_{\ a} - \frac{2}{l}\bar{\psi}\Psi\right) \\
& + \frac{\beta_4}{l}\left(\epsilon_{abc}\left(R^{ab}\tilde{h}^c + D_\omega \tilde{k}^{ab} e^c\right) + 2\bar{\xi}\Psi + 2\bar{\psi}\Xi\right) \\
& + \alpha_4\left(R^a_{\ b} k^b_{\ a} + \frac{1}{l^2} e^a T_a + D_\omega \tilde{k}^a_{\ b}\tilde{k}^b_{\ a} - \frac{2}{l}\bar{\xi}\Psi - \frac{2}{l}\bar{\psi}\Xi\right)\Big],
\end{aligned} \tag{75}
$$

where

$$
\begin{aligned}
R^{ab} &= d\omega^{ab} + \omega^a_{\ c}\omega^{cb},\ T^a = de^a + \omega^a_{\ c} e^c, \\
\Psi &= d\psi + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi, \\
\Xi &= d\xi + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi + \frac{1}{2l} e_a \Gamma^a \psi.
\end{aligned}
$$

Thus, we reproduce the CS action presented in Ref. [22] using a different approach. Unlike the non-standard Maxwell, the minimal Maxwell superalgebra allows one to properly write a three-dimensional CS supergravity action invariant under local Maxwell supersymmetry transformations (up to boundary terms). However, as was shown at the algebraic level, this requires the introduction of an additional Majorana spinor field ξ. The presence of a second spinorial generator was already introduced in Refs. [70,71] in the context of $D = 11$ supergravity and superstring theory, respectively. Subsequently, the introduction of a second spinorial generator in the Maxwell symmetries was proposed in Ref. [61]. More recently, a family of Maxwell superalgebras was presented, which generalize the superalgebra introduced by D 'Auria, Fré and Green and contains the minimal Maxwell one [72–74].

Let us note that the bosonic term reproduces the CS action presented in Refs. [69,75].

5 Inönü–Wigner contraction and $\mathcal{N}$-extended Supergravity

We now present the explicit derivation of the three-dimensional (p, q) Maxwell supergravity action using our approach. In particular, we first show the explicit construction of the $(2, 0)$ *AdS*–Lorentz supergravity using the semigroup expansion method. The $(2, 0)$ Maxwell supergravity is then derived from the $(2, 0)$ *AdS*–Lorentz superalgebra applying the IW contraction to both generators and constants appearing in the invariant tensor.

5.1 $\mathcal{N} = 2$ *AdS*–Lorentz supergravity

A non-trivial $\mathcal{N} = 2$ *AdS*–Lorentz superalgebra can be obtained as an S-expansion of the $\mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2)$ superalgebra. In order to apply the S-expansion procedure, let us first consider a decomposition of the original superalgebra

$$
\mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2) = V_0 \oplus V_1 \oplus V_2,
$$

where V_0 corresponds to a subalgebra generated by the Lorentz generators $\tilde{J}_{ab}$ and by the internal symmetry generator $\tilde{T}^{ij}$ (with $i = 1, 2$), V_1 corresponds to the fermionic subspace and V_2 is generated by $\tilde{P}_a$. In particular, the $\mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2)$ generators satisfy the following (anti)commutation relations:

$$
\left[\tilde{J}_{ab}, \tilde{J}_{cd}\right] = \eta_{bc}\tilde{J}_{ad} - \eta_{ac}\tilde{J}_{bd} - \eta_{bd}\tilde{J}_{ac} + \eta_{ad}\tilde{J}_{bc}, \tag{76}
$$

$$
\left[\tilde{T}^{ij}, \tilde{T}^{kl}\right] = \delta^{jk}\tilde{T}^{il} - \delta^{ik}\tilde{T}^{jl} - \delta^{jl}\tilde{T}^{ik} + \delta^{il}\tilde{T}^{jk}, \tag{77}
$$

$$
\left[\tilde{J}_{ab}, \tilde{P}_c\right] = \eta_{bc}\tilde{P}_a - \eta_{ac}\tilde{P}_b, \tag{78}
$$

$$
\left[\tilde{P}_a, \tilde{P}_b\right] = \tilde{J}_{ab}, \tag{79}
$$

$$
\left[\tilde{T}^{ij}, \tilde{Q}^k_\alpha\right] = \left(\delta^{jk}\tilde{Q}^i_\alpha - \delta^{ik}\tilde{Q}^j_\alpha\right), \tag{80}
$$

$$
\left[\tilde{J}_{ab}, \tilde{Q}^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab}\tilde{Q}^i\right)_\alpha, \left[\tilde{P}_a, \tilde{Q}^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a\tilde{Q}^i\right)_\alpha, \tag{81}
$$

$$
\left\{\tilde{Q}^i_\alpha, \tilde{Q}^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta}\tilde{P}_a\right] + C_{\alpha\beta}\tilde{T}^{ij}. \tag{82}
$$

The subspace structure satisfies

$$
[V_0, V_0] \subset V_0, \quad [V_0, V_1] \subset V_1, \quad [V_0, V_2] \subset V_2, \tag{83}
$$

$$
[V_1, V_1] \subset V_0 \oplus V_2, \quad [V_1, V_2] \subset V_1, \quad [V_2, V_2] \subset V_0. \tag{84}
$$

Let us now consider $S^{(4)}_{\mathcal{M}} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ as the relevant abelian semigroup whose elements satisfy the multiplication law

$$
\lambda_\alpha\lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha+\beta \leq 4, \\ \lambda_{\alpha+\beta-4}, & \text{if } \alpha+\beta > 4. \end{cases} \tag{85}
$$

Let $S^{(4)}_{\mathcal{M}} = S_0 \cup S_1 \cup S_2$ be a subset decomposition with

$$
\begin{aligned}
S_0 &= \{\lambda_0, \lambda_2, \lambda_4\}, \\
S_1 &= \{\lambda_1, \lambda_3\}, \\
S_2 &= \{\lambda_2, \lambda_4\},
\end{aligned}
$$

where S_0, S_1, and S_2 satisfy the resonance condition [compare with Eqs. (83)–(84)]

$$
S_0 \cdot S_0 \subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2, \tag{86}
$$

$$
S_0 \cdot S_1 \subset S_1, \quad S_1 \cdot S_2 \subset S_1, \tag{87}
$$

$$
S_0 \cdot S_2 \subset S_2, \quad S_2 \cdot S_2 \subset S_0. \tag{88}
$$

Then according to Ref. [47],

$$
\mathfrak{G}_R = W_0 \oplus W_1 \oplus W_2,
$$

is a resonant subalgebra of $S_{\mathcal{M}}^{(4)} \times \mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2)$ where

$$W_0 = (S_0 \times V_0) = \{\lambda_0, \lambda_2, \lambda_4\} \times \left\{\tilde{J}_{ab}, \tilde{T}^{ij}\right\},$$
$$W_1 = (S_1 \times V_1) = \{\lambda_1, \lambda_3\} \times \left\{\tilde{Q}_\alpha^i\right\},$$
$$W_2 = (S_2 \times V_2) = \{\lambda_2, \lambda_4\} \times \left\{\tilde{P}_a\right\}.$$

The expanded superalgebra corresponds to a $(2,0)$ *AdS*–Lorentz superalgebra and is generated by the set of generators $\{J_{ab}, P_a, \tilde{Z}_{ab}, \tilde{Z}_a, Z_{ab}, T^{ij}, \tilde{Y}^{ij}, Y^{ij}, Q_\alpha^i, \Sigma_\alpha^i\}$, which are related to the original ones through

$$\begin{aligned}
&J_{ab} = \lambda_0 \tilde{J}_{ab},\ P_a = \lambda_2 \tilde{P}_a,\ Q_\alpha^i = \lambda_1 \tilde{Q}_\alpha^i,\\
&\tilde{Z}_{ab} = \lambda_2 \tilde{J}_{ab},\ \tilde{Z}_a = \lambda_4 \tilde{P}_a,\ \Sigma_\alpha^i = \lambda_3 \tilde{Q}_\alpha^i,\\
&Z_{ab} = \lambda_4 \tilde{J}_{ab},\ T^{ij} = \lambda_0 \tilde{T}^{ij},\ \tilde{Y}^{ij} = \lambda_2 \tilde{T}^{ij},\\
&Y^{ij} = \lambda_4 \tilde{T}^{ij}.
\end{aligned}$$

The explicit (anti)commutation relations can be derived using the multiplication law of the semigroup and the original superalgebra (the $\mathcal{N}$-extended *AdS*–Lorentz superalgebra can be found in Appendix A).

In order to construct the explicit supergravity action let us first derive the invariant tensor for the $(2,0)$ *AdS*–Lorentz superalgebra. According to Theorem VII.2 of Ref. [47], it is possible to show that the non-vanishing components of the invariant tensor for the $\mathcal{N}=2$ *AdS*–Lorentz are, besides those given by Eqs. (53)–(62),

$$\left\langle Q_\alpha^i Q_\beta^j \right\rangle = \left\langle \Sigma_\alpha^i \Sigma_\beta^j \right\rangle = \tilde{\alpha}_2 \left\langle \tilde{Q}_\alpha^i \tilde{Q}_\beta^j \right\rangle = 2(\beta_2 - \alpha_2) C_{\alpha\beta}\, \delta^{ij}, \tag{89}$$

$$\left\langle Q_\alpha^i \Sigma_\beta^j \right\rangle = \tilde{\alpha}_4 \left\langle \tilde{Q}_\alpha^i \tilde{Q}_\beta^j \right\rangle = 2(\beta_4 - \alpha_4) C_{\alpha\beta}\, \delta^{ij}, \tag{90}$$

$$\left\langle T^{ij} T^{kl} \right\rangle = \tilde{\alpha}_0 \left\langle \tilde{T}^{ij} \tilde{T}^{kl} \right\rangle = 2(\alpha_0 - \beta_0)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right), \tag{91}$$

$$\begin{aligned}\left\langle T^{ij} \tilde{Y}^{kl} \right\rangle &= \left\langle \tilde{Y}^{ij} Y^{kl} \right\rangle = \tilde{\alpha}_2 \left\langle \tilde{T}^{ij} \tilde{T}^{kl} \right\rangle\\
&= 2(\alpha_2 - \beta_2)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right),\end{aligned} \tag{92}$$

$$\begin{aligned}\left\langle T^{ij} Y^{kl} \right\rangle &= \left\langle \tilde{Y}^{ij} \tilde{Y}^{kl} \right\rangle = \left\langle Y^{ij} Y^{kl} \right\rangle = \tilde{\alpha}_4 \left\langle \tilde{T}^{ij} \tilde{T}^{kl} \right\rangle\\
&= 2(\alpha_4 - \beta_4)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right),\end{aligned} \tag{93}$$

where $\{\tilde{J}_{ab}, \tilde{P}_a, \tilde{T}^{ij}, \tilde{Q}_\alpha^i\}$ generate the $\mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2)$ superalgebra and where we have defined

$$\begin{aligned}
&\alpha_0 \equiv \tilde{\alpha}_0 \mu_0,\ \alpha_2 \equiv \tilde{\alpha}_2 \mu_0,\ \alpha_4 \equiv \tilde{\alpha}_4 \mu_0,\\
&\beta_0 \equiv \tilde{\alpha}_0 \mu_1,\ \beta_2 \equiv \tilde{\alpha}_2 \mu_1,\ \beta_4 \equiv \tilde{\alpha}_4 \mu_1.
\end{aligned}$$

Here $\tilde{\alpha}_0, \tilde{\alpha}_2, \tilde{\alpha}_4$ are arbitrary constants as well as μ_0 and μ_1. To construct the CS supergravity action we require, in addition to the invariant tensor, the gauge connection one-form:

$$\begin{aligned}A = &\frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{2}\tilde{k}^{ab} \tilde{Z}_{ab} + \frac{1}{2}k^{ab} Z_{ab} + \frac{1}{l} e^a P_a + \frac{1}{l}\tilde{h}^a \tilde{Z}_a\\
&+ \frac{1}{2}A^{ij} T_{ij} + \frac{1}{2}\tilde{B}^{ij} \tilde{Y}_{ij} + \frac{1}{2}B^{ij} Y_{ij} + \frac{1}{\sqrt{l}}\bar{\psi}_i Q^i + \frac{1}{\sqrt{l}}\bar{\xi}_i \Sigma^i.\end{aligned} \tag{94}$$

The associated curvature two-form $F = dA + AA$ is given by

$$\begin{aligned}F = &\frac{1}{2}R^{ab} J_{ab} + \frac{1}{2}\tilde{F}^{ab} \tilde{Z}_{ab} + \frac{1}{2}F^{ab} Z_{ab} + \frac{1}{l} F^a P_a + \frac{1}{l}\tilde{H}^a \tilde{Z}_a\\
&+ \frac{1}{2}F^{ij} T_{ij} + \frac{1}{2}\tilde{G}^{ij} \tilde{Y}_{ij} + \frac{1}{2}G^{ij} Y_{ij} + \frac{1}{\sqrt{l}}\bar{\Psi}_i Q^i + \frac{1}{\sqrt{l}}\bar{\Xi}_i \Sigma^i,\end{aligned} \tag{95}$$

where

$$\begin{aligned}
R^{ab} &= d\omega^{ab} + \omega^a{}_c \omega^{cb},\\
F^a &= de^a + \omega^a{}_b e^b + k^a{}_b e^b + \tilde{k}^a{}_b \tilde{h}^b - \frac{1}{2}\bar{\psi}^i \Gamma^a \psi^i - \frac{1}{2}\bar{\xi}^i \Gamma^a \xi^i,\\
\tilde{H}^a &= d\tilde{h}^a + \omega^a{}_b \tilde{h}^b + \tilde{k}^a{}_b e^b + k^a{}_b \tilde{h}^b - \bar{\psi}^i \Gamma^a \xi^i,\\
\tilde{F}^{ab} &= d\tilde{k}^{ab} + \omega^a{}_c \tilde{k}^{cb} - \omega^b{}_c \tilde{k}^{ca} + k^a{}_c \tilde{k}^{cb} - k^b{}_c \tilde{k}^{ca} + \frac{2}{l^2} e^a \tilde{h}^b\\
&\quad + \frac{1}{2l}\bar{\psi}^i \Gamma^{ab} \psi^i + \frac{1}{2l}\bar{\xi}^i \Gamma^{ab} \xi^i,\\
F^{ab} &= dk^{ab} + \omega^a{}_c k^{cb} - \omega^b{}_c k^{ca} + \tilde{k}^a{}_c \tilde{k}^{cb} + k^a{}_c k^{cb}\\
&\quad + \frac{1}{l^2} e^a e^b + \frac{1}{l^2}\tilde{h}^a \tilde{h}^b + \frac{1}{l}\bar{\xi}^i \Gamma^{ab} \psi^i,
\end{aligned}$$

$$\begin{aligned}
F^{ij} &= dA^{ij} + A^{ik} A^{kj},\\
\tilde{G}^{ij} &= d\tilde{B}^{ij} + A^{ik} \tilde{B}^{kj} + \tilde{B}^{ik} A^{kj}\\
&\quad + \tilde{B}^{ik} B^{kj} + B^{ik} \tilde{B}^{kj} + \bar{\psi}^i \psi^j + \bar{\xi}^i \xi^j,\\
G^{ij} &= dB^{ij} + A^{ik} B^{kj} + B^{ik} A^{kj} + \tilde{B}^{ik} \tilde{B}^{kj} + B^{ik} B^{kj} + 2\bar{\psi}^i \xi^j,\\
\Psi^i &= d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i + \frac{1}{4}k_{ab}\Gamma^{ab}\psi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\xi^i\\
&\quad + \frac{1}{2l} e_a \Gamma^a \xi^i + \frac{1}{2l}\tilde{h}_a \Gamma^a \psi^i\\
&\quad + A^{ij}\psi^j + B^{ij}\psi^j + \tilde{B}^{ij}\xi^j,\\
\Xi^i &= d\xi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}k_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi^i\\
&\quad + \frac{1}{2l} e_a \Gamma^a \psi^i + \frac{1}{2l}\tilde{h}_a \Gamma^a \xi^i\\
&\quad + A^{ij}\xi^j + B^{ij}\xi^j + \tilde{B}^{ij}\psi^j.
\end{aligned}$$

Then considering the connection one-form (94) and the non-vanishing components of the invariant tensor ((53)–(62) and (89)–(93)) in the general three-dimensional CS expression, we find the $(2,0)$ *AdS*–Lorentz CS action supergravity up to a surface term:

$$
\begin{aligned}
I_{\mathrm{CS}}^{(2+1)} = k \int & \frac{\alpha_0}{2}\left(\omega^a_{\ b} d\omega^b_{\ a} + \frac{2}{3}\omega^a_{\ c}\omega^c_{\ b}\omega^b_{\ a}\right) \\
& + (\alpha_0 - \beta_0)\left(A^{ij} dA^{ji} + \frac{2}{3}A^{ik}A^{kj}A^{ji}\right) \\
& + \frac{\beta_2}{l}\epsilon_{abc}\left(R^{ab}e^c + \frac{1}{3l^2}e^a e^b e^c \right.\\
& \left. + K^{ab}e^c + \tilde{K}^{ab}\tilde{h}^c + \frac{1}{l^2}\tilde{h}^a\tilde{h}^b e\right) \\
& + (\alpha_2 - \beta_2)\left[\tilde{B}^{ij}\left(dA^{ji} + A^{jk}A^{ki}\right)\right. \\
& + \tilde{B}^{ij}\left(dB^{ji} + A^{jk}B^{ki} + B^{jk}A^{ki} + \tilde{B}^{jk}\tilde{B}^{ki} + B^{jk}B^{ki}\right) \\
& + \left(A^{ij} + B^{ij}\right)\left(d\tilde{B}^{ji} + A^{jk}\tilde{B}^{ki} + \tilde{B}^{jk}A^{ki}\right. \\
& \left.\left. + \tilde{B}^{jk}B^{ki} + B^{jk}\tilde{B}^{ki}\right) - \frac{2}{l}\bar{\psi}^i\Psi^i - \frac{2}{l}\bar{\xi}^i\Xi^i\right] \\
& + \alpha_2\left[R^a_{\ b}\tilde{k}^b_{\ a} + K^a_{\ b}\tilde{k}^b_{\ a} + \tilde{K}^a_{\ b}k^b_{\ a} + \frac{1}{l^2}e^a H_a + \frac{1}{l^2}\tilde{h}^a K_a\right] \\
& + \frac{\beta_4}{l}\epsilon_{abc}\left(R^{ab}\tilde{h}^c + \frac{1}{3l^2}\tilde{h}^a\tilde{h}^b\tilde{h}^c + \tilde{K}^{ab}e^c + K^{ab}\tilde{h}^c + \frac{1}{l^2}e^a e^b\tilde{h}^c\right) \\
& + (\alpha_4 - \beta_4)\left[\tilde{B}^{ij}\left(d\tilde{B}^{ji} + A^{jk}\tilde{B}^{ki}\right.\right. \\
& \left. + \tilde{B}^{jk}A^{ki} + \tilde{B}^{jk}B^{ki} + B^{jk}\tilde{B}^{ki}\right) + B^{ij}\left(dA^{ji} + A^{jk}A^{ki}\right) \\
& + \left(A^{ij} + B^{ij}\right)\left(dB^{ji} + A^{jk}B^{ki} + B^{jk}A^{ki}\right. \\
& \left.\left. + \tilde{B}^{jk}\tilde{B}^{ki} + B^{jk}B^{ki}\right) - \frac{2}{l}\bar{\xi}^i\Psi^i - \frac{2}{l}\bar{\psi}^i\Xi^i\right] \\
& + \alpha_4\left[R^a_{\ b}k^b_{\ a} + K^a_{\ b}k^b_{\ a} + \tilde{K}^a_{\ b}\tilde{k}^b_{\ a} + \frac{1}{l^2}e^a K_a + \frac{1}{l^2}\tilde{h}^a H_a\right], \qquad (96)
\end{aligned}
$$

where Ψ^i and Ξ^i are the fermionic components of the curvature two-form and

$$
\begin{aligned}
K^{ab} &= Dk^{ab} + k^a_{\ d}k^d_{\ b} + \tilde{k}^a_{\ d}\tilde{k}^d_{\ b},\ \tilde{K}^{ab} = D\tilde{k}^{ab} + k^a_{\ d}\tilde{k}^d_{\ b} + k_b^{\ d}\tilde{k}^a_{\ d}, \\
H^a &= D\tilde{h}^a + k^a_{\ b}\tilde{h}^b + \tilde{k}^a_{\ b}e^b,\ K^a = T^a + k^a_{\ b}e^b + \tilde{k}^a_{\ b}\tilde{h}^b,
\end{aligned}
$$

As in the $\mathcal{N} = 1$ case, the (2, 0) Maxwell supergravity cannot be trivially obtained considering the rescaling of the fields in (96) and applying some limit.

5.2 $\mathcal{N} = 2$ Maxwell supergravity

As the $\mathfrak{osp}(2|2) \otimes \mathfrak{sp}(2)$ superalgebra has its (2, 0) Poincaré limit, the (2, 0) *AdS*–Lorentz superalgebra possesses its proper IW contracted superalgebra. Indeed, after rescaling the generators

$$
\begin{aligned}
&\tilde{Z}_{ab} \to \sigma^2\tilde{Z}_{ab},\ Z_{ab} \to \sigma^4 Z_{ab},\ P_a \to \sigma^2 P_a,\ J_{ab} \to J_{ab}, \\
&\tilde{Z}_a \to \sigma^4\tilde{Z}_a,\ Q^i_\alpha \to \sigma Q^i_\alpha,\ \Sigma^i \to \sigma^3\Sigma^i,\ Y^{ij} \to \sigma^4 Y^{ij}, \\
&\tilde{Y}^{ij} \to \sigma^2\tilde{Y}^{ij},\ T^{ij} \to T^{ij},
\end{aligned}
$$

and applying the limit $\sigma \to \infty$, we obtain the $\mathcal{N} = 2$ Maxwell superalgebra whose (anti)commutation relations can be found in Refs. [72,73] (see Appendix B for $p = 2$, $q = 0$).

As in the previous case, the CS supergravity action for the (2, 0) Maxwell supergroup can be derived combining the IW contraction with the *S*-expanded invariant tensor. Indeed, it is necessary to extend the rescaling of the generators to the α and β constants appearing in the non-vanishing components of the invariant tensor of the (2, 0) *AdS*–Lorentz superalgebra. A rescaling which preserves the curvature structure is given by

$$
\begin{aligned}
&\alpha_4 \to \sigma^4\alpha_4, \quad \alpha_2 \to \sigma^2\alpha_2, \quad \alpha_0 \to \alpha_0, \\
&\beta_4 \to \sigma^4\beta_4, \quad \beta_2 \to \sigma^2\beta_2, \quad \beta_0 \to \beta_0.
\end{aligned}
$$

Then, considering the rescaling of both constants and generators, and applying the limit $\sigma \to \infty$, we obtain the (2, 0) Maxwell non-vanishing components of the invariant tensor,

$$
\begin{aligned}
\langle J_{ab}J_{cd}\rangle_{\mathcal{M}} &= \alpha_0\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), && (97) \\
\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} &= \alpha_2\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), && (98) \\
\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} &= \langle J_{ab}Z_{cd}\rangle = \alpha_4\left(\eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}\right), && (99) \\
\langle J_{ab}P_c\rangle_{\mathcal{M}} &= \beta_2\epsilon_{abc}, && (100) \\
\left\langle \tilde{Z}_{ab}P_c\right\rangle_{\mathcal{M}} &= \left\langle J_{ab}\tilde{Z}_c\right\rangle_{\mathcal{M}} = \beta_4\epsilon_{abc}, && (101) \\
\langle P_a P_b\rangle_{\mathcal{M}} &= \alpha_4\eta_{ab}, && (102)
\end{aligned}
$$

$$
\begin{aligned}
\left\langle Q^i_\alpha Q^j_\beta\right\rangle_{\mathcal{M}} &= 2\left(\beta_2 - \alpha_2\right)C_{\alpha\beta}\,\delta^{ij}, && (103) \\
\left\langle Q^i_\alpha \Sigma^j_\beta\right\rangle_{\mathcal{M}} &= 2\left(\beta_4 - \alpha_4\right)C_{\alpha\beta}\,\delta^{ij}, && (104) \\
\left\langle T^{ij}T^{kl}\right\rangle_{\mathcal{M}} &= 2\left(\alpha_0 - \beta_0\right)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right), && (105) \\
\left\langle T^{ij}\tilde{Y}^{kl}\right\rangle_{\mathcal{M}} &= 2\left(\alpha_2 - \beta_2\right)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right), && (106) \\
\left\langle T^{ij}Y^{kl}\right\rangle_{\mathcal{M}} &= \left\langle \tilde{Y}^{ij}\tilde{Y}^{kl}\right\rangle_{\mathcal{M}} = 2\left(\alpha_4 - \beta_4\right)\left(\delta^{il}\delta^{kj} - \delta^{ik}\delta^{lj}\right), && (107)
\end{aligned}
$$

where $\alpha_0, \alpha_2, \alpha_4, \beta_2$ and β_4 are arbitrary constants and the generators now satisfy the (2, 0) Maxwell (anti)commutation relations. In order to write down a CS action we require the gauge connection one-form given by

$$
\begin{aligned}
A = {} & \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{2}\tilde{k}^{ab}\tilde{Z}_{ab} + \frac{1}{2}k^{ab}Z_{ab} + \frac{1}{l}e^a P_a + \frac{1}{l}\tilde{h}^a\tilde{Z}_a \\
& + \frac{1}{2}A^{ij}T_{ij} + \frac{1}{2}\tilde{B}^{ij}\tilde{Y}_{ij} + \frac{1}{2}B^{ij}Y_{ij} + \frac{1}{\sqrt{l}}\bar{\psi}_i Q^i + \frac{1}{\sqrt{l}}\bar{\xi}_i\Sigma^i. \qquad (108)
\end{aligned}
$$

Considering the non-vanishing components of the invariant tensor, the CS action for the $\mathcal{N} = 2$ Maxwell superalgebra reduces to

$$
I_{\mathcal{M}-\mathrm{CS}}^{(2+1)} = k \int \frac{\alpha_0}{2}\left(\omega^a_{\ b} d\omega^b_{\ a} + \frac{2}{3}\omega^a_{\ c}\omega^c_{\ b}\omega^b_{\ a}\right)
$$

$$
\begin{aligned}
&+ (\alpha_0 - \beta_0)\left(A^{ij}dA^{ji} + \frac{2}{3}A^{ik}A^{kj}A^{ji}\right) \\
&+ \frac{\beta_2}{l}\epsilon_{abc}R^{ab}e^c + \alpha_2 R^a_{\ b}\tilde{k}^b_{\ a} \\
&+ (\alpha_2 - \beta_2)\left[\tilde{B}^{ij}\left(dA^{ji} + A^{jk}A^{ki}\right)\right. \\
&\left.+ A^{ij}\left(d\tilde{B}^{ji} + A^{jk}\tilde{B}^{ki} + \tilde{B}^{jk}A^{ki}\right) - \frac{2}{l}\bar{\psi}^i\Psi^i\right] \\
&+ \frac{\beta_4}{l}\epsilon_{abc}\left(R^{ab}\tilde{h}^c + D_\omega\tilde{k}^{ab}e^c\right) \\
&+ (\alpha_4 - \beta_4)\left[\tilde{B}^{ij}\left(d\tilde{B}^{ji} + A^{jk}\tilde{B}^{ki} + \tilde{B}^{jk}A^{ki}\right)\right. \\
&+ B^{ij}\left(dA^{ji} + A^{jk}A^{ki}\right) \\
&+ \left(A^{ij}\right)\left(dB^{ji} + A^{jk}B^{ki} + B^{jk}A^{ki} + \tilde{B}^{jk}\tilde{B}^{ki}\right) \\
&\left.- \frac{2}{l}\bar{\xi}^i\Psi^i - \frac{2}{l}\bar{\psi}^i\Xi^i\right] \\
&+ \alpha_4\left[R^a_{\ b}k^b_{\ a} + D_\omega\tilde{k}^a_{\ b}\tilde{k}^b_{\ a} + \frac{1}{l^2}e^aT_a\right], \qquad (109)
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi^i &= d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i + A^{ij}\psi^j, \\
\Xi^i &= d\xi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi^i \\
&\quad + \frac{1}{2l}e_a\Gamma^a\psi^i + A^{ij}\xi^j + \tilde{B}^{ij}\psi^j.
\end{aligned}
$$

This CS supergravity action is invariant up to boundary terms under the local gauge transformations of the $\mathcal{N}=2$ Maxwell supergroup. In particular, under the supersymmetric transformations, the fields transform as

$$\delta\omega^{ab} = 0, \delta e^a = \bar{\epsilon}^i\Gamma^a\psi^i, \qquad (110)$$

$$\delta\tilde{k}^{ab} = -\frac{1}{l}\bar{\epsilon}^i\Gamma^{ab}\psi^i, \qquad (111)$$

$$\delta k^{ab} = -\frac{1}{l}\bar{\varrho}^i\Gamma^{ab}\psi^i - \frac{1}{l}\bar{\epsilon}^i\Gamma^{ab}\xi^i, \qquad (112)$$

$$\delta\tilde{h}^a = \bar{\varrho}^i\Gamma^a\psi^i + \bar{\epsilon}^i\Gamma^a\xi^i, \qquad (113)$$

$$\delta A^{ij} = 0, \qquad (114)$$

$$\delta\tilde{B}^{ij} = -\frac{2}{l}\bar{\psi}^{[i}\epsilon^{j]}, \qquad (115)$$

$$\delta B^{ij} = -\frac{2}{l}\bar{\psi}^{[i}\varrho^{j]}, \qquad (116)$$

$$\delta\psi^i = d\epsilon^i + \frac{1}{4}\omega^{ab}\Gamma_{ab}\epsilon^i + A^{ij}\epsilon^j, \qquad (117)$$

$$\delta\xi^i = d\varrho^i + \frac{1}{4}\omega^{ab}\Gamma_{ab}\varrho^i + \frac{1}{2l}e^a\Gamma_a\epsilon^i + \frac{1}{4}\tilde{k}^{ab}\Gamma_{ab}\epsilon^i \qquad (118)$$

$$+ A^{ij}\varrho^j + \tilde{B}^{ij}\epsilon^j, \qquad (119)$$

where the ϵ^i and ϱ^i parameters are related to the Q^i and Σ^i generators, respectively.

We remark that the generalized cosmological constant term appearing in the (2, 0) *AdS*–Lorentz supergravity model is no longer present after the IW contraction. This is analogous to the Poincaré limit from the *AdS* one. However, unlike the Poincaré supergravity theory, the internal symmetry fields appear explicitly in the exotic Lagrangian. Additionally, the spinorial fields contribute to the exotic like part.

5.3 (p, q) *AdS*–Lorentz supergravity and the Maxwell limit

In this section we present the three-dimensional $\mathcal{N} = p + q$ extended *AdS*–Lorentz Supergravity and its Maxwell limit applying the IW contraction not only at the generators level but also to the constants appearing in the invariant tensor. To this purpose we expand the three-dimensional (p, q) exotic supergravity theory [11], in order to obtain the local *AdS*–Lorentz supersymmetric extension. In particular, we generalize the Poincaré limit showed in Sect. 2 to the Maxwell limit.

The (p, q) *AdS*–Lorentz superalgebra can be obtained as an S-expansion of the $\mathfrak{osp}(2|p) \otimes \mathfrak{osp}(2|q)$ superalgebra. Indeed, considering $S^{(4)}_{\mathcal{M}} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ as the relevant semigroup whose elements satisfy

$$
\lambda_\alpha\lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha+\beta \leq 4 \\ \lambda_{\alpha+\beta-4}, & \text{if } \alpha+\beta > 4 \end{cases}
$$

and considering the resonant condition (see $\mathcal{N} = 2$ case), we obtain a new superlagebra generated by $\{J_{ab}, P_a, \tilde{Z}_{ab}, \tilde{Z}_a, Z_{ab}, T^{ij}, \tilde{Y}^{ij}, Y^{ij}, T^{IJ}, \tilde{Y}^{IJ}, Y^{IJ}, Q^i_\alpha, \Sigma^i_\alpha, Q^I_\alpha, \Sigma^J_\alpha\}$ whose generators satisfy the (p, q) *AdS*–Lorentz superalgebra. In particular, besides satisfying the (anti)commutation relations appearing in Appendix A, the I-index generators satisfy

$$\left[T^{IJ}, T^{KL}\right] = \delta^{JK}T^{IL} - \delta^{IK}T^{JL} - \delta^{JL}T^{IK} + \delta^{IL}T^{JK}, \qquad (120)$$

$$\left[T^{IJ}, \tilde{Y}^{KL}\right] = \delta^{JK}\tilde{Y}^{IL} - \delta^{IK}\tilde{Y}^{JL} - \delta^{JL}\tilde{Y}^{IK} + \delta^{IL}\tilde{Y}^{JK}, \qquad (121)$$

$$\left[T^{IJ}, Y^{KL}\right] = \delta^{JK}Y^{IL} - \delta^{IK}Y^{JL} - \delta^{JL}Y^{IK} + \delta^{IL}Y^{JK}, \qquad (122)$$

$$\left[\tilde{Y}^{IJ}, \tilde{Y}^{KL}\right] = \delta^{JK}Y^{IL} - \delta^{IK}Y^{JL} - \delta^{JL}Y^{IK} + \delta^{IL}Y^{JK}, \qquad (123)$$

$$\left[\tilde{Y}^{IJ}, Y^{KL}\right] = \delta^{JK}\tilde{Y}^{IL} - \delta^{IK}\tilde{Y}^{JL} - \delta^{JL}\tilde{Y}^{IK} + \delta^{IL}\tilde{Y}^{JK}, \qquad (124)$$

$$\left[Y^{IJ}, Y^{KL}\right] = \delta^{JK}Y^{IL} - \delta^{IK}Y^{JL} - \delta^{JL}Y^{IK} + \delta^{IL}Y^{JK}, \qquad (125)$$

$$\left[J_{ab},Q^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}Q^I\right)_\alpha,\left[P_a,Q^I_\alpha\right]=\frac{1}{2}\left(\Gamma_a\Sigma^I\right)_\alpha, \tag{126}$$

$$\left[\tilde{Z}_{ab},Q^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}\Sigma^I\right)_\alpha,\left[\tilde{Z}_a,Q^I_\alpha\right]=\frac{1}{2}\left(\Gamma_aQ^I\right)_\alpha, \tag{127}$$

$$\left[Z_{ab},Q^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}Q^I\right)_\alpha,\left[P_a,\Sigma^I_\alpha\right]=\frac{1}{2}\left(\Gamma_aQ^I\right)_\alpha, \tag{128}$$

$$\left[J_{ab},\Sigma^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}\Sigma^I\right)_\alpha,\left[\tilde{Z}_a,\Sigma^I_\alpha\right]=\frac{1}{2}\left(\Gamma_a\Sigma^I\right)_\alpha, \tag{129}$$

$$\left[\tilde{Z}_{ab},\Sigma^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}Q^I\right)_\alpha,\left[Z_{ab},\Sigma^I_\alpha\right]=-\frac{1}{2}\left(\Gamma_{ab}\Sigma^I\right)_\alpha, \tag{130}$$

$$\begin{aligned}\left[T^{IJ},Q^K_\alpha\right]&=(\delta^{JK}Q^I_\alpha-\delta^{IK}Q^J_\alpha),\left[\tilde{Y}^{IJ},Q^K_\alpha\right]\\&=(\delta^{JK}\Sigma^I_\alpha-\delta^{IK}\Sigma^J_\alpha),\end{aligned} \tag{131}$$

$$\begin{aligned}\left[Y^{IJ},Q^K_\alpha\right]&=(\delta^{JK}Q^I_\alpha-\delta^{IK}Q^J_\alpha),\left[T^{IJ},\Sigma^K_\alpha\right]\\&=(\delta^{JK}\Sigma^I_\alpha-\delta^{IK}\Sigma^J_\alpha),\end{aligned} \tag{132}$$

$$\begin{aligned}\left[\tilde{Y}^{IJ},\Sigma^K_\alpha\right]&=(\delta^{JK}Q^I_\alpha-\delta^{IK}Q^J_\alpha),\left[Y^{IJ},\Sigma^K_\alpha\right]\\&=(\delta^{JK}\Sigma^I_\alpha-\delta^{IK}\Sigma^J_\alpha),\end{aligned} \tag{133}$$

$$\left\{Q^I_\alpha,Q^J_\beta\right\}=\frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{Z}_{ab}+2\left(\Gamma^aC\right)_{\alpha\beta}P_a\right]-C_{\alpha\beta}\tilde{Y}^{IJ}, \tag{134}$$

$$\left\{Q^I_\alpha,\Sigma^J_\beta\right\}=\frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}Z_{ab}+2\left(\Gamma^aC\right)_{\alpha\beta}\tilde{Z}_a\right]-C_{\alpha\beta}Y^{IJ}, \tag{135}$$

$$\left\{\Sigma^I_\alpha,\Sigma^J_\beta\right\}=\frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{Z}_{ab}+2\left(\Gamma^aC\right)_{\alpha\beta}P_a\right]-C_{\alpha\beta}\tilde{Y}^{IJ}. \tag{136}$$

Here, the T^{ij}, $\tilde{Y}^{ij}$, Y^{ij}, T^{IJ}, $\tilde{Y}^{IJ}$ and Y^{IJ} generators correspond to internal symmetry generators with $i=1,\ldots,p$ and $I=1,\ldots,q$.

Using Theorem VII.2 of Ref. [47], it is possible to show that the non-vanishing components of the invariant tensor for the $\mathcal{N}=p+q$ AdS–Lorentz are, besides those given by Eqs. (53)–(62),

$$\begin{aligned}\left\langle Q^i_\alpha Q^j_\beta\right\rangle&=\left\langle\Sigma^i_\alpha\Sigma^j_\beta\right\rangle\\&=\tilde{\alpha}_2\left\langle\tilde{Q}^i_\alpha\tilde{Q}^j_\beta\right\rangle=2\left(\beta_2-\alpha_2\right)C_{\alpha\beta}\,\delta^{ij},\end{aligned} \tag{137}$$

$$\begin{aligned}\left\langle Q^I_\alpha Q^J_\beta\right\rangle&=\left\langle\Sigma^I_\alpha\Sigma^J_\beta\right\rangle\\&=\tilde{\alpha}_2\left\langle\tilde{Q}^I_\alpha\tilde{Q}^J_\beta\right\rangle=2\left(\beta_2+\alpha_2\right)C_{\alpha\beta}\,\delta^{IJ},\end{aligned} \tag{138}$$

$$\left\langle Q^i_\alpha\Sigma^j_\beta\right\rangle=\tilde{\alpha}_4\left\langle\tilde{Q}^i_\alpha\tilde{Q}^j_\beta\right\rangle=2\left(\beta_4-\alpha_4\right)C_{\alpha\beta}\,\delta^{ij}, \tag{139}$$

$$\left\langle Q^I_\alpha\Sigma^J_\beta\right\rangle=\tilde{\alpha}_4\left\langle\tilde{Q}^I_\alpha\tilde{Q}^J_\beta\right\rangle=2\left(\beta_4+\alpha_4\right)C_{\alpha\beta}\,\delta^{IJ}, \tag{140}$$

$$\left\langle T^{ij}T^{kl}\right\rangle=\tilde{\alpha}_0\left\langle\tilde{T}^{ij}\tilde{T}^{kl}\right\rangle=2\left(\alpha_0-\beta_0\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right), \tag{141}$$

$$\left\langle T^{IJ}T^{KL}\right\rangle=\tilde{\alpha}_0\left\langle\tilde{T}^{IJ}\tilde{T}^{KL}\right\rangle=2\left(\alpha_0+\beta_0\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right), \tag{142}$$

$$\begin{aligned}\left\langle T^{ij}\tilde{Y}^{kl}\right\rangle&=\left\langle\tilde{Y}^{ij}Y^{kl}\right\rangle=\tilde{\alpha}_2\left\langle\tilde{T}^{ij}\tilde{T}^{kl}\right\rangle\\&=2\left(\alpha_2-\beta_2\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right),\end{aligned} \tag{143}$$

$$\begin{aligned}\left\langle T^{IJ}\tilde{Y}^{KL}\right\rangle&=\left\langle\tilde{Y}^{IJ}Y^{KL}\right\rangle=\tilde{\alpha}_2\left\langle\tilde{T}^{IJ}\tilde{T}^{KL}\right\rangle\\&=2\left(\alpha_2+\beta_2\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right),\end{aligned} \tag{144}$$

$$\begin{aligned}\left\langle T^{ij}Y^{kl}\right\rangle&=\left\langle\tilde{Y}^{ij}\tilde{Y}^{kl}\right\rangle=\left\langle Y^{ij}Y^{kl}\right\rangle=\tilde{\alpha}_4\left\langle\tilde{T}^{ij}\tilde{T}^{kl}\right\rangle\\&=2\left(\alpha_4-\beta_4\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right),\end{aligned} \tag{145}$$

$$\begin{aligned}\left\langle T^{IJ}Y^{KL}\right\rangle&=\left\langle\tilde{Y}^{IJ}\tilde{Y}^{KL}\right\rangle=\left\langle Y^{IJ}Y^{KL}\right\rangle=\tilde{\alpha}_4\left\langle\tilde{T}^{IJ}\tilde{T}^{KL}\right\rangle\\&=2\left(\alpha_4+\beta_4\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right),\end{aligned} \tag{146}$$

where $\left\{\tilde{J}_{ab},\tilde{P}_a,\tilde{T}^{ij},\tilde{T}^{IJ},\tilde{Q}^i_\alpha,\tilde{Q}^I_\alpha\right\}$ correspond to the original $\mathfrak{osp}\left(2|p\right)\otimes\mathfrak{osp}\left(2|q\right)$ generators and where we have defined

$$\begin{aligned}&\alpha_0\equiv\tilde{\alpha}_0\mu_0,\quad\alpha_2\equiv\tilde{\alpha}_2\mu_0,\quad\alpha_4\equiv\tilde{\alpha}_4\mu_0,\\&\beta_0\equiv\tilde{\alpha}_0\mu_1,\quad\beta_2\equiv\tilde{\alpha}_2\mu_1,\quad\beta_4\equiv\tilde{\alpha}_4\mu_1.\end{aligned}$$

Here $\tilde{\alpha}_0,\tilde{\alpha}_2,\tilde{\alpha}_4$ are arbitrary constants as μ_0 and μ_1. Let us now consider the gauge connection one-form of this extended superalgebra:

$$\begin{aligned}A=&\frac{1}{2}\omega^{ab}J_{ab}+\frac{1}{2}\tilde{k}^{ab}\tilde{Z}_{ab}+\frac{1}{2}k^{ab}Z_{ab}+\frac{1}{l}e^aP_a+\frac{1}{l}\tilde{h}^a\tilde{Z}_a\\&+\frac{1}{2}A^{ij}T_{ij}+\frac{1}{2}A^{IJ}T_{IJ}+\frac{1}{2}\tilde{B}^{ij}\tilde{Y}_{ij}\\&+\frac{1}{2}\tilde{B}^{IJ}\tilde{Y}_{IJ}+\frac{1}{2}B^{ij}Y_{ij}+\frac{1}{2}B^{IJ}Y_{IJ}\\&+\frac{1}{\sqrt{l}}\bar{\psi}_iQ^i+\frac{1}{\sqrt{l}}\bar{\psi}_IQ^I+\frac{1}{\sqrt{l}}\bar{\xi}_i\Sigma^i+\frac{1}{\sqrt{l}}\bar{\xi}_I\Sigma^I.\end{aligned} \tag{147}$$

The curvature two-form $F=dA+AA$ is given by

$$\begin{aligned}F=&\frac{1}{2}R^{ab}J_{ab}+\frac{1}{2}\tilde{F}^{ab}\tilde{Z}_{ab}+\frac{1}{2}F^{ab}Z_{ab}+\frac{1}{l}F^aP_a+\frac{1}{l}\tilde{H}^a\tilde{Z}_a\\&+\frac{1}{2}F^{ij}T_{ij}+\frac{1}{2}F^{IJ}T_{IJ}+\frac{1}{2}\tilde{G}^{ij}\tilde{Y}_{ij}\\&+\frac{1}{2}\tilde{G}^{IJ}\tilde{Y}_{IJ}+\frac{1}{2}G^{ij}Y_{ij}+\frac{1}{2}G^{IJ}Y_{IJ}\\&+\frac{1}{\sqrt{l}}\bar{\Psi}_iQ^i+\frac{1}{\sqrt{l}}\bar{\Psi}_IQ^I+\frac{1}{\sqrt{l}}\bar{\Xi}_i\Sigma^i+\frac{1}{\sqrt{l}}\bar{\Xi}_I\Sigma^I,\end{aligned} \tag{148}$$

where

$$\begin{aligned}F^a=&D_\omega e^a+k^a_{\ b}e^b+\tilde{k}^a_{\ b}\tilde{h}^b-\frac{1}{2}\bar{\psi}^i\Gamma^a\psi^i\\&-\frac{1}{2}\bar{\xi}^i\Gamma^a\xi^i-\frac{1}{2}\bar{\psi}^I\Gamma^a\psi^I-\frac{1}{2}\bar{\xi}^I\Gamma^a\xi^I,\end{aligned}$$

$$\begin{aligned}
\tilde{H}^a &= D_\omega \tilde{h}^a + \tilde{k}^a{}_b e^b + k^a{}_b \tilde{h}^b - \bar{\psi}^i \Gamma^a \xi^i - \bar{\psi}^I \Gamma^a \xi^I, \\
\tilde{F}^{ab} &= D_\omega \tilde{k}^{ab} + k^a{}_c \tilde{k}^{cb} - k^b{}_c \tilde{k}^{ca} \\
&\quad + \frac{2}{l^2} e^a \tilde{h}^b + \frac{1}{2l} \bar{\psi}^i \Gamma^{ab} \psi^i - \frac{1}{2l} \bar{\psi}^I \Gamma^{ab} \psi^I \\
&\quad + \frac{1}{2l} \bar{\xi}^i \Gamma^{ab} \xi^i - \frac{1}{2l} \bar{\xi}^I \Gamma^{ab} \xi^I, \\
F^{ab} &= D_\omega k^{ab} + \tilde{k}^a{}_c \tilde{k}^{cb} + k^a{}_c k^{cb} + \frac{1}{l^2} e^a e^b + \frac{1}{l^2} \tilde{h}^a \tilde{h}^b \\
&\quad + \frac{1}{l} \bar{\xi}^i \Gamma^{ab} \psi^i - \frac{1}{l} \bar{\xi}^I \Gamma^{ab} \psi^I,
\end{aligned}$$

$$\begin{aligned}
F^{IJ} &= dA^{IJ} + A^{IK} A^{KJ}, \\
\tilde{G}^{IJ} &= d\tilde{B}^{IJ} + A^{IK} \tilde{B}^{KJ} + \tilde{B}^{IK} A^{KJ} + \tilde{B}^{IK} B^{KJ} \\
&\quad + B^{IK} \tilde{B}^{KJ} - \bar{\psi}^I \psi^J - \bar{\xi}^I \xi^J, \\
G^{IJ} &= dB^{IJ} + A^{IK} B^{KJ} + B^{IK} A^{KJ} + \tilde{B}^{IK} \tilde{B}^{KJ} \\
&\quad + B^{IK} B^{KJ} - 2\bar{\psi}^I \xi^J, \\
\Psi^I &= D_\omega \psi^I + \frac{1}{4} k_{ab} \Gamma^{ab} \psi^I \\
&\quad + \frac{1}{4} \tilde{k}_{ab} \Gamma^{ab} \xi^I - \frac{1}{2l} e_a \Gamma^a \xi^I - \frac{1}{2l} \tilde{h}_a \Gamma^a \psi^I \\
&\quad + A^{IJ} \psi^J + B^{IJ} \psi^J + \tilde{B}^{IJ} \xi^J, \\
\Xi^I &= D_\omega \xi^I + \frac{1}{4} k_{ab} \Gamma^{ab} \xi^I + \frac{1}{4} \tilde{k}_{ab} \Gamma^{ab} \psi^I \\
&\quad - \frac{1}{2l} e_a \Gamma^a \psi^I - \frac{1}{2l} \tilde{h}_a \Gamma^a \xi^I \\
&\quad + A^{IJ} \xi^J + B^{IJ} \xi^J + \tilde{B}^{IJ} \psi^J,
\end{aligned}$$

and R^{ab}, F^{ij}, $\tilde{G}^{ij}$, G^{ij}, Ψ^i and Σ^i are defined as in the $\mathcal{N} = 2$ case (see Eq. (95)).

Considering the connection one-form (147) and the non-vanishing components of the invariant tensor ((53)–(62) and (137)–(146)) in the general three-dimensional CS expression, we find the CS action of $\mathcal{N} = p + q$ *AdS*–Lorentz supergravity up to a surface term:

$$\begin{aligned}
I_{\text{CS}}^{(2+1)} &= k \int \frac{\alpha_0}{2} \left(\omega^a{}_b d\omega^b{}_a + \frac{2}{3} \omega^a{}_c \omega^c{}_b \omega^b{}_a \right) \\
&\quad + (\alpha_0 - \beta_0) A^{ij} F^{ji}(A) + (\alpha_0 + \beta_0) A^{IJ} F^{JI}(A) \\
&\quad + \frac{\beta_2}{l} \epsilon_{abc} \left(R^{ab} e^c + \frac{1}{3l^2} e^a e^b e^c \right. \\
&\quad \left. + K^{ab} e^c + \tilde{K}^{ab} \tilde{h}^c + \frac{1}{l^2} \tilde{h}^a \tilde{h}^b e \right) \\
&\quad + (\alpha_2 - \beta_2) \left[\tilde{B}^{ij} F^{ji}(A) \right. \\
&\quad \left. + \tilde{B}^{ij} F^{ji}(B) + \left(A^{ij} + B^{ij} \right) F^{ji}\left(\tilde{B}\right) \right] \\
&\quad + (\alpha_2 + \beta_2) \left[\tilde{B}^{IJ} F^{JI}(A) \right. \\
&\quad \left. + \tilde{B}^{IJ} F^{JI}(B) + \left(A^{IJ} + B^{IJ} \right) F^{JI}\left(\tilde{B}\right) \right] \\
&\quad + 2(\beta_2 - \alpha_2) \left[\frac{1}{l} \bar{\psi}^i \Psi^i + \frac{1}{l} \bar{\xi}^i \Xi^i \right] \\
&\quad + 2(\beta_2 + \alpha_2) \left[\frac{1}{l} \bar{\psi}^I \Psi^I + \frac{1}{l} \bar{\xi}^I \Xi^I \right] \\
&\quad + \alpha_2 \left[R^a{}_b \tilde{k}^b{}_a + K^a{}_b \tilde{k}^b{}_a + \tilde{K}^a{}_b k^b{}_a \right. \\
&\quad \left. + \frac{1}{l^2} e^a H_a + \frac{1}{l^2} \tilde{h}^a K_a \right] \\
&\quad + \frac{\beta_4}{l} \epsilon_{abc} \left(R^{ab} \tilde{h}^c + \frac{1}{3l^2} \tilde{h}^a \tilde{h}^b \tilde{h}^c \right. \\
&\quad \left. + \tilde{K}^{ab} e^c + K^{ab} \tilde{h}^c + \frac{1}{l^2} e^a e^b \tilde{h}^c \right) \\
&\quad + (\alpha_4 - \beta_4) \left[\tilde{B}^{ij} F^{ji}\left(\tilde{B}\right) \right. \\
&\quad \left. + B^{ij} F^{ji}(A) + \left(A^{ij} + B^{ij} \right) F^{ji}(B) \right] \\
&\quad + (\alpha_4 + \beta_4) \left[\tilde{B}^{IJ} F^{JI}\left(\tilde{B}\right) \right. \\
&\quad \left. + B^{IJ} F^{JI}(A) + \left(A^{IJ} + B^{IJ} \right) F^{JI}(B) \right] \\
&\quad + 2(\beta_4 - \alpha_4) \left[\frac{1}{l} \bar{\xi}^i \Psi^i + \frac{1}{l} \bar{\psi}^i \Xi^i \right] \\
&\quad + 2(\beta_4 + \alpha_4) \left[\frac{1}{l} \bar{\xi}^I \Psi^I + \frac{1}{l} \bar{\psi}^I \Xi^I \right] \\
&\quad + \alpha_4 \left[R^a{}_b k^b{}_a + K^a{}_b k^b{}_a + \tilde{K}^a{}_b \tilde{k}^b{}_a + \frac{1}{l^2} e^a K_a + \frac{1}{l^2} \tilde{h}^a H_a \right],
\end{aligned} \tag{149}$$

where Ψ^i, Ψ^I, Ξ^i and Ξ^I are the fermionic components of the curvature two-form and

$$\begin{aligned}
F^{ij}(A) &= dA^{ij} + A^{ik} A^{kj}, \; F^{IJ}(A) = dA^{IJ} + A^{IK} A^{KJ}, \\
F^{ij}\left(\tilde{B}\right) &= d\tilde{B}^{ij} + A^{ik} \tilde{B}^{kj} + \tilde{B}^{ik} A^{kj} + \tilde{B}^{ik} B^{kj} + B^{ik} \tilde{B}^{kj}, \\
F^{IJ}\left(\tilde{B}\right) &= d\tilde{B}^{IJ} + A^{IK} \tilde{B}^{KJ} + \tilde{B}^{IK} A^{KJ} + \tilde{B}^{IK} B^{KJ} + B^{IK} \tilde{B}^{KJ}, \\
F^{ij}(B) &= dB^{ij} + A^{ik} B^{kj} + B^{ik} A^{kj} + \tilde{B}^{ik} \tilde{B}^{kj} + B^{ik} B^{kj}, \\
F^{IJ}(B) &= dB^{IJ} + A^{IK} B^{KJ} + B^{IK} A^{KJ} + \tilde{B}^{IK} \tilde{B}^{KJ} + B^{IK} B^{KJ}, \\
K^{ab} &= Dk^{ab} + k^a{}_d k^d{}_b + \tilde{k}^a{}_d \tilde{k}^d{}_b, \; \tilde{K}^{ab} = D\tilde{k}^{ab} + k^a{}_d \tilde{k}^d{}_b + k_b{}^d \tilde{k}^a{}_d, \\
H^a &= D\tilde{h}^a + k^a{}_b \tilde{h}^b + \tilde{k}^a{}_b e^b, \; K^a = T^a + k^a{}_b e^b + \tilde{k}^a{}_b \tilde{h}^b.
\end{aligned}$$

As the $(2,0)$ *AdS*–Lorentz superalgebra has its $(2,0)$ Maxwell limit, the IW contraction of the (p,q) *AdS*–Lorentz superalgebra leads to the (p,q) Maxwell superalgebra. Indeed, by rescaling the *AdS*–Lorentz generators as

$$\begin{aligned}
&\tilde{Z}_{ab} \to \sigma^2 \tilde{Z}_{ab}, \; Z_{ab} \to \sigma^4 Z_{ab}, \; P_a \to \sigma^2 P_a, \; J_{ab} \to J_{ab}, \\
&\tilde{Z}_a \to \sigma^4 \tilde{Z}_a, \; Q^i_\alpha \to \sigma Q^i_\alpha, \; Q^I_\alpha \to \sigma Q^I_\alpha, \; \Sigma^i \to \sigma^3 \Sigma^i, \\
&\Sigma^I \to \sigma^3 \Sigma^I, \; Y^{ij} \to \sigma^4 Y^{ij}, \; Y^{IJ} \to \sigma^4 Y^{IJ}, \\
&\tilde{Y}^{ij} \to \sigma^2 \tilde{Y}^{ij}, \; \tilde{Y}^{IJ} \to \sigma^2 \tilde{Y}^{IJ}, \; T^{ij} \to T^{ij}, \; T^{IJ} \to T^{IJ},
\end{aligned}$$

and applying the limit $\sigma \to \infty$, we obtain the $\mathcal{N} = p + q$ Maxwell superalgebra whose explicit (anti)commutation relations can be found in Appendix B. Extending the rescaling of the generators to the α and β constants appearing in the invariant tensor of the (p,q) *AdS*–Lorentz superalgebra as

$$\begin{aligned}
&\alpha_4 \to \sigma^4\alpha_4, \quad \alpha_2 \to \sigma^2\alpha_2, \quad \alpha_0 \to \alpha_0, \\
&\beta_4 \to \sigma^4\beta_4, \quad \beta_2 \to \sigma^2\alpha_1, \quad \beta_0 \to \beta_0,
\end{aligned} \tag{150}$$

and considering the limit $\sigma \to \infty$, we obtain the non-vanishing components of the invariant tensor for the (p,q) Maxwell superalgebra:

$$\langle J_{ab}J_{cd}\rangle_{\mathcal{M}} = \alpha_0\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{151}$$

$$\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} = \alpha_2\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{152}$$

$$\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{\mathcal{M}} = \langle J_{ab}Z_{cd}\rangle = \alpha_4\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{153}$$

$$\langle J_{ab}P_c\rangle_{\mathcal{M}} = \beta_2\epsilon_{abc}, \langle P_aP_b\rangle_{\mathcal{M}} = \alpha_4\eta_{ab}, \tag{154}$$

$$\left\langle \tilde{Z}_{ab}P_c\right\rangle_{\mathcal{M}} = \left\langle J_{ab}\tilde{Z}_c\right\rangle_{\mathcal{M}} = \beta_4\epsilon_{abc}, \tag{155}$$

$$\begin{aligned}
\left\langle Q^i_\alpha Q^j_\beta\right\rangle_{\mathcal{M}} &= 2\left(\beta_2-\alpha_2\right)C_{\alpha\beta}\,\delta^{ij}, \left\langle Q^I_\alpha Q^J_\beta\right\rangle_{\mathcal{M}} \\
&= 2\left(\beta_2+\alpha_2\right)C_{\alpha\beta}\,\delta^{IJ},
\end{aligned} \tag{156}$$

$$\begin{aligned}
\left\langle Q^i_\alpha \Sigma^j_\beta\right\rangle_{\mathcal{M}} &= 2\left(\beta_4-\alpha_4\right)C_{\alpha\beta}\,\delta^{ij}, \left\langle Q^I_\alpha \Sigma^J_\beta\right\rangle_{\mathcal{M}} \\
&= 2\left(\beta_4+\alpha_4\right)C_{\alpha\beta}\,\delta^{IJ},
\end{aligned} \tag{157}$$

$$\left\langle T^{ij}T^{kl}\right\rangle_{\mathcal{M}} = 2\left(\alpha_0-\beta_0\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right), \tag{158}$$

$$\left\langle T^{IJ}T^{KL}\right\rangle_{\mathcal{M}} = 2\left(\alpha_0+\beta_0\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right), \tag{159}$$

$$\left\langle T^{ij}\tilde{Y}^{kl}\right\rangle_{\mathcal{M}} = 2\left(\alpha_2-\beta_2\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right), \tag{160}$$

$$\left\langle T^{IJ}\tilde{Y}^{KL}\right\rangle_{\mathcal{M}} = 2\left(\alpha_2+\beta_2\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right), \tag{161}$$

$$\left\langle T^{ij}Y^{kl}\right\rangle_{\mathcal{M}} = \left\langle \tilde{Y}^{ij}\tilde{Y}^{kl}\right\rangle_{\mathcal{M}} = 2\left(\alpha_4-\beta_4\right)\left(\delta^{il}\delta^{kj}-\delta^{ik}\delta^{lj}\right), \tag{162}$$

$$\left\langle T^{IJ}Y^{KL}\right\rangle_{\mathcal{M}} = \left\langle \tilde{Y}^{IJ}\tilde{Y}^{KL}\right\rangle_{\mathcal{M}} = 2\left(\alpha_4+\beta_4\right)\left(\delta^{IL}\delta^{KJ}-\delta^{IK}\delta^{LJ}\right). \tag{163}$$

Let us note that the generators appearing in the invariant tensor $\langle\ldots\rangle_{\mathcal{M}}$ satisfy the (p,q) Maxwell superalgebra. Additionally, the rescaling of the constants considered here preserves the curvature structure.

Then the three-dimensional (p,q) Maxwell supergravity CS action can be derived considering the (p,q) Maxwell connection one-form (analogous to Eq. (147)) and the non-vanishing components of the invariant tensor (151)–(163):

$$\begin{aligned}
I^{(2+1)}_{\mathcal{M}-\text{CS}} = k\int &\frac{\alpha_0}{2}\left(\omega^a_b d\omega^b_a + \frac{2}{3}\omega^a_c\omega^c_b\omega^b_a\right) + \frac{\beta_2}{l}\epsilon_{abc}R^{ab}e^c \\
&+ \left(\alpha_0-\beta_0\right)A^{ij}F^{ji}\left(A\right) + \left(\alpha_0+\beta_0\right)A^{IJ}F^{JI}\left(A\right) \\
&+ \left(\alpha_2-\beta_2\right)\left[\tilde{B}^{ij}F^{ji}\left(A\right) + A^{ij}F^{ji}\left(\tilde{B}\right)\right] \\
&+ \left(\alpha_2+\beta_2\right)\left[\tilde{B}^{IJ}F^{JI}\left(A\right) + A^{IJ}F^{JI}\left(\tilde{B}\right)\right] \\
&+ 2\left(\beta_2-\alpha_2\right)\frac{1}{l}\bar{\psi}^i\Psi^i + 2\left(\beta_2+\alpha_2\right)\frac{1}{l}\bar{\psi}^I\Psi^I \\
&+ \alpha_2 R^a_b\tilde{k}^b_a + \frac{\beta_4}{l}\epsilon_{abc}\left(R^{ab}\tilde{h}^c + D_\omega\tilde{k}^{ab}e^c\right) \\
&+ \left(\alpha_4-\beta_4\right)\left[\tilde{B}^{ij}F^{ji}\left(\tilde{B}\right) + B^{ij}F^{ji}\left(A\right) + A^{ij}F^{ji}\left(B\right)\right] \\
&+ \left(\alpha_4+\beta_4\right)\left[\tilde{B}^{IJ}F^{JI}\left(\tilde{B}\right) + B^{IJ}F^{JI}\left(A\right) + A^{IJ}F^{JI}\left(B\right)\right] \\
&+ 2\left(\beta_4-\alpha_4\right)\left[\frac{1}{l}\bar{\xi}^i\Psi^i + \frac{1}{l}\bar{\psi}^i\Xi^i\right] \\
&+ 2\left(\beta_4+\alpha_4\right)\left[\frac{1}{l}\bar{\xi}^I\Psi^I + \frac{1}{l}\bar{\psi}^I\Xi^I\right] \\
&+ \alpha_4\left[R^a_bk^b_a + D_\omega\tilde{k}^a_b\tilde{k}^b_a + \frac{1}{l^2}e^aT_a\right],
\end{aligned} \tag{164}$$

where Ψ^i, Ψ^I, Ξ^i and Ξ^I are the fermionic components of the (p,q) Maxwell curvature two-form, given by

$$\begin{aligned}
\Psi^i &= d\psi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^i \\
&\quad + A^{ij}\psi^j, \\
\Psi^I &= d\psi^I + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi^I + A^{IJ}\psi^J, \\
\Xi^i &= d\xi^i + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi^i + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi^i + \frac{1}{2l}e_a\Gamma^a\psi^i \\
&\quad + A^{ij}\xi^j + \tilde{B}^{ij}\psi^j, \\
\Xi^I &= d\xi^I + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi^I + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi^I - \frac{1}{2l}e_a\Gamma^a\psi^I \\
&\quad + A^{IJ}\xi^J + \tilde{B}^{IJ}\psi^J,
\end{aligned}$$

and

$$\begin{aligned}
F^{ij}\left(A\right) &= dA^{ij} + A^{ik}A^{kj}, F^{IJ}\left(A\right) = dA^{IJ} + A^{IK}A^{KJ}, \\
F^{ij}\left(\tilde{B}\right) &= d\tilde{B}^{ij} + A^{ik}\tilde{B}^{kj} + \tilde{B}^{ik}A^{kj}, \\
F^{IJ}\left(\tilde{B}\right) &= d\tilde{B}^{IJ} + A^{IK}\tilde{B}^{KJ} + \tilde{B}^{IK}A^{KJ}, \\
F^{ij}\left(B\right) &= dB^{ij} + A^{ik}B^{kj} + B^{ik}A^{kj} + \tilde{B}^{ik}\tilde{B}^{kj}, \\
F^{IJ}\left(B\right) &= dB^{IJ} + A^{IK}B^{KJ} + B^{IK}A^{KJ} + \tilde{B}^{IK}\tilde{B}^{KJ}.
\end{aligned}$$

One can note that the (p,q) Maxwell supergravity action can be obtained directly from the *AdS*–Lorentz one considering the rescaling of the constant (given by Eq. (150)) and the gauge fields

$$\begin{aligned}
&\omega_{ab} \to \omega_{ab}, \tilde{k}_{ab} \to \sigma^{-2}\tilde{k}_{ab}, k_{ab} \to \sigma^{-4}k_{ab}, \\
&\qquad e_a \to \sigma^{-2}e_a, \tilde{h}_a \to \sigma^{-4}\tilde{h}_a, \\
&\psi^i \to \sigma^{-1}\psi^i, \psi^I \to \sigma^{-1}\psi^I, \\
&\qquad \xi^i \to \sigma^{-3}\xi^i, \xi^I \to \sigma^{-3}\xi^I, A^{ij} \to A^{ij}, \\
&A^{IJ} \to A^{IJ}, \tilde{B}^{ij} \to \sigma^{-2}\tilde{B}^{ij}, \tilde{B}^{IJ} \to \sigma^{-2}\tilde{B}^{IJ}, \\
&\qquad B^{ij} \to \sigma^{-4}B^{ij}, B^{IJ} \to \sigma^{-4}B^{IJ},
\end{aligned}$$

and then the limit $\sigma \to \infty$. Thus the procedure presented here ensures the explicit Maxwell limit considering the rescaling not only of the fields but also of the constants appearing in the invariant tensor. Naturally, the Poincaré limit approach presented in Ref. [9] could be applied here, but it would require the introduction of additional gauge fields. This would lead not only to new Maxwell supergravity theories but also to new AdS–Lorentz ones.

6 Conclusion

We have presented a generalization of the standard Inönü–Wigner contraction combining the rescaling of the generators and the constants appearing in the invariant tensor of a Lie superalgebra. The procedure considered here allows one not only to obtain the invariant tensor of a contracted superalgebra but also to construct the contracted supergravity action.

In particular, we have shown that the Poincaré limit can be applied to a (p, q) AdS supergravity in the presence of the exotic Lagrangian without introducing an $\mathfrak{so}(p) \oplus \mathfrak{so}(q)$ extension. Naturally, the internal symmetries generators of the AdS superalgebra behave as central charges after the contraction and do not contribute explicitly in the construction of the Poincaré action. Additionally, no gravitino fields contribute to the exotic form. It is important to point out that the standard IW contraction does not allow one to apply the Poincaré limit to the (p, q) AdS supergravity in the presence of the Pontryagin–Chern–Simons form.

We have also applied our generalized IW scheme to expanded superalgebras. We have constructed a new class of $D = 2 + 1$ (p, q) Maxwell supergravity theories as a particular limit of an AdS–Lorentz supergravity model. The results presented here show an explicit relation between contraction and expansion. Besides, we have shown that the fermionic and the internal symmetries fields contribute to the exotic CS form. Nevertheless, we have clarified that a supergravity with Maxwell supersymmetry requires the introduction of an additional spinorial field ξ.

The procedure considered here could be useful in higher-dimensional supergravity models in order to derive non-trivial Chern–Simons supergravity theories (work in progress). It would be interesting to explore the expansion and contraction method in the context of infinite-dimensional algebras and hypergravity.

It would also be interesting to extend our study of the Maxwell supergravities to black hole solutions. It is of particular interest to study black hole solutions with torsion for their thermodynamical properties [76–78]. In particular, one could explore the possibility of finding Maxwell "exotic" BTZ type black holes.

Acknowledgements This work was supported by the Newton-Picarte CONICYT Grant No. DPI20140053. (P.K.C. and E.K.R.) and by the Conicyt-PAI Grant No. 79150061 (P.K.C.).

A Appendix

The $\mathcal{N}$-extended AdS–Lorentz superalgebra is generated by the set of generators

$$\left\{ J_{ab}, P_a, \tilde{Z}_{ab}, \tilde{Z}_a, Z_{ab}, T^{ij}, \tilde{Y}^{ij}, Y^{ij}, Q^i_\alpha, \Sigma^i_\alpha \right\}$$

(with $i = 1, \ldots, \mathcal{N}$; $\alpha = 1, \ldots, 4$) whose generators satisfy the (anti)-commutation relations:

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \tag{165}$$

$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{166}$$

$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{167}$$

$$\left[J_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{168}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{169}$$

$$\left[\tilde{Z}_{ab}, Z_{cd}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{170}$$

$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, [Z_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \tag{171}$$

$$\left[\tilde{Z}_{ab}, P_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \left[J_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{172}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_c\right] = \eta_{bc} P_a - \eta_{ac} P_b, \left[Z_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{173}$$

$$[P_a, P_b] = Z_{ab}, \left[\tilde{Z}_a, P_b\right] = \tilde{Z}_{ab}, \left[\tilde{Z}_a, \tilde{Z}_b\right] = Z_{ab}, \tag{174}$$

$$\left[T^{ij}, T^{kl}\right] = \delta^{jk} T^{il} - \delta^{ik} T^{jl} - \delta^{jl} T^{ik} + \delta^{il} T^{jk}, \tag{175}$$

$$\left[T^{ij}, Y^{kl}\right] = \delta^{jk} Y^{il} - \delta^{ik} Y^{jl} - \delta^{jl} Y^{ik} + \delta^{il} Y^{jk}, \tag{176}$$

$$\left[T^{ij}, \tilde{Y}^{kl}\right] = \delta^{jk} \tilde{Y}^{il} - \delta^{ik} \tilde{Y}^{jl} - \delta^{jl} \tilde{Y}^{ik} + \delta^{il} \tilde{Y}^{jk}, \tag{177}$$

$$\left[\tilde{Y}^{ij}, \tilde{Y}^{kl}\right] = \delta^{jk} Y^{il} - \delta^{ik} Y^{jl} - \delta^{jl} Y^{ik} + \delta^{il} Y^{jk}, \tag{178}$$

$$\left[\tilde{Y}^{ij}, Y^{kl}\right] = \delta^{jk} \tilde{Y}^{il} - \delta^{ik} \tilde{Y}^{jl} - \delta^{jl} \tilde{Y}^{ik} + \delta^{il} \tilde{Y}^{jk}, \tag{179}$$

$$\left[Y^{ij}, Y^{kl}\right] = \delta^{jk} Y^{il} - \delta^{ik} Y^{jl} - \delta^{jl} Y^{ik} + \delta^{il} Y^{jk}, \tag{180}$$

$$\left[J_{ab}, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} Q^i\right)_\alpha, \left[P_a, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a \Sigma^i\right)_\alpha, \tag{181}$$

$$\left[\tilde{Z}_{ab}, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^i\right)_\alpha, \left[\tilde{Z}_a, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a Q^i\right)_\alpha, \tag{182}$$

$$\left[Z_{ab}, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} Q^i\right)_\alpha, \left[P_a, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a Q^i\right)_\alpha, \tag{183}$$

$$\left[J_{ab}, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^i\right)_\alpha, \left[\tilde{Z}_a, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a \Sigma^i\right)_\alpha, \tag{184}$$

$$\left[\tilde{Z}_{ab}, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} Q^i\right)_\alpha, \left[Z_{ab}, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^i\right)_\alpha, \tag{185}$$

$$\begin{aligned}\left[T^{ij}, Q^k_\alpha\right] &= (\delta^{jk} Q^i_\alpha - \delta^{ik} Q^j_\alpha), \left[\tilde{Y}^{ij}, Q^k_\alpha\right] \\ &= (\delta^{jk} \Sigma^i_\alpha - \delta^{ik} \Sigma^j_\alpha),\end{aligned} \tag{186}$$

$$\begin{aligned}\left[Y^{ij}, Q^k_\alpha\right] &= (\delta^{jk} Q^i_\alpha - \delta^{ik} Q^j_\alpha), \left[T^{ij}, \Sigma^k_\alpha\right] \\ &= (\delta^{jk} \Sigma^i_\alpha - \delta^{ik} \Sigma^j_\alpha),\end{aligned} \tag{187}$$

$$\begin{aligned}\left[\tilde{Y}^{ij}, \Sigma^k_\alpha\right] &= (\delta^{jk} Q^i_\alpha - \delta^{ik} Q^j_\alpha), \left[Y^{ij}, \Sigma^k_\alpha\right] \\ &= (\delta^{jk} \Sigma^i_\alpha - \delta^{ik} \Sigma^j_\alpha),\end{aligned} \tag{188}$$

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab} C\right)_{\alpha\beta} \tilde{Z}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta} P_a\right] + C_{\alpha\beta} \tilde{Y}^{ij}, \tag{189}$$

$$\left\{Q^i_\alpha, \Sigma^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab} C\right)_{\alpha\beta} Z_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta} \tilde{Z}_a\right] + C_{\alpha\beta} Y^{ij}, \tag{190}$$

$$\left\{\Sigma^i_\alpha, \Sigma^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab} C\right)_{\alpha\beta} \tilde{Z}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta} P_a\right] + C_{\alpha\beta} \tilde{Y}^{ij}. \tag{191}$$

Let us note that the $\mathcal{N} = 1$ case (whose internal symmetries generators T^{ij}, Y^{ij} and $\tilde{Y}^{ij}$ are absent) reproduces the minimal *AdS*–Lorentz superalgebra introduced in Ref. [65].

B Appendix

The (p, q) Maxwell superalgebra is generated by the following set:

$$\left\{J_{ab}, P_a, \tilde{Z}_{ab}, \tilde{Z}_a, Z_{ab}, T^{ij}, T^{IJ}, \tilde{Y}^{ij}, \tilde{Y}^{IJ}, Y^{ij}, Y^{IJ}, Q^i_\alpha, Q^I_\alpha, \Sigma^i_\alpha, \Sigma^I_\alpha\right\}$$

with $i = 1, \ldots, p$ and $I = 1, \ldots, q$. The (p, q) Maxwell generators satisfy the following (anti)commutation relations:

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \tag{192}$$

$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{193}$$

$$\left[J_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{194}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{195}$$

$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, [P_a, P_b] = Z_{ab}, \tag{196}$$

$$\left[\tilde{Z}_{ab}, P_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \left[J_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{197}$$

$$\left[T^{ij}, T^{kl}\right] = \delta^{jk} T^{il} - \delta^{ik} T^{jl} - \delta^{jl} T^{ik} + \delta^{il} T^{jk}, \tag{198}$$

$$\left[T^{ij}, Y^{kl}\right] = \delta^{jk} Y^{il} - \delta^{ik} Y^{jl} - \delta^{jl} Y^{ik} + \delta^{il} Y^{jk}, \tag{199}$$

$$\left[T^{ij}, \tilde{Y}^{kl}\right] = \delta^{jk} \tilde{Y}^{il} - \delta^{ik} \tilde{Y}^{jl} - \delta^{jl} \tilde{Y}^{ik} + \delta^{il} \tilde{Y}^{jk}, \tag{200}$$

$$\left[\tilde{Y}^{ij}, \tilde{Y}^{kl}\right] = \delta^{jk} Y^{il} - \delta^{ik} Y^{jl} - \delta^{jl} Y^{ik} + \delta^{il} Y^{jk}, \tag{201}$$

$$\left[T^{IJ}, T^{KL}\right] = \delta^{JK} T^{IL} - \delta^{IK} T^{JL} - \delta^{JL} T^{IK} + \delta^{IL} T^{JK}, \tag{202}$$

$$\left[T^{IJ}, \tilde{Y}^{KL}\right] = \delta^{JK} \tilde{Y}^{IL} - \delta^{IK} \tilde{Y}^{JL} - \delta^{JL} \tilde{Y}^{IK} + \delta^{IL} \tilde{Y}^{JK}, \tag{203}$$

$$\left[T^{IJ}, Y^{KL}\right] = \delta^{JK} Y^{IL} - \delta^{IK} Y^{JL} - \delta^{JL} Y^{IK} + \delta^{IL} Y^{JK}, \tag{204}$$

$$\left[\tilde{Y}^{IJ}, \tilde{Y}^{KL}\right] = \delta^{JK} Y^{IL} - \delta^{IK} Y^{JL} - \delta^{JL} Y^{IK} + \delta^{IL} Y^{JK}, \tag{205}$$

$$\left[J_{ab}, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} Q^i\right)_\alpha, \left[P_a, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_a \Sigma^i\right)_\alpha, \tag{206}$$

$$\left[\tilde{Z}_{ab}, Q^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^i\right)_\alpha, \left[J_{ab}, \Sigma^i_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^i\right)_\alpha, \tag{207}$$

$$\left[T^{ij}, Q^k_\alpha\right] = (\delta^{jk} Q^i_\alpha - \delta^{ik} Q^j_\alpha), \left[\tilde{Y}^{ij}, Q^k_\alpha\right] = (\delta^{jk} \Sigma^i_\alpha - \delta^{ik} \Sigma^j_\alpha), \tag{208}$$

$$\left[T^{ij}, \Sigma^k_\alpha\right] = (\delta^{jk} \Sigma^i_\alpha - \delta^{ik} \Sigma^j_\alpha), \tag{209}$$

$$\left[J_{ab}, Q^I_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} Q^I\right)_\alpha, \left[P_a, Q^I_\alpha\right] = \frac{1}{2}\left(\Gamma_a \Sigma^I\right)_\alpha, \tag{210}$$

$$\left[\tilde{Z}_{ab}, Q^I_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^I\right)_\alpha, \left[J_{ab}, \Sigma^I_\alpha\right] = -\frac{1}{2}\left(\Gamma_{ab} \Sigma^I\right)_\alpha, \tag{211}$$

$$\begin{aligned}\left[T^{IJ}, Q^K_\alpha\right] &= (\delta^{JK} Q^I_\alpha - \delta^{IK} Q^J_\alpha), \left[\tilde{Y}^{IJ}, Q^K_\alpha\right] \\ &= (\delta^{JK} \Sigma^I_\alpha - \delta^{IK} \Sigma^J_\alpha),\end{aligned} \tag{212}$$

$$\left[T^{IJ}, \Sigma^K_\alpha\right] = (\delta^{JK} \Sigma^I_\alpha - \delta^{IK} \Sigma^J_\alpha), \tag{213}$$

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{Z}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta}P_a\right] + C_{\alpha\beta}\tilde{Y}^{ij}, \tag{214}$$

$$\left\{Q^i_\alpha, \Sigma^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}Z_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta}\tilde{Z}_a\right] + C_{\alpha\beta}Y^{ij}, \tag{215}$$

$$\left\{Q^I_\alpha, Q^J_\beta\right\} = \frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{Z}_{ab} + 2\left(\Gamma^a C\right)_{\alpha\beta}P_a\right] - C_{\alpha\beta}\tilde{Y}^{IJ}, \tag{216}$$

$$\left\{Q^I_\alpha, \Sigma^J_\beta\right\} = \frac{1}{2}\delta^{IJ}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}Z_{ab} + 2\left(\Gamma^a C\right)_{\alpha\beta}\tilde{Z}_a\right] - C_{\alpha\beta}Y^{IJ}. \tag{217}$$

One can note that when $q = 0$, we recover the usual $\mathcal{N}$-extended Maxwell superalgebra whose (anti)commutation relations can be found in Refs. [72,73].

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