

On the structure of anomalous composite Higgs models

Ben Gripaios^{1,a}, Marco Nardecchia^{1,2,3,b}, Tevong You^{1,2,c}

¹ Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge CB3 0HE, UK

² DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

³ CERN, Theoretical Physics Department, Geneva, Switzerland

Received: 30 November 2016 / Accepted: 1 January 2017 / Published online: 13 January 2017
© The Author(s) 2017. This article is published with open access at Springerlink.com

Abstract We describe the anomaly structure of a composite Higgs model in which the $SO(5)/SO(4)$ coset structure of the minimal model is extended by an additional, non-linearly realised $U(1)_\eta$. In addition, we show that the effective Lagrangian admits a term that, like the Wess–Zumino–Witten term in the chiral Lagrangian for QCD, is not invariant under the non-linearly realised symmetries, but rather changes by a total derivative. This term is unlike the Wess–Zumino–Witten term in that it does not arise from anomalies. If present, it may give rise to the rare decay $\eta \rightarrow hW^+W^-Z$. The phenomenology of the singlet in this model differs from that in a model based on $SO(6)/SO(5)$, in that couplings to both gluons and photons, arising via anomalies, are present. We show that while some tuning is needed to accommodate flavour and electroweak precision constraints, the model is no worse than the minimal model in this regard.

Contents

1	Introduction	1
2	The model	2
3	Anomalies and WZW-like terms	3
4	Couplings to fermions and flavour violation	6
5	The scalar potential	8
6	Conclusion	10
	Appendix A: Scalar potential computations	10
	Appendix B: Higher-order contributions to the anomalous effective action	13
	References	13

^a e-mail: gripaios@hep.phy.cam.ac.uk

^b e-mail: m.nardecchia@damtp.cam.ac.uk

^c e-mail: tty20@cam.ac.uk

1 Introduction

In recent years, theorists have devoted much attention to models in which the electroweak hierarchy problem is solved by postulating that the Higgs boson arises as a composite pseudo-Goldstone boson of some new, TeV-scale strong dynamics [1–3].

If this is really what happens in Nature, then it is interesting to ask how we might go about figuring out what the underlying UV dynamics is, given our current rather poor theoretical understanding of strongly coupled dynamics.

One way in which we may do so is via triangle anomalies, which are not renormalised and so, if present in the UV, must be reproduced in the IR, either by massless fermions or by terms involving the pseudo-Goldstone bosons. Such anomalies are not only not renormalised, but they are also topological in nature. This means that by measuring them in the IR, we may gain concrete information as regards the UV dynamics. The classic example, of course, is in QCD, where the measurement of the decay rate $\pi^0 \rightarrow \gamma\gamma$ (which arises via the electromagnetic anomaly [4,5]) enables us to infer that $N_c = 3$.

In order to make such spectacular inferences, one must be lucky enough to have a low-energy Lagrangian that admits a non-trivial anomaly structure. The minimal, and by far the most popular, composite Higgs model, based on $SO(5)/SO(4)$ [6] does not feature anomalies. However, the ‘next-to-minimal’ model based on $SO(6)/SO(5)$ [7], which is just as good from the phenomenological point of view, does. Compared to the minimal model, it features only an additional electroweak singlet scalar, which couples to electroweak gauge bosons via a single $SO(6)^3$ triangle anomaly.

Here we wish to describe yet another model, based on $SO(5) \times U(1)/SO(4)$. It is just as minimal as the $SO(6)/SO(5)$ model, in the sense that it features only an additional electroweak singlet scalar. But it turns out to have a much richer anomaly structure, with several novel features.

A first novel feature is that there are now three distinct triangle anomalies, which give rise, at leading order, to couplings of the singlet to both gluons and electroweak bosons.

A second novel feature is that the higher-order structure of the anomalous effective action is not unique. Indeed, we exhibit two solutions to the Wess–Zumino consistency conditions. As far as we are aware, this phenomenon has not been observed before in the literature on sigma models.

A third novel feature is that the effective Lagrangian admits a term that is not invariant, but rather changes by a total derivative, under the non-linearly realised symmetries. Such a term is much like the Wess–Zumino–Witten (WZW) term in the chiral Lagrangian of QCD, which allows processes violating a putative internal symmetry under which Goldstone bosons change sign, such as $K + \bar{K} \rightarrow 3\pi$ [8,9]. But there is one noteworthy distinction between the WZW-like term presented here and WZW term in QCD. In the latter, the presence of the anomaly implies the presence of the WZW term, in the sense that the low-energy effective action reproducing the anomaly reduces to the WZW term when the gauge fields vanish. In the model presented here, this is not so. This phenomenon is also, we believe, unknown in the sigma-model literature. The WZW-like term is also of phenomenological interest, in that it may lead to a rare decay of the singlet via $\eta \rightarrow hW^+W^-Z$.

The outline is as follows. In the next section, we present the pattern of symmetry breaking and sketch the concomitant anomalies. We then present a full discussion of the anomaly structure and the WZW-like term in Sect. 3. In Sect. 4, we describe the couplings to fermions and the implications for flavour physics. In Sect. 5, we discuss the form of the scalar potential that is induced by the couplings to gauge fields and fermions. We conclude in Sect. 6. Two more technical discussions are relegated to appendices.

2 The model

We wish to consider composite Higgs models based on a homogeneous space G/H that feature triangle anomalies.¹

The minimal model [6], based on $SO(5)/SO(4)$ (or $SO(5)/O(4)$ with custodial protection of $Z \rightarrow b\bar{b}$ [11]), features no triangle anomalies. The ‘next-to-minimal’ model based on $SO(6)/SO(5)$ [7] does, however, feature triangle anomalies. Indeed the Goldstone bosons transform as the 5-d irrep of $SO(5)$, which, on restriction to the $SO(4)$ subgroup, yields both a 4-d irrep (viz. the Higgs field) and a singlet. Moreover, since $SO(6)$ is locally isomorphic to $SU(4)$, we

have the possibility of an $SU(4)^3$ triangle anomaly.² This anomaly leads to an interaction, at leading order, of the form $\frac{1}{16\pi^2} \frac{\eta}{f} (g_2^2 W_{\mu\nu} \tilde{W}^{\mu\nu} - g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu})$, with neither a coupling to gluons nor to photons [12].

The absence of a coupling to photons in this model is something of a group-theoretical accident, in that there are couplings to ZZ , γZ , and WW . But the absence of a coupling to gluons looks, at first sight, to be a generic problem in a composite Higgs model, given that the rôle of the new strong dynamics is to break the electroweak symmetry, independently of the $SU(3)_C$ dynamics. In fact, this is not so, since a consequence of partial compositeness is that the new strong sector must be charged under $SU(3)_C$ [13]. So it seems quite plausible that the elementary fermions of the UV theory could generate an anomaly involving $SU(3)_C$.

One way to get couplings of the singlet to both electroweak gauge bosons and to gluons via anomalies is to include both $SU(3)_C$ and $SU(2)_L$ or $U(1)_Y$ in some simple subgroup of G . But such a strategy will lead to additional coloured Goldstone bosons, with potentially dangerous phenomenological implications.³ A safer, and simpler, strategy is to modify the minimal model by adding a non-linearly realised $U(1)_\eta$ factor, such that the symmetry breaking pattern in the strong sector becomes

$$\frac{G}{H} = \frac{SU(3)_C \times SO(5) \times U(1)_X \times U(1)_\eta}{SU(3)_C \times SO(4) \times U(1)_X}, \tag{2.1}$$

where $U(1)_X$ denotes the usual $U(1)$ needed in composite Higgs models to give the correct hypercharge assignments to SM fermions. This model features an additional SM singlet compared to the minimal composite Higgs model. We remark that, unlike the $SO(6)/SO(5)$ model, this coset space allows for two distinct decay constants, f and f_η , associated with the Higgs boson and the η , respectively. We assume henceforth that these are generated by the same strong dynamics, and hence are of the same order of magnitude.

Let us now consider the possible triangle anomalies in this model. As we shall see in Sect. 3, triangle anomalies in G are admissible only if they vanish on restriction to H . Thus, our model admits three possible sources of triangle anomalies, namely $SU(3)_C^2 U(1)_\eta$ and $SO(5)^2 U(1)_\eta$ anomalies, and anomalies involving $U(1)_\eta$ and $U(1)_X$.

The leading contributions to the resulting low-energy effective action arise at dimension-5, taking the form

$$\mathcal{L}_{eff} = \frac{1}{16\pi^2} \frac{\eta}{f_\eta} (c_3 g_3^2 G_{\mu\nu} \tilde{G}^{\mu\nu} + c_5 (g_2^2 W_{\mu\nu} \tilde{W}^{\mu\nu} + g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu}) + c_1 g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu}), \tag{2.2}$$

² Since $H_{dR}^5(SO(6)/SO(5)) = H_{dR}^5(S^5) = \mathbb{R}$, there is also a possible WZW term. As explained in the next sections, H_{dR}^5 denotes the fifth de Rham cohomology group.

³ Such states may also have desirable phenomenological implications, however [14,15].

¹ See [7,10] for earlier discussions of anomalies in composite Higgs models.

where the coefficients are real, but otherwise arbitrary (corresponding to the freedom to arbitrarily choose the $U(1)_\eta$ irreps of fermions in the UV theory that contribute to the anomaly).

3 Anomalies and WZW-like terms

We now discuss the anomaly structure of the model in more detail, together with the phenomenological consequences. Let us begin with a general discussion. A theory with internal global symmetry group G may be anomalous, in the sense⁴ that there is no way to regularise the theory such that the divergences of 3-point functions of conserved currents are all vanishing. Such anomalies are not renormalised and must be reproduced at all energies, with consequences for low-energy physics.

One consequence is a consistency condition on the possible pattern of symmetry breaking at low energy: if a subgroup $H \subset G$ is linearly realised at low energy, then H must be anomaly free. The reason [16] is that we could perturb the theory in an arbitrarily small way by gauging the whole of G , but choosing the gauge coupling to be arbitrarily small. If there were anomalies in H , the gauge bosons in H could get masses via a loop diagram formed out of two anomalous vertices, implying that H could not be linearly realised.

Once this restriction has been taken into account, it can be shown that the remaining anomalies can be reproduced satisfactorily at low-energies by Goldstone boson contributions [17] and an explicit formula for the anomalous contribution to the low-energy effective action for a reductive homogeneous space G/H can be found (see also [18]). As in [17], in this Section we employ the language of differential forms and omit normalisation factors, giving the result only for the special case of a symmetric space, which is sufficient for our needs. The formula is most conveniently written in the fully gauged case; the result for gauging a subset $F \subset G$ can be obtained by setting the corresponding gauge fields to zero in the formula.⁵ Let \mathfrak{g} and \mathfrak{h} be the Lie algebras of G and H . Since G/H is reductive and symmetric, there exists a split, $\mathfrak{g} = \mathfrak{h} + \mathfrak{k}$, such that $[\mathfrak{h}, \mathfrak{k}] \subseteq \mathfrak{k}$ and $[\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{h}$, together with an ‘internal parity’ automorphism of \mathfrak{g} given by $\mathfrak{h} \rightarrow \mathfrak{h}$ and $\mathfrak{k} \rightarrow -\mathfrak{k}$. Letting A be a \mathfrak{g} -valued 1-form representing the gauge fields and letting the coset representative be e^ξ , with $\xi \in \mathfrak{k}$, we have

$$W[\xi, A] = \sum_{\pm} \int_0^1 dt \int d^4x c_{\pm} \text{tr}[\xi G^{\pm}[A_t]], \tag{3.1}$$

⁴ We consider only triangle anomalies here.

⁵ We caution the reader that the symmetry group of the resulting theory is not G , even at the classical level, but rather it is the normaliser of F in G [19].

where $A_t = e^{t\xi}(A + d)e^{-t\xi} \implies F_t = e^{t\xi} F e^{-t\xi}$, c_{\pm} are arbitrary coefficients and

$$G^+[A] = 3F_h^2 + F_k^2 - 4(A_k^2 F_h + A_k F_h A_k + F_h A_k^2) + 8A_k^4, \tag{3.2}$$

$$G^-[A] = \frac{3}{2}(F_h F_k + F_k F_h - F_k A_k^2 - A_k F_k A_k - A_k^2 F_k). \tag{3.3}$$

Here, G^{\pm} are the positive/negative eigenstates with respect to the internal parity and the subscripts h and k denote projections onto the corresponding subspaces, such that $F_h = dA_h + A_h^2 + A_k^2$, $F_k = dA_k + A_h A_k + A_k A_h$.

The action (3.1) is unique in the sense that it is the only action which vanishes when the Goldstone bosons vanish and whose anomaly is given by $\delta_\alpha \Gamma = \sum_{\pm} c_{\pm} \alpha G^{\pm}[A]$ [18]. But it is not unique in the sense that the anomaly can take many forms, corresponding to the addition of local counterterms to the effective action. (For a counterexample, it suffices to choose $H = 0$, for which any form $G[A]$ for the anomaly is reproduced by the effective action $\Gamma = \int_0^1 dt \int_x \text{tr} \xi G[A_t]$.) The action (3.1) is the one obtained by starting from the canonical form of the anomaly (which is symmetric with respect to G) and subtracting a counterterm that enforces the vanishing of the anomaly on H [17]. Hadronic data suggest that this is the option chosen by the strong interactions, but we are unaware of an argument that it is the only consistent option.

Even though its *raison d'être* is to reproduce anomalies that arise due to gauging, (3.1) may not vanish in the limit that gauge fields vanish. In that limit, we obtain

$$W[\xi, 0] = \int_0^1 dt \int d^4x c_+ \text{tr}[\xi (e^{t\xi} d e^{-t\xi})_k^4]. \tag{3.4}$$

Such a term, which contains an undifferentiated Goldstone boson at leading order is not invariant under a G transformation, but rather changes by a total derivative. We will call such non-invariant Lagrangian terms ‘WZW terms’, in honour of their prototype in the chiral Lagrangian. It was shown in [20] that, for compact G in $d = 4$, and for field configurations in the trivial fourth homotopy class,⁶ such terms are in 1-1 correspondence with the generators of the fifth de Rham cohomology group of G/H .

We caution the reader that not all such terms can arise from effective actions reproducing triangle anomalies. By way of a counterexample, consider the homogeneous space $SU(2) \times SU(2)/U(1)$, where the $U(1)$ is included in one of the $SU(2)$ s. This space is equivalent as a smooth manifold to $S^3 \times S^2$ and a straightforward generalisation of the arguments presented below shows that $H_{dR}^5(S^3 \times S^2) = \mathbb{R}$. Thus, there

⁶ As usual, we identify spacetime, with fields thereon tending to a constant value at infinity, with S^4 .

is a WZW term in this case, but since $SU(2)$ has no triangle anomalies, it cannot arise from reproducing them.⁷

Composite Higgs model anomalies

For the $SO(5) \times SU(3) \times U(1) \times U(1)/SO(4) \times SU(3) \times U(1)$ model, it is straightforward to show that the effective action (3.1) reduces, at leading order, to (2.2). For $SU(3)^2U(1)$ and the anomalies involving $U(1)$ s, there are no higher-order corrections to the effective action. There are, however, higher-order corrections for the $SO(5)^2U(1)$ anomaly, the detailed calculation of which we relegate to Appendix 1. The next-to-leading order corrections arise at dimension 7, up to which order the effective action is given, in the operator basis of [21], by

$$\int c_5 \eta (W^i W^i + B^2 - \frac{16}{9f^2} (H^\dagger H (W^i W^i + B^2) + 2H^\dagger \sigma^i H W^i B)) + \dots, \tag{3.5}$$

where W^i and B are the field strength 2-forms and f is the non-linear scale.

These corrections to the leading-order action appear to constitute a definite prediction of the model, once c_5 has been determined from measurements at leading order. Unfortunately, the issue of non-uniqueness discussed above now rears its ugly head. Indeed, it is easy to check that the leading-order action (2.2) alone also provides a solution of the Wess–Zumino consistency conditions that vanishes on $SO(4)$ and so is, *ceteris paribus*, just as good a candidate for the anomalous action. It corresponds to a regularisation of the $SO(5)^2U(1)$ anomaly such that it appears entirely in the $U(1)$ symmetry, whereas our action corresponds to an anomaly that is symmetric with respect to the broken generators in $U(1)$ and $SO(5)$. Whether there exist yet more consistent effective actions is an open question.

The two anomalous effective actions that we have found differ structurally only in their higher-order terms. But this does not mean that the non-uniqueness is phenomenologically inconsequential. Indeed, different choices of regulator for the anomaly lead to different values of the coefficient of the leading-order term. In particular, the coefficient that corresponds to an anomaly that is symmetrised amongst all three currents is $\frac{1}{3}$ that of the coefficient that corresponds to the anomaly that is contained wholly in a single current. So the resolution of the non-uniqueness issue will be crucial, if we want to make inferences about the UV structure of the theory (in particular its fermionic representation content), using experimental data.

Even if this non-uniqueness can be resolved, one should also bear in mind that the couplings of the Goldstone bosons

⁷ Moreover, since $\pi_4(S^3 \times S^2) = \mathbb{Z}/2$, one cannot use Witten’s trick to write the WZW term as an integral over a 5-disk in this case.

to SM fermions will also generate loop contributions to the couplings in the anomalous effective action.

The WZW term

There is a possible WZW term in the model, as we can see by computing $H_{dR}^5(SO(5) \times U(1)/SO(4))$. Recalling that $SO(n+1)/SO(n)$ and S^n are equivalent as smooth manifolds, we thus have $H_{dR}^5(SO(5) \times U(1)/SO(4)) = H_{dR}^5(S^4 \times S^1) = H_{dR}^4(S^4) \otimes H_{dR}^1(S^1) \simeq \mathbb{R} \otimes \mathbb{R} \simeq \mathbb{R}$ (where we used the Künneth formula and the fact that $H_{dR}^i(S^n)$ vanishes unless $i = 0$ or $i = n$, in which case it is isomorphic to \mathbb{R}). Thus, the theory admits a WZW term.

We may easily find the form of the WZW term, at least for field configurations that correspond to the trivial class of the fourth homotopy group. These may be written [20] as the integral over a 5-ball, whose boundary is the spacetime S^4 , of a G -invariant 5-form, whose existence is guaranteed by the non-vanishing fifth de Rham cohomology group.⁸ For $G/H \simeq S^4 \times S^1$ it is just the product of the usual volume forms on the hyperspheres. At leading order in the fields, we can integrate over the 5-ball to get

$$\int_{S^4} \epsilon_{ijkl} \eta dh^i dh^j dh^k dh^l, \tag{3.6}$$

where h^i are co-ordinates in the neighbourhood of the identity on S^4 .⁹ In $SU(2) \times U(1)$ language, the LO WZW term is $\eta dH^\dagger \sigma^i dH dH^\dagger \sigma^i dH$.

As expected, the leading-order term is invariant under the linearly realised subgroup $SO(4)$ and changes by a total derivative under a shift of the Goldstone bosons, corresponding to an infinitesimal $SO(5) \times U(1)$ transformation.

As we see in Appendix 1, the WZW term does not arise from (3.1), which vanishes when the gauge fields vanish. Thus, unlike in QCD, the WZW term and the anomaly are independent, at least for this choice of regularisation of the anomaly.

The WZW term is in fact the leading-order term coupling all 5 Goldstone bosons to each other. This can be seen by forming Lagrangian invariants of the sigma model in the usual way out of the objects $d\eta$ and $e^\xi de^{-\xi}$, which transform as adjoints under H . By Lorentz invariance, all terms involve an even number of derivatives. Terms with no derivatives are forbidden by the non-linearly realised symmetry, while terms with two derivatives are forbidden, because such a term must take the form $\partial_\mu \eta \text{tr} e^\xi \partial_\mu e^{-\xi} = 0$. A possible term with 4 derivatives takes the form $\partial_\mu \eta \text{tr} (e^\xi \partial_\nu e^{-\xi})(e^\xi \partial_\sigma e^{-\xi})(e^\xi \partial_\rho e^{-\xi})$. Since $SO(5)$ is free

⁸ Unfortunately, this trick does not work for a general field configuration, because the fourth homotopy is $\pi_4(S^4 \times S^1) \simeq \pi^4(S^4) \oplus \pi^4(S^1) \simeq \mathbb{Z} \oplus \{e\} \simeq \mathbb{Z} \neq 0$.

⁹ This term was also singled out in [22], but for different reasons.

of triangle anomalies, the trace term must be antisymmetric in its three entries and so a non-vanishing Lorentz invariant can be obtained only by contracting with $\epsilon^{\mu\nu\sigma\rho}$, such that we can revert to the language of differential forms. We have $e^\xi de^{-\xi} = d\xi + \frac{1}{2}[\xi, d\xi] + \dots$, such that the leading-order term involving all Goldstone bosons takes the form $\frac{3}{2}d\eta \text{tr} d\xi d\xi[\xi, d\xi]$. We need this to be non-vanishing when each ξ corresponds to a distinct Goldstone boson and one easily check using the basis in (B.11) that this is not so.

To explore the physics of the WZW term, we first gauge the SM subgroup. Since this is a subgroup of H , under which the WZW term transforms linearly, we may follow the usual prescription of promoting derivatives to covariant derivatives, obtaining $\eta DH^\dagger \sigma^i DHDH^\dagger \sigma^i DH$.

Being of high dimension, the WZW leads to small contributions to low-energy physics. They may, nevertheless, be observable at a future high-precision collider, if sufficiently exotic. As an example, by the Goldstone boson equivalence theorem and by the antisymmetry in the fields, the WZW term leads, after electroweak-symmetry breaking, to a coupling involving $\eta, h, W^+, W^-,$ and Z and hence a possible decay mode $\eta \rightarrow hW^+W^-Z$.

We remark that, whilst the WZW term is the leading-order term coupling all five Goldstone bosons to one another, this does not necessarily imply that it gives the dominant contribution to this decay mode. Indeed, once we switch on the gauging and other symmetry-breaking couplings, we may well get contributions to this decay at lower orders, albeit paying the price of small, symmetry breaking couplings instead.

Discrete symmetries and $Z \rightarrow b\bar{b}$

As we have already remarked, the fact that $SO(5) \times U(1)/SO(4)$ is a symmetric space means that the Lie algebra possesses the ‘internal parity’ automorphism $\mathfrak{h} \rightarrow \mathfrak{h}, \mathfrak{k} \rightarrow -\mathfrak{k}$. The terms in the effective action giving rise to production and decay of the η are odd under this automorphism, so it could only be a symmetry of the dynamics if it were accompanied by a spatial inversion. In any case, the internal parity is broken in the vacuum by the Higgs VEV.

A more desirable symmetry to have, perhaps, is one that protects the decay rate for $Z \rightarrow b\bar{b}$ [11]. In the minimal model based on $G = SO(5)$, this is achieved by enlarging the linearly realised subgroup from $SO(4)$ to $O(4)$.¹⁰ The same enlargement could, of course, be carried out in the model

¹⁰ In fact, if we wish to include matter fields in the theory in spinor representations, then we should consider not $SO(5)$ but rather its universal cover $Sp(2)$. As described in [23], the relevant homogeneous spaces without and with custodial protection of $Z \rightarrow b\bar{b}$ are $Sp(2)/(Sp(1) \times Sp(1))$ and $Sp(2)/(Sp(1) \times Sp(1) \times \mathbb{Z}_2)$, where the homomorphism in the semi-direct product maps the non-trivial element in \mathbb{Z}_2 to the outer automorphism of $Sp(1) \times Sp(1)$ that interchanges the two $Sp(1)$ s. The homogeneous spaces are homeomorphic

described here, but it has the consequence that the WZW term is forced to vanish. Indeed, the usual action of $SO(5)$ on \mathbb{R}^5 gives rise to transitive actions on both S^4 (included in \mathbb{R}^5 as the set of points equidistant from the origin) and $\mathbb{R}P^4$ (given as the set of lines through the origin in \mathbb{R}^5 and which we may also think of as the sphere with antipodal points identified). The stability subgroup in the former case is isomorphic to $SO(4)$, while in the latter case it is $O(4)$. Thus $SO(5)/SO(4)$ is homeomorphic to S^4 , while $SO(5)/O(4)$ is homeomorphic to $\mathbb{R}P^4$. Now, $H_{dR}^4(\mathbb{R}P^4)$ vanishes,¹¹ as do its other de-Rham cohomology groups (excepting of course H_{dR}^0), and so the Künneth formula tells us that with $O(4)$ included in this way, $H_{dR}^5(SO(5) \times SO(2)/O(4)) = 0$, such that there can be no WZW term.

The WZW term may, however, be resurrected by changing the inclusion of the custodial $O(4)$ in G . To understand this, it is useful to see more explicitly why the leading-order WZW term is forbidden in the standard implementation. To this end, choose co-ordinates $(\mathbf{h}, 1)$ on the unit 4-sphere included in \mathbb{R}^5 in the neighbourhood of the stability point $(\mathbf{0}, 1)$. The stability group of the sphere is then $\left\{ \begin{pmatrix} O^+ & 0 \\ 0 & +1 \end{pmatrix} \right\}$, where O^+ is any 4x4 orthogonal matrix of determinant $+1$, and hence is isomorphic to $SO(4)$. But if we identify antipodal points, $(-\mathbf{h}, -1) \sim (\mathbf{h}, 1)$, then the stability subgroup is enhanced to $\left\{ \begin{pmatrix} O^\pm & 0 \\ 0 & \pm 1 \end{pmatrix} \right\}$, where O^- is any 4x4 orthogonal matrix of determinant ± 1 , and hence is indeed isomorphic to $O(4)$, as we claimed earlier. Now, under the action of an element of $O(4)$ that is disconnected from the identity, $(\mathbf{h}, 1) \rightarrow (O^-\mathbf{h}, -1) \sim (-O^-\mathbf{h}, +1)$. Thus the putative leading-order WZW term, which is proportional to $\epsilon_{ijkl}h^i h^j h^k h^l$ is sent to $(-1)^4 \det O^- = -1$ times itself, and is not invariant under such transformations. But the leading-order WZW term should be invariant under $O(4)$ and so must vanish.

Clearly, we can resurrect the WZW term, at least at leading order, by arranging for the $O(4)$ custodial group to be included in G in such a way that the action of elements in $O(4)$ disconnected from the identity also sends $\eta \rightarrow -\eta$. To achieve this, set $G = SO(5) \times O(2)$ and let H be the subgroup

$$\left\{ \left(\begin{pmatrix} O^+ & 0 \\ 0 & +1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right), \left(\begin{pmatrix} O^- & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \right\} \quad (3.7)$$

H is still isomorphic to $O(4)$, but now the action of elements in $O(4)$ disconnected from the identity sends $\eta \rightarrow -\eta$. We

to $SO(5)/SO(4)$ and $SO(5)/O(4)$, respectively, and the discussion given here can be carried over straightforwardly.

¹¹ The reason for this is that the volume form on S^4 , which is given by the pull-back to S^4 via the inclusion map $i : S^4 \rightarrow \mathbb{R}^5$ of the form $\sum_{i=1}^4 (-1)^i x^i dx^1 \dots dx^4$ (where in the ellipsis we omit dx^i), is not identical at antipodal points; this is consistent with the non-orientability of $\mathbb{R}P^4$.

Mixing Parameter	Value
$\epsilon_1^q = \lambda^3 \epsilon_3^q$	$1.15 \times 10^{-2} \epsilon_3^q$
$\epsilon_2^q = \lambda^2 \epsilon_3^q$	$5.11 \times 10^{-2} \epsilon_3^q$
$\epsilon_1^u = \frac{m_u}{v g_\rho} \frac{1}{\lambda^3 \epsilon_3^q}$	$5.48 \times 10^{-4} / (g_\rho \epsilon_3^q)$
$\epsilon_2^u = \frac{m_c}{v g_\rho} \frac{1}{\lambda^2 \epsilon_3^q}$	$5.96 \times 10^{-2} / (g_\rho \epsilon_3^q)$
$\epsilon_3^u = \frac{m_t}{v g_\rho} \frac{1}{\epsilon_3^q}$	$0.866 / (g_\rho \epsilon_3^q)$
$\epsilon_1^d = \frac{m_d}{v g_\rho} \frac{1}{\lambda^3 \epsilon_3^q}$	$1.24 \times 10^{-3} / (g_\rho \epsilon_3^q)$
$\epsilon_2^d = \frac{m_s}{v g_\rho} \frac{1}{\lambda^2 \epsilon_3^q}$	$5.29 \times 10^{-3} / (g_\rho \epsilon_3^q)$
$\epsilon_3^d = \frac{m_b}{v g_\rho} \frac{1}{\epsilon_3^q}$	$1.40 \times 10^{-2} / (g_\rho \epsilon_3^q)$
$\epsilon_1^\ell = \epsilon_1^e = \left(\frac{m_e}{g_\rho v}\right)^{1/2}$	$1.67 \times 10^{-3} / g_\rho^{1/2}$
$\epsilon_2^\ell = \epsilon_2^e = \left(\frac{m_\mu}{g_\rho v}\right)^{1/2}$	$2.43 \times 10^{-2} / g_\rho^{1/2}$
$\epsilon_3^\ell = \epsilon_3^e = \left(\frac{m_\tau}{g_\rho v}\right)^{1/2}$	$0.101 / g_\rho^{1/2}$

Fig. 1 Partial compositeness mixing parameters and values. The input running masses of the SM particles are taken at the renormalisation scale of 1 TeV, with $v = 174$ GeV

conjecture therefore that $H_{dR}^5 \neq 0$ in this case, such that there is a WZW term.

4 Couplings to fermions and flavour violation

We now discuss the couplings of the η singlet to SM fermions. We postulate that the SM fermion Yukawa couplings are generated via the paradigm of partial compositeness (PC) [24]. The basic assumption is that elementary states f^i (where $f \in \{Q_L, u_R, d_R, L_L, e_R\}$ and i is the family index) couple linearly to fermionic operators \bar{O}_i^f of the strong sector:

$$\mathcal{L}_{PC} = g_\rho \epsilon_i^q \bar{O}_i^q Q_L^i + g_\rho \epsilon_i^u \bar{O}_i^u u_R^i + g_\rho \epsilon_i^d \bar{O}_i^d d_R^i + g_\rho \epsilon_i^\ell \bar{O}_i^\ell L_L^i + g_\rho \epsilon_i^e \bar{O}_i^e e_R^i + \text{h.c.}$$

We simplify the description of the strong sector as in [25], assuming a single strong coupling g_ρ , and a single mass scale m_ρ . The linear mixing parameters ϵ_i^a are taken to be hierarchical in order to reproduce the pattern of masses and mixing of the SM fermions. In particular, it can be shown that the Yukawa couplings of up and down quarks and of charged leptons are given by

$$Y_{ij}^U \sim g_\rho \epsilon_i^q \epsilon_j^u, \quad Y_{ij}^D \sim g_\rho \epsilon_i^q \epsilon_j^d \quad \text{and} \quad Y_{ij}^E \sim g_\rho \epsilon_i^\ell \epsilon_j^e. \tag{4.1}$$

Throughout this section, we use the symbol \sim to indicate a relation that holds up to an unknown $\mathcal{O}(1)$ complex coefficient whose value is determined by the unknown strong-sector dynamics. As in [15, 26], a viable choice of the mixing parameters is given in Fig. 1.

We remark that we have tacitly assumed, for simplicity, that every elementary field f_i^a couples to a *single* operator of

the strong sector. In that case, it is easy to derive the coupling of the goldstone boson η to the fermions f^i . Indeed, it is enough to replace $f^i \rightarrow f^i \exp\left(i \frac{\sqrt{2}}{f_\eta} \eta Z_{f_i}\right)$ in the EFT of the usual composite Higgs model based on $SO(5) \times U(1)_X$, where Z_{f_i} is the $U(1)_\eta$ charge. As we shall see in Sect. 5, there is a price to be paid for this assumption, namely that one then requires an additional source of explicit $U(1)_\eta$ breaking in the model in order to generate a potential for the singlet. We expect, however, that relaxing this assumption will lead to comparable bounds.

Without specifying the details and quantum numbers of the composite operators under $SO(5) \times U(1)_X$, integrating away the heavy sector at the scale m_ρ and keeping the leading term in H/f and η/f_η , we get (in complete generality) that

$$\mathcal{L}_{\text{yuk}} = -Y_{ij}^U H \bar{Q}_L^i u_R^j \left[1 + i \frac{\sqrt{2}}{f_\eta} \eta (Z_{Q_L^i} - Z_{u_R^j}) \right] + \text{h.c.} \tag{4.2}$$

$$- Y_{ij}^D H^c \bar{Q}_L^i d_R^j \left[1 + i \frac{\sqrt{2}}{f_\eta} \eta (Z_{Q_L^i} - Z_{d_R^j}) \right] + \text{h.c.} \tag{4.3}$$

$$- Y_{ij}^E H^c \bar{L}_L^i e_R^j \left[1 + i \frac{\sqrt{2}}{f_\eta} \eta (Z_{L_L^i} - Z_{e_R^j}) \right] + \text{h.c.} \tag{4.4}$$

The Yukawa couplings are specified in a basis where the SM fields have specific charge assignments under $U(1)_\eta$. The Yukawa matrices are diagonalised by bi-unitary transformations:

$$\hat{Y}^U = L_U Y^U R_U^\dagger = \frac{1}{v} \text{diag}(m_u, m_c, m_t), \tag{4.5}$$

$$\hat{Y}^D = L_D Y^D R_D^\dagger = \frac{1}{v} \text{diag}(m_d, m_s, m_b), \tag{4.6}$$

$$\hat{Y}^E = L_E Y^E R_E^\dagger = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau). \tag{4.7}$$

The expected size of the entries of these unitary matrices are linked to the mixing parameters in the following way:

$$(L_U)_{ij} \sim (L_D)_{ij} \sim \min\left(\frac{\epsilon_i^q}{\epsilon_j^q}, \frac{\epsilon_j^q}{\epsilon_i^q}\right)$$

$$(R_U)_{ij} \sim \min\left(\frac{\epsilon_i^u}{\epsilon_j^u}, \frac{\epsilon_j^u}{\epsilon_i^u}\right)$$

$$(R_D)_{ij} \sim \min\left(\frac{\epsilon_i^d}{\epsilon_j^d}, \frac{\epsilon_j^d}{\epsilon_i^d}\right) \tag{4.8}$$

and similarly for the leptonic sector.

Rewriting the Lagrangian in the mass basis and replacing the Higgs doublet with its VEV, one may deduce the flavour- and CP-violating couplings of the η to SM fermions:

$$\begin{aligned} \mathcal{L}_{\text{yuk}} \supset & - \sum_{u_i, u_j = u, c, t} Y_{u_i u_j} \eta \bar{u}_i P_R u_j \\ & - \sum_{d, d_j = d, s, b} Y_{d_i d_j} \eta \bar{d}_i P_R d_j \\ & - \sum_{\ell_i, \ell_j = e, \mu, \tau} Y_{\ell_i \ell_j} \eta \bar{\ell}_i P_R \ell_j + \text{h.c.} \end{aligned}$$

The typical size of the induced flavour violating Yukawa couplings depends on the structure dictated by partial compositeness and by the $U(1)_\eta$ charge assignment of the different fields. It is easy to show that

$$Y_{u_i u_j} = i \frac{\sqrt{2} v}{f_\eta} \left[L_U \hat{Z}_{Q_L} L_U^\dagger \hat{Y}_U + \hat{Y}_U R_U \hat{Z}_{U_R} R_U^\dagger \right]_{ij}, \quad (4.9)$$

$$Y_{d_i d_j} = i \frac{\sqrt{2} v}{f_\eta} \left[L_D \hat{Z}_{Q_L} L_D^\dagger \hat{Y}_D + \hat{Y}_D R_D \hat{Z}_{D_R} R_D^\dagger \right]_{ij}, \quad (4.10)$$

$$Y_{e_i e_j} = i \frac{\sqrt{2} v}{f_\eta} \left[L_E \hat{Z}_{L_L} L_E^\dagger \hat{Y}_E + \hat{Y}_E R_E \hat{Z}_{E_R} R_E^\dagger \right]_{ij}. \quad (4.11)$$

The \hat{Z} matrices are diagonal and contain the Z -charges of the fields; in particular we have defined $\hat{Z}_f \equiv \text{diag}(Z_{f^1}, Z_{f^2}, Z_{f^3})$.

With these expressions in hand, let us consider to what extent the suppression provided by the partial compositeness ansatz is sufficient to protect the model from dangerous flavour- and CP-violating contributions to physical observables.

If we assume the ‘worst-case scenario’ of an anarchic charge assignment ($Z_{f^i} = \mathcal{O}(Z)$ for every field $f^i = \{Q_L^i, u_R^i, d_R^i, L_L^i, E_R^j\}$), we obtain couplings of the following sizes:

$$\begin{aligned} (Y_\eta^U)_{ij} \equiv Y_{u_i u_j} & \sim g_\rho \epsilon_i^q \epsilon_j^u \frac{\sqrt{2} v}{f_\eta} Z = \frac{\sqrt{2} v}{f_\eta} Z \\ & \times \begin{pmatrix} 6.3 \times 10^{-6} & 6.8 \times 10^{-4} & 9.9 \times 10^{-3} \\ 2.8 \times 10^{-5} & 3.0 \times 10^{-3} & 4.4 \times 10^{-2} \\ 5.4 \times 10^{-4} & 6.0 \times 10^{-2} & 0.87 \end{pmatrix}, \end{aligned} \quad (4.12)$$

$$\begin{aligned} (Y_\eta^D)_{ij} \equiv Y_{d_i d_j} & \sim g_\rho \epsilon_i^q \epsilon_j^d \frac{\sqrt{2} v}{f_\eta} Z = \frac{\sqrt{2} v}{f_\eta} Z \\ & \times \begin{pmatrix} 1.4 \times 10^{-5} & 6.1 \times 10^{-5} & 1.6 \times 10^{-4} \\ 6.3 \times 10^{-5} & 2.7 \times 10^{-4} & 7.1 \times 10^{-4} \\ 1.2 \times 10^{-3} & 5.3 \times 10^{-3} & 1.4 \times 10^{-2} \end{pmatrix}, \end{aligned} \quad (4.13)$$

$$\begin{aligned} (Y_\eta^E)_{ij} \equiv Y_{e_i e_j} & \sim g_\rho \epsilon_i^e \epsilon_j^e \frac{\sqrt{2} v}{f_\eta} Z = \frac{\sqrt{2} v}{f_\eta} Z \\ & \times \begin{pmatrix} 2.8 \times 10^{-6} & 4.1 \times 10^{-5} & 1.7 \times 10^{-4} \\ 4.1 \times 10^{-5} & 5.9 \times 10^{-4} & 2.5 \times 10^{-3} \\ 1.7 \times 10^{-4} & 2.5 \times 10^{-3} & 1.0 \times 10^{-2} \end{pmatrix} \end{aligned} \quad (4.14)$$

These couplings are subject to phenomenological constraints. Bounds derived from flavour- and CP-violating processes induced by the exchange of the η boson can be found in the model independent analysis of [27]. We translate these into bounds on the combination $\frac{Z}{f_\eta}$, as reported in Fig. 2.

It is clear from these results that, in order to pass the bounds imposed by observables involving the first two families of quarks, we need $Z \left(\frac{f_\eta}{700 \text{ GeV}}\right)^{-1} \left(\frac{M_\eta}{750 \text{ GeV}}\right)^{-1} \lesssim 10^{-2}$. The values of Z and f_η are unknown and depend on the details of the strongly coupled sector. However, the most natural expectation is that $f_\eta \sim f$ and $Z \sim 1$, because the composite Higgs and the composite η are generated from the same strong dynamics. If this is the case, an extra source of flavour protection is required. An easy fix to this problem is to assume that the η PNCB couples to flavour in a universal way. More specifically, we can impose $Z_{f^i} = Z_f$ for $i = \{1, 2, 3\}$. In this case the η and the Higgs boson couplings to fermions are aligned in each sector, such that

$$(Y_\eta^U)_{ij} = i \delta_{ij} \frac{m_i^U}{f_\eta} (Z_{Q_L} - Z_{u_R}) \quad m_i^U = \{m_u, m_c, m_t\}, \quad (4.15)$$

$$(Y_\eta^D)_{ij} = i \delta_{ij} \frac{m_i^D}{f_\eta} (Z_{Q_L} - Z_{d_R}) \quad m_i^D = \{m_d, m_s, m_b\}, \quad (4.16)$$

$$(Y_\eta^E)_{ij} = i \delta_{ij} \frac{m_i^E}{f_\eta} (Z_{L_L} - Z_{e_R}) \quad m_i^E = \{m_e, m_\mu, m_\tau\} \quad (4.17)$$

All the flavour and CP problems are solved, since this pattern is flavour diagonal.¹² It is, moreover, rather predictive. Indeed the η , like the Higgs, couples predominantly to the third generation. This could have important implications for the production and decay mechanisms of the singlet, as we now discuss.

In the narrow width approximation the prompt η production at the LHC can be expressed in terms of the relevant decay widths

$$\sigma(pp \rightarrow \eta) = \frac{1}{M_\eta s} \sum_P C_{P\bar{P}}(M_\eta, s) \Gamma_{P\bar{P}}, \quad (4.18)$$

where \sqrt{s} is the center of mass energy of the collider and $C_{P\bar{P}}(M_\eta, s)$ parametrise the relevant parton luminosities. In our framework the relevant partons to be taken into account are expected to be the gluons [if the associated anomalous term in Eq. (2.2) is present] and the bottom quarks. The explicit expressions for the partial widths are given by

¹² There remain, however, sub-dominant flavour-violating contributions from possible derivative operators, analogous to those described in [28].

Fig. 2 Constraints on η couplings to SM fermions (*first column*) derived from low energy precision observables (*second column*). The limits on $\frac{Z}{f_\eta M_\eta}$ are presented in the *third column*

Bound on $Y_{f,f'}$	Observable	$Z \left(\frac{f_\eta}{700 \text{ GeV}}\right)^{-1} \left(\frac{M_\eta}{750 \text{ GeV}}\right)^{-1}$
$\sqrt{\text{Re}[(Y_{sd})^2]}, \sqrt{\text{Re}[(Y_{ds})^2]} < 1.3 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_K	< 5.9
$\sqrt{\text{Re}[(Y_{sd}Y_{ds}^*)]} < 4.6 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_K	< 2.1
$\sqrt{\text{Im}[(Y_{sd})^2]}, \sqrt{\text{Im}[(Y_{ds})^2]} < 3.4 \times 10^{-6} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	ϵ_K	< 0.15
$\sqrt{\text{Im}[(Y_{sd}Y_{ds}^*)]} < 1.6 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	ϵ_K	$< 5.2 \times 10^{-2}$
$\sqrt{\text{Re}[(Y_{cu})^2]}, \sqrt{\text{Re}[(Y_{uc})^2]} < 3.3 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	x_D	< 1.4
$\sqrt{\text{Re}[(Y_{cu}Y_{uc}^*)]} < 3.9 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	x_D	< 0.17
$\sqrt{\text{Im}[(Y_{cu})^2]}, \sqrt{\text{Im}[(Y_{uc})^2]} < 4.0 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	$(q/p)_D, \phi_D$	< 0.17
$\sqrt{\text{Im}[(Y_{cu}Y_{uc}^*)]} < 4.0 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	$(q/p)_D, \phi_D$	$< 2.0 \times 10^{-2}$
$\sqrt{\text{Re}[(Y_{bd})^2]}, \sqrt{\text{Re}[(Y_{db})^2]} < 4.1 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_d	< 7.3
$\sqrt{\text{Re}[(Y_{bd}Y_{db}^*)]}, \sqrt{\text{Re}[(Y_{bd})^2]} < 1.4 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_d	< 2.4
$\sqrt{\text{Im}[(Y_{bd})^2]}, \sqrt{\text{Im}[(Y_{db})^2]} < 2.3 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	$\sin 2\beta$	< 4.1
$\sqrt{\text{Im}[(Y_{bd}Y_{db}^*)]} < 7.6 \times 10^{-5} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	$\sin 2\beta$	< 1.4
$ Y_{bs} , Y_{sb} < 1.7 \times 10^{-3} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_s	< 0.91
$\sqrt{ Y_{bs}Y_{sb}^* } < 5.7 \times 10^{-4} \left(\frac{M_\eta}{750 \text{ GeV}}\right)$	Δm_s	< 0.31

$$\Gamma(\eta \rightarrow gg) = c_3^2 \frac{\alpha_s^2}{8\pi^3} \frac{M_\eta^3}{f_\eta^2}, \tag{4.19}$$

$$\Gamma(\eta \rightarrow b\bar{b}) = \frac{3M_\eta}{8\pi} (Y_\eta^D)_{33}^2. \tag{4.20}$$

The mechanism of partial compositeness also allows one to predict the dominant decay mode to be into top-quarks if $m_\eta > 2m_t$. Depending on the value of the mass of the PNGB, phase space could be important and the expression for the decay width in this channel is given by

$$\Gamma(\eta \rightarrow t\bar{t}) = \frac{3M_\eta}{8\pi} (Y_\eta^U)_{33}^2 \left(1 - \frac{4m_t^2}{m_\eta^2}\right)^{3/2}. \tag{4.21}$$

The large coupling of η with the heaviest fermion allows for its production at LHC in association with top quarks. A recent analysis from the ATLAS collaboration [29], using data at $\sqrt{s} = 13$ TeV, leads to the following bound:

$$\sigma(pp \rightarrow \eta + t\bar{t}) \times Br(\eta \rightarrow t\bar{t}) \lesssim \mathcal{O}(10^{-1}) \text{ pb} \tag{4.22}$$

for a mass of the PNGB $m_\eta < 1$ TeV.

We conclude this section noticing that the simple flavour structure that we have just described, while guaranteeing immunity from flavour problems, does not allow one to generate a scalar potential (and hence a mass) for the singlet from fermionic couplings. As we discuss in the next Section, to do so requires that at least one of the elementary fermions in the partial compositeness scenario mixes with multiple strong-sector operators. Even if one tries to do so in a way that is as safe as possible (for example by allowing the right-handed up quarks to couple to strong-sector operators with just two values of the $U(1)_\eta$ charge), one ends up re-introducing flavour violation in the right-handed up sector at a level comparable

to that obtained with anarchic charge assignments in Fig. 2, which is itself comparable to that obtained in the minimal composite Higgs model. Thus, if one wishes to generate the scalar potential from fermionic couplings, it would seem that either a mild tuning or some kind of flavour-alignment mechanism (such as those advanced in [30]) is required.

5 The scalar potential

Since the η singlet is protected by a shift symmetry, its mass and non-derivative interactions must be proportional to $U(1)_\eta$ -breaking couplings. The elementary fermion couplings to the strong sector are the main source of such global symmetry violations, and the η singlet then obtains a potential via the same Coleman–Weinberg mechanism that radiatively generates the pseudo-Goldstone Higgs potential in one loop. This must originate from fermion couplings, since no potential is generated by gauge couplings in the absence of anomalies, because $U(1)_\eta$ commutes with the rest of G .¹³ The particular form of the symmetry breaking from Yukawa couplings is, in general, model-dependent.

To illustrate the mechanism in a minimal phenomenological model, we take an elementary top-right coupling to two strong-sector operators with different $U(1)_\eta$ charges such that the symmetry is explicitly broken by a collective mechanism.¹⁴ The doubling of the top-right operator is necessary

¹³ In the presence of anomalies and without other sources of $U(1)_\eta$ -breaking, the η plays the role of an electroweak axion. The resulting contributions to its mass are thus completely negligible compared to those considered here.

¹⁴ This is in contrast to various composite Higgs models where the top right is a fully composite state.

to break the $U(1)_\eta$ symmetry, since with only one operator, we can restore it by assigning a suitable $U(1)_\eta$ charge to the elementary fermion.

The simplest realisation of the model is to extend the minimal composite Higgs with the elementary fermions $q_L, u_R,$ and d_R uplifted to a spinorial representation of $SO(5)$, under which the corresponding composite operators $\mathcal{O}_q, \mathcal{O}_{u_1}, \mathcal{O}_{u_2}$ and \mathcal{O}_d transform. Summing implicitly over three flavours, the relevant Lagrangian terms may be written as

$$\mathcal{L} \supset g_\rho \epsilon^q \bar{q}_L \mathcal{O}_q + g_\rho \epsilon^{u_1} \bar{u}_R \mathcal{O}_{u_1} + g_\rho \epsilon^{u_2} \bar{u}_R \mathcal{O}_{u_2} + g_\rho \epsilon^d \bar{d}_R \mathcal{O}_d + \text{h.c.} \tag{5.1}$$

We see that if one of the two top-right couplings is set to zero then a $U(1)_\eta$ symmetry may be restored. The doubling of the corresponding u_R operator thereby provides a collective mechanism for breaking the symmetry. The elementary fermions embedded in complete spinorial representations of $SO(5)$ decompose as $\mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$ under $SU(2)_L \times SU(2)_R$. By completing the representation with spurious fermions they can be represented by fields transforming under this symmetry as

$$\Psi_q = \begin{pmatrix} q_L \\ 0 \end{pmatrix}_{\frac{1}{6}}^{Z_q}, \quad \Psi_{u_1} = \begin{pmatrix} 0 \\ u_R \\ 0 \end{pmatrix}_{\frac{1}{6}}^{Z_{u_1}},$$

$$\Psi_{u_2} = \begin{pmatrix} 0 \\ u_R \\ 0 \end{pmatrix}_{\frac{1}{6}}^{Z_{u_2}}, \quad \Psi_d = \begin{pmatrix} 0 \\ 0 \\ d_R \end{pmatrix}_{\frac{1}{6}}^{Z_d},$$

where the superscript Z represents the $U(1)_\eta$ charges and the subscript is the $U(1)_X$ charge assigned by requiring $Y = T_R^3 + X$. We have set to zero the non-dynamical spurions that complete the $SU(2)_L$ ($SU(2)_R$) representation in the upper (lower) two components of the multiplet, though they are formally required to restore the global $SO(5)$ symmetry.

The Coleman–Weinberg effective potential may be derived by writing the most general $SO(5) \times U(1)_X \times U(1)_\eta$ -invariant effective action then setting the spurions to zero to recover the effective Lagrangian, as detailed in Appendix 1. The quadratic terms in the background of the Higgs and singlet are then responsible for the one-loop effective action. Assuming real CP-conserving form factors, we obtain for the third-generation $q_L = (t_L, b_L), t_R$ sector, in momentum space,

$$\mathcal{L} = \bar{q}_L \not{p} [\Pi_0^q(p) + \Pi_1^q(p)c_h] q_L + \bar{t}_R \not{p} \left[\Pi_0^{12}(p) + \Pi_1^{u_{12}}(p)c_\eta^{12} - (\Pi_1^{12}(p) + \Pi_1^{u_{12}}(p)c_\eta^{12})c_h \right] t_R + \text{h.c.} + \bar{q}_L [M_1^{u_1}(p)U_{q1} + M_1^{u_2}(p)U_{q2}] s_h H^c t_R + \text{h.c.}, \tag{5.2}$$

where $H^c \equiv i\sigma^2 H$ with H the usual complex Higgs doublet and

$$U_{rs} \equiv e^{i\frac{\sqrt{2}}{f_\eta}(Z_r - Z_s)\eta} = c_\eta^{rs} + i s_\eta^{rs}.$$

We have also defined $c_h \equiv \cos(h/f), s_h \equiv \sin(h/f), c_\eta^{rs} \equiv \cos(\sqrt{2}(Z_r - Z_s)\eta/f_\eta)$, and $s_\eta^{rs} \equiv \sin(\sqrt{2}(Z_r - Z_s)\eta/f_\eta)$, with $h \equiv \sqrt{h^a h^a}, a = 1, 2, 3, 4$. The $\Pi_{0,1}, M_1$ functions are form factors that encapsulate effects from strong dynamics. The resulting potential is detailed in Appendix 1 with the leading-order approximation found to be of the form

$$V(h, \eta) \simeq (\alpha + \alpha_{12}c_\eta^{12})c_h - (\beta + \beta_{12}c_\eta^{12})s_h^2, \tag{5.3}$$

where $\alpha, \beta, \alpha_{12}$, and β_{12} are coefficients related to momentum integrals of the $\pi_{0,1}, M_1$ form factors. Thus, the resulting potential is almost identical to that obtained in the minimal model in [6], but with the coefficients replaced by $\alpha, \beta \rightarrow \alpha + \alpha_{12}c_\eta^{12}, \beta + \beta_{12}c_\eta^{12}$.

The potential has extrema occurring at $s_\eta^{12} = 0 \implies c_\eta^{12} = \pm 1$ ¹⁵ and $c_h = -\frac{1}{2} \frac{\alpha \pm \alpha_{12}}{\beta \pm \beta_{12}}$. As is usual in composite Higgs models, we find that with $O(1)$ values for the coefficients, $v \sim f$ is expected and so a slight tuning is needed to obtain the required suppression of the weak scale for compatibility with electroweak precision tests.

There is no mixing between the Higgs and η , so there is no risk of running into bounds from existing observations in the Higgs sector. The non-vanishing second derivatives are given by

$$\frac{\partial V}{\partial \eta^2} = \mp \frac{1}{f_\eta^2} (\alpha_{12}c_h + \beta_{12}s_h^2), \tag{5.4}$$

$$\frac{\partial V}{\partial h^2} = -\frac{1}{f^2} [(\alpha \pm \alpha_{12})c_h - 2(\beta \pm \beta_{12})c_2h] = \frac{2}{f^2} (\beta \pm \beta_{12})s_h^2 = \frac{2}{f^2} \frac{v^2}{f^2} (\beta \pm \beta_{12}) \tag{5.5}$$

Thus, once we have tuned the electroweak vev to be small compared to f , we will also obtain a corresponding suppression of the Higgs mass-squared, exactly as one finds in [6]. The mass of η , however, is unsuppressed, so we obtain a hierarchy of scales, of parametric size v/f (assuming $f_\eta \sim f$) between the η mass and either the electroweak scale or the mass of the Higgs boson.

An identical conclusion is reached if we instead embed the elementary fermions in the fundamental, 5-d representation of $SO(5)$, as we describe in Appendix 1. (In this case, as we discuss in Appendix 3, we can also protect the

¹⁵ We remark that a vacuum with $c_\eta^{12} = -1$ does not imply spontaneous violation of CP, because CP sends $\eta \rightarrow -\eta$ and because physics is periodic in the argument of the cosine.

$Zb_L\bar{b}_L$ coupling by a custodial symmetry.) The hierarchy of scales is, in fact, generic and follows from the fact that the scalar potential is an even function of h . Indeed, with $V(h, \eta) = f(h^2, \eta)$, we obtain $\partial V/\partial h^2 = 4v^2\partial f/\partial h^2|_v$, at an electroweak-symmetry-breaking minimum. One can also see that any mixing between the mass eigenstates of the singlet and the Higgs will also be v/f suppressed.

Although this hierarchy of scales is generic, it may be affected by the well-known difficulty (see e.g. [31] for a comprehensive discussion) of accommodating a Higgs mass as low as 125 GeV in composite Higgs models, given the size of contributions to the Higgs potential from top quark loops. If the required additional suppression is an accidental tuning, then we expect no corresponding suppression in the η mass. But if it is achieved by the presence of light top partners that cut off all contributions to the scalar potential, then one should find a corresponding suppression of the η mass.

6 Conclusion

Composite Higgs models remain viable possibilities for solving the electroweak hierarchy problem. Here we introduced the most minimal extension of the coset structure allowing a non-trivial anomaly structure and discussed the details of the low-energy action reproducing the anomalies. We showed that there can be higher-order corrections, beyond dimension 5, to the action reproducing the $SO(5)^2U(1)$ anomaly, but also pointed out that the effective action is not unique. We also showed that the structure of the coset space admits a possible Wess–Zumino–Witten term, by which we mean a term in the effective Lagrangian which is not invariant under the non-linearly realised symmetries, but rather shifts by a total derivative. Unlike in QCD, this term is not contained in the anomalous effective action that we consider. If present, the term leads to an exotic phenomenological signature in the form of the singlet decay $\eta \rightarrow hW^+W^-Z$.

The discussion of the anomaly structure in this specific model highlights three questions that it would be interesting to resolve in models based on a general coset space, G/H . Firstly: is there a way to resolve the non-uniqueness issue of the low-energy anomalous effective action? Secondly: do Wess–Zumino–Witten terms that are not required to reproduce triangle anomalies have some other purpose? Thirdly, is there an elegant way to write the Wess–Zumino–Witten term for coset spaces whose fourth homotopy group is non-vanishing?

The anomaly-induced production and decays of the singlet may induce flavour violation of its couplings to fermions and we have shown how they can be kept under control without fine-tuning if the η couples in a flavour-universal way through the mechanism of partial compositeness. For natural $\mathcal{O}(1)$ charge assignments, this pattern of coupling predicts a large decay width through the $t\bar{t}$ final state.

We also showed how the potential for the PNGB Higgs and singlet can be generated by elementary fermion couplings to the strong sector that break the global symmetry, though this requires a slight departure from the flavour-universal pattern of couplings, because of the need for a collective breaking mechanism to give mass to the singlet. We find that the singlet mass is naturally unsuppressed relative to the Higgs mass and electroweak scale, thus requiring no additional tuning beyond the usual ones needed for a small electroweak scale and light Higgs mass in composite models. Since the form of the potential contains no mixing between the Higgs and the singlet there are no further bounds from the Higgs sector.

Should the Higgs arise as a pseudo-Nambu–Goldstone boson, it will be imperative to determine the new strong sector responsible for it. Given our current limited understanding of strongly coupled theories, the anomaly structure, if present, may be crucial in gaining some insight as to the nature of the underlying UV dynamics. We hope that the model described here, or some variant thereof, may be useful in this regard.

Acknowledgements BG acknowledges the support of the Science and Technology Facilities Council (Grant ST/L000385/1) and King’s College, Cambridge and thanks O. Randal-Williams for discussions. TY is supported by a Research Fellowship from Gonville and Caius College, Cambridge.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

Appendix A: Scalar potential computations

Higgs-singlet potential in the extension of the MCHM4

The elementary fermions are uplifted to a $\mathbf{4}$ of $SO(5)$, which decomposes as $\mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$ under $SU(2)_L \times SU(2)_R$. The most general $SO(5) \times U(1)_X \times U(1)_\eta$ -invariant effective action up to quadratic order can then be written in momentum space as

$$\begin{aligned} \mathcal{L} = & \sum_{r=q,u_1,u_2,d} \bar{\Psi}_r \not{p} \left[\Pi_0^r(p) + \Pi_1^r(p)\Gamma^i \Sigma_i \right] \Psi_r \\ & + \left\{ \bar{\Psi}_{u_1} \not{p} \left[\Pi_0^{u_{12}}(p) + \Pi_1^{u_{12}}(p)\Gamma^i \Sigma_i \right] U_{12} \Psi_{u_2} + \text{h.c.} \right\} \\ & + \sum_{r=u_1,u_2,d} \left\{ \bar{\Psi}_q \left[M_0^r(p) + M_1^r(p)\Gamma^i \Sigma_i \right] U_{qr} \Psi_r + \text{h.c.} \right\}, \end{aligned} \tag{A.1}$$

where the pseudo-Goldstone singlet η and Higgs doublet $h^a = (h^1, h^2, h^3, h^4)$ are given here by

$$\Sigma^i = \frac{S_h}{h} \left(h^1, h^2, h^3, h^4, h \frac{c_h}{s_h} \right)$$

$$U_{rs} = e^{i \frac{\sqrt{2}}{f_\eta} (Z_r - Z_s) \eta} = c_\eta^{rs} + i s_\eta^{rs}, \tag{A.2}$$

We recall the definitions $h \equiv \sqrt{h^a h^a}$, $c_h \equiv \cos(h/f)$, $s_h \equiv \sin(h/f)$, $c_\eta^{rs} \equiv \cos(\sqrt{2}(Z_r - Z_s)\eta/f_\eta)$, and $s_\eta^{rs} \equiv \sin(\sqrt{2}(Z_r - Z_s)\eta/f_\eta)$. Explicit expressions for the $SO(5)$ gamma matrices Γ^i can be found in Ref. [6]. The $\Pi(p)$, $M(p)$ functions are form factors that encapsulate information from the strong sector.

Setting to zero the non-dynamical spurions that complete the Ψ representation, we obtain the quadratic terms in the Lagrangian for the third-generation $q_L = (t_L, b_L), t_R$ sector,

$$\begin{aligned} \mathcal{L} = & \bar{q}_L \not{p} [\Pi_0^q(p) + \Pi_1^q(p)c_h] q_L \\ & + \bar{t}_R \not{p} [\Pi_0^{u_1} + \Pi_0^{u_2} - (\Pi_1^{u_1} + \Pi_1^{u_2})c_h] t_R \\ & + \bar{t}_R \not{p} [\Pi_0^{u_{12}} - \Pi_1^{u_{12}}c_h] U_{Z_1 - Z_2} t_R + \text{h.c.} \\ & + \bar{q}_L (M_1^{u_1} U_{Z_q - Z_q} + M_1^{u_2} U_{Z_q - Z_2}) s_h \hat{H}^c t_R + \text{h.c.}, \end{aligned} \tag{A.3}$$

where $H^c \equiv i\sigma^2 H$ and H is the complex Higgs doublet. Assuming real CP-conserving form factors, this becomes

$$\begin{aligned} \mathcal{L} = & \bar{q}_L \not{p} [\Pi_0^q(p) + \Pi_1^q(p)c_h] q_L \\ & + \bar{t}_R \not{p} [\Pi_0^{12}(p) + \Pi_0^{u_{12}}(p)c_\eta^{12}] \\ & - (\Pi_1^{12}(p) + \Pi_1^{u_{12}}(p)c_\eta^{12}) c_h t_R + \text{h.c.} \\ & + \bar{q}_L [M_1^{u_1}(p)U_{q1} + M_1^{u_2}(p)U_{q2}] s_h H^c t_R + \text{h.c.}, \end{aligned} \tag{A.4}$$

where $\Pi_{0,1}^{12} \equiv \Pi_{0,1}^{u_1} + \Pi_{0,1}^{u_2}$. Including the $SU(2)_L$ gauge field contributions with form factors Π_0 and Π_1 as defined in Ref. [6], the resulting Coleman–Weinberg potential generated at one loop is given by

$$\begin{aligned} V(h, \eta) = & -2N_c \int \frac{d^4 p}{(2\pi)^4} \left\{ 2 \log \left(1 + \frac{\Pi_1^q}{\Pi_0^q} c_h \right) + \log \left(1 - \frac{\Pi_1^{12} + \Pi_1^{u_{12}} c_\eta^{12}}{\Pi_0^{12} + \Pi_0^{u_{12}} c_\eta^{12}} c_h \right) \right. \\ & \left. + \log \left(1 - \frac{|M_1^{u_1} U_{q1} + M_1^{u_2} U_{q2}|^2}{p^2 (\Pi_0^q + \Pi_1^q c_h) [(\Pi_0^{12} - \Pi_1^{12} c_h) + (\Pi_0^{u_{12}} - \Pi_1^{u_{12}} c_h) c_\eta^{12}]} s_h^2 \right) \right\} + \frac{9}{2} \int \frac{d^4 p}{(2\pi)^4} \log \left(1 + \frac{1}{4} \frac{\Pi_1}{\Pi_0} s_h^2 \right). \end{aligned} \tag{A.5}$$

Assuming the form factors decrease fast enough with increasing momentum, the logarithm may be expanded to give the leading-order approximation for the potential,

$$V(h, \eta) \simeq (\alpha + \alpha_{12} c_\eta^{12}) c_h - (\beta + \beta_{12} c_\eta^{12}) s_h^2. \tag{A.6}$$

The coefficients are related to the form factor integrals as

$$\begin{aligned} \alpha = & 2N_c \int \frac{d^4 p}{(2\pi)^4} \left(\frac{\Pi_1^{12}}{\Pi_0^{12} + \Pi_0^{u_{12}}} - 2 \frac{\Pi_1^q}{\Pi_0^q} \right), \\ \alpha_{12} = & 2N_c \int \frac{d^4 p}{(2\pi)^4} \left(\frac{\Pi_1^{u_{12}}}{\Pi_0^{12} + \Pi_0^{u_{12}}} \right), \\ \beta_V = & - \int \frac{d^4 p}{(2\pi)^4} \frac{9}{8} \frac{\Pi_1}{\Pi_0}, \\ \beta_{1,2} = & 2N_c \int \frac{d^4 p}{(2\pi)^4} \left(\frac{(M_1^{u_{1,2}})^2}{(-p^2) (\Pi_0^q + \Pi_1^q) (\Pi_0^{12} + \Pi_0^{u_{12}} - \Pi_1^{12} - \Pi_1^{u_{12}})} \right), \\ \beta_{12} = & 2N_c \int \frac{d^4 p}{(2\pi)^4} \left(\frac{2M_1^{u_1} M_1^{u_2}}{(-p^2) (\Pi_0^q + \Pi_1^q) (\Pi_0^{12} + \Pi_0^{u_{12}} - \Pi_1^{12} - \Pi_1^{u_{12}})} \right), \end{aligned}$$

with $\beta \equiv \beta_V + \beta_1 + \beta_2$.

Higgs-singlet potential in the extension of the MCHM5

The elementary fermions may instead be embedded in the fundamental representation of $SO(5)$. Such a setup can also be extended to protect the $Z b_L \bar{b}_L$ coupling by a custodial symmetry if we assume that q_L is embedded such that it couples to two operators with different $U(1)_X$ charges. The resulting Lagrangian of the effective coupling to the composite operators can be written as

$$\begin{aligned} \mathcal{L} = & g_\rho \epsilon^{q_1} \bar{q}_L \mathcal{O}_{q_1} + g_\rho \epsilon^{q_2} \bar{q}_L \mathcal{O}_{q_2} + g_\rho \epsilon^{u_1} \bar{u}_R \mathcal{O}_{u_1} \\ & + g_\rho \epsilon^{u_2} \bar{u}_R \mathcal{O}_{u_2} + g_\rho \epsilon^d \bar{d}_R \mathcal{O}_d + \text{h.c.} \end{aligned}$$

The fields transforming under the **5** of $SO(5)$ with non-dynamical spurions completing the representation (which we again set here to zero) are chosen to be

$$\Psi_{1L} = \frac{1}{\sqrt{2}} \begin{pmatrix} -b_L \\ ib_L \\ t_L \\ -it_L \\ 0 \end{pmatrix}_{\frac{2}{3}}^{Z_{q1}}, \quad \Psi_{2L} = \frac{1}{\sqrt{2}} \begin{pmatrix} t_L \\ -it_L \\ b_L \\ -ib_L \\ 0 \end{pmatrix}_{-\frac{1}{3}}^{Z_{q2}},$$

$$\Psi_{1,2R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_R \end{pmatrix}_{\frac{2}{3}}^{Z_{1,2}}, \quad \Psi_{dR} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ d_R \end{pmatrix}_{-\frac{2}{3}}^{Z_d}.$$

The superscripts and subscripts denote the $U(1)_\eta$ and $U(1)_X$ charges, respectively. It might initially seem that an explicit breaking of $U(1)_\eta$ from q_L coupling to two different operators will generate a potential for the singlet, thus making the doubling of the top-right couplings redundant, but it turns out that the unbroken $U(1)_X$ symmetry forbids the necessary η coupling in the effective action. For this reason we minimally extend the top-right sector as in the previous model and fix $Z_{q1} = Z_{q2} = Z_q$.

The most general effective action under $SO(5) \times U(1)_X \times U(1)_\eta$ is then

$$\begin{aligned} \mathcal{L} = & \sum_{r=1,2} \bar{\Psi}_{rL}^i \not{p} \left(\delta^{ij} \hat{\Pi}_0^{rL} + \Sigma^i \Sigma^j \hat{\Pi}_1^{rL} \right) \Psi_{rL}^j \\ & + \sum_{r=1,2,d} \bar{\Psi}_{rR}^i \not{p} \left(\delta^{ij} \hat{\Pi}_0^{rR} + \Sigma^i \Sigma^j \hat{\Pi}_1^{rR} \right) \Psi_{rR}^j \\ & + \left[\sum_{r=1,2} \bar{\Psi}_{1L}^i \left(\delta^{ij} \hat{M}_0^{1rL} + \Sigma_i \Sigma^j \hat{M}_1^{1rL} \right) U_{qr} \Psi_{rR}^j \right. \\ & + \bar{\Psi}_{2L}^i \left(\delta^{ij} \hat{M}_0^{2bL} + \Sigma^i \Sigma^j \hat{M}_1^{2bL} \right) U_{qb} \Psi_{dR}^j \\ & \left. + \bar{\Psi}_{1R}^i \not{p} \left(\delta^{ij} \hat{\Pi}_0^{12R} + \Sigma^i \Sigma^j \hat{\Pi}_1^{12R} \right) U_{12} \Psi_{2R}^j + \text{h.c.} \right]. \end{aligned} \tag{A.7}$$

Setting the non-dynamical spurions to zero to keep the relevant terms for computing the Coleman–Weinberg effective potential, omitting the bottom contributions, we find

$$\begin{aligned} \mathcal{L} = & \bar{q}_L \not{p} \left[\Pi_0^q + \frac{1}{2} s_h^2 \left(\Pi_1^{q1} \hat{H}^c \hat{H}^{c\dagger} + \Pi_1^{q2} \hat{H} \hat{H}^\dagger \right) \right] q_L \\ & + \bar{u}_R \not{p} \left[\Pi_0^u + \frac{1}{2} s_h^2 \Pi_1^u \right] u_R \\ & + \left[\frac{1}{\sqrt{2}} c_h s_h \left(M_1^{qu1} U_{q1} + M_1^{qu2} U_{q2} \right) \bar{q}_L \hat{H}^c u_R + \text{h.c.} \right] \\ & + \bar{u}_R \not{p} \text{Re} \left\{ \left(\Pi_0^{uu} + \frac{1}{2} s_h^2 \Pi_1^{uu} \right) U_{12} \right\} u_R, \end{aligned} \tag{A.8}$$

where

$$\begin{aligned} \Pi_0^q & \equiv \hat{\Pi}_0^{1L} + \hat{\Pi}_0^{2L}, \quad \Pi_1^{q1,q2} \equiv \hat{\Pi}_1^{1L,2L}, \\ \Pi_1^u & \equiv -2 \left(\hat{\Pi}_1^{1R} + \hat{\Pi}_1^{2R} \right), \quad \Pi_1^{uu} \equiv -2 \hat{\Pi}_1^{12R}, \\ \Pi_0^u & \equiv \hat{\Pi}_0^{1R} + \hat{\Pi}_0^{2R} + \hat{\Pi}_1^{1R} + \hat{\Pi}_1^{2R}, \\ M_1^{qu1,q2} & \equiv \hat{M}_1^{11L,12L}, \quad \Pi_0^{uu} \equiv \hat{\Pi}_0^{12R} + \hat{\Pi}_1^{12R}. \end{aligned} \tag{A.9}$$

Assuming real form factors with CP conservation, in the unitary gauge this gives for the top quark sector the quadratic Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{t}_L \not{p} \left[\Pi_0^q + \frac{1}{2} s_h^2 \Pi_1^{q1} \right] t_L \\ & + \bar{t}_R \not{p} \left[\Pi_0^u + \Pi_0^{uu} + \frac{1}{2} s_h^2 \left(\Pi_1^u + \Pi_1^{uu} c_\eta^{12} \right) \right] t_R \\ & + \left[\frac{1}{\sqrt{2}} \left(M_1^{qu1} U_{q1} + M_1^{qu2} U_{q2} \right) c_h s_h \bar{t}_L t_R + \text{h.c.} \right]. \end{aligned} \tag{A.10}$$

The resulting Coleman–Weinberg potential is

$$\begin{aligned} V(h, \eta) = & \frac{9}{2} \int \frac{d^4 p}{(2\pi)^4} \log \left(1 + \frac{1}{4} \frac{\Pi_1}{\Pi_0} s_h^2 \right) \\ & - 2N_c \int \frac{d^4 p}{(2\pi)^4} \left\{ \log \left(1 + \frac{1}{2} \frac{\Pi_1^{q1}}{\Pi_0^q} s_h^2 \right) + \log \left(1 + \frac{1}{2} \frac{\Pi_1^{q2}}{\Pi_0^q} s_h^2 \right) + \log \left(1 + \frac{1}{2} \frac{\left(\Pi_1^u + \Pi_1^{uu} c_\eta^{12} \right) s_h^2}{\Pi_0^u + \Pi_0^{uu}} \right) \right. \\ & \left. + \log \left(1 - \frac{\frac{1}{2} \left| M_1^{qu1} U_{q1} + M_1^{qu2} U_{q2} \right|^2 c_h^2 s_h^2}{p^2 \left[\Pi_0^u + \Pi_0^{uu} + \frac{1}{2} s_h^2 \left(\Pi_1^u + \Pi_1^{uu} c_\eta^{12} \right) \right] \left[\Pi_0^q + \frac{1}{2} s_h^2 \Pi_1^{q1} \right]} \right)} \right\}, \end{aligned} \tag{A.11}$$

which may be simplified to the form

$$V(h, \eta) \simeq (\alpha + \alpha_{12}c_\eta^{12})s_h^2 - (\beta + \beta_{12}c_\eta^{12})c_h^2s_h^2. \tag{A.12}$$

Appendix B: Higher-order contributions to the anomalous effective action

To compute higher-order contributions to the anomalous effective action (3.1) for the $SO(5) \times U(1)/SO(4)$ model, it is useful to consider what happens if we start from some G/H and add an additional, broken, ungauged, $U(1)$ factor, along with a $G^2U(1)$ triangle anomaly. We thus need to add a Goldstone boson η to the existing Goldstone bosons, ξ , and to make the replacements $A_t \rightarrow A_t - t d\eta \implies F_t \rightarrow F_t, A_t^2 \rightarrow A_t^2$.

We observe that η can appear in G^\pm in (3.1) only in the terms $(A_t)_k(F_t)_{h,k}(A_t)_k \rightarrow (A_t)_k(F_t)_{h,k}(A_t)_k - t d\eta[(F_t)_{h,k}, (A_t)_k]$. Since we must take the trace of this with a Goldstone boson ξ in G/H in order to get a non-vanishing contribution via the anomaly, and since the generators in \mathfrak{g} are orthogonal, the sole such contribution to the action is given by

$$\Gamma \supset 4c_+ \int t d\eta \text{tr} \xi [(F_t)_h, (A_t)_k]. \tag{B.1}$$

In addition, we get contributions where we take terms in G^\pm not involving η , of the form

$$\Gamma \supset \sum_{\pm} c_{\pm} \int \eta \text{tr} G^\pm [A_t]. \tag{B.2}$$

These simplify dramatically. Indeed, orthogonality of generators, together with $\text{tr}(A_k^2 F_k + A_k F_k A_k + F_k A_k^2) = \text{tr} A_k^2 F_k = 0$, implies that $G^- = 0$. Moreover, since $\text{tr}(A_t)_k^4 = 0$, we see that there can be no WZW term arising from our anomalous effective action in the ungauged limit.

All in all, we find that the anomalous action can be simplified to

$$\Gamma = c_+ \int (\eta \text{tr}[3(F_t)_h^2 + (F_t)_k^2 - 4(A_t)_k^2(F_t)_h] + 4t d\eta \text{tr} \xi [(F_t)_h, (A_t)_k]). \tag{B.3}$$

We now consider the contributions of each of the triangle anomalies in turn. For $SU(3)^2U(1)$ and the anomalies involving $U(1)$ s, the effective action just reduces to

$$\int c_3 \eta \text{tr} GG + c_1 \eta BB \tag{B.4}$$

to all orders.

Things are somewhat more complicated for the $SO(5)^2U(1)$ anomaly. Let us content ourselves with computing the action at the next-to-leading order. Evidently, we have

$$F_k = t[\xi, F] + \dots, \tag{B.5}$$

$$F_h = F + \frac{t^2}{2}[\xi, [\xi, F]] + \dots, \tag{B.6}$$

$$A_k = -t(d\xi - [\xi, A]) + \dots \equiv -t(D\xi) + \dots. \tag{B.7}$$

From which it is clear that the first corrections arise not at dimension 6, but at dimension 7. Explicitly, we find¹⁶

$$\int \frac{c_5}{3} \eta \text{tr} (3F^2 + \frac{1}{2}(F[\xi, [\xi, F]] + [\xi, [\xi, F]]F + 2[\xi, F]^2) - \frac{4}{3}(D\xi)^2 F) + \frac{4}{3} d\eta \text{tr} \xi [F, D\xi] + \dots \tag{B.8}$$

The last term may be integrated by parts, to get

$$\int \frac{4}{3} d\eta \text{tr} \xi [F, D\xi] = \int \frac{4}{3} \eta \text{tr} (2(D\xi)^2 F - \xi[F, [F, \xi]]). \tag{B.9}$$

Finally, we obtain

$$\int \frac{c_5}{3} \eta \text{tr} \left(3F^2 + \frac{1}{2}(F[\xi, [\xi, F]] + [\xi, [\xi, F]]F + 2[\xi, F]^2) + \frac{4}{3}(D\xi)^2 F \right) - \frac{4}{3} \xi[F, [F, \xi]] + \dots \tag{B.10}$$

To convert this into an explicit formula in terms of $SU(2) \times U(1)$ invariant operators in the basis of [21], we use the basis for $\mathfrak{so}(5) \simeq \mathfrak{sp}(2)$ [7], wherein¹⁷

$$F = \frac{1}{2} \begin{pmatrix} W^i \sigma^i & 0 \\ 0 & B \sigma^3 \end{pmatrix}, \quad \xi = \begin{pmatrix} 0 & (H^c H) \\ (H^c H)^\dagger & 0 \end{pmatrix}. \tag{B.11}$$

The only non-vanishing term at next-to-leading order is the last one, for which

$$\text{tr} \xi [F, [F, \xi]] = H^\dagger H (W^i W^i + B^2) + 2H^\dagger \sigma^i H W^i B, \tag{B.12}$$

where W^i and B are the field strength 2-forms.

Putting everything together, we obtain the expression in Eq. (3.5).

References

1. D.B. Kaplan, H. Georgi, $SU(2) \times U(1)$ Breaking by vacuum misalignment. Phys. Lett. B **136**, 183 (1984)
2. H. Georgi, D.B. Kaplan, Composite Higgs and custodial $SU(2)$. Phys. Lett. B **145**, 216 (1984)
3. M.J. Dugan, H. Georgi, D.B. Kaplan, Anatomy of a composite Higgs model. Nucl. Phys. B **254**, 299 (1985)

¹⁶ The term multiplied by $\frac{1}{2}$ simplifies to $[[F^2, \xi], \xi]$, but we prefer to write it in a form that leaves the Lie algebra structure manifest.

¹⁷ We have removed erroneous factors of $\pm i$, which appear in [7].

4. J.S. Bell, R. Jackiw, A.P.C.A.C. Puzzle, $\pi \rightarrow \gamma\gamma$ in the sigma model. *Nuovo Cim. A* **60**, 47–61 (1969)
5. S.L. Adler, Axial-vector vertex in spinor electrodynamics. *Phys. Rev.* **177**, 2426–2438 (1969)
6. K. Agashe, R. Contino, A. Pomarol, The minimal composite Higgs model. *Nucl. Phys. B* **719**, 165–187 (2005). [arXiv:hep-ph/0412089](#)
7. B. Gripaios, A. Pomarol, F. Riva, J. Serra, Beyond the minimal composite Higgs model. *JHEP* **04**, 070 (2009). [arXiv:0902.1483](#)
8. J. Wess, B. Zumino, Consequences of anomalous ward identities. *Phys. Lett. B* **37**, 95–97 (1971)
9. E. Witten, Global aspects of current algebra. *Nuclear Phys. B* **223**, 422–432 (1983)
10. B. Gripaios, Anomaly holography, the Wess–Zumino–Witten term, and electroweak symmetry breaking. *Phys. Lett. B* **663**, 419–423 (2008). [arXiv:0803.0497](#)
11. K. Agashe, R. Contino, L. Da Rold, A. Pomarol, A custodial symmetry for $Z b$ anti- b . *Phys. Lett. B* **641**, 62–66 (2006). [arXiv:hep-ph/0605341](#)
12. B. Bellazzini, R. Franceschini, F. Sala, J. Serra, Goldstones in diphotons. [arXiv:1512.0533](#)
13. B. Gripaios, Composite leptoquarks at the LHC. *JHEP* **02**, 045 (2010). [arXiv:0910.1789](#)
14. B. Gripaios, A. Papaefstathiou, K. Sakurai, B. Webber, Searching for third-generation composite leptoquarks at the LHC. *JHEP* **01**, 156 (2011). [arXiv:1010.3962](#)
15. B. Gripaios, M. Nardecchia, S.A. Renner, Composite leptoquarks and anomalies in B -meson decays. *JHEP* **05**, 006 (2015). [arXiv:1412.1791](#)
16. J. Preskill, Gauge anomalies in an effective field theory. *Ann. Phys.* **210**, 323–379 (1991)
17. C.-S. Chu, P.-M. Ho, B. Zumino, NonAbelian anomalies and effective actions for a homogeneous space G/H . *Nucl. Phys. B* **475**, 484–504 (1996). [arXiv:hep-th/9602093](#)
18. S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 2005)
19. B. Gripaios, *Lectures on Effective Field Theory*. [arXiv:1506.0503](#)
20. E. D’Hoker, S. Weinberg, General effective actions. *Phys. Rev. D* **50**, 6050–6053 (1994). [arXiv:hep-ph/9409402](#)
21. B. Gripaios, D. Sutherland, An operator basis for the Standard Model with an added scalar singlet. [arXiv:1604.0736](#)
22. R. Franceschini, G.F. Giudice, J.F. Kamenik, M. McCullough, A. Pomarol, R. Rattazzi, M. Redi, F. Riva, A. Strumia, R. Torre, What is the $\gamma\gamma$ resonance at 750 GeV? *JHEP* **03**, 144 (2016). [arXiv:1512.0493](#)
23. B. Gripaios, T. Mller, M.A. Parker, D. Sutherland, Search strategies for top partners in composite Higgs models. *JHEP* **08**, 171 (2014). [arXiv:1406.5957](#)
24. D.B. Kaplan, Flavor at SSC energies: a new mechanism for dynamically generated fermion masses. *Nucl. Phys. B* **365**, 259–278 (1991)
25. G.F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, The strongly-interacting light Higgs. *JHEP* **06**, 045 (2007). [arXiv:hep-ph/0703164](#)
26. B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi et al., On partial compositeness and the CP asymmetry in charm decays. *Nucl. Phys. B* **867**, 394–428 (2013). [arXiv:1205.5803](#)
27. F. Goertz, J.F. Kamenik, A. Katz, M. Nardecchia, Indirect constraints on the scalar di-photon resonance at the LHC. [arXiv:1512.0850](#)
28. K. Agashe, R. Contino, Composite Higgs-mediated FCNC. *Phys. Rev. D* **80**, 075016 (2009). [arXiv:0906.1542](#)
29. ATLAS Collaboration, search for new phenomena in $t\bar{t}$ final states with additional heavy-flavour jets in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector. ATLAS-CONF-2016-104
30. M. Redi, A. Weiler, Flavor and CP invariant composite Higgs models. *JHEP* **11**, 108 (2011). [arXiv:1106.6357](#)
31. G. Panico, M. Redi, A. Tesi, A. Wulzer, On the tuning and the mass of the composite Higgs. *JHEP* **03**, 051 (2013). [arXiv:1210.7114](#)