Tetraquark states with open flavors

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Abstract In this work, we estimate the masses of tetraquark states with four different flavors by virtue of QCD sum rules, in both b and c sectors. We construct four $[8_c]_{\bar{b}s} \otimes [8_c]_{\bar{d}u}$ tetraquark currents with $J^P = 0^+$, and then we perform an analytic calculation up to dimension eight in the operator product expansion. We keep terms which are linear in the strange quark mass m_s, and in the end we find two possible tetraquark states with masses (5.57 ± 0.15) and (5.58 ± 0.15) GeV. We find that their charmed-partner masses lie in (2.54 ± 0.13) and (2.55 ± 0.13) GeV, respectively, and are hence accessible in experiments like BESIII and Belle.

1 Introduction

Recently, the DØ Collaboration has reported the first observation of a narrow structure, called $X(5568)$, in the decay chain $X(5568) \rightarrow B_s^0 \pi^{\pm}$, $B_s^0 \rightarrow J/\psi \phi$, $J/\psi \rightarrow \mu^+ \mu^-$, $\phi \rightarrow K^+ K^-$ based on the $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV collected at the Fermilab Tevatron collider [1]. Its mass and width were, respectively, measured to be $M_X = 5567.8 \pm 2.9^{+0.9}_{-1.9}$ MeV and $\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5}$ MeV, and the favored quantum number is $J^P = 0^+$. The statistical significance including the look-elsewhere effect and systematic errors is about 5.1 σ. The decay final state $B_s^0 \pi^+$ indicates that the component of X(5568) has to be $su\bar{b}\bar{d}$, and this is the first observation of a hadronic state with four different flavors. However, more recently a preliminary analysis on data collected $\sqrt{s} = 7$ and 8 TeV by the LHCb Collaboration does not confirm the existence of state X(5568) [2]. The contradictory information from DØ and LHCb Collaborations on X(5568) is very interesting and urges more investigations of related topics.

The observation of X(5568) has immediately inspired extensive discussions on the possibility of its internal structure. Very recently, the authors investigated the $X(5568)$ as a scalar tetraquark state using the approach of QCD sum rules [3–7]. Meanwhile, in Ref. [8], the authors constructed a series of tetraquark currents to calculate the corresponding mass in the framework of QCD sum rules, and their results support the X(5568) as a tetraquark state with quantum numbers $J^P = 0^+$ or 1^+. Wang and Zhu [9] employed the effective Hamiltonian approach to calculate the mass of the tetraquark state. The molecular picture of X_b was carried out in Ref. [10] using the QCD sum rules, where the $X_b(5568)$ was taken as the $B\bar{K}$ bound states. Assuming the X(5568) as the S-wave $B\bar{K}$ molecular state, Xiao and Chen discussed the decay of $X(5568) \rightarrow B_s^0 \pi^+$ in Ref. [11]. Besides, a number of works analyzing the structure of X(5568) were accomplished based on other theoretical methods [12–15], and its decay property was investigated in Refs. [6,7,16].

It is to be noted that according to quantum chromodynamics (QCD), there exists a tetraquark configuration, which is composed of two color-octet parts. Since there exists a QCD interaction, it is different from the molecular state with two color-singlet mesons. That is to say, it could decay to two mesons by exchanging one or more gluons. Therefore, the study of the color-octet tetraquark state is very important for possible new hadron states. In this work, we will construct four color-octet tetraquark currents, and we calculate their masses by means of QCD sum rules.

After the introduction, we present the primary formulas of the QCD sum rules in Sect. 2. The numerical analyses and related figures are shown in Sect. 3. Finally, we give a short summary of the tetraquark states with open flavors in Sect. 4.

2 Formalism

In this work, we study the color-octet tetraquark state with $J^P = 0^+$ via the approach of the QCD sum rules [17–21]. In

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the literature, the color-octet–octet type currents have been used to study the scalar, vector, and axial-vector tetraquark states [22–25]. The starting point of the QCD sum rules is the two-point correlation function constructed from two hadronic currents. For a scalar state as considered in this work, the two-point correlation function has the following form:

$$\Pi(q^2) = i \int \mathrm{d}^4 x e^{iq\cdot x} \langle 0|T\{j(x)j^\dagger(0)\}|0\rangle. \tag{1}$$

Here $j(x)$ and $j(0)$ are the above-mentioned hadronic currents with $J^P = 0^+$, and they are constructed as follows:

$$j_A(x) = [i\,\bar{b}^j(x)\gamma_5(t^a)_{jk}s^k(x)][i\,\bar{d}^m(x)\gamma_5(t^a)_{mn}u^n(x)], \tag{2}$$

$$j_B(x) = [\bar{b}^j(x)(t^a)_{jk}s^k(x)][\bar{d}^m(x)(t^a)_{mn}u^n(x)], \tag{3}$$

$$j_C(x) = [\bar{b}^j(x)\gamma^\mu(t^a)_{jk}s^k(x)][\bar{d}^m(x)\gamma_\mu(t^a)_{mn}u^n(x)], \tag{4}$$

$$j_D(x) = [\bar{b}^j(x)\gamma^\mu\gamma_5(t^a)_{jk}s^k(x)][\bar{d}^m(x)\gamma_\mu\gamma_5(t^a)_{mn}u^n(x)], \tag{5}$$

where j, k, m, and n are color indices, t^a is the generator of the group $SU_c(3)$. Here, the subscripts A, B, C, and D represent the currents composed of two 0^- color-octet constituents, two 0^+, two 1^-, and two 1^+, respectively. We will take into account all these currents in the following calculation.

The principle of quark–hadron duality is the basic assumption to employ the approach of the QCD sum rules, as shown in Refs. [21,26]. Hence, on the one hand, the correlation function $\Pi(q^2)$ can be calculated at the quark–gluon level, where the operator product expansion (OPE) is employed; on the other hand, it can be expressed at the hadron level, in which the coupling constant and mass of the hadron are introduced.

In order to calculate the spectral density of the OPE side, the full propagators $S_{ij}^q(x)$ and $S_{ij}^Q(p)$ of a light quark ($q = u$, d or s) and a heavy quark ($Q = c$ or b) are used:

$$S_{jk}^q(x) = \frac{i\delta_{jk}\slashed{x}}{2\pi^2 x^4} - \frac{\delta_{jk}m_q}{4\pi^2 x^2} - \frac{it_{jk}^a}{32\pi^2 x^2}(\sigma^{\alpha\beta}\slashed{x} + \slashed{x}\sigma^{\alpha\beta})$$
$$- \frac{\delta_{jk}}{12}\langle\bar{q}q\rangle + \frac{i\delta_{jk}\slashed{x}}{48}m_q\langle\bar{q}q\rangle - \frac{\delta_{jk}x^2}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle$$
$$+ \frac{i\delta_{jk}x^2\slashed{x}}{1152}m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle - \frac{t_{jk}^a\sigma_{\alpha\beta}}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle$$
$$+ \frac{it_{jk}^a}{768}(\sigma_{\alpha\beta}\slashed{x} + \slashed{x}\sigma_{\alpha\beta})m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle, \tag{6}$$

$$S_{jk}^Q(p) = \frac{i\delta_{jk}(\slashed{p} + m_Q)}{p^2 - m_Q^2} - \frac{i}{4}\frac{t_{jk}^a}{(p^2 - m_Q^2)^2}[\sigma^{\alpha\beta}(\slashed{p} + m_Q)$$
$$+ (\slashed{p} + m_Q)\sigma^{\alpha\beta}] + \frac{i\delta_{jk}m_Q\langle g_s^2 G^2\rangle}{12(p^2 - m_Q^2)^3}\left[1 + \frac{m_Q(\slashed{p} + m_Q)}{p^2 - m_Q^2}\right]$$

$$+ \frac{i\delta_{jk}}{48}\left\{\frac{(\slashed{p} + m_Q)[\slashed{p}(p^2 - 3m_Q^2) + 2m_Q(2p^2 - m_Q^2)]}{(p^2 - m_Q^2)^6}\right.$$
$$\left.\times (\slashed{p} + m_Q)\right\}\langle g_s^3 G^3\rangle, \tag{7}$$

where the vacuum condensates are clearly displayed. Note that these full propagators of QCD for both light and heavy quarks have been explained in Ref. [27].

Accordingly, based on the dispersion relation, the correlation function $\Pi(q^2)$ at the quark–gluon level can be obtained:

$$\Pi_i^{\mathrm{OPE}}(q^2) = \int_{(m_b+m_s)^2}^\infty \mathrm{d}s \frac{\rho_i^{\mathrm{OPE}}(s)}{s - q^2} + \Pi_i^{\langle G^3\rangle}(q^2)$$
$$+ \Pi_i^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(q^2) + \Pi_i^{\langle G^2\rangle^2}(q^2), \tag{8}$$

where $\rho_i^{\mathrm{OPE}}(s) = \mathrm{Im}[\Pi_i^{\mathrm{OPE}}(s)]/\pi$ and

$$\rho_i^{\mathrm{OPE}}(s) = \rho_i^{\mathrm{pert}}(s) + \rho_i^{\langle\bar{s}s\rangle}(s) + \rho_i^{\langle G^2\rangle}(s) + \rho_i^{\langle\bar{s}Gs\rangle}(s)$$
$$+ \rho_i^{\langle\bar{q}q\rangle^2}(s) + \rho_i^{\langle G^3\rangle}(s) + \rho_i^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) + \rho_i^{\langle G^2\rangle^2}(s), \tag{9}$$

in which the subscript i runs from A to D, and $\Pi_i^{\langle G^3\rangle}(q^2)$, $\Pi_i^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(q^2)$ and $\Pi_i^{\langle G^2\rangle}(q^2)$ represent those contributions of the correlation function that do not have imaginary parts but have nontrivial values after the Borel transform.

Applying the Borel transform to the quark–gluon side, we have

$$\Pi_i^{\mathrm{OPE}}(M_B^2) = \int_{(m_b+m_s)^2}^\infty \mathrm{d}s\rho^{\mathrm{OPE}}(s)e^{-s/M_B^2} + \Pi_i^{\langle G^3\rangle}(M_B^2)$$
$$+ \Pi_i^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) + \Pi_i^{\langle G^2\rangle}(M_B^2). \tag{10}$$

In order to take into account the effects induced by the mass of the strange quark, we keep terms which are linear in the strange quark mass m_s in the following calculations. For all the tetraquark states considered in this article, we put the detailed formulas of spectral densities in Eq. (10) into the appendix.

On the hadron side, after isolating the ground state contribution of the tetraquark state, we obtain the correlation function $\Pi(q^2)$, which is expressed as a dispersion integral over a physical regime,

$$\Pi_i(q^2) = \frac{(\lambda_X^i)_2}{(M_X^i)^2 - q^2} + \frac{1}{\pi}\int_{s_0}^\infty \mathrm{d}s \frac{\rho_X^i(s)}{s - q^2}, \tag{11}$$

where M_X^i is the mass of the tetraquark state with $J^P = 0^+$, and $\rho_X^i(s)$ is the spectral density that contains the contributions from the higher excited states and the continuum states, s_0 is the threshold of the higher excited states and continuum states. The coupling constant λ_X is defined by $\langle 0|j_X^i|X\rangle = \lambda_X^i$, where X is the lowest lying tetraquark state.

Performing the Borel transform on the hadron side (Eq. (11)) and then matching it to Eq. (10), we can obtain the mass of the scalar tetraqark state with open flavors:

$$M_X^i(s_0, M_B^2) = \sqrt{-\frac{R_1^i(s_0, M_B^2)}{R_0^i(s_0, M_B^2)}}, \tag{12}$$

where X denotes the tetraquark state and

$$R_0^i(s_0, M_B^2) = \int_{(m_b + m_s)^2}^{s_0} ds \, \rho^{\mathrm{OPE}}(s) e^{-s/M_B^2} + \Pi_i^{\langle G^3 \rangle}(M_B^2)$$
$$+ \Pi_i^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(M_B^2) + \Pi_i^{\langle G^2 \rangle^2}(M_B^2), \tag{13}$$

$$R_1^i(s_0, M_B^2) = \frac{\partial}{\partial \frac{1}{M_B^2}} R_0(s_0, M_B^2). \tag{14}$$

3 Numerical results

The expressions of the QCD sum rules contain various input parameters, such as the condensates and the quark masses. As shown in Refs. [5,8,17–21,28], we take these values as $m_u = m_d = 0$, $m_s(2\mathrm{GeV}) = (95 \pm 5)$ MeV, $m_c(m_c) = \overline{m}_c = (1.275 \pm 0.025)$ GeV, $m_b(m_b) = \overline{m}_b = (4.18 \pm 0.03)$ GeV, $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3$ GeV³, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$, $\langle g_s^2 G^2 \rangle = 0.88$ GeV⁴, $\langle \bar{s}g_s \sigma \cdot Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $\langle g_s^3 G^3 \rangle = 0.045$ GeV⁶, and $m_0^2 = 0.8$ GeV². Here, $\overline{m}_c$ and $\overline{m}_b$ are the running masses of the heavy quarks in the $\overline{MS}$ scheme.

Moreover, there exist two additional parameters M_B^2 and s_0 introduced by the QCD sum rules, which should be fixed in accordance with the standard procedures. In Refs. [17–19,21], there are two criteria to constrain the parameter M_B^2 and the threshold s_0. The first criterion is the convergence of the OPE. That is, we need to compare the relative contribution of each term to the total contributions of the OPE side, and choose a reliable region of M_B^2 to retain their convergence. Second, the pole contribution (PC) defined as the pole contribution (corresponding to the contribution of the ground state) divided by the total contribution (pole plus continuum) should be larger than 50 % [21,29]. Thus, we can safely eliminate the contributions of the higher excited and continuum states.

Meanwhile, in order to find a proper value of $\sqrt{s_0}$, we perform a similar analysis as in Refs. [30,31]. Because the continuum threshold s_0 is connected to the mass of the ground state by the relation $\sqrt{s_0} \sim (M_X + \delta)$ GeV, in which δ lies in the range of 0.4–0.8 GeV, various $\sqrt{s_0}$ satisfying this constraint should be taken into account in the numerical analyses. Among these values, we need then to pick out the proper one that has an optimal window for Borel parameter M_B^2. That is to say, in this optimal window, the tetraquark mass M_X is independent of the Borel parameter M_B^2 as much as possible. Finally, the value of $\sqrt{s_0}$ corresponding to the optimal mass curve is the central value of $\sqrt{s_0}$. In practice, it is nor-
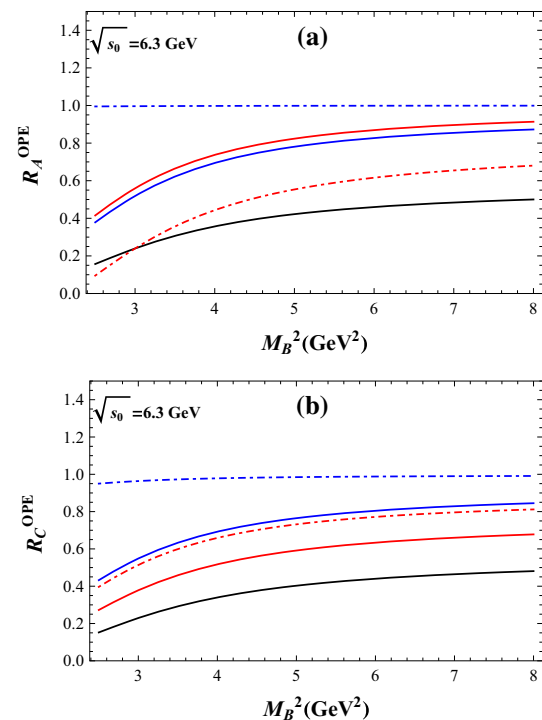
Fig. 1 **a** The OPE convergence R_A^{OPE} as a function of the Borel parameter M_B^2 in the region $2.5 \leq M_B^2 \leq 8.0$ GeV² for the tetraquark state of case A, where $\sqrt{s_0} = 6.3$ GeV. **b** The OPE convergence R_C^{OPE} as a function of the Borel parameter M_B^2 in the region $2.5 \leq M_B^2 \leq 8.0$ GeV² for the tetraquark state of case C, where $\sqrt{s_0} = 6.3$ GeV. The *black line* represents the fraction of perturbative contribution, and each subsequent line stands for the addition of one extra condensate, i.e., $+\langle \bar{s}s \rangle$ (*red line*), $+\langle g_s^2 G^2 \rangle$ (*blue line*), $+\langle g_s \bar{s}\sigma \cdot Gs \rangle$ (*red dotted line*), $+\langle \bar{q}q \rangle^2$ (*blue dotted line*). Since the curves that add the condensate terms of $\langle g_s^3 G^3 \rangle$, $\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma \cdot Gq \rangle$, and $\langle g_s^2 G^2 \rangle^2$ one by one are just straight lines, respectively, we do not show them here

mally acceptable to vary the $\sqrt{s_0}$ by 0.2 GeV [32,33] in the QCD sum rules calculation, which determines the upper and lower bounds of $\sqrt{s_0}$. Hence, these bounds give rise to the uncertainties of $\sqrt{s_0}$.

We illustrate the OPE convergences in Fig. 1a, b, respectively for case A and C. Performing the first criterion, we find the lower limit constraint of M_B^2 is $M_B^2 \gtrsim 3.0$ GeV² with $\sqrt{s_0} = 6.3$ GeV for both case A and C. The curves of the pole contribution R^{PC} are drawn in Fig. 2a, b, which indicate the upper limit constraint of M_B^2 is $M_B^2 \lesssim 4.0$ GeV² with $\sqrt{s_0} = 6.3$ GeV for both case A and C. It should be noted that the limit constraints of M_B^2 also depend on the threshold parameter $\sqrt{s_0}$. That is, there are different limit constraints of M_B^2 for different $\sqrt{s_0}$. For determining the value of $\sqrt{s_0}$, we carry out an analysis similar to Ref. [29]. The masses M_X^A and M_X^C as a function of the Borel parameter M_B^2 for different values $\sqrt{s_0}$ are drawn in Fig. 3a, b. The Borel parameters, continuum threshold parameters, the pole contributions, and
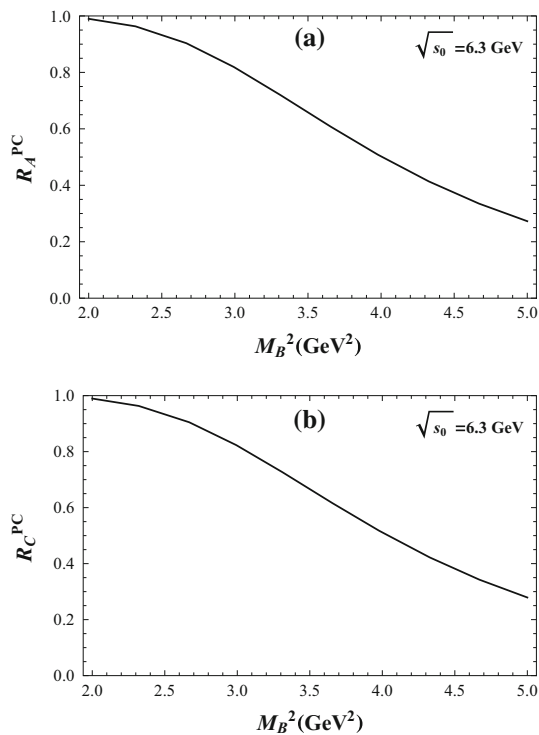
Fig. 2 **a** The pole contribution R_A^{PC} for the tetraquark state as a function of the Borel parameter M_B^2 in case A with $\sqrt{s_0} = 6.3$ GeV. **b** The pole contribution R_C^{PC} for the tetraquark state as a function of the Borel parameter M_B^2 in case C with $\sqrt{s_0} = 6.3$ GeV

Fig. 3 **a** The mass of the tetraquark state in case A as a function of the Borel parameter M_B^2, for different values of $\sqrt{s_0}$. **b** The mass of the tetraquark state in case C as a function of the Borel parameter M_B^2, for different values of $\sqrt{s_0}$

the predicted masses for both case A and C are shown explicitly in Table 1.

However, we find there does not exist a reasonable region of the parameter M_B^2 for both case B and D. Therefore, we can conclude that these two cases do not correspond to any tetraquark states.

Eventually, the masses of the tetraquark states with currents A and C are determined to be
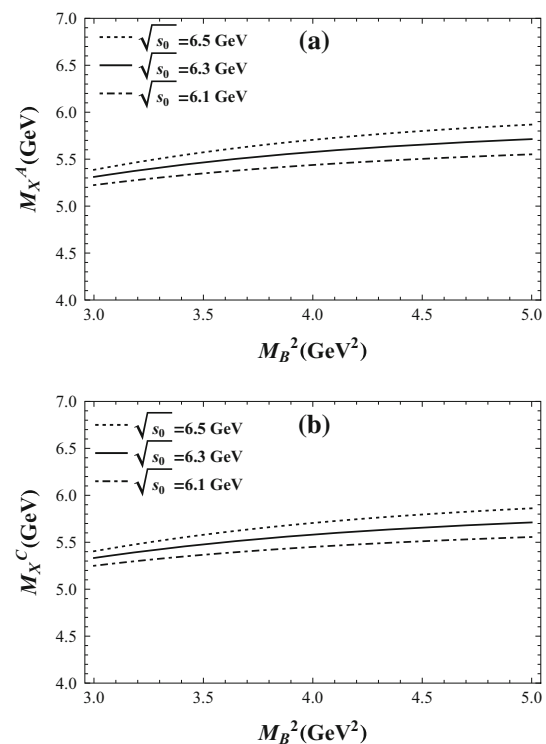
$$M_X^A = (5.57 \pm 0.15) \text{ GeV}, \tag{15}$$

$$M_X^C = (5.58 \pm 0.15) \text{ GeV}, \tag{16}$$

where the central value of the mass M_X corresponds to the result with the optimal stability of M_B^2, and the errors stem from the uncertainties of the condensates, the quark mass, the threshold parameter $\sqrt{s_0}$, and the Borel parameter M_B^2.

Moreover, we predict their charmed partners with masses of (2.54 ± 0.13) and (2.55 ± 0.13) GeV, respectively.

4 Summary

In this work, we estimate the masses of the tetraquark states with four different flavors by virtue of QCD sum rules, in both b and c sectors. We construct four $[8_c]_{\bar{b}s} \otimes [8_c]_{\bar{d}u}$ tetraquark

Table 1 The Borel parameters, continuum threshold parameters, pole contributions, and predicted masses of the tetraquark states with currents A and C

	M_B^2 (GeV^2)	$\sqrt{s_0}$ (GeV)	Pole	M_X (GeV)
Case A	3.0–4.0	6.3 ± 0.1	(50–83) %	5.57 ± 0.15
Case C	3.0–4.0	6.3 ± 0.1	(50–82) %	5.58 ± 0.15

currents with $J^P = 0^+$, and then we perform an analytic calculation up to dimension eight in the OPE. We keep terms which are linear in the strange quark mass m_s.

The numerical results are, respectively, (5.57 ± 0.15) GeV and (5.58 ± 0.15) GeV for case A and C. However, due to the lack of reasonable windows of the Borel parameter M_B^2, case B and D do not correspond to any hadron states. Our results imply that two S-wave octet parts can form a resonance, whereas two P-wave octet parts cannot form a resonance. Therefore, we can speculate that two $[8_c]_{\bar{b}s} \otimes [8_c]_{\bar{d}u}$ tetraquark states with $J^P = 1^+$ should exist and have a degenerate mass. We will present detailed analyses of tetraquark states with $J^P = 1^+$ in our next work.

In conclusion, we find in b-quark sector two possible open-flavor tetraquark states with masses (5.57 ± 0.15) and (5.58 ± 0.15) GeV may exist, while their charmed partners lie in (2.54 ± 0.13) and (2.55 ± 0.13) GeV, respectively, and

are hence accessible in experiments like BESIII and Belle. Though a preliminary analysis performed by the LHCb Collaboration does not favor the existence of $X(5568)$ [2], the tetraquark state with open flavors is still an interesting target deserving more explorations.

Appendix A: The spectral densities for cases A–D

For case A where the current is composed of two 0^- color-octet parts, we obtain the spectral density as follows:

$$\rho_A^{\text{pert}}(s) = \frac{1}{2^{11} \times 3^2 \pi^6} \int_0^\lambda d\alpha \frac{H_\alpha^3(H_\alpha - 4m_b m_s \alpha)}{\alpha(1-\alpha)^3}, \qquad (A1)$$

$$\rho_A^{\langle \bar{s}s \rangle}(s) = \frac{\langle \bar{s}s \rangle}{2^7 \times 3\pi^4} \int_0^\lambda d\alpha \frac{H_\alpha^2(m_s - \alpha(m_b + m_s))}{\alpha(1-\alpha)^2}, \qquad (A2)$$

$$\rho_A^{\langle GG \rangle}(s) = -\frac{\langle g_s^2 GG \rangle}{2^{15} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{1}{\alpha(1-\alpha)^3}[9H_\alpha^2(\alpha^2 - 3\alpha + 2)$$
$$-4H_\alpha m_b \alpha(4m_b\alpha^2 - 3m_s(\alpha^2 + 6\alpha - 3)) + 16m_b^3 m_s \alpha^4], \qquad (A3)$$

$$\rho_A^{\langle \bar{s}Gs \rangle}(s) = \frac{\langle \bar{s}g_s\sigma \cdot Gs \rangle}{2^{10} \times 3^2 \pi^4}$$
$$\times \int_0^\lambda d\alpha \frac{H_\alpha[3m_b\alpha(7\alpha - 8) + m_s(5\alpha^2 - 13\alpha + 8)]}{\alpha(1-\alpha)^2}, \qquad (A4)$$

$$\rho_A^{\langle G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{2^{13} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{\alpha^2(H_\alpha + 2m_b\alpha(m_b - 3m_s))}{(1-\alpha)^3}, \qquad (A5)$$

$$\rho_A^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle}{72\pi^2} \int_0^\lambda d\alpha \frac{(m_b m_s \alpha - H_\alpha)}{\alpha}, \qquad (A6)$$

$$\rho_A^{\langle G^2 \rangle^2}(s) = \frac{\langle g_s^2 G^2 \rangle^2}{2^{16} \times 3\pi^6}\lambda, \qquad (A7)$$

$$\rho_A^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(s) = 0, \qquad (A8)$$

$$\Pi_A^{\langle G^3 \rangle}(M_B^2) = \frac{\langle g_s^3 G^3 \rangle}{2^{12} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b^3 m_s \alpha^3}{(1-\alpha)^4} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \quad (A9)$$

$$\Pi_A^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(M_B^2) = -\frac{\langle \bar{q}q \rangle \langle \bar{q}g_s\sigma \cdot Gq \rangle}{2^4 \times 3^2 \pi^2}$$

$$\times \int_0^1 d\alpha \frac{1}{\alpha^2(1-\alpha)} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \qquad (A10)$$

$$\Pi_A^{\langle G^2 \rangle^2}(M_B^2) = \frac{\langle g_s^2 G^2 \rangle^2}{2^{16} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b \alpha}{(1-\alpha)^3 M_B^2}$$

$$\times \left[m_b^2 m_s + M_B^2(1-\alpha)(m_b - 3m_s)\right] e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \qquad (A11)$$

where M_B^2 is the Borel parameter, $H_\alpha = m_b^2\alpha - \alpha(1-\alpha)s$, and $\lambda = 1 - m_b^2/s$.

For case B where the current is composed of two 0^+ color-octet parts, we obtain the spectral density as follows:

$$\rho_B^{\text{pert}}(s) = \frac{1}{2^{11} \times 3^2 \pi^6} \int_0^\lambda d\alpha \frac{H_\alpha^3(H_\alpha + 4m_b m_s \alpha)}{\alpha(1-\alpha)^3}, \qquad (A12)$$

$$\rho_B^{\langle \bar{s}s \rangle}(s) = \frac{\langle \bar{s}s \rangle}{2^7 \times 3\pi^4} \int_0^\lambda d\alpha \frac{H_\alpha^2(m_s + \alpha(m_b - m_s))}{\alpha(1-\alpha)^2}, \quad (A13)$$

$$\rho_B^{\langle GG \rangle}(s) = -\frac{\langle g_s^2 GG \rangle}{2^{15} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{1}{\alpha(1-\alpha)^3}[9H_\alpha^2(\alpha^2 - 3\alpha + 2)$$
$$-4H_\alpha m_b \alpha(4m_b\alpha^2 + 3m_s(\alpha^2 + 6\alpha - 3)) - 16m_b^3 m_s \alpha^4], \qquad (A14)$$

$$\rho_B^{\langle \bar{s}Gs \rangle}(s) = \frac{\langle \bar{s}g_s\sigma \cdot Gs \rangle}{2^{10} \times 3^2 \pi^4}$$
$$\times \int_0^\lambda d\alpha \frac{H_\alpha[3m_b(8 - 7\alpha)\alpha + m_s(5\alpha^2 - 13\alpha + 8)]}{\alpha(1-\alpha)^2}, \qquad (A15)$$

$$\rho_B^{\langle G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{2^{13} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{\alpha^2(H_\alpha + 2m_b\alpha(m_b + 3m_s))}{(1-\alpha)^3}, \qquad (A16)$$

$$\rho_B^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle}{72\pi^2} \int_0^\lambda d\alpha \frac{(m_b m_s \alpha + H_\alpha)}{\alpha}, \qquad (A17)$$

$$\rho_B^{\langle G^2 \rangle^2}(s) = \frac{\langle g_s^2 G^2 \rangle^2}{2^{16} \times 3\pi^6}\lambda, \qquad (A18)$$

$$\rho_B^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(s) = 0, \qquad (A19)$$

$$\Pi_B^{\langle G^3 \rangle}(M_B^2) = -\frac{\langle g_s^3 G^3 \rangle}{2^{12} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b^3 m_s \alpha^3}{(1-\alpha)^4} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \qquad (A20)$$

$$\Pi_B^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(M_B^2) = \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s\sigma \cdot Gq \rangle}{2^4 \times 3^2 \pi^2}$$

$$\times \int_0^1 d\alpha \frac{1}{\alpha^2(1-\alpha)} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \qquad (A21)$$

$$\Pi_B^{\langle G^2 \rangle^2}(M_B^2) = -\frac{\langle g_s^2 G^2 \rangle^2}{2^{16} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b \alpha}{(1-\alpha)^3 M_B^2}$$

$$\times [m_b^2 m_s - M_B^2(1-\alpha)(m_b + 3m_s)] e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}. \qquad (A22)$$

For case C where the current is composed of two 1^- color-octet parts, we obtain the spectral density as follows:

$$\rho_C^{\text{pert}}(s) = \frac{1}{2^9 \times 3^2 \pi^6} \int_0^\lambda d\alpha \frac{H_\alpha^3(H_\alpha - 2m_b m_s \alpha)}{\alpha(1-\alpha)^3}, \quad (A23)$$

$$\rho_C^{\langle \bar{s}s \rangle}(s) = \frac{\langle \bar{s}s \rangle}{2^6 \times 3\pi^4} \int_0^\lambda d\alpha \frac{H_\alpha^2(2m_s(1-\alpha) - m_b \alpha)}{\alpha(1-\alpha)^2}, \quad (A24)$$

$$\rho_C^{\langle GG \rangle}(s) = -\frac{\langle g_s^2 GG \rangle}{2^{14} \times 3^3 \pi^6}$$
$$\times \int_0^\lambda d\alpha \frac{1}{\alpha(1-\alpha)^3}[-81H_\alpha^2(\alpha^2 - 3\alpha + 2)$$
$$+ 2H_\alpha m_b \alpha(-16m_b \alpha^2$$
$$+ m_s(-39\alpha^2 + 45\alpha + 18)) + 16m_b^3 m_s \alpha^4], \quad (A25)$$

$$\rho_C^{\langle \bar{s}Gs \rangle}(s) = \frac{\langle \bar{s}g_s \sigma \cdot Gs \rangle}{2^9 \times 3^2 \pi^4} \int_0^\lambda d\alpha \frac{H_\alpha(3m_b \alpha + 11m_s(\alpha - 1))}{\alpha(1-\alpha)}, \quad (A26)$$

$$\rho_C^{\langle G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{2^{11} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{\alpha^2(H_\alpha + m_b \alpha(2m_b - 3m_s))}{(1-\alpha)^3}, \quad (A27)$$

$$\rho_C^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle}{36\pi^2} \int_0^\lambda d\alpha \frac{(2m_b m_s \alpha - H_\alpha)}{\alpha}, \quad (A28)$$

$$\rho_C^{\langle G^2 \rangle^2}(s) = \frac{139\langle g_s^2 G^2 \rangle^2}{2^{17} \times 3^3 \pi^6} \lambda, \quad (A29)$$

$$\rho_C^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(s) = \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma \cdot Gq \rangle}{128\pi^2} \lambda, \quad (A30)$$

$$\Pi_C^{\langle G^3 \rangle}(M_B^2) = \frac{\langle g_s^3 G^3 \rangle}{2^{11} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b^3 m_s \alpha^3}{(1-\alpha)^4} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \quad (A31)$$

$$\Pi_C^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(M_B^2) = \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma \cdot Gq \rangle}{2^7 \times 3^2 \pi^2}$$
$$\times \int_0^1 d\alpha \frac{(18m_b m_s \alpha^2 - 7)}{\alpha^2(1-\alpha)} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \quad (A32)$$

$$\Pi_C^{\langle G^2 \rangle^2}(M_B^2) = -\frac{\langle g_s^2 G^2 \rangle^2}{2^{15} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b \alpha}{(1-\alpha)^3 M_B^2}$$
$$\times [m_b^2 m_s + M_B^2(1-\alpha)(9m_b - 3m_s)]e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}. \quad (A33)$$

For case D where the current is composed of two 1^+ color-octet parts, we obtain the spectral density as follows:

$$\rho_D^{\text{pert}}(s) = \frac{1}{2^9 \times 3^2 \pi^6} \int_0^\lambda d\alpha \frac{H_\alpha^3(H_\alpha + 2m_b m_s \alpha)}{\alpha(1-\alpha)^3}, \quad (A34)$$

$$\rho_D^{\langle \bar{s}s \rangle}(s) = \frac{\langle \bar{s}s \rangle}{2^6 \times 3\pi^4} \int_0^\lambda d\alpha \frac{H_\alpha^2(2m_s(1-\alpha) + m_b \alpha)}{\alpha(1-\alpha)^2}, \quad (A35)$$

$$\rho_D^{\langle GG \rangle}(s) = \frac{\langle g_s^2 GG \rangle}{2^{14} \times 3^3 \pi^6}$$
$$\times \int_0^\lambda d\alpha \frac{1}{\alpha(1-\alpha)^3}[81H_\alpha^2(\alpha^2 - 3\alpha + 2)$$
$$+ 2H_\alpha m_b \alpha(16m_b \alpha^2$$
$$+ m_s(-39\alpha^2 + 45\alpha + 18)) + 16m_b^3 m_s \alpha^4], \quad (A36)$$

$$\rho_D^{\langle \bar{s}Gs \rangle}(s) = -\frac{\langle \bar{s}g_s \sigma \cdot Gs \rangle}{2^9 \times 3^2 \pi^4} \int_0^\lambda d\alpha \frac{H_\alpha(3m_b \alpha - 11m_s(\alpha - 1))}{\alpha(1-\alpha)}, \quad (A37)$$

$$\rho_D^{\langle G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{2^{11} \times 3^3 \pi^6} \int_0^\lambda d\alpha \frac{\alpha^2(H_\alpha + m_b \alpha(2m_b + 3m_s))}{(1-\alpha)^3}, \quad (A38)$$

$$\rho_D^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle}{36\pi^2} \int_0^\lambda d\alpha \frac{(2m_b m_s \alpha + H_\alpha)}{\alpha}, \quad (A39)$$

$$\rho_D^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(s) = -\frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma \cdot Gq \rangle}{128\pi^2} \lambda, \quad (A40)$$

$$\rho_D^{\langle G^2 \rangle^2}(s) = \frac{139\langle g_s^2 G^2 \rangle^2}{2^{17} \times 3^3 \pi^6} \lambda, \quad (A41)$$

$$\Pi_D^{\langle G^3 \rangle}(M_B^2) = -\frac{\langle g_s^3 G^3 \rangle}{2^{11} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b^3 m_s \alpha^3}{(1-\alpha)^4} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \quad (A42)$$

$$\Pi_D^{\langle \bar{q}q \rangle \langle \bar{q}Gq \rangle}(M_B^2) = \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma \cdot Gq \rangle}{2^7 \times 3^2 \pi^2}$$
$$\times \int_0^1 d\alpha \frac{(18m_b m_s \alpha^2 + 7)}{\alpha^2(1-\alpha)} e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}, \quad (A43)$$

$$\Pi_D^{\langle G^2 \rangle^2}(M_B^2) = \frac{\langle g_s^2 G^2 \rangle^2}{2^{15} \times 3^3 \pi^6} \int_0^1 d\alpha \frac{m_b \alpha}{(1-\alpha)^3 M_B^2}$$
$$\times [m_b^2 m_s - M_B^2(1-\alpha)(9m_b + 3m_s)]e^{-\frac{m_b^2}{\alpha(1-\alpha)M_B^2}}. \quad (A44)$$

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