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Geometric back-reaction in pre-inflation from relativistic quantum geometry

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Abstract The pre-inflationary evolution of the universe describes the beginning of the expansion from a static initial state, such that the Hubble parameter is initially zero, but increases to an asymptotic constant value, in which it could achieve a de Sitter (inflationary) expansion. The expansion is driven by a background phantom field. The back-reaction effects at this moment should describe vacuum geometrical excitations, which are studied in detail in this work using relativistic quantum geometry.

1 Introduction

The inflationary model is a very well-tested description of how the universe can provide a physical mechanism to generate primordial energy density fluctuations on cosmological scales [1–3], below Planckian scales. During this stage, the primordial scalar perturbations drove the seeds of large scale structure which had then gradually formed today's galaxies. This is being tested in current observations of cosmic microwave background (CMB) [4]. These fluctuations are today larger than a 1000 times the size of a typical galaxy, but during inflation they were very much larger than the size of the causal horizon [5]. According to this scenario, the almost constant potential depending of a minimal coupling to a gravity inflation field, ϕ, called the inflaton, caused the accelerated expansion of the very early universe. During this epoch, the potential energy density was dominant, so that the kinetic energy can be neglected. This is known as the slow-roll condition for the inflaton field dynamics. In this framework the

problem of nonlinear (scalar) perturbative corrections to the metric has been studied in [6,7].

Geometrodynamics [8,9] is a picture of general relativity that studies the evolution of the spacetime geometry. The significant advantages of geometrodynamics usually have come at the expense of manifest local Lorentz symmetry [10]. During the 1970s and 1980s a method of quantization was developed in order to deal with some unresolved problems of quantum field theory in curved spacetimes [11–13]. In this context, recently we have introduced a new method to study the scalar perturbations of the metric in a non-perturbative manner [14] by introducing relativistic quantum geometry (RQG). This formalism is non-perturbative and serves to describe the dynamics of the geometric departure of a background Riemann spacetime with the help of a quantum geometrical scalar field [15–17]. The dynamics of the geometrical scalar field is defined on a Weyl-integrable manifold that preserves the gauge-invariance under the transformations of the Einstein's equations, which involves the cosmological constant. Our approach is different from quantum gravity. The natural way to construct quantum gravity models is to apply quantum field theory methods to the theories of classical gravitational fields interacting with matter. In spite of numerous efforts the general problems of quantum gravity still remain unsolved. Our approach is different because our subject is the dynamics of the geometrical quantum fields. This dynamics is obtained from the Einstein–Hilbert action, and not by using the standard effective action used in various models of quantum gravity [18]. There are no non-linearities or high-derivative problems in the dynamical description, so our formalism is much easier to apply to different physical systems like inflation [14], or pre-inflation. This primordial epoch is of significant interest in cosmology and deserves a detailed study. Presently, we cannot understand completely the first epoch of the universal evolution. How the did universe begin

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to expand and how must we understand the first stage of this evolution? The theory that describes this epoch is called pre-inflation [19–21]. The existence of a pre-inflationary epoch with fast-roll of the inflaton field would introduce an infrared depression in the primordial power spectrum. This depression might have left an imprint in the CMB anisotropy [22]. It is supposed that during pre-inflation the universe began to expand from some Planck-size initial volume, to thereafter pass to an inflationary epoch. In this framework RQG should be very useful when we try to study the evolution of the geometrical back-reaction effects given that we are dealing with Planck energetic scales, and back-reaction effects should be very intense at these scales.

In this work we shall describe a model in which the universe begins to expand from nothing (the initial energy density is null). The Hubble parameter is initially zero and thereafter increases to an asymptotic constant value, in which it could achieve a de Sitter (inflationary) expansion. In our model the background expansion is driven by a phantom scalar field ϕ, in which the equation of state of the universe during pre-inflation is $P_{\mathrm{pi}}/\rho_{\mathrm{pi}} < -1$ [23–25]. The back-reaction effects at this moment should describe vacuum geometrical excitations, which are the main subject in this work.

2 Relativistic quantum geometry: the structure of spacetime in an expanding universe

We shall consider a metric tensor in the Riemann manifold with a null covariant derivative (we denote by a semicolon the Riemann-covariant derivative): $\Delta \bar{g}_{\alpha\beta} = \bar{g}_{\alpha\beta;\gamma} \, \mathrm{d}x^\gamma = 0$, such that the Weyl [26] covariant derivative $\bar{g}_{\alpha\beta|\gamma} = \theta_\gamma \, \bar{g}_{\alpha\beta}$, described with respect to the Weyl connections,[1]

$$\Gamma^\alpha_{\beta\gamma} = \begin{Bmatrix} \alpha \\ \beta \ \gamma \end{Bmatrix} + \theta^\alpha \bar{g}_{\beta\gamma}, \tag{1}$$

is nonzero; we have

$$\delta \bar{g}_{\alpha\beta} = \bar{g}_{\alpha\beta|\gamma} \, \mathrm{d}x^\gamma = - \left[\theta_\beta \bar{g}_{\alpha\gamma} + \theta_\alpha \bar{g}_{\beta\gamma} \right] \mathrm{d}x^\gamma. \tag{2}$$

In the case of an expanding universe, the Riemann manifold will be described by the background geometry characterized with a FRW metric. Of course, all the variations with respect to the expanding background are in the Weyl geometrical representation. As was demonstrated in [15] the Einstein tensor can be written

$$\bar{G}_{\alpha\beta} = G_{\alpha\beta} + \theta_{\alpha;\beta} + \theta_\alpha \theta_\beta + \frac{1}{2} g_{\alpha\beta} \left[(\theta^\mu)_{;\mu} + \theta_\mu \theta^\mu \right]$$
$$\equiv G_{\alpha\beta} - \bar{g}_{\alpha\beta} \Lambda, \tag{3}$$

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[1] To simplify the notation we shall denote $\theta_\alpha \equiv \theta_{,\alpha}$, where the comma denotes the partial derivative. Furthermore, we shall denote by a bar the quantities represented on the Riemann background manifold.

and we can obtain the semi-Riemann invariant (the cosmological constant) Λ

$$\Lambda = -\frac{3}{4} \left[\theta_\alpha \theta^\alpha + \bar{\Box} \theta \right]. \tag{4}$$

Notice that $\Lambda(\theta, \theta^\alpha)$ is a Riemann invariant, but not a Weyl invariant. Hence, one can define a geometrical Weyl quantum action $\mathcal{W} = \int \mathrm{d}^4 x \sqrt{-\bar{g}} \, \Lambda(\theta, \theta^\alpha)$, such that the dynamics of the geometrical field, after imposing $\delta W = 0$, is described by the Euler–Lagrange equations, which take the form

$$\bar{\nabla}_\alpha \Pi^\alpha = 0, \quad \text{or} \quad \bar{\Box} \theta = 0. \tag{5}$$

The canonical momentum components are $\Pi^\alpha \equiv -\frac{3}{4} \theta^\alpha$ and the relativistic quantum algebra is given by [15]

$$[\theta(x), \theta^\alpha(y)] = -i\Theta^\alpha \, \delta^{(4)}(x - y),$$
$$[\theta(x), \theta_\alpha(y)] = i\Theta_\alpha \, \delta^{(4)}(x - y), \tag{6}$$

with $\Theta^\alpha = i\hbar \, \bar{U}^\alpha$ and $\Theta^2 = \Theta_\alpha \bar{\Theta}^\alpha = \hbar^2 \bar{U}_\alpha \bar{U}^\alpha$ for the Riemann components of velocities $\bar{U}^\alpha$.

3 Pre-inflation and back-reaction

One of the most important paradigms in cosmology consists in providing an explanation of the initial moment of the expansion of the universe. This implies a model of how the universe begun its expansion before the inflationary accelerated expansion with a Hubble parameter very close to a constant and $P_i/\rho_i \gtrsim -1$. A possible scenario is pre-inflation, in which the Hubble parameter is initially null, to thereafter increase to an asymptotically constant value. During the beginning of the expansion the universe has an equation of state with $P_{\mathrm{pi}}/\rho_{\mathrm{pi}} < -1$, which implies that the expansion is driven by a minimally coupled to gravity scalar phantom field ϕ. During pre-inflation the action is

$$\mathcal{I} = \int_V \mathrm{d}^4 x \sqrt{|\bar{g}|} \left[\frac{\bar{R}}{2\kappa} + \frac{\lambda}{2} \dot{\phi}^2 - V(\phi) \right], \tag{7}$$

where $\kappa = 8\pi G$, G is the gravitational constant, $\sqrt{|\bar{g}|} = a^3(t)$ is the volume of the manifold $\mathcal{M}$, and $\bar{g}_{\mu\nu} = \mathbf{diag}[1, -a^2, -a^2, -a^2]$ are the components of the diagonal tensor metric. With the aim to describe pre-inflation, we shall use $\lambda = -1$, which describes the dynamics of a fast-rolling phantom field. However, this epoch would be followed by an inflationary expansion driven by the slow-rolling inflaton field, for which the dynamics is obtained when $\lambda = 1$. Here, $\phi(t)$ is the background solution that describe the dynamics of an isotropic and homogeneous background metric that char-

acterizes a semi-Riemann manifold. We have

$$\ddot{\phi} + 3H\dot{\phi} + \lambda V'(\phi) = 0, \tag{8}$$

where $V(\phi)$ is the potential and the prime denotes the derivative with respect to ϕ. The semi-Riemann (background) Einstein equations are

$$\bar{G}_{\alpha\beta} \equiv \bar{R}_{\alpha\beta} - \frac{1}{2}\bar{g}_{\alpha\beta}\bar{R} = -\kappa\bar{T}_{\alpha\beta}, \tag{9}$$

where the components of the background stress tensor are $\bar{T}_{\alpha\beta} = \frac{\delta\bar{\mathcal{L}}}{\delta\bar{g}^{\alpha\beta}} - \bar{g}_{\alpha\beta}\bar{\mathcal{L}}$. For a background FRW metric the Einstein equations result,

$$3H^2 = \kappa\rho_{\text{pi}} = \kappa\left[\lambda\frac{\dot{\phi}^2}{2} + V(\phi)\right], \tag{10}$$

$$-\left(3H^2 + 2\dot{H}\right) = \kappa P_{\text{pi}} = \kappa\left[\lambda\frac{\dot{\phi}^2}{2} - V(\phi)\right]. \tag{11}$$

From the two Einstein equations we obtain $\dot{\phi} = -\frac{2H'}{\kappa\lambda}$, and the time dependent potential can be written as a function of the Hubble parameter and its time derivative:

$$V(t) = \frac{1}{\kappa}\left[3H^2 + \dot{H}\right]. \tag{12}$$

This expression can be re-written taking into account the ϕ-dependence

$$V(\phi) = \frac{1}{\kappa}\left[3H^2(\phi) - \frac{2}{\kappa\lambda}(H')^2\right]. \tag{13}$$

3.1 The pre-inflationary model with a phantom field

We consider a model in which the Hubble parameter is initially zero and tends asymptotically to $H|_{t\gg 1/(\sqrt{2}A)} \to H_0$,

$$H(t) = H_0 \tanh\left[2 H_0 t\right], \tag{14}$$

where the cosmological constant is related to H_0: $\Lambda = 3H_0^2$. The scale factor of the universe during this stage is

$$a(t) = \frac{a_0}{\left[1 - \tanh^2(2 H_0 t)\right]^{1/4}}, \tag{15}$$

with $a_0 = H_0^{-1}$. Notice that this solution describes a universe in which $\dot{H} > 0$. In other words, the model describes a universe which began to expand since we have an initial scale factor $a(t = 0) \equiv H_0^{-1}$. Furthermore, the Hubble parameter increases super-exponentially from a null value to an asymptotically constant value. The scalar potential can be written as a function of t,

$$V(t) = \frac{H_0^2}{\kappa}\left[\tanh^2(2H_0t) + 2\right], \tag{16}$$

so that for sufficiently large times, we obtain $V(t)|_{t\to\infty} \to \frac{3H_0^2}{\kappa}$. From the Einstein equations (10) and (11), we obtain the time dependence of $\dot{\phi}$

$$\dot{\phi} = \frac{2H_0}{\sqrt{\kappa}}\left[1 - \tanh^2(2H_0t)\right]^{1/2}. \tag{17}$$

Using the fact that $V' = \frac{\dot{V}}{\dot{\phi}}$ in the equation of motion (8), we obtain the time dependence of the background scalar field,

$$\phi(t) = \frac{2}{\sqrt{\kappa}}\arctan\left(e^{2H_0t}\right) - \frac{\pi}{2\sqrt{\kappa}}, \tag{18}$$

where $0 \leq \phi \leq \frac{\pi}{2\sqrt{\kappa}}$. Notice that the phantom field increases during pre-inflation. Therefore, if we use this expression in Eqs. (14) and (16), we obtain the ϕ-dependence of the Hubble parameter and the scalar potential

$$H(\phi) = H_0\left[1 - 2\cos^2\left[\frac{\sqrt{\kappa}}{2}\left(\phi + \frac{\pi}{2\sqrt{\kappa}}\right)\right]\right], \tag{19}$$

$$V(\phi) = \frac{H_0^2}{\kappa}\left\{\left[1 - 2\cos^2\left[\frac{\sqrt{\kappa}}{2}\left(\phi + \frac{\pi}{2\sqrt{\kappa}}\right)\right]\right]^2 + 2\right\}, \tag{20}$$

such that $V[\phi(t = 0)] = \frac{2H_0^2}{\kappa} \leq V(\phi) \leq V[\phi(t \to \infty)] = \frac{3H_0^2}{\kappa}$. Notice that $\rho_{\text{pi}}(t = 0) = 0$, so that in this model the universe is created from nothing.

3.2 Back-reaction effects in pre-inflation

The geometrical scalar field θ can be expressed as a Fourier expansion

$$\theta(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left[A_k e^{i\vec{k}.\vec{x}}\xi_k(t) + A_k^\dagger e^{-i\vec{k}.\vec{x}}\xi_k^*(t)\right], \tag{21}$$

where $A_k^\dagger$ and A_k are the creation and annihilation operators. From the point of view of the metric tensor, an example in power-law inflation can be illustrated by

$$g_{\mu\nu} = \text{diag}\left[e^{2\theta}, -a^2(t)e^{-2\theta}, -a^2(t)e^{-2\theta}, -a^2(t)e^{-2\theta}\right], \tag{22}$$

where the scale background scale factor $a(t)$ is given by (15). The quantum volume of the manifold described by (22) is $V_q = a^3(t)e^{-2\theta} = \sqrt{-g}\,e^{-2\theta}$. The dynamics for θ is governed by the equation

$$\ddot{\theta} + 3\frac{\dot{a}}{a}\dot{\theta} - \frac{1}{a^2}\nabla^2\theta = 0, \tag{23}$$

and the momentum components are $\Pi^\alpha \equiv -\frac{3}{4}\theta^\alpha$, so that the relativistic quantum algebra is given by the expressions (6) with co-moving relativistic velocities $U^0 = 1$, $U^i = 0$.

Furthermore, as was calculated in a previous work [14], the variation of the energy density fluctuations is given by the expression

$$\frac{1}{\bar\rho}\frac{\delta\bar\rho}{\delta S} = -2\theta_0 = -2\dot\theta, \tag{24}$$

such that $\dot\theta \equiv \langle B|\dot\theta^2|B\rangle^{1/2}$. To understand what the line element S is in a quantum context, we can define the operator

$$\check x^\alpha(t, \vec x) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3k\,\check e^\alpha\left[b_k\,\check x_k(t, \vec x) + b_k^\dagger\,\check x_k^*(t, \vec x)\right], \tag{25}$$

where $b_k^\dagger$ and b_k are the creation and annihilation operators of spacetime, such that $\langle B|[b_k, b_{k'}^\dagger]|B\rangle = \delta^{(3)}(\vec k - \vec k')$ and $\check e^\alpha = \epsilon^\alpha_{\beta\gamma\delta}\check e^\beta\check e^\gamma\check e^\delta$, where $\epsilon^\alpha_{\beta\gamma\delta}$ are the Levi-Civita symbols. In this framework, we must understand that the exact differential related to a coordinate x^α,

$$\mathrm{d}x^\alpha|B\rangle = \bar U^\alpha \mathrm{d}S|B\rangle = \delta\check x^\alpha(x^\beta)|B\rangle, \tag{26}$$

is the eigenvalue that results when we apply the operator $\delta\check x^\alpha(x^\beta)$ on the background quantum state $|B\rangle$, defined as a Fock space on the Riemann manifold. The Weyl line element is given by

$$\mathrm{d}S^2\,\delta_{BB'} = \left(\bar U_\alpha\bar U^\alpha\right)\mathrm{d}S^2\,\delta_{BB'} = \langle B\left|\delta\check x_\alpha\delta\check x^\alpha\right|B'\rangle. \tag{27}$$

Hence, the differential Weyl line element $\mathrm{d}S$ provides the displacement of the quantum trajectories with respect to the "classical" (Riemann) ones: $\mathrm{d}\bar S^2 = \bar g_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta$.

3.3 Quantization of modes

The equation of motion for the modes $\xi_k(t)$ is

$$\ddot\xi_c(t) + 3\frac{\dot a}{a}\dot\xi_c(t) + \frac{k^2}{a(t)^2}\xi_c(t) = 0. \tag{28}$$

The annihilation and creation operators B_k and $B_k^\dagger$ satisfy the usual commutation algebra

$$\left[A_k, A_{k'}^\dagger\right] = \delta^{(3)}(\vec k - \vec k'), \quad [A_k, A_{k'}] = \left[A_k^\dagger, A_{k'}^\dagger\right] = 0. \tag{29}$$

Using the commutation relation (29) and the Fourier expansions (21), we obtain the normalization condition for the modes. For convenience we shall re-define the dimensionless time: $\tau = b\,t$, where $b = \sqrt{\frac{2\Lambda}{3}} = \frac{1}{a_0}$, so that the normalization condition for $\xi_c(\tau)$ is

$$\xi_k(t)\frac{\mathrm{d}\xi_k^*(\tau)}{\mathrm{d}\tau} - \xi_k^*(\tau)\frac{\mathrm{d}\xi_k(\tau)}{\mathrm{d}\tau} = i\left(\frac{a_0}{a(\tau)}\right)^3, \tag{30}$$

where the asterisk denotes the complex conjugated. The general solution for the modes $\xi_k(\tau)$ is

$$\begin{aligned}
\xi_k(\tau) = {} &C_1\frac{\sinh(\tau)}{\sqrt{2\cosh^2(\tau) - 1}}\\
&\times\mathrm{Hn}\left[-1, \frac{k^2 - 1}{4}; 0, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}; -\tanh^2(\tau)\right]\\
&+C_2\frac{\cosh(\tau)}{\sqrt{2\cosh^2(\tau) - 1}}\\
&\times\mathrm{Hn}\left[-1, \frac{k^2 + 1}{4}; -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}; -\tanh^2(\tau)\right],
\end{aligned} \tag{31}$$

where $\mathrm{Hn}[a, q; \alpha, \beta, \gamma, \delta; z] = \sum_{j=0}^\infty c_j\, z^j$ is the Heun function. Since the Heun functions are written as infinite series, we can make a series expansion in both sides of (30), in order to obtain the restrictions for the coefficients C_1 and C_2, and the wavenumber values k. The polynomial expansion of $\xi_k(t)\left[\xi_k^*(\tau)\right]' - \xi_k^*(t)\left[\xi_k(\tau)\right]' = i\left(\frac{a_0}{a(\tau)}\right)^3$ can be written as a series expansion,

$$\begin{aligned}
&\xi_k(t)\left(\xi_k^*(\tau)\right)' - \xi_k^*(t)\left(\xi_k(\tau)\right)' - i\left(\frac{a_0}{a(\tau)}\right)^3\\
&= \sum_{N=1}^\infty f_N(k)\,\tau^N = 0,
\end{aligned} \tag{32}$$

where $f_N(k_n^{(N)}) = 0$, for each N. To simplify the notation we denote the τ-derivative with a prime. There are $2N$ modes for each Nth order of the expansion, which comes from the roots of each equation. These roots provide us with the discrete quantum modes coming from the quantization of θ. From the zeroth order of the expansion (in τ), we obtain $C_2 = -i\,C_1/2$. Hence, we shall choose $C_1 = 1$ and $C_2 = -i/2$ in the general solution (31). The first eight terms of the series are

$$\begin{aligned}
\sum_{N=1}^\infty f_N(k)\,\tau^N = {} &\frac{2}{3}i\left(k^2 + 2\right)\tau^2 + 4/3\,i\left(k^2 + 2\right)\tau^3\\
&+\frac{2}{45}i\left(-43k^2 - 95 + 3k^4\right)\tau^4\\
&+\frac{4}{15}i\left(-21k^2 - 45 + k^4\right)\tau^5\\
&+\frac{1}{945}i\left(3667k^2 + 8996 - 474k^4 + 12k^6\right)\tau^6\\
&+\frac{2}{315}i\left(2263k^2 + 5248 - 214k^4 + 4k^6\right)\tau^7\\
&+\frac{1}{14175}i\left(-99563k^2 - 268995\right.\\
&\left.+17513k^4 - 820k^6 + 10k^8\right)\tau^8 + \cdots = 0.
\end{aligned} \tag{33}$$

From each k-dependent polynomial we obtain the roots, which provide us the permitted modes that guarantee the quantization of θ. There are infinite discrete permitted modes. The expectation value for $\dot{\theta}^2$ on the quantum state $|B\rangle >$, calculated on the background semi-Riemann hypersurface, is

$$\left\langle B|\dot{\theta}^2|B \right\rangle = \frac{a_0^2}{(2\pi^2)} \sum_{n=1}^{\infty} \left(k_n\, k_n^* \right) \left[\left[\xi_{k_n}(\tau) \right]' \left[\xi_{k_n}^*(\tau) \right]' \right], \quad (34)$$

such that $\left(\xi_{k_n}(\tau) \right)'$ is a τ-derivative of ξ_{k_n} evaluated at $k = k_n$: $(\xi_k(\tau))'|_{k=k_n}$ and k_n^2 are the complex roots of the polynomials $f_N(k) = 0$ in (33). Equation (34) can be alternatively written for each mode k_n, thus:

$$\left\langle B|\dot{\theta}^2|B \right\rangle_{k_n} = \frac{a_0^2}{(2\pi^2)} \left(k_n\, k_n^* \right) \left[\left[\xi_{k_n}(\tau) \right]' \left[\xi_{k_n}^*(\tau) \right]' \right], \quad (35)$$

which takes into account the contribution of each k_n-mode in $\left\langle B|\dot{\theta}^2|B \right\rangle$. We see that the first modes have roots in $k_{1,2} = \pm\sqrt{2}\,i$. The modes for these roots have the same contribution in the expression for $\left\langle B|\dot{\theta}^2|B \right\rangle_{k_{1,2}}$. The modes of the second polynomial in (33) are the same. The modes of the third polynomial come from the roots of $3\,k^4 - 43\,k^2 - 95 = 0$, they are $k_{3,4,5,6} = 1.394736996\,i, -1.394736996\,i, 4.034677759, -4.034677759$. The modes of the fourth polynomial have roots in $k_{7,8,9,10} = 1.399977069\,i, -1.399977069\,i, 4.791652721, -4.791652721$. In Fig. 1 we have drawn the contributions of the modes k_1 (red), k_3 (blue), k_5 (black), and k_7 (green), to $\left\langle B|\dot{\theta}^2|B \right\rangle_{k_n}$, for $a_0 = G^{1/2}$. Notice that all the contributions tend asymptotically to zero for a few Planck times ($t_{\rm p} \simeq 10^{-43}\,sec$). In other words, the excitations of the background (i.e., the Riemann vacuum), are significant at the moment of the big bang, but decrease to zero when $\dot{H}/H^2 \to 0$. This corresponds just to the approximation to the de Sitter (inflationary) regime.

4 Final comments

We have studied back-reaction effects in a pre-inflationary universe using RQG. This formalism makes possible the non-perturbative treatment of the vacuum fluctuations of the spacetime, by making a displacement from a semi-Riemann description to a Weyl one. In this framework the Einstein equations are exactly valid on the Riemann manifold, but the quantum effects are described on the Weyl one by the field θ. In the Weyl manifold the cosmological constant is not an invariant, but a Lagrangian density $\Lambda(\theta, \theta^\alpha)$ with which we define the quantum action $\mathcal{W}$. The dynamics of the geometrical field θ is that of a free scalar field and describes the dynamics of the geometrical quantum fluctuations with respect to
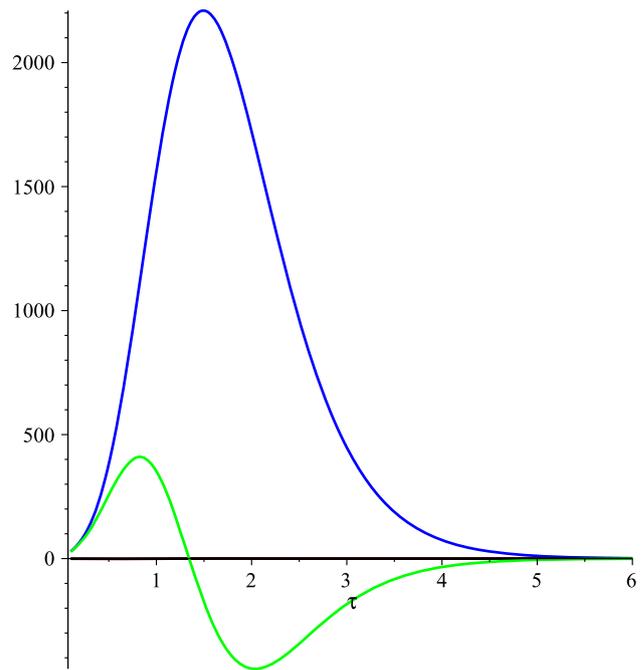
Fig. 1 Contributions to $\left\langle B|\dot{\theta}^2|B \right\rangle_{k_n}$ drawn for $a_0 = G^{1/2}$, due to the modes $k_1 = 1.414213562\,i$ (*red*), $k_3 = 1.394736996\,i$ (*blue*), $k_5 = 4.034677761$ (*black*), and $k_7 = -4.082914929 + 0.6506152090\,i$ (*green*)

the Riemann (classical) background. When we apply this formalism to the pre-inflationary scenario, which describes the beginning of the universal expansion from nothing, we see that the modes of the geometrical field θ are discrete, but infinite in number. The contribution of some of these to the variation of the energy density were drawn in Fig. 1. Notice that all the modes' contributions become asymptotically null for $\tau \gg 1$. Another distinctive characteristic of these modes is that they are not unstable, as in the case of the modes of θ during inflation [14]. A subsequent study of how to describe the transition from pre-inflation to inflation remains pendent. This issue will be the subject of future work.

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2 k_n^* is the complex conjugate.

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