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A 3-3-1 model with right-handed neutrinos based on the Δ (27) family symmetry

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Abstract We present the first multiscalar singlet extension of the original 3-3-1 model with right-handed neutrinos, based on the Δ (27) family symmetry, supplemented by the $Z_4 \otimes Z_8 \otimes Z_{14}$ flavor group, consistent with current low energy fermion flavor data. In the model under consideration, the light active neutrino masses are generated from a double seesaw mechanism and the observed pattern of charged fermion masses and quark mixing angles is caused by the breaking of the Δ (27) $\otimes Z_4 \otimes Z_8 \otimes Z_{14}$ discrete group at very high energy. Our model has only 14 effective free parameters, which are fitted to reproduce the experimental values of the 18 physical observables in the quark and lepton sectors. The obtained physical observables for the quark sector agree with their experimental values, whereas those for the lepton sector also do, only for the inverted neutrino mass hierarchy. The normal neutrino mass hierarchy scenario of the model is disfavored by the neutrino oscillation experimental data. We find an effective Majorana neutrino mass parameter of neutrinoless double beta decay of $m_{\beta\beta} = 22$ meV, a leptonic Dirac CP violating phase of 34°, and a Jarlskog invariant of about 10^{-2} for the inverted neutrino mass spectrum.

1 Introduction

The observation of the 125 GeV Higgs boson at the LHC [1,2], confirmed the great success of the Standard Model (SM) as the right theory of electroweak interactions. Despite the couplings of this scalar state with the SM particles being very consistent with the properties expected of the SM Higgs boson, the possibility that new scalar states may exist and play a role in the Electroweak Symmetry Breaking (EWSB) mechanism is still open. The current priority of the LHC experiments will be to make very precise measurements of the Higgs boson selfcouplings as well as of its couplings to the SM particles with the aim to shed light on the underlying theory behind Electroweak Symmetry Breaking (EWSB). Furthermore, despite its great experimental success, there are several aspects not explained in the context of the SM, such as, for example, the smallness of the neutrino masses, the observed pattern of fermion masses and mixing angles, and the existence of three generations of fermions. The SM does not explain why in the quark sector the mixing angles are small, whereas in the lepton sector two of the mixing angles are large and one is small. The Daya Bay [3], T2K [4], MINOS [5], Double CHOOZ [6], and RENO [7] neutrino oscillation experiments clearly indicate that at least two of the light active neutrinos have non-vanishing masses. These experiments have provided important constraints on the neutrino mass squared splittings and leptonic mixing parameters [8]. Furthermore, the SM does not provide an explanation for the charged fermion mass hierarchy, which is extended over a range of about 11 orders of magnitude, from the neutrino mass scale up to the top quark mass.

The unexplained SM fermion mass and mixing pattern motivates us to consider models with extended symmetry and larger scalar and/or fermion content, addressed to explain the fermion mass and mixing pattern. There are two approaches to describe the observed fermion mass and mixing pattern: assuming Yukawa textures [9–35] and implementing discrete flavor groups in extensions of the SM (see Refs. [36–39] for recent reviews on flavor symmetries). Recently, extensions of the SM with A_4 [40–61], S_3 [62–76], S_4 [77–86], D_4 [87–96], T_7 [97–106], T_{13} [107–110], T' [111–116], and $\Delta(27)$ [117–130] family symmetries have been considered to address the flavor puzzle of the SM.

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On the other hand, the existence of three fermion families, which is not explained in the context of the SM, can be understood in the framework of models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry, called 3-3-1 models for short, where $U(1)_X$ is a nonuniversal family symmetry that distinguishes the third fermion family from the first and second ones [25,59,60,72,73,103,105,131–161]. These models have several interesting features. First, the existence of three generations of fermions is a consequence of the chiral anomaly cancellation and the asymptotic freedom in QCD. Second, the large mass hierarchy between the heaviest quark family and the two lighter ones can be understood from the fact that the former has a different $U(1)_X$ charge from the latter. Third, these models include a natural Peccei–Quinn symmetry, which addresses the strong-CP problem [162–165]. Finally, versions with heavy sterile neutrinos include cold dark matter candidates as weakly interacting massive particles (WIMPs) [166]. Besides that, the 3-3-1 models can explain the 750 GeV diphoton excess recently reported by ATLAS and CMS [167–170] as well as the 2 TeV diboson excess found by ATLAS [171].

In the 3-3-1 models, the electroweak gauge symmetry is broken in two steps as follows. First the $SU(3)_L \otimes U(1)_X$ symmetry is broken down to the SM electroweak group $SU(2)_L \otimes U(1)_Y$ by one heavy $SU(3)_L$ triplet field acquiring a Vacuum Expectation Value (VEV) at high energy scale v_χ, thus giving masses to non SM fermions and gauge bosons. Second, the usual EWSB mechanism is triggered by the remaining lighter triplets, with VEVs at the electroweak scale v_ρ and v_η, thus providing SM fermions and gauge bosons with masses [25].

In Ref. [130] we have proposed a 3-3-1 model with Δ (27) flavor symmetry supplemented by the $\mathrm{U}(1)_{\mathcal{L}}$ new lepton global symmetry that enforces to have different scalar fields in the Yukawa interactions for the charged lepton, neutrino, and quark sectors, thus allowing us to treat these sectors independently. The scalar sector of that model includes 10 $SU(3)_L$ scalar triplets and three $SU(3)_L$ scalar antisextets. The $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathcal{L}} \otimes \Delta$ (27) assignments of the fermion sector of our previous model require that these 10 $SU(3)_L$ scalar triplets be distributed as follows: four for the quark sector, three for the charged lepton sector and three for the neutrino sector. Furthermore the three $SU(3)_L$ scalar antisextets are needed to implement a type-I seesaw mechanism. In that model, light active neutrino masses are generated from type-I and type-II seesaw mechanisms, mediated by three heavy right-handed Majorana neutrinos and three $SU(3)_L$ scalar antisextets, respectively. Since the Yukawa terms in that model are renormalizable, to explain the charged fermion mass pattern one needs to impose a strong hierarchy among the charged fermion Yukawa couplings of the model.

It is interesting to find an alternative and better explanation for the SM fermion mass and mixing hierarchy, by formulating a 3-3-1 model with less scalar content than our previous model of Ref. [130]. To this end, we propose an alternative and improved version of the 3-3-1 model based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \Delta\,(27) \otimes Z_4 \otimes Z_8 \otimes Z_{14}$ symmetry that successfully describes the observed fermion mass and mixing pattern and is consistent with the current low energy fermion flavor data. The particular role of each discrete group factor is explained in detail in Sect. 2. The scalar sector of our model includes three $SU(3)_L$ scalar triplets and 22 $SU(3)_L$ scalar singlets, assigned into triplet and singlet irreducible representations of the $\Delta(27)$ discrete group. This scalar sector of our current $\Delta(27)$ flavor 3-3-1 model is more minimal than that one of our previous model of Ref. [130] and does not include $SU(3)_L$ scalar antisextets. Furthermore, our current model does not include the $\mathrm{U}(1)_{\mathcal{L}}$ new lepton global symmetry presented in our previous $\Delta(27)$ flavor 3-3-1 model. Unlike our previous $\Delta(27)$ flavor 3-3-1 model of Ref. [130], in our current 3-3-1 model, the charged fermion mass and quark mixing pattern can successfully be accounted for, by having all Yukawa couplings of order unity, and arises from the breaking of the $\Delta\,(27) \otimes Z_4 \otimes Z_8 \otimes Z_{14}$ discrete group at very high energy, triggered by $SU(3)_L$ scalar singlets acquiring vacuum expectation values much larger than the TeV scale.

In the following we summarize the most important differences of our current $\Delta(27)$ flavor 3-3-1 model with our previous 3-3-1 model also based on the $\Delta(27)$ family symmetry. First of all, the scalar sector of our current 3-3-1 model has three $SU(3)_L$ scalar triplets plus 22 very heavy $SU(3)_L$ scalar singlets. On the other hand, our previous $\Delta(27)$ flavor 3-3-1 model has a scalar sector composed of 10 $SU(3)_L$ scalar triplets and three $SU(3)_L$ scalar antisextets. Second, the charged fermion mass and quark mixing pattern can successfully be accounted for in our current 3-3-1 model with $\Delta(27)$ family symmetry by having the Yukawa couplings of order unity, whereas in our previous $\Delta(27)$ flavor 3-3-1 model, a strong hierarchy of the Yukawa couplings is needed to accommodate the current pattern of charged fermion masses and the CKM quark mixing matrix is predicted to be equal to the identity matrix. Third, in our current 3-3-1 model with $\Delta(27)$ family symmetry the light active neutrino masses arise from a double seesaw mechanism whereas in our previous $\Delta(27)$ flavor 3-3-1 model, type-I and type-II seesaw mechanisms generate the masses for the light active neutrinos. Finally, our current 3-3-1 model with $\Delta(27)$ family symmetry, does not include the $\mathrm{U}(1)_{\mathcal{L}}$ new lepton global symmetry presented in our previous $\Delta(27)$ flavor 3-3-1 model, but it has instead a $Z_4 \otimes Z_8 \otimes Z_{14}$ discrete symmetry, whose breaking at very high energy gives rise to the observed pattern of charged fermion masses and quark mixing angles.

It is noteworthy that our previous $\Delta(27)$ flavor 3-3-1 model corresponds to an extension of the original 3-3-1 model with right-handed Majorana neutrinos (which includes 3 $SU(3)_L$ scalar triplets in its scalar spectrum), where seven extra $SU(3)_L$ scalar triplets and 3 $SU(3)_L$ scalar antisextets are added to build the charged fermion and neutrino Yukawa terms needed to give masses to SM charged fermions and light active neutrinos. On the other hand, in our current $\Delta(27)$ flavor 3-3-1 model, one preserves the content of particles of the 3-3-1 model with right-handed Majorana neutrinos, but we add additional very heavy $SU(3)_L$ singlet scalar fields with quantum numbers that allow one to build Yukawa terms invariant under the local and discrete groups. Consequently our current model corresponds to the first multiscalar singlet extension of the original 3-3-1 model with right-handed neutrinos, based on the $\Delta(27)$ family symmetry. As these singlet scalars fields are assumed to be much heavier than the 3 $SU(3)_L$ scalar triplets, our model at low energies reduces to the 3-3-1 model with right-handed neutrinos.

In this paper we propose the first implementation of the $\Delta(27)$ flavor symmetry in a multiscalar singlet extension of the original 3-3-1 model with right-handed neutrinos. In our model, light active neutrino masses arise from a double seesaw mechanism mediated by three heavy right-handed Majorana neutrinos. This paper is organized as follows. In Sect. 2 we outline the proposed model. In Sect. 3 we discuss the implications of our model in masses and mixings in the lepton sector. In Sect. 4 we present a discussion of quark masses and mixings, followed by a numerical analysis. Finally we conclude in Sect. 5. Appendix A provides a description of the Δ (27) discrete group. Appendix B includes a discussion of the scalar potential for two $\Delta(27)$ scalar triplets and its minimization equations.

2 The model

The first 3-3-1 model with right-handed Majorana neutrinos in the $SU(3)_L$ lepton triplet was considered in [134]. However, that model cannot describe the observed pattern of fermion masses and mixings, due to the unexplained hierarchy among the large number of Yukawa couplings in the model. Below we consider a multiscalar singlet extension of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) model with right-handed neutrinos, which successfully describes the SM fermion mass and mixing pattern. In our model the full symmetry $\mathcal{G}$ experiences the following three-step spontaneous breaking:

$$\begin{aligned}\mathcal{G} &= SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes \Delta(27)\\ &\otimes Z_4 \otimes Z_8 \otimes Z_{14} \xrightarrow{\Lambda_{\text{int}}} SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{v_\chi}\\ &SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v_\eta, v_\rho} SU(3)_C \otimes U(1)_Q ,\end{aligned} \tag{2.1}$$

and the symmetry breaking scales obey the relation $\Lambda_{\text{int}} \gg v_\chi \gg v_\eta, v_\rho$.

We define the electric charge of our 3-3-1 model as a combination of the $SU(3)$ generators and the identity, as follows:

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + XI, \tag{2.2}$$

with $I = \text{diag}(1, 1, 1)$, $T_3 = \frac{1}{2}\text{diag}(1, -1, 0)$, and $T_8 = (\frac{1}{2\sqrt{3}})\text{diag}(1, 1, -2)$ for the triplet.

From the requirement of anomaly cancellation, it follows that the fermions of our model are assigned in the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$\begin{aligned}
Q_L^{1,2} &= \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), \quad \begin{cases} D_R^{1,2} : (3, 1, -1/3), \\ U_R^{1,2} : (3, 1, 2/3), \\ J_R^{1,2} : (3, 1, -1/3), \end{cases}\\
Q_L^3 &= \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), \quad \begin{cases} U_R^3 : (3, 1, 2/3), \\ D_R^3 : (3, 1, -1/3), \\ T_R : (3, 1, 2/3), \end{cases}\\
L_L^{1,2,3} &= \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), \quad \begin{cases} e_R^{1,2,3} : (1, 1, -1), \\ N_R^{1,2,3} : (1, 1, 0), \end{cases}
\end{aligned} \tag{2.3}$$

where U_L^i and D_L^i for $i = 1, 2, 3$ are three up- and down-type quark components in the flavor basis, while ν_L^i and e_L^i are the neutral and charged lepton families. The right-handed fermions transform as singlets under $SU(3)_L$ with $U(1)_X$ quantum numbers equal to their electric charges. Furthermore, the model has the following heavy fermions: a single flavor quark T with electric charge $2/3$, two flavor quarks $J^{1,2}$ with charge $-1/3$, three neutral Majorana leptons $(\nu^{1,2,3})_L^c$, and three right-handed Majorana leptons $N_R^{1,2,3}$.

Regarding the scalar sector of the 3-3-1 model with right-handed Majorana neutrinos, we assign the scalar fields to the following $[SU(3)_L, U(1)_X]$ representations:

$$\begin{aligned}
\chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3),\\
\rho &= \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\upsilon_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3),\\
\eta &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\upsilon_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3, -1/3).
\end{aligned} \tag{2.4}$$

We extend the scalar sector of the 3-3-1 model with right-handed Majorana neutrinos by adding the following $SU(3)_L$ scalar singlets:

$$\begin{aligned}&\phi:(1,0),\sigma\sim(1,0),\xi_n:(1,0),n=1,2,\\&\tau_j:(1,0),\Xi_j:(1,0),S_j:(1,0),j=1,2,3,\\&\Phi_j:(1,0),\Omega_j:(1,0),\Theta_j:(1,0),j=1,2,3.\end{aligned}\tag{2.5}$$

The scalar fields are assigned to different singlet and triplet representations of the $\Delta(27)$ discrete group, as follows:

$$\begin{aligned}&\eta\sim\mathbf{1_{1,0}},\rho\sim\mathbf{1_{2,0}},\chi\sim\mathbf{1_{0,0}},\sigma\sim\mathbf{1_{0,0}},\\&\phi\sim\mathbf{1_{0,0}},\tau_1\sim\mathbf{1_{0,0}},\tau_2\sim\mathbf{1_{0,2}},\\&\tau_3\sim\mathbf{1_{0,2}},\xi_1\sim\mathbf{1_{0,0}},\xi_2\sim\mathbf{1_{0,0}},\\&S\sim\mathbf{3},\Xi\sim\mathbf{3},\Phi\sim\mathbf{3},\Omega\sim\mathbf{3},\Theta\sim\mathbf{3}.\end{aligned}\tag{2.6}$$

The $Z_4\otimes Z_8\otimes Z_{14}$ assignments of the scalar fields are

$$\begin{aligned}&\eta\sim(1,1,1),\rho\sim(1,1,1),\chi\sim\left(i,1,e^{-\frac{i\pi}{7}}\right),\\&\sigma\sim\left(1,1,e^{-\frac{i\pi}{7}}\right),\phi\sim(-i,-1,-1),\\&\tau_1\sim\left(-1,-i,e^{\frac{3i\pi}{7}}\right),\tau_2\sim\left(1,e^{-\frac{i\pi}{4}},e^{\frac{4i\pi}{7}}\right),\\&\tau_3\sim\left(1,-i,e^{\frac{5i\pi}{7}}\right),\xi_1\sim\left(1,e^{-\frac{i\pi}{4}},e^{\frac{i\pi}{7}}\right),\\&\xi_2\sim\left(-i,1,e^{\frac{2i\pi}{7}}\right),S\sim\left(1,e^{-\frac{i\pi}{4}},1\right),\\&\Xi\sim\left(1,1,e^{-\frac{2i\pi}{7}}\right),\Phi\sim\left(1,1,e^{\frac{i\pi}{7}}\right),\\&\Omega\sim(-1,1,1),\Theta\sim\left(-1,1,e^{-\frac{i\pi}{7}}\right).\end{aligned}\tag{2.7}$$

Regarding leptons, we group left-handed leptons and right-handed Majorana neutrinos in $\Delta(27)$ triplets, whereas right-handed charged leptons are assigned as $\Delta(27)$ triplets, as follows:

$$\begin{aligned}&L_L\sim\mathbf{3},e_R\sim\mathbf{1_{1,0}},\mu_R\sim\mathbf{1_{2,0}},\\&\tau_R\sim\mathbf{1_{0,0}},N_R\sim\mathbf{3}.\end{aligned}\tag{2.8}$$

The $Z_4\otimes Z_8\otimes Z_{14}$ assignments for the leptons are

$$\begin{aligned}&L_L\sim(i,1,1),e_R\sim\left(i,e^{\frac{i\pi}{4}},-1\right),\mu_R\sim\left(i,e^{\frac{i\pi}{4}},e^{\frac{4i\pi}{7}}\right),\\&\tau_R\sim\left(i,e^{\frac{i\pi}{4}},e^{\frac{2i\pi}{7}}\right),N_R\sim\left(1,1,e^{\frac{i\pi}{7}}\right).\end{aligned}\tag{2.9}$$

Regarding quarks, we assign quark fields into different singlet representations of the $\Delta(27)$ discrete group, as follows:

$$\begin{aligned}&Q_L^1\sim\mathbf{1_{0,0}},Q_L^2\sim\mathbf{1_{0,0}},Q_L^3\sim\mathbf{1_{0,0}},\\&U_R^1\sim\mathbf{1_{2,0}},U_R^2\sim\mathbf{1_{2,0}},U_R^3\sim\mathbf{1_{2,0}},\\&D_R^1\sim\mathbf{1_{1,0}},D_R^2\sim\mathbf{1_{1,1}},D_R^3\sim\mathbf{1_{1,0}},\\&T_R\sim\mathbf{1_{0,0}},J_R^1\sim\mathbf{1_{0,0}},J_R^2\sim\mathbf{1_{0,0}}.\end{aligned}\tag{2.10}$$

The $Z_4\otimes Z_8\otimes Z_{14}$ assignments for the quarks are

$$\begin{aligned}&Q_L^1\sim(1,-i,1),Q_L^2\sim\left(1,e^{-\frac{i\pi}{4}},1\right),Q_L^3\sim(1,1,1),\\&U_R^1\sim(i,i,1),U_R^2\sim\left(-1,e^{\frac{i\pi}{4}},1\right),U_R^3\sim(1,1,1),\\&D_R^1\sim\left(i,i,e^{-\frac{i\pi}{7}}\right),D_R^2\sim(1,1,1),D_R^3\sim(i,1,1),\\&T_R\sim\left(-i,1,e^{\frac{i\pi}{7}}\right),\\&J_R^1\sim\left(i,-i,e^{-\frac{i\pi}{7}}\right),J_R^2\sim\left(i,e^{-\frac{i\pi}{4}},e^{-\frac{i\pi}{7}}\right).\end{aligned}\tag{2.11}$$

Here the dimensions of the $\Delta(27)$ irreducible representations are specified by the numbers in boldface. As regards the lepton sector, we recall that the left- and right-handed leptons are grouped into $\Delta(27)$ triplet and $\Delta(27)$ singlet irreducible representations, respectively, whereas the right-handed Majorana neutrinos are unified into a $\Delta(27)$ triplet. Regarding the quark sector, we assign the quarks fields into $\Delta(27)$ singlet representations. Specifically, we assign the left-handed $SU(3)_L$ quark triplets and right-handed exotic quarks as $\Delta(27)$ trivial singlets, whereas the right-handed SM quarks are assigned as $\Delta(27)$ nontrivial singlets. Furthermore, it is worth mentioning that the $SU(3)_L$ scalar triplets are assigned to one $\Delta(27)$ trivial and two $\Delta(27)$ nontrivial singlet representations, whereas the $SU(3)_L$ scalar singlets are accommodated into five $\Delta(27)$ triplets, six $\Delta(27)$ trivial singlets and four $\Delta(27)$ nontrivial singlets. Out of the five $SU(3)_L$ scalar singlets $\Delta(27)$ triplets, only one is charged under the Z_8 discrete symmetry whereas the remaining ones are Z_8 neutral. As we will see in the following, the Z_8 discrete symmetry separates the $\Delta(27)$ scalar triplets participating in the charged lepton Yukawa interactions from those one appearing in the neutrino Yukawa terms. Furthermore, as regards the Z_8 neutral $\Delta(27)$ scalar triplets participating in the neutrino Yukawa interactions, it is worth mentioning that they are distinguished by their Z_4 charges. Those Z_8 neutral $\Delta(27)$ scalar triplets, transforming trivially under the Z_4 symmetry, contribute to the right-handed Majorana neutrino mass matrix, whereas the remaining Z_8 neutral $\Delta(27)$ triplet scalar fields are Z_4 charged and give rise to the Dirac neutrino mass matrix.

With the above particle content, the following Yukawa terms for the quark and lepton sectors arise:

$$\begin{aligned}-\mathcal{L}_Y^{(Q)}=&\,y_{11}^{(U)}\overline{Q}_L^1\rho^*U_R^1\frac{\phi\sigma^7}{\Lambda^8}+y_{22}^{(U)}\overline{Q}_L^2\rho^*U_R^2\frac{\tau_1\sigma^3}{\Lambda^4}\\&+y_{23}^{(U)}\overline{Q}_L^2\rho^*U_R^3\frac{\xi_1\sigma}{\Lambda^2}+y_{13}^{(U)}\overline{Q}_L^1\rho^*U_R^3\frac{\xi_1^2\sigma^2}{\Lambda^4}\\&+y_{33}^{(U)}\overline{Q}_L^3\eta U_R^3+y_{11}^{(D)}\overline{Q}_L^1\eta^*D_R^1\frac{\phi\sigma^6}{\Lambda^7}\\&+y_{22}^{(D)}\overline{Q}_L^2\eta^*D_R^2\frac{\tau_2\sigma^4}{\Lambda^5}+y_{12}^{(D)}\overline{Q}_L^1\eta^*D_R^2\frac{\tau_3\sigma^5}{\Lambda^6}\end{aligned}$$

$$+y_{33}^{(D)}\overline{Q}_L^3\rho D_R^3\frac{\xi_2\sigma^2}{\Lambda^3}+y_1^{(J)}\overline{Q}_L^1\chi^* J_R^1$$
$$+y_2^{(J)}\overline{Q}_L^2\chi^* J_R^2+y^{(T)}\overline{Q}_L^3\chi T_R+H.c., \quad (2.12)$$
$$-\mathcal{L}_Y^{(L)}=h_{\rho e}^{(L)}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{0,0}}}e_R\frac{\sigma^7}{\Lambda^8}+h_{\rho\mu}^{(L)}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{1,0}}}\mu_R\frac{\sigma^4}{\Lambda^5}$$
$$+h_{\rho\tau}^{(L)}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{2,0}}}\tau_R\frac{\sigma^2}{\Lambda^3}+h_\chi^{(L)}\left(\overline{L}_L\chi N_R\right)_{\mathbf{1_{0,0}}}$$
$$+\frac{h_{1N}}{2}\left(\overline{N}_R N_R^C\right)_{\mathbf{3}_{S_1}}\Xi^*+\frac{h_{2N}}{2}\left(\overline{N}_R N_R^C\right)_{\mathbf{3}_{S_1}}\Phi^*\frac{\sigma}{\Lambda}$$
$$+h_{3N}\left(\overline{N}_R N_R^C\right)_{\mathbf{3}_{S_2}}\Xi^*+h_{4N}\left(\overline{N}_R N_R^C\right)_{\mathbf{3}_{S_2}}\Phi^*\frac{\sigma}{\Lambda}$$
$$+h_\rho^{(1)}\varepsilon_{abc}\left(\overline{L}_L^a\left(L_L^C\right)^b\right)_{\mathbf{3}_{S_2}}\left(\rho^*\right)^c\frac{\Omega^*}{\Lambda}$$
$$+h_\rho^{(2)}\varepsilon_{abc}\left(\overline{L}_L^a\left(L_L^C\right)^b\right)_{\mathbf{3}_{S_2}}\left(\rho^*\right)^c\frac{\Theta^*\sigma}{\Lambda^2}+H.c., \quad (2.13)$$

where $y_{ij}^{(U,D)}$ $(i,j=1,2,3)$, $y^{(T)}$, $y_m^{(J)}$, $h_\rho^{(m)}$ $(m=1,2)$, h_{sN} $(s=1,2,3,4)$, $h_\chi^{(L)}$, $h_{\rho e}^{(L)}$, $h_{\rho\mu}^{(L)}$, and $h_{\rho\tau}^{(L)}$ are $\mathcal{O}(1)$ dimensionless couplings. We assume that all of these dimensionless couplings are real, except for $y_{13}^{(U)}$, $h_{\rho\mu}^{(L)}$, and $h_{\rho\tau}^{(L)}$, taken to be complex. In the following we provide a justification for the aforementioned assumption. As shown in Sect. 3, having $h_{\rho\mu}^{(L)}$ and $h_{\rho\tau}^{(L)}$ complex is required to yield leptonic mixing angles consistent with the current neutrino oscillation experimental data. Furthermore, as shown in Sect. 4, the quark assignments under the different group factors of our model will give rise to the SM quark mass texture where the Cabbibo mixing arise from the down-type quark sector, whereas the up-type quark sector contributes to the remaining mixing angles. As indicated by the current low energy quark flavor data encoded in the standard parametrization of the quark mixing matrix, the complex phase responsible for CP violation in the quark sector is associated with the quark mixing angle in the 1–3 plane. Consequently, in order to reproduce the experimental values of the quark mixing angles and the CP violating phase, $y_{13}^{(U)}$ is required to be complex.

An explanation of the role of each discrete group factor of our model is provided in the following. The $\Delta(27)$, Z_4, and Z_8 discrete groups are crucial for reducing the number of model parameters, thus increasing the predictivity of our model and giving rise to predictive and viable textures for the fermion sector, consistent with the observed pattern of fermion masses and mixings, as will be shown later in Sects. 3 and 4. The Z_4 and Z_{14} symmetries reduce the number of parameters in the neutrino sector. Besides that, the Z_4 and Z_8 discrete group determine the allowed entries of the SM quark mass matrices. As a result of the $Z_4\otimes Z_8$ charge assignments for the SM quark sector given by Eq. (2.10), the Cabbibo mixing will arise from the down-type quark sector, whereas the up sector will contribute to the remaining mixing angles. Furthermore, thanks to the $\Delta(27)$ discrete symmetry, SM quarks do not mix with the exotic ones. This arises from the fact that the right-handed SM and exotic quarks are assigned as nontrivial and trivial $\Delta(27)$ singlets, respectively. The Z_{14} symmetry give rises to the hierarchical structure of the charged fermion mass matrices that yields the observed charged fermion mass and quark mixing pattern. Let us note that the five dimensional Yukawa operators $\frac{1}{\Lambda}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{0,0}}}e_R$, $\frac{1}{\Lambda}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{1,0}}}\mu_R$, and $\frac{1}{\Lambda}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{2,0}}}\tau_R$ are invariant under the $\Delta(27)$ family symmetry but do not respect the Z_{14} symmetry, as the right-handed charged leptons transform nontrivially under the Z_{14} cyclic group. Let us note that the Z_{14} symmetry is the smallest lowest cyclic symmetry, from which a charged lepton Yukawa term of dimension 12 can be built, by inserting $\frac{\sigma^7}{\Lambda^7}$ on the $\frac{1}{\Lambda}\left(\overline{L}_L\rho S\right)_{\mathbf{1_{0,0}}}e_R$ operator. It is noteworthy that the small value of the electron mass can naturally arise from the aforementioned charged lepton Yukawa term of dimension 12.

Furthermore, since the breaking of the $\Delta(27)\otimes Z_4\otimes Z_8\otimes Z_{14}$ discrete group gives rise to the charged fermion mass and quark mixing pattern, we set the VEVs of the $SU(3)_L$ singlet scalar fields ϕ, ξ_n $(n=1,2)$, τ_j, S_j $(j=1,2,3)$, and σ, with respect to the Wolfenstein parameter $\lambda=0.225$ and the model cutoff Λ, as follows:

$$v_S\sim v_\phi\sim v_{\tau_1}\sim v_{\tau_2}\sim v_{\tau_3}\sim v_{\xi_1}\sim v_{\xi_2}\sim v_\sigma\sim\Lambda_{\text{int}}=\lambda\Lambda. \quad (2.14)$$

Let us note that the $SU(3)_L$ singlet scalar fields ϕ, ξ_n $(n=1,2)$, τ_j, S_j, Ω_j, Θ_j $(j=1,2,3)$, and σ having the VEVs of the same order of magnitude are the ones that appear in the SM charged fermion Yukawa terms, thus playing an important role in generating the SM charged fermion masses and quark mixing angles.

As we will explain in the following, we are going to implement a double seesaw mechanism for the generation of the light active neutrino masses. To implement a double seesaw mechanism, we need very heavy right-handed Majorana neutrinos, which implies that the $SU(3)_L$ singlet scalars should acquire very large vacuum expectation values. In addition, in order to simplify our analysis of the scalar potential for the $\Delta(27)$ scalar triplets, we need that the $\Delta(27)$ scalar triplets Ξ and Φ contributing to the right-handed Majorana neutrino masses should acquire much lower VEVs than the $\Delta(27)$ scalar triplet S that gives rise to the charged lepton masses. That hierarchy in their VEVs will allow to neglect the mixings between these fields as follows from the method of recursive expansion of Ref. [172] and to treat their scalar potentials independently. Because of these reasons, we assume that the VEVs of $SU(3)_L$ singlet scalar fields Ξ_j, Φ_j $(j=1,2,3)$ satisfy the following hierarchy:

$$v_\chi\ll v_\Xi\sim v_\Phi\ll\Lambda_{\text{int}}. \quad (2.15)$$

Furthermore, implementing a double seesaw mechanism also requires that the $\Delta(27)$ scalar triplets Ω and Θ, contributing to the Dirac neutrino Yukawa terms, should acquire VEVs much lower than the electroweak symmetry breaking scale $v = 246$ GeV. Consequently, the scalar fields of our model obey the following hierarchy:

$$v_\Omega \sim v_\Theta \ll v_\rho \sim v_\eta \sim v \ll v_\chi \ll v_\Xi \sim v_\Phi \ll \Lambda_{\text{int}}. \tag{2.16}$$

Thus, the $SU(3)_L$ scalar singlets, presented in the right-handed Majorana neutrino Yukawa interactions, acquire very large vacuum expectation values, which implies that the Majorana neutrinos acquire very large masses, hence allowing one to implement a double seesaw mechanism to generate the light active neutrino masses. Consequently, the neutrino spectrum is composed of very light active neutrinos as well as heavy and very heavy sterile neutrinos.

In summary, for the reasons mentioned above and considering a very high model cutoff $\Lambda \gg v_\chi$, we set the vacuum expectation values (VEVs) of the $SU(3)_L$ scalar singlets at a very high energy, much larger than $v_\chi \approx \mathcal{O}(1)$ TeV, with the exception of the VEVs of Ω_j and Θ_j ($j = 1, 2, 3$), taken to be much smaller than the electroweak symmetry breaking scale $v = 246$ GeV. It is noteworthy the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes \Delta(27) \otimes Z_4 \otimes Z_8 \otimes Z_{14}$ symmetry is broken down to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, at the scale Λ_{int}, by the vacuum expectation values of the $SU(3)_L$ singlet scalar fields ϕ, ξ_n ($n = 1, 2$), τ_j, S_j ($j = 1, 2, 3$), and σ.

In the following we comment on the possible VEV patterns for the $\Delta(27)$ scalar triplets S, Ξ, Φ, Ω, and Θ. Since the VEVs of the $\Delta(27)$ scalar triplets satisfy the following hierarchy: $v_\Omega \sim v_\Theta \ll v_\Xi \sim v_\Phi \ll v_S$, the mixing angles of S and Ξ with Φ, Ω, and v_S are very small since they are suppressed by the ratios of their VEVs, which is a consequence of the method of recursive expansion proposed in Ref. [172]. Thus, the scalar potential for the $\Delta(27)$ scalar triplet S can be treated independently from the scalar potentials for the two sets of $\Delta(27)$ scalar triplets Ξ, Φ, and Ω and Θ. Furthermore, because of the reason mentioned above, one can treat the scalar potential for Ξ, Φ independently from the one that involves Ω and Θ. As shown in detail in Appendix B, the following VEV patterns for the $\Delta(27)$ scalar triplets are consistent with the scalar potential minimization equations for a large region of parameter space:

$$\begin{aligned}
&\langle S\rangle = \frac{v_S}{\sqrt{3}}(1,1,1)\,,\ \langle \Xi\rangle = v_\Xi(1,0,0)\,,\\
&\langle \Phi\rangle = v_\Phi(0,0,1)\,,\ \langle \Omega\rangle = v_\Omega(1,0,0)\,,\\
&\langle \Theta\rangle = v_\Theta(0,0,1)\,.
\end{aligned} \tag{2.17}$$

3 Lepton masses and mixings

From the lepton Yukawa terms given by Eq. (2.13), we find that the mass matrix for charged leptons takes the form

$$\begin{aligned}
&M_l = R_{lL}^\dagger P_l \,\mathrm{diag}\left(m_e, m_\mu, m_\tau\right),\\
&R_{lL} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1\\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega\end{pmatrix},\\
&P_l = \begin{pmatrix} 1 & 0 & 0\\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta}\end{pmatrix},\ \omega = e^{\frac{2\pi i}{3}},
\end{aligned} \tag{3.1}$$

α and β being the complex phases of $h_{\rho\mu}^{(L)}$ and $h_{\rho\tau}^{(L)}$, respectively, and the charged lepton masses given by

$$m_e = a_1^{(l)}\lambda^8\frac{v}{\sqrt{2}},\ m_\mu = a_2^{(l)}\lambda^5\frac{v}{\sqrt{2}},\ m_\tau = a_3^{(l)}\lambda^3\frac{v}{\sqrt{2}}. \tag{3.2}$$

$\lambda = 0.225$ is one of the Wolfenstein parameters, $v = 246$ GeV the electroweak symmetry breaking scale, and $a_i^{(l)}$ ($i = 1, 2, 3$) $\mathcal{O}(1)$ dimensionless parameters. Let us note that the charged lepton masses are connected with the scale of electroweak symmetry breaking, through their power dependence on the Wolfenstein parameter $\lambda = 0.225$, with $\mathcal{O}(1)$ coefficients.

Regarding the neutrino sector, we see that the neutrino mass terms take the form

$$-\mathcal{L}_{\text{mass}}^{(\nu)} = \frac{1}{2}\left(\overline{\nu_L^C}\ \ \overline{\nu_R}\ \ \overline{N_R}\right) M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C\end{pmatrix} + H.c., \tag{3.3}$$

where the $\Delta(27)$ family symmetry constrains the neutrino mass matrix to be of the form

$$\begin{aligned}
&M_\nu = \begin{pmatrix} 0_{3\times3} & M_D & 0_{3\times3}\\ M_D^T & 0_{3\times3} & M_\chi \\ 0_{3\times3} & M_\chi^T & M_R\end{pmatrix},\\
&M_D = \frac{h_\rho^{(1)} v_\rho v_\Omega}{2\Lambda}\begin{pmatrix} 0 & 1 & 0\\ -1 & 0 & -a \\ 0 & a & 0\end{pmatrix},\\
&M_\chi = h_\chi^{(L)}\frac{v_\chi}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0 \\ 0 & 0 & 1\end{pmatrix},\ M_R = h_{1N} v_\Xi \begin{pmatrix} 1 & y & 0\\ y & 0 & x \\ 0 & x & z\end{pmatrix},\\
&x = \frac{h_{3N}}{h_{1N}},\ y = \frac{h_{4N} v_\Phi}{h_{1N} v_\Xi},\ z = \frac{h_{2N} v_\Phi v_\sigma}{h_{1N} v_\Xi \Lambda},\ a = \frac{h_\rho^{(2)} v_\Theta v_\sigma}{h_\rho^{(1)} v_\Omega \Lambda}.
\end{aligned} \tag{3.4}$$

As the $SU(3)_L$ scalar singlets presented in the right-handed Majorana neutrino Yukawa interactions acquire very large vacuum expectation values, the Majorana neutrinos are very heavy, thus giving rise to a double seesaw mechanism that generates small masses for the active neutrinos.

The neutrino mass matrix is diagonalized by a rotation matrix, which is approximately given by [152]

$$\mathbb{U}=\begin{pmatrix} R_{\nu} & B_2U_{\chi} & 0 \\ -B_2^{\dagger}R_{\nu} & U_{\chi} & B_1U_R \\ 0 & B_1^{\dagger}U_{\chi} & U_R \end{pmatrix}, \tag{3.5}$$

with

$$B_1^{\dagger}=M_R^{-1}M_{\chi}^T,\; B_2^{\dagger}=M_D\left(M_{\chi}^T\right)^{-1}M_RM_{\chi}^{-1}, \tag{3.6}$$

and the neutrino mass matrices for the physical states take the form

$$M_{\nu}^{(1)}=M_D\left(M_{\chi}^T\right)^{-1}M_RM_{\chi}^{-1}M_D^T, \tag{3.7}$$

$$M_{\nu}^{(2)}=-M_{\chi}M_R^{-1}M_{\chi}^T, \tag{3.8}$$

$$M_{\nu}^{(3)}=M_R, \tag{3.9}$$

where $M_{\nu}^{(1)}$ is the light active neutrino mass matrix, whereas $M_{\nu}^{(2)}$ and $M_{\nu}^{(3)}$ are the heavy and very heavy sterile neutrino mass matrices, respectively. Thus, the double seesaw mechanism produces a neutrino spectrum composed of light active neutrinos, and heavy and very heavy sterile neutrinos. Furthermore, let us note that the neutrino mass matrices $M_{\nu}^{(1)}$, $M_{\nu}^{(2)}$, and $M_{\nu}^{(3)}$ are diagonalized by the rotation matrices R_{ν}, U_R and U_{χ}, respectively [152].

Using Eq. (3.7), we find that the light active neutrino mass matrix takes the form

$$\begin{aligned} M_{\nu}^{(1)} &= \frac{h_{1N}\left(h_{\rho}^{(1)}\right)^2 v_{\rho}^2 v_{\Omega}^2 v_{\Xi}}{2\left(h_{\chi}^{(L)}\right)^2 v_{\chi}^2\Lambda^2}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -a \\ 0 & a & 0 \end{pmatrix}\begin{pmatrix} 1 & y & 0 \\ y & 0 & x \\ 0 & x & z \end{pmatrix} \\ &\quad\times\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & a \\ 0 & -a & 0 \end{pmatrix} \\ &= \frac{h_{1N}\left(h_{\rho}^{(1)}\right)^2 v_{\rho}^2 v_{\Omega}^2 v_{\Xi}}{2\left(h_{\chi}^{(L)}\right)^2 v_{\chi}^2\Lambda^2} \\ &\quad\times\begin{pmatrix} 0 & -y-ax & 0 \\ -y-ax & za^2+1 & -a\left(y+ax\right) \\ 0 & -a\left(y+ax\right) & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & A & 0 \\ A & C & B \\ 0 & B & 0 \end{pmatrix}. \end{aligned} \tag{3.10}$$

Then we find that, for the normal (NH) and inverted (IH) neutrino mass hierarchies, the light active neutrino mass matrix is diagonalized by a rotation matrix R_{ν}, according to

$$\begin{aligned} &R_{\nu}^T M_{\nu}^{(1)} R_{\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix},\; m_{\nu_1}=0, \\ &m_{\nu_{2,3}}=\frac{C}{2}\mp\frac{1}{2}\sqrt{K},\ \text{for NH}, \\ &R_{\nu}=\begin{pmatrix} -\frac{B}{\sqrt{A^2+B^2}} & \frac{\sqrt{2}A}{\sqrt{K-C\sqrt{K}}} & \frac{\sqrt{2}A}{\sqrt{K+C\sqrt{K}}} \\ 0 & \frac{1}{\sqrt{2}}\frac{C-\sqrt{K}}{\sqrt{K-C\sqrt{K}}} & \frac{1}{\sqrt{2}}\frac{C+\sqrt{K}}{\sqrt{K+C\sqrt{K}}} \\ \frac{A}{\sqrt{A^2+B^2}} & \frac{\sqrt{2}B}{\sqrt{K-C\sqrt{K}}} & \frac{\sqrt{2}B}{\sqrt{K+C\sqrt{K}}} \end{pmatrix}, \end{aligned} \tag{3.11}$$

$$\begin{aligned} &R_{\nu}^T M_{\nu}^{(1)} R_{\nu}=\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & 0 \end{pmatrix},\; m_{\nu_{1,2}}=\frac{C}{2}\mp\frac{1}{2}\sqrt{K}, \\ &m_{\nu_3}=0,\ \text{for IH}, \\ &R_{\nu}=\begin{pmatrix} \frac{\sqrt{2}A}{\sqrt{K-C\sqrt{K}}} & \frac{\sqrt{2}A}{\sqrt{K+C\sqrt{K}}} & -\frac{B}{\sqrt{A^2+B^2}} \\ \frac{1}{\sqrt{2}}\frac{C-\sqrt{K}}{\sqrt{K-C\sqrt{K}}} & \frac{1}{\sqrt{2}}\frac{C+\sqrt{K}}{\sqrt{K+C\sqrt{K}}} & 0 \\ \frac{\sqrt{2}B}{\sqrt{K-C\sqrt{K}}} & \frac{\sqrt{2}B}{\sqrt{K+C\sqrt{K}}} & \frac{A}{\sqrt{A^2+B^2}} \end{pmatrix}, \end{aligned} \tag{3.12}$$

where

$$K=4A^2+4B^2+C^2. \tag{3.13}$$

Using the rotation matrices in the charged lepton sector V_L, given by Eq. (3.1), and in the neutrino sector R_{ν}, given by Eqs. (3.11) and (3.12) for the normal (NH) and inverted (IH) neutrino mass hierarchies, respectively, we find that the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix takes the form

$$\begin{aligned} U &= R_{lL}^{\dagger}P_lR_{\nu} \\ &= \begin{cases} \begin{pmatrix} -\frac{B-Ae^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} & \frac{2A+2Be^{i\beta}+e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2Be^{i\beta}+e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} \\ -\frac{B-A\omega e^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} & \frac{2A+2B\omega e^{i\beta}+\omega^2e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2B\omega e^{i\beta}+\omega^2e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} \\ -\frac{B-A\omega^2e^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} & \frac{2A+2B\omega^2e^{i\beta}+\omega e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2B\omega^2e^{i\beta}+\omega e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} \end{pmatrix} & \text{for NH}, \\ \begin{pmatrix} \frac{2A+2Be^{i\beta}+e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2Be^{i\beta}+e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} & -\frac{B-Ae^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} \\ \frac{2A+2B\omega e^{i\beta}+\omega^2e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2B\omega e^{i\beta}+\omega^2e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} & -\frac{B-A\omega e^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} \\ \frac{2A+2B\omega^2e^{i\beta}+\omega e^{i\alpha}\left(C-\sqrt{K}\right)}{\sqrt{6}\sqrt{K-C\sqrt{K}}} & \frac{2A+2B\omega^2e^{i\beta}+\omega e^{i\alpha}\left(C+\sqrt{K}\right)}{\sqrt{6}\sqrt{K+C\sqrt{K}}} & -\frac{B-A\omega^2e^{i\beta}}{\sqrt{3}\sqrt{A^2+B^2}} \end{pmatrix} & \text{for IH}. \end{cases} \end{aligned} \tag{3.14}$$

Let us note that, according to Eqs. (3.1), (3.11), and (3.12), the lepton sector of our model is described by eight effective free parameters that are fitted to reproduce the experimental values of the eight physical observables in the lepton sector, i.e., the three charged lepton masses, the two neutrino mass squared splittings and the three leptonic mixing

Table 1 Experimental ranges of neutrino squared mass differences and leptonic mixing angles, from Ref. [8], for the case of inverted neutrino mass spectrum

Parameter	$\Delta m_{21}^2(10^{-5}\text{eV}^2)$	$\Delta m_{13}^2(10^{-3}\text{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\text{exp}}$	$\left(\sin^2\theta_{23}\right)_{\text{exp}}$	$\left(\sin^2\theta_{13}\right)_{\text{exp}}$
Best fit	7.60	2.38	0.323	0.573	0.0240
1σ range	7.42 – 7.79	2.32 – 2.43	0.307 – 0.339	0.530 – 0.598	0.0221 – 0.0259
2σ range	7.26 – 7.99	2.26 – 2.48	0.292 – 0.357	0.432 – 0.621	0.0202 – 0.0278
3σ range	7.11 – 8.11	2.20 – 2.54	0.278 – 0.375	0.403 – 0.640	0.0183 – 0.0297

angles. Despite this parametric freedom, we found that the normal hierarchy scenario of our model leads to a large value of the reactor mixing angle, not consistent with the experimental data on neutrino oscillations. On the contrary, for the case of inverted hierarchy, as we will see in the following, our obtained physical parameters in the lepton sector are in excellent agreement with the experimental data. We fit the parameters A, B, C, α, and β to reproduce the experimental values of the neutrino mass squared splittings and three leptonic mixing angles. By varying the parameters A, B, C, α, and β, we find the following best fit result:

$$\begin{aligned}
&m_{\nu_2} = \sqrt{\Delta m_{21}^2 + \Delta m_{13}^2} \approx 50\,\text{meV},\\
&m_{\nu_1} = \sqrt{\Delta m_{13}^2} \approx 49\,\text{meV}, \quad \alpha \simeq -60^\circ, \quad \beta \simeq -165^\circ,\\
&\sin^2\theta_{12} = 0.323, \quad \sin^2\theta_{23} = 0.573, \quad \sin^2\theta_{13} = 0.0240,\\
&\delta \simeq 34^\circ,\, J \simeq 1.96\times10^{-2},\, A \simeq -2.94\times10^{-2}\,\text{meV},\\
&B \simeq 3.92\times10^{-2}\,\text{meV},\, C \simeq 7.76\times10^{-4}\,\text{meV}\,. \qquad (3.15)
\end{aligned}$$

Comparing Eq. (3.15) with Table 1 we see that the leptonic mixing parameters $\sin^2\theta_{12}$, $\sin^2\theta_{13}$, and $\sin^2\theta_{23}$ and the neutrino mass squared splittings are in excellent agreement with the experimental data. We found a leptonic Dirac CP violating phase close to 34° and a Jarlskog invariant of about 10^{-2}.

Now we compute the effective Majorana neutrino mass parameter, which is proportional to the neutrinoless double beta ($0\nu\beta\beta$) decay amplitude. The effective Majorana neutrino mass parameter is given by

$$m_{\beta\beta} = \left|\sum_j U_{ek}^2 m_{\nu_k}\right|, \qquad (3.16)$$

U_{ej}^2 being the PMNS mixing matrix elements and m_{ν_k} the Majorana neutrino masses.

From Eqs. (3.14), (3.15), and (3.16), we obtain the following value for the effective Majorana neutrino mass parameter, for the case of an inverted mass hierarchy:

$$m_{\beta\beta} \approx 22\ \text{meV}. \qquad (3.17)$$

Then we get a value for the Majorana neutrino mass parameter within the declared reach of the next-generation bolometric CUORE experiment [173] or, more realistically, of the next-to-next-generation ton-scale $0\nu\beta\beta$-decay experiments. It is worth mentioning that the upper limit of the Majorana neutrino mass parameter is $m_{\beta\beta} \leq 160$ meV, which corresponds to $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.6\times10^{25}$ yr at 90 % C.L, as follows from the EXO-200 experiment [174]. There is expected an improvement of this bound within a not too far future. The GERDA "phase-II" experiment [175,176] is expected to reach $T_{1/2}^{0\nu\beta\beta}(^{76}\text{Ge}) \geq 2\times10^{26}$ yr, corresponding to $m_{\beta\beta} \leq 100$ meV. A bolometric CUORE experiment, using ^{130}Te [173], is currently under construction and has an estimated sensitivity close to $T_{1/2}^{0\nu\beta\beta}(^{130}\text{Te}) \sim 10^{26}$ yr, which corresponds to $m_{\beta\beta} \leq 50$ meV. Besides that, there are proposals for ton-scale next-to-next-generation $0\nu\beta\beta$ experiments with ^{136}Xe [177,178] and ^{76}Ge [175,179], which claim sensitivities over $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr, corresponding to $m_{\beta\beta} \sim 12 - 30$ meV. For a recent review, see for example Ref. [180]. Consequently, our model predicts $T_{1/2}^{0\nu\beta\beta}$, which is at the level of the sensitivities of the next-generation or next-to-next-generation $0\nu\beta\beta$ experiments.

4 Quark masses and mixings

From the quark Yukawa terms of Eq. (2.12), it follows that the SM quark mass matrices take the form

$$\begin{aligned}
M_U &= \frac{v}{\sqrt{2}}\begin{pmatrix} c_1\lambda^8 & 0 & a_1\lambda^4 \\ 0 & b_1\lambda^4 & a_2\lambda^2 \\ 0 & 0 & a_3 \end{pmatrix},\\
M_D &= \frac{v}{\sqrt{2}}\begin{pmatrix} e_1\lambda^7 & f_1\lambda^6 & 0 \\ 0 & f_2\lambda^5 & 0 \\ 0 & 0 & g_1\lambda^3 \end{pmatrix}, \qquad (4.1)
\end{aligned}$$

where a_k ($k = 1, 2, 3$), b_1, c_1, g_1, f_1, f_2, and e_1 are $\mathcal{O}(1)$ parameters. Here $\lambda = 0.225$ is one of the Wolfenstein parameters and $v = 246$ GeV the scale of electroweak symmetry breaking. From the SM quark mass textures given above, it follows that the Cabbibo mixing emerges from the down-type quark sector, whereas the up-type quark sector generates the remaining mixing angles. Besides that, the low energy quark flavor data indicates that the CP violating phase in the quark sector is associated with the quark mixing angle in the 1–3 plane, as follows from the standard parametrization of the quark mixing matrix. Consequently, in order to get quark mixing angles and a CP violating phase consistent with the experimental data, we assume that all dimensionless parameters given in Eq. (4.1) are real, except for a_1, taken to be complex.

Furthermore, as follows from the different $\Delta(27)$ singlet assignments for the quark fields, the exotic quarks do not mix with the SM quarks. We find that the exotic quark masses are:

$$m_T = y^{(T)}\frac{v_\chi}{\sqrt{2}},$$
$$m_{J^1} = y_1^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_1^{(J)}}{y^{(T)}}m_T, m_{J^2} = y_2^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_2^{(J)}}{y^{(T)}}m_T. \quad (4.2)$$

Since the breaking of the $\Delta(27) \otimes Z_4 \otimes Z_8 \otimes Z_{14}$ discrete group gives rise to the observed pattern of charged fermion masses and quark mixing angles, and in order to simplify the analysis, we set $e_1 = f_1$ as well as $c_1 = a_3 = 1$ and $g_1 = b_1$, motivated by naturalness arguments and by the relation $m_c \sim m_b$, respectively. Consequently, there are only six effective free parameters in the SM quark sector of our model, i.e., $|a_1|, a_2, b_1, f_1, f_2$, and the phase γ_q. We fit these six parameters to reproduce the 10 physical observables of the quark sector, i.e., the six quark masses, the three mixing angles, and the CP violating phase. By varying the parameters $|a_1|, a_2, b_1, f_1, f_2$ and γ_q, we find the quark masses, the three quark mixing angles, and the CP violating phase δ reported in Table 2, which correspond to the best fit values:

$$|a_1| \simeq 1.36, \quad a_2 \simeq 0.80, \quad b_1 \simeq 1.43,$$
$$f_1 \simeq 0.58, \quad f_2 \simeq 0.57, \quad \gamma_q = -112°. \quad (4.3)$$

In Table 2 we show the model and experimental values for the physical observables of the quark sector. We use the M_Z-scale experimental values of the quark masses given by Ref. [181] (which are similar to those in [182]). The experimental values of the CKM parameters are taken from Ref. [183]. As indicated by Table 2, the obtained quark masses, quark mixing angles, and CP violating phase are highly consistent with the experimental low energy quark flavor data. Note that in our previous paper [130], the CKM matrix, at the tree level, is the identity, which should be improved by higher order loop corrections.

Table 2 Model and experimental values of the quark masses and CKM parameters

Observable	Model value	Experimental value
$m_u(MeV)$	1.16	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	641	635 ± 86
$m_t(GeV)$	174	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	59.2	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.85	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}$	0.225	0.225
$\sin\theta_{23}$	0.0407	0.0412
$\sin\theta_{13}$	0.00352	0.00351
δ	68°	68°

5 Conclusions

We constructed the first multiscalar singlet extension of the original 3-3-1 model with right-handed neutrinos, based on the $\Delta(27)$ family symmetry supplemented by the $Z_4 \otimes Z_8 \otimes Z_{14}$ discrete group. Contrary to the previous $\Delta(27)$ flavor 3-3-1 model [130], where the CKM matrix is the identity, this model provides an excellent description of the observed SM fermion mass and mixing pattern. The $\Delta(27)$, Z_4, and Z_8 symmetries allow one to reduce the number of parameters in the Yukawa terms, increasing the predictivity power of the model, whereas the Z_{14} symmetry causes the charged fermion mass and quark mixing pattern. In the model under consideration, the light active neutrino masses are generated from a double seesaw mechanism and the observed pattern of charged fermion masses and quark mixing angles is caused by the breaking of the $\Delta(27) \otimes Z_4 \otimes Z_8 \otimes Z_{14}$ discrete group at very high energy. The resulting the neutrino spectrum of our model is composed of light active neutrinos, heavy and very heavy sterile neutrinos. The smallness of the active neutrino masses arises from their scaling with inverse powers of the large model cutoff Λ and by their quadratic dependence on the very small vacuum expectation value of the $\Delta(27)$ scalar triplets Ω and Θ participating in the Dirac neutrino Yukawa interactions. The SM Yukawa sector of our predictive $\Delta(27)$ flavor 3-3-1 model has in total only 14 effective free parameters (eight and six effective free parameters in the lepton and quark sectors, respectively), which are fitted to reproduce the experimental values of the 18 physical observables in the quark and lepton sectors, i.e., nine charged fermion masses, two neutrino mass squared splittings, three lepton mixing parameters, three quark mixing angles, and one CP violating phase of the CKM quark mixing matrix. The obtained physical observables for the quark sector are consistent with the experimental data, whereas the ones for the lepton also do but only for the inverted neutrino mass hierarchy. The normal neutrino mass hierarchy scenario of our model is disfavored by the neutrino oscillation experimental data. We find an effective Majorana neutrino mass parameter of neutrinoless double beta decay of $m_{\beta\beta} = 22$ meV, a leptonic Dirac CP violating phase of 34°, and a Jarlskog invariant of about 10^{-2} for the inverted neutrino mass spectrum. Our obtained value of 22 meV for the effective Majorana neutrino mass is within the declared reach of the next-generation bolometric CUORE experiment [173] or, more realistically, of the next-to-next-generation ton-scale $0\nu\beta\beta$-decay experiments.

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A Appendices

A.1 The product rules of the $\Delta(27)$ discrete group

The $\Delta(27)$ discrete group is a subgroup of $SU(3)$; it has 27 elements divided into 11 conjugacy classes. Then the $\Delta(27)$ discrete group contains the following 11 irreducible representations: two triplets, i.e., $\mathbf{3}[0][1]$ (which we denote by $\mathbf{3}$) and its conjugate $\mathbf{3}_{[0][2]}$ (which we denote by $\overline{\mathbf{3}}$) and nine singlets, i.e., $\mathbf{1}_{k,l}$ $(k, l = 0, 1, 2)$, where k and l correspond to the Z_3 and Z_3' charges, respectively [36]. The $\Delta(27)$ discrete group, which is a simple group of the type $\Delta(3n^2)$ with $n = 3$, is isomorphic to the semi-direct product group $(Z_3' \times Z_3'') \rtimes Z_3$ [36]. It is worth mentioning that the simplest group of the type $\Delta(3n^2)$ is $\Delta(3) \equiv Z_3$. The next group is $\Delta(12)$, which is isomorphic to A_4. Consequently the $\Delta(27)$ discrete group is the simplest nontrivial group of the type $\Delta(3n^2)$. Any element of the $\Delta(27)$ discrete group can be expressed as $b^k a^m a'^n$, b, a, and a' being the generators of the Z_3, Z_3', and Z_3'' cyclic groups, respectively. These generators fulfill the relations

$$\begin{aligned} &a^3 = a'^3 = b^3 = 1, aa' = a'a, \\ &bab^{-1} = a^{-1}a'^{-1},\ \ ba'b^{-1} = a. \end{aligned} \tag{A.1}$$

The characters of the $\Delta(27)$ discrete group are shown in Table 3. Here n is the number of elements, h is the order of each element, and $\omega = e^{\frac{2\pi i}{3}} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ is the cube root of unity, which satisfies the relations $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. The conjugacy classes of $\Delta(27)$ are given by

Table 3 Characters of $\Delta(27)$

	h	$\chi_{1_{(r,s)}}$	$\chi_{3_{[0,1]}}$	$\chi_{3_{[0,2]}}$
$1C_1$	1	1	3	3
$1C_1^{(1)}$	1	1	$3\omega^2$	3ω
$1C_1^{(2)}$	1	1	3ω	$3\omega^2$
$3C_1^{(0,1)}$	3	ω^s	0	0
$3C_1^{(0,2)}$	3	ω^{2s}	0	0
$C_3^{(1,p)}$	3	ω^{r+sp}	0	0
$C_3^{(2,p)}$	3	ω^{2r+sp}	0	0

$$\begin{aligned}
&C_1: && \{e\}, && h = 1,\\
&C_1^{(1)}: && \{a, a'^2\}, && h = 3,\\
&C_1^{(2)}: && \{a^2, a'\}, && h = 3,\\
&C_3^{(0,1)}: && \{a'^2 a'^2\}, && h = 3,\\
&C_3^{(0,2)}: && \{a'^2, a^2, aa'\}, && h = 3,\\
&C_3^{(1,p)}: && \{ba^p, ba^{p-1}a'^{p-2}a'^2\}, && h = 3,\\
&C_3^{(2,p)}: && \{ba^p, ba^{p-1}a'^{p-2}a'^2\}, && h = 3.
\end{aligned}$$

The tensor products between $\Delta(27)$ triplets are described by the following relations [36]:

$$\begin{aligned}
&\begin{pmatrix} x_{1,-1}\\ x_{0,1}\\ x_{-1,0}\end{pmatrix}_{\mathbf{3}_{[0][1]}} \otimes \begin{pmatrix} y_{1,-1}\\ y_{0,1}\\ y_{-1,0}\end{pmatrix}_{\mathbf{3}_{[0][1]}}\\
&= \begin{pmatrix} x_{1,-1}y_{1,-1}\\ x_{0,1}y_{0,1}\\ x_{-1,0}y_{-1,0}\end{pmatrix}_{\mathbf{3}^{(S_1)}_{[0][2]}} \oplus \frac{1}{2}\begin{pmatrix} x_{0,1}y_{-1,0}+x_{-1,0}y_{0,1}\\ x_{-1,0}y_{1,-1}+x_{1,-1}y_{-1,0}\\ x_{1,-1}y_{0,1}+x_{0,1}y_{1,-1}\end{pmatrix}_{\mathbf{3}^{(S_2)}_{[0][2]}}\\
&\oplus \frac{1}{2}\begin{pmatrix} x_{0,1}y_{-1,0}-x_{-1,0}y_{0,1}\\ x_{-1,0}y_{1,-1}-x_{1,-1}y_{-1,0}\\ x_{1,-1}y_{0,1}-x_{0,1}y_{1,-1}\end{pmatrix}_{\mathbf{3}^{(A)}_{[0][2]}},
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
&\begin{pmatrix} x_{2,-2}\\ x_{0,2}\\ x_{-2,0}\end{pmatrix}_{\mathbf{3}_{[0][2]}} \otimes \begin{pmatrix} y_{2,-2}\\ y_{0,2}\\ y_{-2,0}\end{pmatrix}_{\mathbf{3}_{[0][2]}} = \begin{pmatrix} x_{2,-2}y_{2,-2}\\ x_{0,2}y_{0,2}\\ x_{-2,0}y_{-2,0}\end{pmatrix}_{\mathbf{3}^{(S_1)}_{[0][1]}}\\
&\oplus \frac{1}{2}\begin{pmatrix} x_{0,2}y_{-2,0}+x_{-2,0}y_{0,2}\\ x_{-2,0}y_{2,-2}+x_{2,-2}y_{-2,0}\\ x_{2,-2}y_{0,2}+x_{0,2}y_{2,-2}\end{pmatrix}_{\mathbf{3}^{(S_2)}_{[0][1]}}\\
&\oplus \frac{1}{2}\begin{pmatrix} x_{0,2}y_{-2,0}-x_{-2,0}y_{0,2}\\ x_{-2,0}y_{2,-2}-x_{2,-2}y_{-2,0}\\ x_{2,-2}y_{0,2}-x_{0,2}y_{2,-2}\end{pmatrix}_{\mathbf{3}^{(A)}_{[0][1]}},
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
&\begin{pmatrix} x_{1,-1}\\ x_{0,1}\\ x_{-1,0}\end{pmatrix}_{\mathbf{3}_{[0][1]}} \otimes \begin{pmatrix} y_{-1,1}\\ y_{0,-1}\\ y_{1,0}\end{pmatrix}_{\mathbf{3}_{[0][2]}}\\
&= \sum_r (x_{1,-1}y_{-1,1} + \omega^{2r}x_{0,1}y_{0,-1} + \omega^r x_{-1,0}y_{1,0})\mathbf{1}_{(r,0)}\\
&\oplus \sum_r (x_{1,-1}y_{0,-1} + \omega^{2r}x_{0,1}y_{1,0} + \omega^r x_{-1,0}y_{-1,1})\mathbf{1}_{(r,1)}\\
&\oplus \sum_r (x_{1,-1}y_{1,0} + \omega^{2r}x_{0,1}y_{-1,1} + \omega^r x_{-1,0}y_{0,-1})\mathbf{1}_{(r,2)}.
\end{aligned} \tag{A.4}$$

Table 4 The singlet multiplications of the group $\Delta(27)$

Singlets	$\mathbf{1}_{01}$	$\mathbf{1}_{02}$	$\mathbf{1}_{10}$	$\mathbf{1}_{11}$	$\mathbf{1}_{12}$	$\mathbf{1}_{20}$	$\mathbf{1}_{21}$	$\mathbf{1}_{22}$
$\mathbf{1}_{01}$	$\mathbf{1}_{02}$	$\mathbf{1}_{00}$	$\mathbf{1}_{11}$	$\mathbf{1}_{12}$	$\mathbf{1}_{10}$	$\mathbf{1}_{21}$	$\mathbf{1}_{22}$	$\mathbf{1}_{20}$
$\mathbf{1}_{02}$	$\mathbf{1}_{00}$	$\mathbf{1}_{01}$	$\mathbf{1}_{12}$	$\mathbf{1}_{10}$	$\mathbf{1}_{11}$	$\mathbf{1}_{22}$	$\mathbf{1}_{20}$	$\mathbf{1}_{21}$
$\mathbf{1}_{10}$	$\mathbf{1}_{11}$	$\mathbf{1}_{12}$	$\mathbf{1}_{20}$	$\mathbf{1}_{21}$	$\mathbf{1}_{22}$	$\mathbf{1}_{00}$	$\mathbf{1}_{01}$	$\mathbf{1}_{02}$
$\mathbf{1}_{11}$	$\mathbf{1}_{12}$	$\mathbf{1}_{10}$	$\mathbf{1}_{21}$	$\mathbf{1}_{22}$	$\mathbf{1}_{20}$	$\mathbf{1}_{01}$	$\mathbf{1}_{02}$	$\mathbf{1}_{00}$
$\mathbf{1}_{12}$	$\mathbf{1}_{10}$	$\mathbf{1}_{11}$	$\mathbf{1}_{22}$	$\mathbf{1}_{20}$	$\mathbf{1}_{21}$	$\mathbf{1}_{02}$	$\mathbf{1}_{00}$	$\mathbf{1}_{01}$
$\mathbf{1}_{20}$	$\mathbf{1}_{21}$	$\mathbf{1}_{22}$	$\mathbf{1}_{00}$	$\mathbf{1}_{01}$	$\mathbf{1}_{02}$	$\mathbf{1}_{10}$	$\mathbf{1}_{11}$	$\mathbf{1}_{12}$
$\mathbf{1}_{21}$	$\mathbf{1}_{22}$	$\mathbf{1}_{20}$	$\mathbf{1}_{01}$	$\mathbf{1}_{02}$	$\mathbf{1}_{00}$	$\mathbf{1}_{11}$	$\mathbf{1}_{12}$	$\mathbf{1}_{10}$
$\mathbf{1}_{22}$	$\mathbf{1}_{20}$	$\mathbf{1}_{21}$	$\mathbf{1}_{02}$	$\mathbf{1}_{00}$	$\mathbf{1}_{01}$	$\mathbf{1}_{12}$	$\mathbf{1}_{10}$	$\mathbf{1}_{11}$

The multiplication rules between the $\Delta(27)$ singlets and the $\Delta(27)$ triplets are given by [36]

$$\begin{pmatrix} x_{(1,-1)} \\ x_{(0,1)} \\ x_{(-1,0)} \end{pmatrix}_{\mathbf{3}_{[0][1]}} \otimes (z)_{1_{k,l}} = \begin{pmatrix} x_{(1,-1)}z \\ \omega^r x_{(0,1)}z \\ \omega^{2r} x_{(-1,0)}z \end{pmatrix}_{\mathbf{3}_{[l][1+l]}}, \tag{A.5}$$

$$\begin{pmatrix} x_{(2,-2)} \\ x_{(0,2)} \\ x_{(-2,0)} \end{pmatrix}_{\mathbf{3}_{[0][2]}} \otimes (z)_{1_{k,l}} = \begin{pmatrix} x_{(2,-2)}z \\ \omega^r x_{(0,2)}z \\ \omega^{2r} x_{(-2,0)}z \end{pmatrix}_{\mathbf{3}_{[l][2+l]}}. \tag{A.6}$$

The tensor products of the $\Delta(27)$ singlets $\mathbf{1}_{k,\ell}$ and $\mathbf{1}_{k',\ell'}$ take the form [36]

$$\mathbf{1}_{k,\ell} \otimes \mathbf{1}_{k',\ell'} = \mathbf{1}_{k+k'\mathrm{mod}3,\ell+\ell'\mathrm{mod}3}. \tag{A.7}$$

From the equation given above, we obtain explicitly the singlet multiplication rules of the $\Delta(27)$ group, which are given in Table 4.

A.2 Scalar potential for two $\Delta(27)$ scalar triplets

The scalar potential for two $\Delta(27)$ scalar triplets, i.e., U and W having different Z_N charges can be written as follows:

$$V = V_U + V_W + V_{U,W} \tag{A.8}$$

where V_U and V_w are the scalar potentials for the $\Delta(27)$ scalar triplets U and W, respectively, whereas $V_{U,W}$ include the interaction terms involving both $\Delta(27)$ scalar triplets U and W. The different parts of the scalar potential for the two $\Delta(27)$ scalar triplets are given by

$$\begin{aligned} V_U =& -\mu_U^2 \left(UU^*\right)_{\mathbf{1_{0,0}}} + \kappa_{U,1}\left(UU^*\right)_{\mathbf{1_{0,0}}}\left(UU^*\right)_{\mathbf{1_{0,0}}} \\ &+\kappa_{U,2}\left(UU^*\right)_{\mathbf{1_{1,0}}}\left(UU^*\right)_{\mathbf{1_{2,0}}} \\ &+\kappa_{U,3}\left(UU^*\right)_{\mathbf{1_{0,1}}}\left(UU^*\right)_{\mathbf{1_{0,2}}} \\ &+\kappa_{U,4}\left[\left(UU^*\right)_{\mathbf{1_{1,1}}}\left(UU^*\right)_{\mathbf{1_{2,2}}}+H.c\right] \\ &+\kappa_{U,5}\left(UU\right)_{\mathbf{\bar{3}_{S_1}}}\left(U^*U^*\right)_{\mathbf{3}_{S_1}} \\ &+\kappa_{U,6}\left(UU\right)_{\mathbf{\bar{3}_{S_2}}}\left(U^*U^*\right)_{\mathbf{3}_{S_2}} \\ &+\kappa_{U,7}\left[\left(UU\right)_{\mathbf{\bar{3}_{S_1}}}\left(U^*U^*\right)_{\mathbf{3}_{S_2}}+H.c\right], \end{aligned} \tag{A.9}$$

$$V_W = V_U\left(U\to W, \mu_U\to\mu_W, \kappa_{U,j}\to\kappa_{W,j}\right), \tag{A.10}$$

$$\begin{aligned} V_{U,W} =& \gamma_{UW,1}\left(UU^*\right)_{\mathbf{1_{0,0}}}\left(WW^*\right)_{\mathbf{1_{0,0}}} \\ &+\kappa_{UW,1}\left(UW^*\right)_{\mathbf{1_{0,0}}}\left(U^*W\right)_{\mathbf{1_{0,0}}} \\ &+\gamma_{UW,2}\left[\left(UU^*\right)_{\mathbf{1_{1,0}}}\left(WW^*\right)_{\mathbf{1_{2,0}}}+H.c\right] \\ &+\kappa_{UW,2}\left[\left(UW^*\right)_{\mathbf{1_{1,0}}}\left(UW^*\right)_{\mathbf{1_{2,0}}}+H.c\right] \\ &+\gamma_{UW,3}\left[\left(UU^*\right)_{\mathbf{1_{0,1}}}\left(WW^*\right)_{\mathbf{1_{0,2}}}+H.c\right] \\ &+\kappa_{UW,3}\left[\left(UW^*\right)_{\mathbf{1_{0,1}}}\left(UW^*\right)_{\mathbf{1_{0,2}}}+H.c\right] \\ &+\gamma_{UW,4}\left[\left(UU^*\right)_{\mathbf{1_{1,1}}}\left(WW^*\right)_{\mathbf{1_{2,2}}}+H.c\right] \\ &+\kappa_{UW,4}\left[\left(UW^*\right)_{\mathbf{1_{1,1}}}\left(UW^*\right)_{\mathbf{1_{2,2}}}+H.c\right] \\ &+\gamma_{UW,5}\left[\left(UU\right)_{\mathbf{\bar{3}_{S_1}}}\left(W^*W^*\right)_{\mathbf{3}_{S_1}}+H.c\right] \\ &+\gamma_{UW,6}\left[\left(UU\right)_{\mathbf{\bar{3}_{S_2}}}\left(W^*W^*\right)_{\mathbf{3}_{S_2}}+H.c\right] \\ &+\kappa_{UW,5}\left(UW\right)_{\mathbf{\bar{3}_{S_1}}}\left(U^*W^*\right)_{\mathbf{3}_{S_1}} \\ &+\kappa_{UW,6}\left(UW\right)_{\mathbf{\bar{3}_{S_2}}}\left(U^*W^*\right)_{\mathbf{3}_{S_2}} \\ &+\gamma_{UW,7}\left[\left(UU\right)_{\mathbf{\bar{3}_{S_1}}}\left(W^*W^*\right)_{\mathbf{3}_{S_2}}+H.c\right] \\ &+\kappa_{UW,7}\left[\left(UW\right)_{\mathbf{\bar{3}_{S_1}}}\left(U^*W^*\right)_{\mathbf{3}_{S_2}}+H.c\right] \\ &+\kappa_{UW,8}\left[\left(UW\right)_{\mathbf{\bar{3}_{A}}}\left(U^*W^*\right)_{\mathbf{3}_{A}}+H.c\right] \\ &+\kappa_{UW,9}\left[\left(UW\right)_{\mathbf{\bar{3}_{A}}}\left(U^*W^*\right)_{\mathbf{3}_{S_1}}+H.c\right] \\ &+\kappa_{UW,10}\left[\left(UW\right)_{\mathbf{\bar{3}_{A}}}\left(U^*W^*\right)_{\mathbf{3}_{S_2}}+H.c\right] \end{aligned} \tag{A.11}$$

where the $\Delta(27)$ scalar triplets U and W acquire the following VEV pattern:

$$\langle U\rangle = (u_1, u_2, u_3), \langle W\rangle = (w_1, w_2, w_3). \tag{A.12}$$

Then, from the previous expressions, the following relations are obtained:

$$\begin{aligned} \frac{\partial V_U}{\partial u_1} =& -2u_1\mu_U^2 + 4\kappa_{U,1}u_1\left(u_1^2+u_2^2+u_3^2\right) \\ &+2\kappa_{U,2}u_1\left(2u_1^2-u_2^2-u_3^2\right) \\ &+2\kappa_{U,3}\left(u_2+u_3\right)\left[u_1\left(u_2+u_3\right)+u_2u_3\right] \\ &+\kappa_{U,4}\left[2u_1u_2u_3+u_2u_3\left(u_2+u_3\right)-2u_1\left(u_2^2+u_3^2\right)\right] \\ &+2\kappa_{U,5}u_1^3+2\kappa_{U,6}u_1\left(u_2^2+u_3^2\right) \\ &+2\kappa_{U,7}u_2u_3\left(2u_1+u_2+u_3\right), \end{aligned} \tag{A.13}$$

$$\begin{aligned} \frac{\partial V_U}{\partial u_2} =& -2u_2\mu_U^2 + 4\kappa_{U,2}u_2\left(u_1^2+u_2^2+u_3^2\right) \\ &+2\kappa_{U,2}u_2\left(2u_2^2-u_1^2-u_3^2\right) \\ &+2\kappa_{U,3}\left(u_1+u_3\right)\left[u_2\left(u_1+u_3\right)+u_1u_3\right] \end{aligned}$$

$$+\kappa_{U,4}\left[2u_1u_2u_3+u_1u_3\left(u_1+u_3\right)-2u_2\left(u_1^2+u_3^2\right)\right] + 2\kappa_{U,5}u_2^3+2\kappa_{U,6}u_2\left(u_1^2+u_3^2\right) + 2\kappa_{U,7}u_1u_3\left(2u_2+u_1+u_3\right), \tag{A.14}$$

$$\frac{\partial V_U}{\partial u_3} = -2u_3\mu_U^2+4\kappa_{U,2}u_3\left(u_1^2+u_2^2+u_3^2\right) + 2\kappa_{U,2}u_3\left(2u_3^2-u_2^2-u_3^2\right) + 2\kappa_{U,3}\left(u_1+u_2\right)\left[u_3\left(u_1+u_2\right)+u_1u_2\right] + \kappa_{U,4}\left[2u_1u_2u_3+u_1u_2\left(u_1+u_2\right)-2u_3\left(u_1^2+u_2^2\right)\right] + 2\kappa_{U,5}u_3^3+2\kappa_{U,6}u_3\left(u_1^2+u_2^2\right) + 2\kappa_{U,7}u_1u_2\left(2u_3+u_1+u_2\right), \tag{A.15}$$

$$\frac{\partial V_W}{\partial w_n} = \lim_{\mu_U\to\mu_W,\kappa_{U,j}\to\kappa_{W,j},u_n\to w_n}\frac{\partial V_U}{\partial u_n}, \quad n=1,2,3,\ j=1,\ldots,10, \tag{A.16}$$

$$\frac{\partial V_{U,W}}{\partial u_1} = 2\gamma_{UW,1}u_1\left(w_1^2+w_2^2+w_3^2\right) + 2\kappa_{UW,1}w_1\left(u_1w_1+u_2w_2+u_3w_3\right) + 2\gamma_{UW,2}u_1\left(2w_1^2-w_2^2-w_3^2\right) + 2\kappa_{UW,2}w_1\left(2u_1w_1-u_2w_2-u_3w_3\right) + 2\gamma_{UW,3}\left(u_2+u_3\right)\left[w_2w_3+w_1\left(w_2+w_3\right)\right] + 2\kappa_{UW,3}\left[2u_1w_2w_3+u_2\left(w_1w_2+w_3^2\right) + u_3\left(w_1w_3+w_2^2\right)\right] + \gamma_{UW,4}\left\{u_2\left[2w_1w_3-w_2\left(w_1+w_3\right)\right] + u_3\left[2w_2w_3-w_1\left(w_2+w_3\right)\right]\right\} + \kappa_{UW,4}\left\{4u_1w_2w_3-u_2\left(w_1w_2+w_3^2\right) - u_3\left(w_1w_3+w_2^2\right)\right\} + 4\gamma_{UW,5}u_1w_1^2+2\gamma_{UW,6}w_1\left(u_2w_2+u_3w_3\right) + 2\kappa_{UW,5}u_1w_1^2+4\gamma_{UW,7}u_1w_2w_3 + \frac{\kappa_{UW,6}}{2}\left[u_1\left(w_2^2+w_3^2\right)+w_1\left(u_2w_2+u_3w_3\right)\right] + \kappa_{UW,7}\left[u_2w_3\left(w_1+w_2\right)+u_3w_2\left(w_1+w_3\right)\right] + \kappa_{UW,8}\left[u_1\left(w_2^2+w_3^2\right)-w_1\left(u_2w_2+u_3w_3\right)\right] + \kappa_{UW,9}\left[u_2w_3\left(w_1-w_2\right)+u_3w_2\left(w_3-w_1\right)\right] + \kappa_{UW,10}u_1\left(w_2^2-w_3^2\right), \tag{A.17}$$

$$\frac{\partial V_{U,W}}{\partial u_2} = 2\gamma_{UW,1}u_2\left(w_1^2+w_2^2+w_3^2\right) + 2\kappa_{UW,1}w_2\left(u_1w_1+u_2w_2+u_3w_3\right) + 2\gamma_{UW,2}u_2\left(2w_2^2-w_1^2-w_3^2\right) + 2\kappa_{UW,2}w_2\left(2u_2w_2-u_1w_1-u_3w_3\right) + 2\gamma_{UW,3}\left(u_1+u_3\right)\left[w_2w_3+w_1\left(w_2+w_3\right)\right] + 2\kappa_{UW,3}\left[2u_2w_1w_3+u_1\left(w_1w_2+w_3^2\right) + u_3\left(w_2w_3+w_1^2\right)\right] + \gamma_{UW,4}\left\{u_3\left[2w_1w_2-w_3\left(w_1+w_2\right)\right] + u_1\left[2w_1w_3-w_2\left(w_1+w_3\right)\right]\right\} + \kappa_{UW,4}\left\{4u_2w_1w_3-u_1\left(w_1w_2+w_3^2\right) - u_3\left(w_2w_3+w_1^2\right)\right\} + 4\gamma_{UW,5}u_2w_2^2+2\gamma_{UW,6}w_2\left(u_1w_1+u_3w_3\right) + 2\kappa_{UW,5}u_2w_2^2+4\gamma_{UW,7}u_2w_1w_3 + \frac{\kappa_{UW,6}}{2}\left[u_2\left(w_1^2+w_3^2\right)+w_2\left(u_1w_1+u_3w_3\right)\right] + \kappa_{UW,7}\left[u_1w_3\left(w_1+w_2\right)+u_3w_1\left(w_2+w_3\right)\right] + \kappa_{UW,8}\left[u_2\left(w_1^2+w_3^2\right)-w_2\left(u_1w_1+u_3w_3\right)\right] + \kappa_{UW,9}\left[u_1w_3\left(w_1-w_2\right)+u_3w_1\left(w_2-w_3\right)\right] + \kappa_{UW,10}u_2\left(w_3^2-w_1^2\right), \tag{A.18}$$

$$\frac{\partial V_{U,W}}{\partial u_3} = 2\gamma_{UW,1}u_3\left(w_1^2+w_2^2+w_3^2\right) + 2\kappa_{UW,1}w_3\left(u_1w_1+u_2w_2+u_3w_3\right) + 2\gamma_{UW,2}u_3\left(2w_3^2-w_1^2-w_2^2\right) + 2\kappa_{UW,2}w_3\left(2u_3w_3-u_1w_1-u_2w_2\right) + 2\gamma_{UW,3}\left(u_1+u_2\right)\left[w_2w_3+w_1\left(w_2+w_3\right)\right] + 2\kappa_{UW,3}\left[2u_3w_1w_2+u_1\left(w_1w_3+w_2^2\right) + u_2\left(w_2w_3+w_1^2\right)\right] + \gamma_{UW,4}\left\{u_2\left[2w_1w_2-w_3\left(w_1+w_2\right)\right] + u_1\left[2w_2w_3-w_1\left(w_2+w_3\right)\right]\right\} + \kappa_{UW,4}\left\{4u_3w_1w_2-u_1\left(w_1w_3+w_2^2\right) - u_2\left(w_2w_3+w_1^2\right)\right\} + 4\gamma_{UW,5}u_3w_3^2+2\gamma_{UW,6}w_3\left(u_1w_1+u_2w_2\right) + 2\kappa_{UW,5}u_3w_3^2+2\gamma_{UW,7}u_3w_1w_2 + \frac{\kappa_{UW,6}}{2}\left[w_1\left(u_2^2+u_3^2\right)+u_1\left(u_2w_2+u_3w_3\right)\right] + \kappa_{UW,7}\left[u_1w_2\left(w_1+w_3\right)+u_2w_1\left(w_2+w_3\right)\right] + \kappa_{UW,8}\left[u_3\left(w_1^2+w_2^2\right)-w_3\left(u_1w_1+u_2w_2\right)\right] + \kappa_{UW,9}\left[u_1w_2\left(w_3-w_1\right)+u_2w_1\left(w_2-w_3\right)\right] + \kappa_{UW,10}u_3\left(w_1^2-w_2^2\right), \tag{A.19}$$

$$\frac{\partial V_{U,W}}{\partial w_1} = 2\gamma_{UW,1}w_1\left(u_1^2+u_2^2+u_3^2\right) + 2\kappa_{UW,1}u_1\left(u_1w_1+u_2w_2+u_3w_3\right) + 2\gamma_{UW,2}w_1\left(2u_1^2-u_2^2-u_3^2\right) + 2\kappa_{UW,2}u_1\left(2u_1w_1-u_2w_2-u_3w_3\right)$$

$$
\begin{aligned}
&+2\gamma_{UW,3}\left(w_2+w_3\right)\left[u_2u_3+u_1\left(u_2+u_3\right)\right]\\
&+2\kappa_{UW,3}\left[2w_1u_2u_3+w_2\left(u_1u_2+u_3^2\right)\right.\\
&\left.+\ w_3\left(u_1u_3+u_2^2\right)\right]\\
&+\gamma_{UW,4}\left\{u_2u_3\left(2w_2-w_3\right)+u_1u_2\left(2w_3-w_2\right)\right.\\
&\left.-\ u_1u_3\left(w_2+w_3\right)\right\}\\
&+\kappa_{UW,4}\left\{u_3\left(2u_3w_2-u_1w_3\right)+2u_2^2w_3\right.\\
&\left.-\ u_2\left(2u_3w_1+u_1w_2\right)\right\}\\
&+4\gamma_{UW,5}w_1u_1^2+2\gamma_{UW,6}u_1\left(u_2w_2+u_3w_3\right)\\
&+2\kappa_{UW,5}w_1u_1^2+2\gamma_{UW,7}\left(u_3^2w_2+u_2^2w_3\right)\\
&+\frac{\kappa_{UW,6}}{2}\left[w_1\left(u_2^2+u_3^2\right)+u_1\left(u_2w_2+u_3w_3\right)\right]\\
&+\kappa_{UW,7}\left[w_2u_3\left(u_1+u_2\right)+w_3u_2\left(u_1+u_3\right)\right]\\
&+\kappa_{UW,8}\left[w_1\left(u_2^2+u_3^2\right)-u_1\left(u_2w_2+u_3w_3\right)\right]\\
&+\kappa_{UW,9}\left[w_2u_3\left(u_2-u_1\right)+w_3u_2\left(u_1-u_3\right)\right]\\
&+\kappa_{UW,10}w_1\left(u_3^2-u_2^2\right),
\end{aligned}
\tag{A.20}
$$

$$
\begin{aligned}
\frac{\partial V_{U,W}}{\partial w_2}=&\ 2\gamma_{UW,1}w_2\left(u_1^2+u_2^2+u_3^2\right)\\
&+2\kappa_{UW,1}u_2\left(u_1w_1+u_2w_2+u_3w_3\right)\\
&+2\gamma_{UW,2}w_2\left(2u_2^2-u_1^2-u_3^2\right)\\
&+2\kappa_{UW,2}u_2\left(2u_2w_2-u_1w_1-u_3w_3\right)\\
&+2\gamma_{UW,3}\left(w_1+w_3\right)\left[u_2u_3+u_1\left(u_2+u_3\right)\right]\\
&+2\kappa_{UW,3}\left[2w_2u_1u_3+w_1\left(u_1u_2+u_3^2\right)\right.\\
&\left.+\ w_3\left(u_2u_3+u_1^2\right)\right]\\
&+\gamma_{UW,4}\left\{u_2u_3\left(2w_1-w_3\right)+u_1u_3\left(2w_3-w_1\right)\right.\\
&\left.-u_1u_2\left(w_1+w_3\right)\right\}\\
&+\kappa_{UW,4}\left\{u_3\left(2u_3w_1-u_2w_3\right)+2u_1^2w_3\right.\\
&\left.-u_1\left(2u_3w_2+u_2w_1\right)\right\}\\
&+4\gamma_{UW,5}w_2u_2^2+2\gamma_{UW,6}u_2\left(u_1w_1+u_3w_3\right)\\
&+2\kappa_{UW,5}w_2u_2^2+2\gamma_{UW,7}\left(u_3^2w_1+u_1^2w_3\right)\\
&+\frac{\kappa_{UW,6}}{2}\left[w_2\left(u_1^2+u_3^2\right)+u_2\left(u_1w_1+u_3w_3\right)\right]\\
&+\kappa_{UW,7}\left[w_1u_3\left(u_1+u_2\right)+w_3u_1\left(u_2+u_3\right)\right]\\
&+\kappa_{UW,8}\left[w_2\left(u_1^2+u_3^2\right)-u_2\left(u_1w_1+u_3w_3\right)\right]\\
&+\kappa_{UW,9}\left[w_1u_3\left(u_2-u_1\right)+w_3u_1\left(u_3-u_2\right)\right]\\
&+\kappa_{UW,10}w_2\left(u_1^2-u_3^2\right),
\end{aligned}
\tag{A.21}
$$

$$
\begin{aligned}
\frac{\partial V_{U,W}}{\partial w_3}=&\ 2\gamma_{UW,1}w_3\left(u_1^2+u_2^2+u_3^2\right)\\
&+2\kappa_{UW,1}u_3\left(u_1w_1+u_2w_2+u_3w_3\right)\\
&+2\gamma_{UW,2}w_3\left(2u_3^2-u_1^2-u_2^2\right)\\
&+2\kappa_{UW,2}u_3\left(2u_3w_3-u_1w_1-u_2w_2\right)\\
&+2\gamma_{UW,3}\left(w_1+w_2\right)\left[u_2u_3+u_1\left(u_2+u_3\right)\right]\\
&+2\kappa_{UW,3}\left[2w_3u_1u_2+w_1\left(u_1u_3+u_2^2\right)\right.\\
&\left.+\ w_2\left(u_2u_3+u_1^2\right)\right]\\
&+\gamma_{UW,4}\left\{u_1u_3\left(2w_2-w_1\right)+u_1u_2\left(2w_1-w_2\right)\right.\\
&\left.-\ u_2u_3\left(w_1+w_2\right)\right\}\\
&+\kappa_{UW,4}\left\{u_1\left(2u_1w_2-u_3w_1\right)+2u_2^2w_1\right.\\
&\left.-u_2\left(2u_1w_3+u_3w_2\right)\right\}\\
&+4\gamma_{UW,5}w_3u_3^2+2\gamma_{UW,6}u_3\left(u_1w_1+u_2w_2\right)\\
&+2\kappa_{UW,5}w_3u_3^2+2\gamma_{UW,7}\left(u_2^2w_1+u_1^2w_2\right)\\
&+\frac{\kappa_{UW,6}}{2}\left[w_3\left(u_1^2+u_2^2\right)+u_3\left(u_1w_1+u_2w_2\right)\right]\\
&+\kappa_{UW,7}\left[w_1u_2\left(u_1+u_3\right)+w_2u_1\left(u_2+u_3\right)\right]\\
&+\kappa_{UW,8}\left[w_3\left(u_1^2+u_2^2\right)-u_3\left(u_1w_1+u_2w_2\right)\right]\\
&+\kappa_{UW,9}\left[w_1u_2\left(u_1-u_3\right)+w_2u_1\left(u_3-u_2\right)\right]\\
&+\kappa_{UW,10}w_3\left(u_2^2-u_1^2\right).
\end{aligned}
\tag{A.22}
$$

Considering the VEV configuration:

$$u_1=u,\ u_2=u_3=0,\ w_1=w_2=0,\ w_3=w. \tag{A.23}$$

From the expressions given above, we find that the scalar potential minimization equations take the form

$$
\begin{aligned}
\frac{\partial V}{\partial u_1}=&\ \frac{u}{2}\left[-4\mu_U^2+8\left(\kappa_{U,1}+\kappa_{U,2}+\kappa_{U,5}\right)u^2+\kappa_{UW,6}w^2\right.\\
&\left.+\left(4\gamma_{UW,1}-4\gamma_{UW,2}+2\kappa_{UW,8}-2\kappa_{UW,10}\right)w^2\right]\\
=&\ 0,
\end{aligned}
\tag{A.24}
$$

$$\frac{\partial V}{\partial u_2}=uw^2\left(2\kappa_{UW,3}-\kappa_{UW,4}\right)=0, \tag{A.25}$$

$$\frac{\partial V}{\partial u_3}=0, \tag{A.26}$$

$$\frac{\partial V}{\partial w_1}=0, \tag{A.27}$$

$$\frac{\partial V}{\partial w_2}=2u^2w\left(\gamma_{UW,7}+\kappa_{UW,3}+\kappa_{UW,4}\right)=0, \tag{A.28}$$

$$
\begin{aligned}
\frac{\partial V}{\partial w_3}=&\ \frac{w}{2}\left[-4\mu_W^2+8\left(\kappa_{W,1}+\kappa_{W,2}+\kappa_{W,5}\right)w^2+\kappa_{UW,6}u^2\right.\\
&\left.+\left(4\gamma_{UW,1}-4\gamma_{UW,2}+2\kappa_{UW,8}-2\kappa_{UW,10}\right)u^2\right]\\
=&\ 0.
\end{aligned}
\tag{A.29}
$$

Then, from the scalar potential minimization equations, we find the following relations:

$$\kappa_{UW,4}=2\kappa_{UW,3}, \tag{A.30}$$

$$\gamma_{UW,7}=-\left(\kappa_{UW,3}+\kappa_{UW,4}\right), \tag{A.31}$$

$$\mu_U^2 = 2\left(\kappa_{U,1}+\kappa_{U,2}+\kappa_{U,5}\right)u^2+\left(4\gamma_{UW,1}-4\gamma_{UW,2}+\kappa_{UW,6}+2\kappa_{UW,8}-2\kappa_{UW,10}\right)\frac{w^2}{4}, \quad (A.32)$$

$$\mu_W^2 = 2\left(\kappa_{W,1}+\kappa_{W,2}+\kappa_{W,5}\right)w^2+\left(4\gamma_{UW,1}-4\gamma_{UW,2}+\kappa_{UW,6}+2\kappa_{UW,8}-2\kappa_{UW,10}\right)\frac{u^2}{4}. \quad (A.33)$$

These results show that the VEV directions for the two $\Delta(27)$ triplets, i.e., U and W scalars in Eq. (A.23), are consistent with a global minimum of the scalar potential given in Eq. (A.8) for a large region of parameter space. Furthermore, let us note that if one only considers one $\Delta(27)$ scalar triplet, by setting Eqs. (A.13)–(A.15) to zero, it follows that the VEV pattern for the $\Delta(27)$ triplet S, pointing in the (1, 1, 1) $\Delta(27)$ direction, is a natural solution of the scalar potential minimization equations.

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