Eur. Phys. J. C (2015) 75:123
DOI 10.1140/epjc/s10052-015-3340-x

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

Vaidya–Tikekar type superdense star admitting conformal motion in presence of quintessence field

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Received: 29 August 2014 / Accepted: 25 February 2015 / Published online: 17 March 2015

Abstract To explain the accelerated expansion of our universe, dark energy is a suitable candidate. Motivated by this concept in the present paper we have obtained a new model of an anisotropic superdense star which admits conformal motions in the presence of a quintessence field which is characterized by a parameter ω_q with $-1 < \omega_q < -\frac{1}{3}$. The model has been developed by choosing the Vaidya–Tikekar ansatz (J Astrophys Astron 3:325, 1982). Our model satisfies all the physical requirements. We have analyzed our result analytically as well as with the help of a graphical representation.

1 Introduction

The study of dark matter and dark energy has become a topic of considerable interest in present decades. This study is not only important from a theoretical point of view but also from a physical point of view. The reason is that much observational evidence suggests that the expansion of our universe is accelerating. Dark energy is the most acceptable hypothesis to explain this. Work done based on the cosmic microwave background (CMB) estimated that our universe is made up of 68.3 % dark energy, 26.8 % dark matter and 4.9 % ordinary matter. Dark matter cannot be seen by telescopes but one can infer its evidence from gravitational effects on visible matter and gravitational lensing of background radiation. On the other hand, the evidence of dark energy may be inferred from measures of large scale wave patterns of the mass density. One notable feature of dark energy is that it has a strong negative pressure, i.e., the ratio of pressure to density, which is termed the equation of state parameter (ω), is negative. The dark energy equation of state is given by $p = \omega\rho$ with $\omega < -\frac{1}{3}$. The dark energy star model has been studied by several authors [1–7]. If we choose $\omega = -1$ we will get the model of a gravastar [8–13], which is also a dark energy star. $\omega < -1$ is for the phantom energy and it violates the null energy condition. Several authors have used a phantom equation of state to describe the wormhole model [14–17]. Motivated by this previous work we have chosen quintessence dark energy to develop our present model. Here the quintessence field is characterized by a parameter ω_q with $-1 < \omega_q < -\frac{1}{3}$. We have assumed that the underlying fluid is a mixture of ordinary matter and a still unknown form of matter i.e. of dark energy type, which is repulsive in nature. These two fluids are non-interacting and we have considered the combined effect of these two fluids in our model. Let us assume that the pressure distribution inside the fluid sphere is not isotropic in nature, but that it can be decomposed into two parts: the radial pressure p_{r} and the transverse pressure p_{t}. Here p_{t} is in the perpendicular direction to p_{r} and $\Delta = p_{\mathrm{t}} - p_{\mathrm{r}}$ is defined as an anisotropic factor. The choice of an anisotropic pressure is inspired by the fact that at the core of the superdense star, where the density $\sim 10^{15}$ gm/cc, the matter distribution shows anisotropy.

In 1982 Vaidya and Tikekar [18] proposed a static spherically symmetric model of a superdense star based on an exact solution of Einstein's equations. The physical 3-space $\{t = \text{constant}\}$ of the star is spheroidal, the density of the star is $\sim 2 \times 10^{14}$ gm/cc, and the mass is about four times the solar mass. Several studies have been performed by using the Vaidya–Tikekar ansatz. Gupta and Kumar have studied a charged Vaidya–Tikekar star in [19]. In this paper the authors have considered a particular form of electric field intensity, which has a positive gradient. This particular form of the electric field intensity was used by Sharma et al. [20]. Komathiraj and Maharaj [21] have also assumed the same expression to model a new type of Vaidya–Tikekar type star. Some new closed form solutions of Vaidya–Tikekar type star were obtained by Gupta et al. [22]. Bijalwan and Gupta [23] have taken a more general form of the electric intensity to obtain a new solution of Vaidya–Tikekar type stars with a charge analog. In this paper the authors have matched their interior solution to the exterior R-N metric and they have analyzed their

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result numerically by assuming suitable values of the chosen parameter. Some other works on Vaidya–Tikekar stars can be found in [24–26]. Very recently Bhar [27,28] proposed a new model of a strange star in the presence of a quintessence field.

In the recent past many researchers have worked on conformal motion. Anisotropic stars admitting conformal motion have been studied by Rahaman et al. [29]. A charged gravastar admitting conformal motion has been studied by Usmani et al. [8]. Relativistic stars admitting conformal motion have been analyzed in [30]. Isotropic and anisotropic charged spheres admitting a one parameter group of conformal motions were analyzed in [31–33]. A charged fluid sphere with a linear equation of state admitting conformal motion has been studied in [34]. In this paper the authors have also discussed the dynamical stability analysis of the system. Ray et al. [35,36] have given an electromagnetic mass model admitting a conformal Killing vector. By assuming the existence of a one parameter group of conformal motion Mak and Harko [37] have described a charged strange quark star model. The authors have also discussed conformally symmetric vacuum solutions of the gravitational field equations in brane-world models [38]. In a very recent work Rahaman et al. [39] and Bhar [40] have described conformal motion in higher dimensional spacetimes.

To search the natural relationship between geometry and matter through the Einstein field equations, we generally use inheritance symmetry. The well known inheritance symmetry is the symmetry under conformal Killing vectors (CKVs), i.e.,

$$L_{\xi} g_{ik} = \psi g_{ik} \tag{1}$$

where L is the Lie derivative of the metric tensor which describes the interior gravitational field of a compact star with respect to the vector field ξ, and ψ is the conformal factor. It is supposed that the vector ξ generates the conformal symmetry and the metric g is conformally mapped onto itself along ξ. Neither ξ nor ψ need to be static, even though one considers a static metric [41,42]. If $\psi = 0$ then (1) gives the Killing vector; for $\psi =$ constant it gives a homothetic vector and if $\psi = \psi(\mathbf{x}, t)$ then it yields conformal vectors. Moreover, note that if $\psi = 0$ the underlying spacetime is asymptotically flat, which further implies that the Weyl tensor will also vanish. So CKV provides a deeper insight in the spacetime geometry.

The plan of our paper is as follows: In Sect. 2 we discuss the interior solution and the Einstein field equation. The conformal Killing vector and solution of the system are given in Sects. 3 and 4, respectively. Some physical properties of the model is given in Sects. 5, 6, 7, 8, 9, 10 and 11 and we make some concluding remarks in Sect. 12.

2 Interior solutions and Einstein field equation

To describe a static spherically symmetry spacetime let us consider the line element in the standard form,

$$\mathrm{d}s^2 = -\mathrm{e}^{\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2). \tag{2}$$

Here λ and ν are functions of the radial co-ordinate r only.

Now let us assume that our model contains a quintessence-like field along with anisotropic pressure. The Einstein equations can be written as

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \tau_{\mu\nu}). \tag{3}$$

Here $\tau_{\mu\nu}$ is the energy-momentum tensor of the quintessence-like field, which is characterized by a parameter ω_q with $-1 < \omega_q < -\frac{1}{3}$. Now Kiselev [43] has shown that the components of this tensor need to satisfy the conditions of additivity and linearity. Considering the different signature used in the line elements, the components can be stated as follows:

$$\tau_{\mathrm{t}}^{\mathrm{t}} = \tau_{\mathrm{r}}^{\mathrm{r}} = -\rho_q, \tag{4}$$

$$\tau_{\theta}^{\theta} = \tau_{\phi}^{\phi} = \frac{1}{2}(3\omega_q + 1)\rho_q, \tag{5}$$

and the corresponding energy-momentum tensor can be written as

$$T_{\nu}^{\mu} = (\rho + p_{\mathrm{r}})u^{\mu}u_{\nu} - p_{\mathrm{t}}g_{\nu}^{\mu} + (p_{\mathrm{r}} - p_{\mathrm{t}})\eta^{\mu}\eta_{\nu}, \tag{6}$$

with $u^i u_j = -\eta^i \eta_j = 1$ and $u^i \eta_j = 0$. Here the vector u_i is the fluid 4-velocity and η^i is the spacelike vector which is orthogonal to u^i, ρ is the energy density; p_{r} and p_{t} are, respectively, the radial and the transversal pressure of the fluid.

The Einstein field equation assuming $G = 1 = c$ can be written as

$$\mathrm{e}^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi(\rho + \rho_q), \tag{7}$$

$$\mathrm{e}^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi(p_{\mathrm{r}} - \rho_q), \tag{8}$$

$$\frac{1}{2}\mathrm{e}^{-\lambda}\left[\frac{1}{2}\nu'^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi\left(p_{\mathrm{t}} + \frac{3\omega_q + 1}{2}\rho_q\right). \tag{9}$$

3 Conformal Killing equation

The conformal Killing equation (1) becomes

$$L_{\xi} g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik}. \tag{10}$$

Now using the conformal Killing equation for the line element (2) we get the following equations:

$$\xi^1\nu' = \psi, \tag{11}$$
$$\xi^4 = C_1, \tag{12}$$
$$\xi^1 = \frac{\psi r}{2}, \tag{13}$$
$$\xi^1\lambda' + 2\xi^1{}_{,1} = \psi. \tag{14}$$

Here C_1 is a constant.

The above four equations consequently give

$$\mathrm{e}^{\nu} = C_2^2 r^2, \tag{15}$$

where 1 stands for the spatial co-ordinates r, 'prime' and ',' denotes the partial derivative with respect to r.

$$\mathrm{e}^{\lambda} = \left(\frac{C_3}{\psi}\right)^2, \tag{16}$$

$$\xi^i = C_1\delta_4^i + \left(\frac{\psi r}{2}\right)\delta_1^i. \tag{17}$$

Here C_2 and C_3 are constants of integrations.

Now using Eqs. (15)–(17) into the Einstein field equations (7)–(9) one can obtain

$$\frac{1}{r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{2\psi\psi'}{rC_3^2} = 8\pi(\rho + \rho_q), \tag{18}$$

$$\frac{1}{r^2}\left[\frac{3\psi^2}{C_3^2} - 1\right] = 8\pi(p_{\mathrm{r}} - \rho_q), \tag{19}$$

$$\frac{\psi^2}{C_3^2 r^2} + \frac{2\psi\psi'}{rC_3^2} = 8\pi\left(p_{\mathrm{t}} + \frac{3\omega_q + 1}{2}\rho_q\right). \tag{20}$$

4 Solution

To solve Eqs. (18)–(20) we consider the Vaidya–Tikekar ansatz [18]

$$\mathrm{e}^{\lambda} = \frac{1 - K\left(\frac{r^2}{R^2}\right)}{1 - \frac{r^2}{R^2}}. \tag{21}$$

It may be noted that the physical 3-space $\{t = \text{constant}\}$ of a Vaidya–Tikekar type star is spheroidal and the geometry of the 3-spheroid is governed by the parameters R and K. Here $K < 1$. For $K = 0$, the hypersurfaces $\{t = \text{constant}\}$ becomes spherical and it gives a Schwarzschild interior solution and for $K = 1$ the hypersurfaces $\{t = \text{constant}\}$ become flat. The metric function e^{λ} is regular at the center and it is well behaved for $r < R$.

From Eqs. (16) and (21) we get

$$\psi^2 = C_3^2\frac{R^2 - r^2}{R^2 - Kr^2}. \tag{22}$$

Using the value of ψ given in Eq. (22) we can write Eqs. (18)–(20) as follows:

$$(1-K)\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} = 8\pi(\rho + \rho_q), \tag{23}$$

$$\frac{1}{r^2}\left[\frac{2R^2 - (3-K)r^2}{R^2 - Kr^2}\right] = 8\pi(p_{\mathrm{r}} - \rho_q), \tag{24}$$

$$\frac{1}{r^2}\frac{R^2 - r^2}{R^2 - Kr^2} - \frac{2(1-K)R^2}{(R^2 - Kr^2)^2} = 8\pi\left(p_{\mathrm{t}} + \frac{3\omega_q + 1}{2}\rho_q\right). \tag{25}$$

One can note from Eqs. (23)–(25) that we have three equations with four unknowns, namely ρ, p_{r}, p_{t}, ρ_q.

To solve the above three Eqs. (23)–(25) let us assume that the radial pressure p_{r} is proportional to the matter density ρ, i.e.,

$$p_{\mathrm{r}} = m\rho, \qquad 0 < m < 1. \tag{26}$$

Here m is the equation of state parameter.

Solving Eqs. (23)–(25) with the help of Eq. (26) one can obtain

$$\rho = \frac{1}{8\pi(1+m)}\left[(1-K)\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} + \frac{2R^2 - (3-K)r^2}{r^2(R^2 - Kr^2)}\right], \tag{27}$$

$$p_{\mathrm{r}} = \frac{m}{8\pi(1+m)}\left[(1-K)\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} + \frac{2R^2 - (3-K)r^2}{r^2(R^2 - Kr^2)}\right], \tag{28}$$

$$\rho_q = \frac{1-K}{8\pi}\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} - \frac{1}{8\pi(1+m)}\left[(1-K)\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} + \frac{2R^2 - (3-K)r^2}{r^2(R^2 - Kr^2)}\right], \tag{29}$$

$$p_{\mathrm{t}} = \frac{1}{8\pi}\left[\frac{R^2 - r^2}{r^2(R^2 - Kr^2)} + \frac{2(K-1)R^2}{(R^2 - Kr^2)^2}\right] - \frac{(3\omega_q + 1)}{2} \times \frac{(1-K)}{8\pi}\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2} - \frac{3\omega_q + 1}{16\pi(1+m)}\frac{(1-K)(3R^2 - Kr^2)}{(R^2 - Kr^2)^2} + \frac{3\omega_q + 1}{16\pi(1+m)}\frac{2R^2 - (3-K)r^2}{r^2(R^2 - Kr^2)}. \tag{30}$$

We denote

$$\rho_{\mathrm{eff}} = \frac{1}{8\pi}\left[(1-K)\frac{3R^2 - Kr^2}{(R^2 - Kr^2)^2}\right], \tag{31}$$

$$p_{\mathrm{r\,eff}} = \frac{1}{8\pi}\frac{1}{r^2}\left[\frac{2R^2 - (3-K)r^2}{R^2 - Kr^2}\right], \tag{32}$$

$$p_{\mathrm{t\,eff}} = \frac{1}{8\pi}\left[\frac{1}{r^2}\frac{R^2 - r^2}{R^2 - Kr^2} - \frac{2(1-K)R^2}{(R^2 - Kr^2)^2}\right]. \tag{33}$$

The plot of effective density and effective radial and transverse pressure are shown in Figs. 1 and 2, respectively. From these two figures we see that density, radial and transverse pressure all are monotonic decreasing function of r and approach zero at the surface of the star.

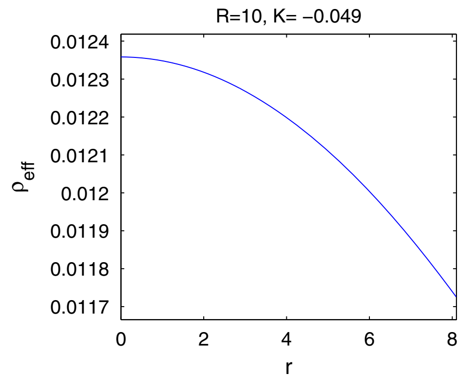

Fig. 1 Effective density plotted against r

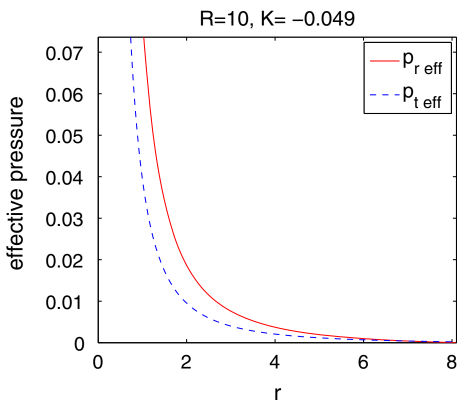

Fig. 2 Effective radial and transverse pressure plotted against r

5 Physical analysis

The effective central density of the quintessence star is given by

$$\rho_{0\ \mathrm{eff}} = \rho_{\mathrm{eff}}(r=0) = \frac{3(1-K)}{8\pi R^2}, \tag{34}$$

$$\frac{\mathrm{d}\rho_{\mathrm{eff}}}{\mathrm{d}r} = \frac{K(1-K)}{4\pi}\frac{r(5R^2-Rr^2)}{(R^2-Kr^2)^3}, \tag{35}$$

$$\frac{\mathrm{d}^2\rho_{\mathrm{eff}}}{\mathrm{d}r^2}|_{r=0} = \frac{5K(1-K)}{4\pi R^4}. \tag{36}$$

So we see that the effective density is regular at the center. The plot of $\frac{\mathrm{d}\rho_{\mathrm{eff}}}{\mathrm{d}r}$ vs. r is shown in Fig. 3. Both the expressions given in Eqs. (35) and (36) are negative, since we have chosen $K<0$ for our model. This tells us that the effective density has a maximum value at the center of the star.

From the expression of $p_{\mathrm{r\ eff}}$ given in Eq. (32) we see that effective radial pressure is not regular at the center. However,

$$\frac{\mathrm{d}p_{\mathrm{r\ eff}}}{\mathrm{d}r} = -\frac{2\left[2R^4+K(3-K)r^4-4Kr^2R^2\right]}{r^3(R^2-Kr^2)^2} < 0. \tag{37}$$

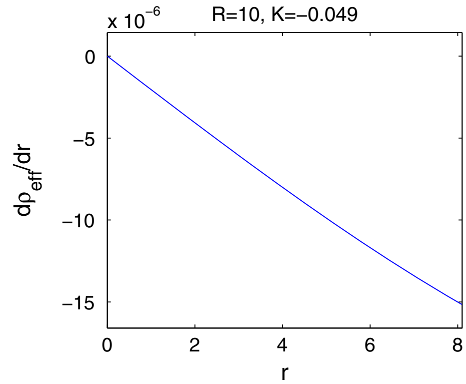

Fig. 3 $\frac{\mathrm{d}\rho_{\mathrm{eff}}}{\mathrm{d}r}$ plotted against r

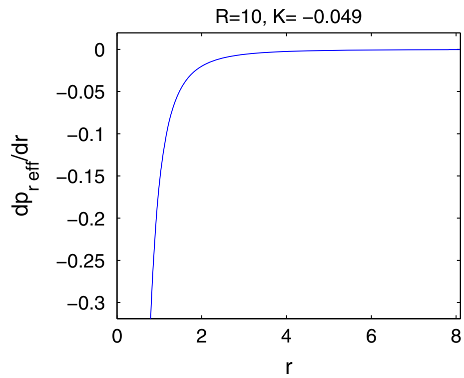

Fig. 4 $\frac{\mathrm{d}p_{\mathrm{r\ eff}}}{\mathrm{d}r}$ plotted against r

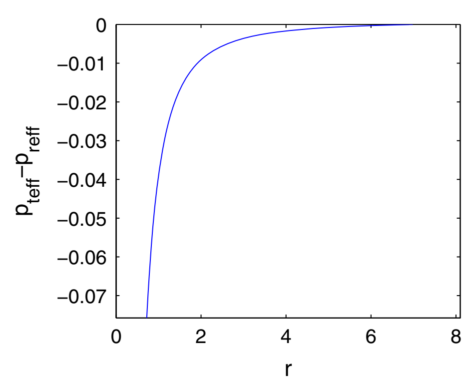

Fig. 5 Anisotropic factor plotted against r using $R = 10$ and $K = -0.049$

The profile of $\frac{\mathrm{d}p_{\mathrm{r\ eff}}}{\mathrm{d}r}$ vs. r is shown in Fig. 4. This figure once again verifies that $\frac{\mathrm{d}p_{\mathrm{r\ eff}}}{\mathrm{d}r} < 0$.

The anisotropic factor of our model of the quintessence star is defined by

$$\Delta = (p_{\mathrm{t\ eff}} - p_{\mathrm{r\ eff}}) = \frac{1}{8\pi}\left[\frac{1}{r^2}\frac{-R^2+(2-K)r^2}{R^2-Kr^2} - \frac{2(1-K)R^2}{(R^2-Kr^2)^2}\right] \tag{38}$$

and $\frac{2\Delta}{r}$ is termed the anisotropic force.

The profile of anisotropic factor is given in Fig. 5, which shows that $\Delta < 0$; this implies that $p_t < p_r$. From this we can conclude that the force is attractive in nature. One may note that generally the quintessence field is repulsive in nature but for our model, where we have considered the combined effect of ordinary matter and the quintessence field, the effective anisotropic force turns out to be attractive.

6 Exterior spacetime and matching condition

In this section we will match our interior solution of the quintessence star to the Schwarzschild exterior solution at the boundary $r = a$ outside the event horizon, i.e. $a > 2M$, where the exterior spacetime is described by the metric

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \quad (39)$$

Using the matching condition at the boundary we have

$$1 - \frac{2M}{a} = C_2^2 a^2 \quad (40)$$

and

$$\left(1-\frac{2M}{a}\right)^{-1} = \frac{R^2 - Ka^2}{R^2 - a^2}. \quad (41)$$

Solving the above two equations we get

$$C_2^2 = \frac{1}{a^2}\left(1-\frac{2M}{a}\right), \quad (42)$$

$$K = \frac{1 - \left(\frac{2M}{a}\right)\left(\frac{R^2}{a^2}\right)}{1-\frac{2M}{a}}. \quad (43)$$

7 TOV equation

To describe the static equilibrium let us consider the generalized Tolman–Oppenheimer–Volkov (TOV) equation, which is represented by the formula

$$-\frac{M_G(\rho + p_r)}{r^2}\mathrm{e}^{\frac{\lambda-\nu}{2}} - \frac{\mathrm{d}p_r}{\mathrm{d}r} + \frac{2}{r}(p_t - p_r) = 0. \quad (44)$$

Here $M_G = M_G(r)$ is termed the effective gravitational mass inside the fluid sphere of radius r and is defined by

$$M_G(r) = \frac{1}{2}r^2\mathrm{e}^{\frac{\nu-\lambda}{2}}\nu'. \quad (45)$$

The above expression of $M_G(r)$ can be derived from the Tolman–Whittaker mass formula.

Using the expression of Eq. (45) in (44) we obtain the modified TOV equation,

$$F_g + F_h + F_a = 0 \quad (46)$$

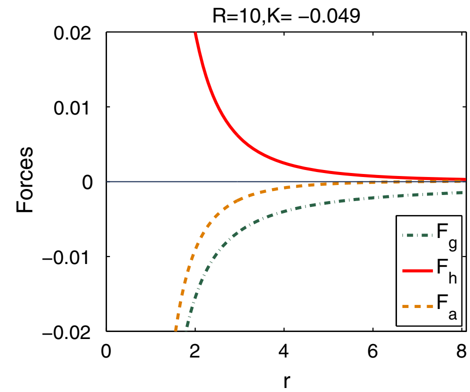

Fig. 6 The system is in static equilibrium under three forces

where

$$F_g = -\frac{\nu'}{2}(\rho_{\mathrm{eff}} + p_{r\,\mathrm{eff}}), \quad (47)$$

$$F_h = -\frac{\mathrm{d}p_r}{\mathrm{d}r}, \quad (48)$$

$$F_a = \frac{2}{r}(p_{t\,\mathrm{eff}} - p_{r\,\mathrm{eff}}). \quad (49)$$

Here F_g, F_h, and F_a are termed gravitational, hydrostatic, and anisotropic forces, respectively, of the system. From Fig. 6 we see that the system is in equilibrium under the above three forces.

8 Energy condition

All the energy conditions, namely, the null energy condition (NEC), the weak energy condition (WEC), the strong energy condition (SEC) are satisfied for our model if the following inequalities hold in the interior of the fluid sphere:

$$\rho_{\mathrm{eff}} \geq 0, \quad (50)$$

$$\rho_{\mathrm{eff}} + p_{r\,\mathrm{eff}} \geq 0, \quad (51)$$

$$\rho_{\mathrm{eff}} + p_{t\,\mathrm{eff}} \geq 0, \quad (52)$$

$$\rho_{\mathrm{eff}} + p_{r\,\mathrm{eff}} + 2p_{t\,\mathrm{eff}} \geq 0. \quad (53)$$

We will prove these inequalities with the help of a graphical representation by choosing some suitable values to the parameters, given in Fig. 7. From the figure we see that WEC, NEC, and SEC are satisfied by our model. Since SEC is satisfied by our model, we can conclude that our spacetime does not contain any black hole.

9 Stability

For a physically acceptable model one must have the velocity of sound in the range $0 < v^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} \leq 1$.

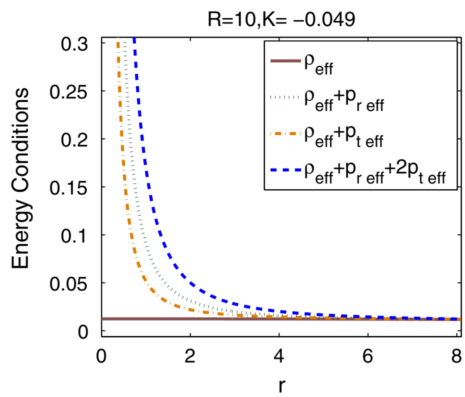

Fig. 7 Energy conditions plotted against r

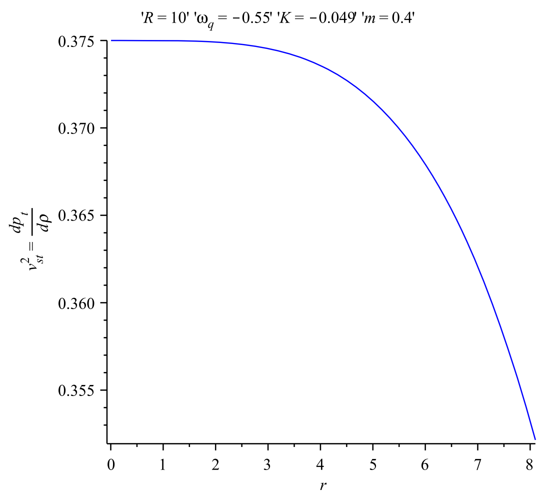

Fig. 8 Transverse velocity shown against r

In the case of anisotropy the radial (v_{sr}^2) and transverse (v_{st}^2) sound velocity can be obtained as

$$v_{\mathrm{sr}}^2 = \frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}\rho} = m = 0.4 < 1, \tag{54}$$

$$v_{\mathrm{st}}^2 = \frac{\mathrm{d}p_{\mathrm{t}}}{\mathrm{d}\rho}. \tag{55}$$

From Eq. (54) we have $v_{\mathrm{sr}}^2 < 1$ and Fig. 8 shows that $0 < v_{\mathrm{st}}^2 < 1$ everywhere within the stellar configuration. According to Herrera's [44] cracking (or overturning) theorem for a potentially stable region one must have $v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2 < 0$. From Fig. 9 it is clear that our model satisfies this condition. So we conclude that our model is potentially stable. Moreover, $0 < v_{\mathrm{sr}}^2 \leq 1$ and $0 < v_{\mathrm{st}}^2 < 1$ therefore according to Andréasson [45], $|v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2| \leq 1$ which is also clear from Fig. 10.

10 Some features

10.1 Mass radius relation

The mass function within the radius r can be obtained as

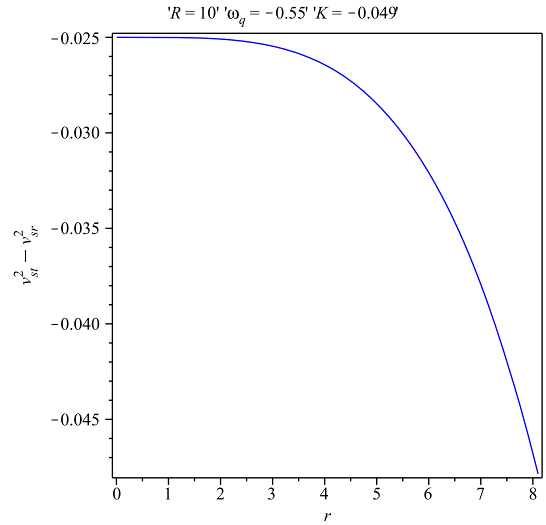

Fig. 9 $v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2$ plotted against r

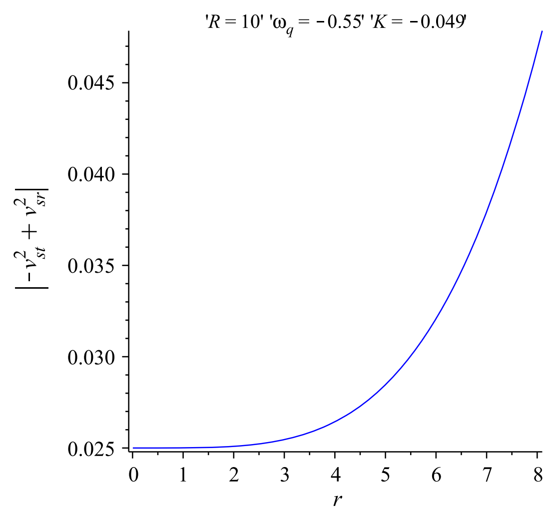

Fig. 10 $|-v_{\mathrm{st}}^2 + v_{\mathrm{sr}}^2|$ plotted against r

$$M_{\mathrm{eff}}(r) = \int_0^r 4\pi r^2 \rho_{\mathrm{eff}} \mathrm{d}r = \frac{1-K}{2} \frac{r^3}{R^2 - Kr^2}. \tag{56}$$

The profile of the effective mass function is given in Fig. 11. For $r \to 0$, $M_{\mathrm{eff}}(r) \to 0$, which implies that the mass function is regular at the center; moreover, the effective mass function is monotonic increasing function of r and it is positive inside the stellar configuration.

10.2 Compactness

The effective compactness of the star $u_{\mathrm{eff}}(r)$ can be defined by

$$u_{\mathrm{eff}} = \frac{M_{\mathrm{eff}}(r)}{r} = \frac{1-K}{2} \frac{r^2}{R^2 - Kr^2}. \tag{57}$$

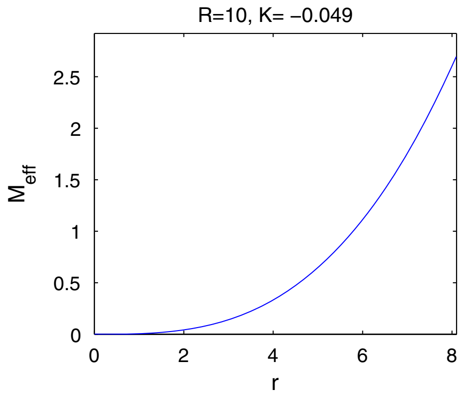

Fig. 11 Effective mass function plotted against r

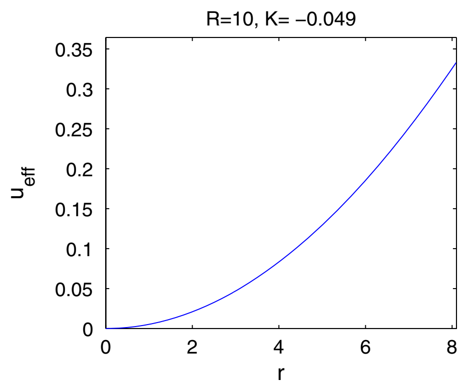

Fig. 12 Effective compactness plotted against r

The profile of the effective compactness of the star is depicted in Fig. 12.

10.3 Surface redshift

The redshift function $Z_{\mathrm{s\ eff}}$ can be defined by

$1 + z_{\mathrm{s\ eff}} = (1 - 2u_{\mathrm{eff}})^{-\frac{1}{2}}$;

using the above formula we obtain

$$z_{\mathrm{s\ eff}} = (1 - 2u_{\mathrm{eff}})^{-\frac{1}{2}} - 1 = \left(\frac{R^2 - r^2}{R^2 - Kr^2}\right)^{-\frac{1}{2}} - 1. \tag{58}$$

The profile of the effective redshift function is given in Fig. 13, which is a monotonic increasing function of r.

11 Junction condition

In Sect. 6 we have matched our interior spacetime to the exterior Schwarzschild at the boundary $r = a$. It is obvious that the metric coefficients are continuous at $r = a$, but this does not ensure that their derivatives are also continuous at the junction surface. To take care of this let us consider the

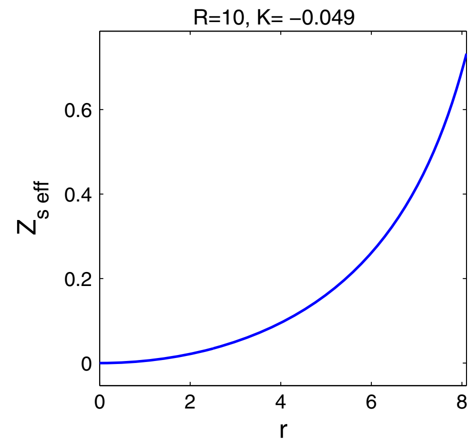

Fig. 13 Effective surface redshift plotted against r

Darmois–Israel [46,47] formation to determine the surface stresses at the junction boundary. The intrinsic surface stress energy tensor S_{ij} is given by the Lancozs equations in the following form:

$$S^i_j = -\frac{1}{8\pi}(\kappa^i_j - \delta^i_j \kappa^k_k). \tag{59}$$

The second fundamental form is given by

$$K^{\pm}_{ij} = -n^{\pm}_{\nu}\left[\frac{\partial^2 X_\nu}{\partial \xi^1 \partial \xi^j} + \Gamma^{\nu}_{\alpha\beta}\frac{\partial X^\alpha}{\partial \xi^i}\frac{\partial X^\beta}{\partial \xi^j}\right]\Bigg|_S, \tag{60}$$

and the discontinuity in the second fundamental form is given by

$$\kappa_{ij} = K^{+}_{ij} - K^{-}_{ij} \tag{61}$$

where $n^{\pm}_{\nu}$ are the unit normal vector defined by

$$n^{\pm}_{\nu} = \pm\left|g^{\alpha\beta}\frac{\partial f}{\partial X^\alpha}\frac{\partial f}{\partial X^\beta}\right|^{-\frac{1}{2}}\frac{\partial f}{\partial X^\nu} \tag{62}$$

with $n^\nu n_\nu = 1$. Here ξ^i is the intrinsic coordinate on the shell. $+$ and $-$ correspond to exterior, i.e., Schwarzschild spacetime and interior (our) spacetime, respectively.

The non-trivial components of the extrinsic curvature are given by

$$K^{\tau\ +}_{\tau} = \frac{\frac{M}{a^2} + \ddot{a}}{\sqrt{1 - \frac{2M}{a} + \dot{a}^2}}, \tag{63}$$

$$K^{\tau\ -}_{\tau} = \frac{\frac{-a(1-K)R^2}{(R^2-Ka^2)^2} + \ddot{a}}{\sqrt{\frac{R^2-a^2}{R^2-Ka^2} + \dot{a}^2}}, \tag{64}$$

and

$$K^{\theta\ +}_{\theta} = \frac{1}{a}\sqrt{1 - \frac{2M}{a} + \dot{a}^2}, \tag{65}$$

$$K_\theta^{\theta\,-} = \frac{1}{a}\sqrt{\frac{R^2 - a^2}{R^2 - Ka^2} + \dot{a}^2}. \tag{66}$$

Considering the spherical symmetry of the spacetime, the surface stress energy tensor can be written as $S_j^i = \mathrm{diag}(-\sigma, \mathcal{P})$. Here σ and $\mathcal{P}$ are the surface energy density and surface pressure, respectively. We have

$$\sigma = -\frac{1}{4\pi a}[\sqrt{\mathrm{e}^{-\lambda}}]_-^+ = -\frac{1}{4\pi a}\left[\sqrt{1 - \frac{2M}{a} + \dot{a}^2} - \sqrt{\frac{R^2 - a^2}{R^2 - Ka^2} + \dot{a}^2}\right], \tag{67}$$

$$\mathcal{P} = \frac{1}{8\pi a}\left[\left\{1 + \frac{a\nu'}{2}\right\}\sqrt{\mathrm{e}^{-\lambda}}\right]_-^+ = \frac{1}{8\pi a}\left[\frac{1 - \frac{M}{a}}{\sqrt{1 - \frac{2M}{a}}} - 2\sqrt{\frac{R^2 - a^2}{R^2 - Ka^2}}\right]. \tag{68}$$

Hence we can match our interior spacetime to the exterior Schwarzschild spacetime in the presence of a thin shell.

The mass of the thin shell is given by

$$m_{\mathrm{s}} = 4\pi a^2 \sigma. \tag{69}$$

From (67) and (69) one can obtain

$$M = \frac{a^3}{2}\left[\frac{1 - K}{R^2 - Ka^2} - m_{\mathrm{s}}^2\right] + 2am_{\mathrm{s}}\sqrt{\frac{R^2 - a^2}{R^2 - Ka^2}}. \tag{70}$$

This gives the mass of the quintessence star in terms of the thin shell mass.

Next we will discuss the evolution identity given by $[T_{\mu\nu}n^\mu n^\nu]_-^+ = \overline{K}_j^i S_i^j$ where

$$\overline{K}_j^i = \frac{1}{2}(K_j^{i+} + K_j^{i-}),$$

which gives

$$p_{\mathrm{r}} + \frac{(\rho + p_{\mathrm{r}})\dot{a}^2}{\frac{R^2 - a^2}{R^2 - Ka^2}} = -\frac{1}{2a}\left(\sqrt{1 - \frac{2M}{a} + \dot{a}^2} + \sqrt{\frac{R^2 - a^2}{R^2 - Ka^2} + \dot{a}^2}\right)\mathcal{P} + \frac{1}{2}\left(\frac{\frac{M}{a^2} + \ddot{a}}{\sqrt{1 - \frac{2M}{a} + \dot{a}^2}} + \frac{\frac{-a(1-K)R^2}{(R^2 - Ka^2)^2} + \ddot{a}}{\sqrt{\frac{R^2 - a^2}{R^2 - Ka^2} + \dot{a}^2}}\right)\sigma. \tag{71}$$

For a static solution a_0 from Eq. (71) (assuming $\dot{a} = 0 = \ddot{a}$) one can obtain

$$p_{\mathrm{r}}(a_0) = -\frac{1}{2a_0}\left(\sqrt{1 - \frac{2M}{a_0}} + \sqrt{\frac{R^2 - a_0^2}{R^2 - Ka_0^2}}\right)\mathcal{P} + \frac{1}{2}\left(\frac{\frac{M}{a_0^2}}{\sqrt{1 - \frac{2M}{a_0}}} + \frac{\frac{-a_0(1-K)R^2}{(R^2 - Ka_0^2)^2}}{\sqrt{\frac{R^2 - a_0^2}{R^2 - Ka_0^2}}}\right)\sigma. \tag{72}$$

The above equation relates the radial pressure (p_{r}) of the quintessence star with the surface pressure ($\mathcal{P}$) and surface density (σ) of the thin shell.

12 Discussion and concluding remarks

In the present paper we have proposed a new model of a superdense star by choosing the Vaidya–Tikekar spacetime which admits CKV in the presence of a quintessence field which is characterized by a parameter ω_q with $-1 < \omega_q < -\frac{1}{3}$. For our model the effective density is regular at the center but the radial and transverse pressure suffers from a central singularity like other CKV models. The profile of both the density function and the radial pressure are monotonically decreasing, which indicates that the density and radial pressure of the star is maximum at the center and it decreases from the center to the surface of the star. By taking $R = 10$ and $K = -0.049$ from the relation $p_{\mathrm{r\,eff}}(a) = 0$ (where a is the radius of the star) we obtain $a = 8.1$ km. Plugging in G and c, the central density of the star is calculated as $\rho_{\mathrm{c}} = 1.69 \times 10^{15}$ gm/cc and $M_{\mathrm{eff}} = 1.83M_\odot$. The mass function is regular at the center and the maximum allowable ratio of mass to radius is $0.333 < \frac{4}{9}$, which lies in the Buchdahl [48] limit; and the maximum value of the surface redshift is calculated as 0.732. For our model the radial and transverse speed of sound is less than 1, which gives the stability condition. According to Herrera's [44] concept if for a model the radial speed of sound is greater than the transverse speed of sound, the model is potentially stable. With the help of a graphical representation we have shown that $v_{\mathrm{sr}}^2 - v_{\mathrm{st}}^2 > 0$. So our model is potentially stable. All the energy conditions are satisfied inside the fluid sphere. Our model is also in static equilibrium under anisotropic, gravitational, and hydrostatic forces. We have matched our interior solution to the exterior Schwarzschild metric in the presence of a thin shell and also obtained the mass of the quintessence star in terms of the thin shell mass. A relation among the radial pressure, surface pressure, and surface density has been obtained.

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