# $f_{J}(2220)$ and hadronic $\bar{B}_{s}^{0}$ decays 

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#### Abstract

We study the hadronic $\bar{B}_{s}^{0}$ decays based on the existence of the resonant state $f_{J}(2220)$. In particular, we are able to explain the unexpected large experimental result of $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)=\left(3.0_{-1.1}^{+1.2} \pm 0.52 \pm 0.03\right) \times 10^{-6}$ measured recently by the LHCb collaboration due to the resonant contribution in $\bar{B}_{s}^{0} \rightarrow J / \psi f_{J}(2220)$ with $f_{J}(2220) \rightarrow p \bar{p}$, while it is estimated to be at most of order $10^{-9}$ in terms of the OZI rule without the resonance. In addition, we find that $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) p \bar{p}\right)=(4.70 \pm 2.89) \times 10^{-7}$, $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) \pi \pi\right)=(15.6 \pm 15.2) \times 10^{-6}$, and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) \pi \pi\right)=(24.5 \pm 24.4) \times 10^{-7}$, while $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) K \bar{K}\right)<1.6 \times 10^{-5}$ and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) K \bar{K}\right)<2.5 \times 10^{-6}$. Moreover, we predict that the decay branching ratios of $\bar{B}_{s}^{0} \rightarrow\left(J / \psi, D^{* 0}\right) \Lambda \bar{\Lambda}$ are $(2.68 \pm 1.23) \times 10^{-7}$ and $(2.25 \pm 0.80) \times 10^{-6}$. Some of the predicted $\bar{B}_{s}^{0}$ decays are accessible to the experiments at the LHCb.


## 1 Introduction

In some three-body $B$ meson decay of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$, with $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ a baryon pair and $M$ a recoiled meson or photon, the partial decay width as the function of $m_{\mathbf{B}_{\mathbf{B}^{\prime}}}=p_{\mathbf{B}}+p_{\overline{\mathbf{B}}^{\prime}}$ is observed to have a peak near $m_{\mathbf{B}^{\prime}} \simeq m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}$ of the threshold area. This is the so-called threshold enhancement, which dominates the decay branching ratio of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$. The examples of these decays include $B \rightarrow p \bar{p} M$ with $M=\left(D^{(*)}, K^{(*)}, \pi, \rho\right)$ and $B \rightarrow \Lambda \bar{p} M^{\prime}$ with $M^{\prime}=(\pi, \rho, \gamma)$. Theoretically, the threshold effect has been realized as the result of the perturbative QCD (pQCD) effect [1-4]. Consequently, many experimental data on the baryonic B decays can be well explained [5-8].

However, it is not the case for $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$. The branching ratio of $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$ presented by the LHCb collaboration is given by [9]

[^0]\[

$$
\begin{equation*}
\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)=\left(3.0_{-1.1}^{+1.2} \pm 0.52 \pm 0.03\right) \times 10^{-6} \tag{1}
\end{equation*}
$$

\]

where the first and second uncertainties are statistical and systematic, respectively, while the third one originates from the control channel branching fraction measurement. Note that $\mathcal{B}\left(\bar{B}^{0} \rightarrow J / \psi p \bar{p}\right)=\left(2.0_{-1.7}^{+1.9} \pm 0.9 \pm 0.1\right) \times 10^{-7}$ has also been given by the LHCb [9]. With $\bar{B}^{0} \rightarrow(c \bar{c})(d \bar{d}) \rightarrow$ $J / \psi p \bar{p}$, the $p \bar{p}$ production has the direct transition from $\bar{B}^{0} \rightarrow d \bar{d} \rightarrow p \bar{p}$, which associates with the threshold enhancement, such that theoretical prediction of $(11.4 \pm$ 5.0) $\times 10^{-7}$ in Ref. [8] ${ }^{1}$ can be seen to be consistent with observation. On the contrary, $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$ via $\bar{B}_{s}^{0} \rightarrow$ $s \bar{s} \rightarrow p \bar{p}$ leads to the OZI suppression, while $s \bar{s}$ should be first annihilated to produce $p \bar{p}$. With the OZI suppression of $\mathcal{B}(\phi \rightarrow \pi \pi) / \mathcal{B}(\phi \rightarrow K \bar{K}) \simeq 10^{-4}$ [10], one expects that $\mathcal{B}\left(\bar{B}_{s_{-}}^{0} \rightarrow J / \psi p \bar{p}\right) \leq 10^{-4} \mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}\right)$, resulting in $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right) \leq 10^{-9}$, while $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}\right)$ is considered to be at the same level as $\mathcal{B}\left(B^{-} \rightarrow J / \psi \Lambda \bar{p}\right) \simeq$ $1.18 \times 10^{-5}$ [10]. Therefore, to understand the large branching ratio of around $3 \times 10^{-6}$ for $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$ in Eq. (1), a new theoretical study on this decay is clearly needed.

To explain $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)$, one possible solution is to have a resonant state between the $s \bar{s}$ annihilation and $p \bar{p}$ production in $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$, so that the process through its mass shell allows an on-shell enhancement for the decay branching ratio. Indeed, it is common to observe resonant peaks in $B \rightarrow p \bar{p} M$. For example, one finds the $c \bar{c}$ mesons, where the resonant $\eta_{c} \rightarrow p \bar{p}$ and $J / \psi \rightarrow p \bar{p}$ raise the $m_{p \bar{p}}$ spectrum of $B^{-} \rightarrow K^{-} p \bar{p}$ [11], as well as those identified as the charmed baryons and the glueball from $D^{(*)} p$ and $p \bar{p}$ spectra in $\bar{B}^{0} \rightarrow D^{(*) 0} p \bar{p}$ [12-14], respectively. According to Refs. $[10,15-18]$, since $f_{J}(2220) \equiv f_{J}$ with the quantum numbers $J^{P C}=2^{++}$or $4^{++}$has the channel of $f_{J} \rightarrow$ $p \bar{p}$, particularly, with its mass and decay width within the allowed region of the $m_{p \bar{p}}$ spectrum in $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$, it is

[^1]Fig. 1 The decays of $\bar{B}_{S_{-}}^{0} \rightarrow V \mathbf{B} \overline{\mathbf{B}}^{\prime}$ with $\mathbf{B} \overline{\mathbf{B}}^{\prime}=(\mathrm{a})$ $\Lambda \bar{\Lambda}$ and (b) $p \bar{p}$, produced by the pQCD effect and the resonance $f_{J}$, respectively, where the block pairs represent the integrated-over $W$ boson in the effective Hamiltonian

(a)

(b)
reasonable that $f_{J}$ can be our candidate as the resonant state in $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$.

The experimental status of $f_{J}$ is reported in Ref. [19], where its evidence comes from the Mark III collaboration [20] and the BES collaboration [21], also being supported by $\pi^{-}\left(K^{-}\right) p$ collisions [22-24]. However, the direct confirmations from $p \bar{p}$ collisions $[25,26]$ and $2 \gamma$ processes $[27,28]$ are inconclusive. Hence, it leaves the room for the $\bar{B}_{s}^{0}$ meson decays to provide the new scenario for the $f_{J}$ study. Moreover, according to the QCD models [29], such as the Lattice QCD (LQCD) calculation [30,31], $f_{J}$ has the mass close to that of the tensor glueball $\left(G_{2^{++}}\right)$with $J^{P C}=2^{++}$. Moreover, the theoretical prediction of [32]
$\mathcal{B}\left(J / \psi \rightarrow \gamma G_{2^{++}}\right)=(1.1 \pm 0.2 \pm 0.1) \times 10^{-2}$
agrees with the lower bound of $\mathcal{B}\left(J / \psi \rightarrow \gamma f_{J}\right)>2.5 \times$ $10^{-3}$ [10]. With $f_{J}(2220)$ being identified as $G_{2^{++}}$, Eq. (2) can be related to the radiative $J / \psi$ decays by the BABAR collaboration [33], given by [10],

$$
\begin{align*}
& \mathcal{B}\left(J / \psi \rightarrow \gamma f_{J}\right) \mathcal{B}\left(f_{J} \rightarrow p \bar{p}, \pi \pi\right) \\
& \quad=(1.5 \pm 0.8,8 \pm 5) \times 10^{-5} \\
& \mathcal{B}\left(J / \psi \rightarrow \gamma f_{J}\right) \mathcal{B}\left(f_{J} \rightarrow K \bar{K}\right)<3.6 \times 10^{-5}, \tag{3}
\end{align*}
$$

so that we obtain
$\mathcal{B}\left(f_{J} \rightarrow p \bar{p}, \pi \pi\right)=(1.4 \pm 0.8,7.3 \pm 3.9) \times 10^{-3}$,
$\mathcal{B}\left(f_{J} \rightarrow K \bar{K}\right)<4.1 \times 10^{-3}$,
where the limit is based on the $1 \sigma$ error of the measured value on $J / \psi \rightarrow \gamma G_{2^{++}}$. We remark that the results in Eq. (4) are consistent with the ratios: $\mathcal{B}\left(f_{J} \rightarrow p \bar{p}, \pi \pi\right) / \mathcal{B}\left(f_{J} \rightarrow\right.$ $K \bar{K})=(0.17 \pm 0.09,1.0 \pm 0.5)$ in the PDG [10].

In this paper, we shall explain $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$ with $f_{J}(2220)$ as the resonant state to $p \bar{p}$. Due to this resonant state, we will also study the other hadronic decays of $\bar{B}_{s}^{0}$, such as $\bar{B}_{s}^{0} \rightarrow J / \psi(\pi \pi, K \bar{K}, \Lambda \bar{\Lambda})$ and $\bar{B}_{s}^{0} \rightarrow$ $D^{0 *}(\pi \pi, K \bar{K}, p \bar{p}, \Lambda \bar{\Lambda})$.

## 2 Formalism

In the effective Hamiltonian [34], the amplitude of $\bar{B}_{s}^{0} \rightarrow$ $V \mathbf{B} \overline{\mathbf{B}}^{\prime}$ with the baryon pair $\mathbf{B} \overline{\mathbf{B}}^{\prime}=p \bar{p}$ or $\Lambda \bar{\Lambda}$ can be factorized as

$$
\begin{align*}
& \mathcal{A}\left(\bar{B}_{s}^{0} \rightarrow V \mathbf{B} \overline{\mathbf{B}}^{\prime}\right) \\
& \quad=\frac{G_{F}}{\sqrt{2}} V_{c b} V_{q s}^{*} a_{2}^{V}\langle V|(\bar{c} q)_{V-A}|0\rangle\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right|(\bar{s} b)_{V-A}\left|\bar{B}_{s}^{0}\right\rangle \tag{5}
\end{align*}
$$

where $G_{F}$ is the Fermi constant, $V_{q_{1} q_{2}}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $a_{2}^{V}$ is the coefficient studied in Ref. [8], while $\left(\bar{q}_{1} q_{2}\right)_{V-A}$ denotes $\bar{q}_{1} \gamma_{\mu}\left(1-\gamma_{5}\right) q_{2}$ and $V$ stands for the vector meson $J / \psi\left(D^{* 0}\right)$ with $q=c(u)$. In Eq. (5), the matrix element of the vector meson production is defined by

$$
\begin{equation*}
\langle V|(\bar{q} c)_{V-A}|0\rangle=m_{V} f_{V} \varepsilon_{\mu}^{*} \tag{6}
\end{equation*}
$$

where $m_{V}, f_{V}$, and $\varepsilon_{\mu}^{*}$ are the mass, decay constant, and polarization of the vector meson $V$, respectively.

For $\bar{B}_{s}^{0} \rightarrow V \Lambda \bar{\Lambda}$, since the $s \bar{s}$ pair can have a direct transition to be a part of the internal quarks in $\Lambda \bar{\Lambda}$ as seen in Fig. 1a, the most general matrix elements of the $\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda}$ transition are given by [7]

$$
\begin{align*}
& \langle\Lambda \bar{\Lambda}|(\bar{s} b)_{V}\left|\bar{B}_{s}^{0}\right\rangle=i \bar{u}\left[g_{1} \gamma_{\mu}+g_{2} i \sigma_{\mu \nu} p^{\nu}\right. \\
& \left.\quad+g_{3} p_{\mu}+g_{4}\left(p_{\bar{\Lambda}}+p_{\Lambda}\right)_{\mu}+g_{5}\left(p_{\bar{\Lambda}}-p_{\Lambda}\right)_{\mu}\right] \gamma_{5} v, \\
& \langle\Lambda \bar{\Lambda}|(\bar{s} b)_{A}\left|\bar{B}_{s}^{0}\right\rangle=i \bar{u}\left[f_{1} \gamma_{\mu}+f_{2} i \sigma_{\mu \nu} p^{\nu}+f_{3} p_{\mu}\right. \\
& \left.\quad+f_{4}\left(p_{\bar{\Lambda}}+p_{\Lambda}\right)_{\mu}+f_{5}\left(p_{\bar{\Lambda}}-p_{\Lambda}\right)_{\mu}\right] v \tag{7}
\end{align*}
$$

with $p=p_{\bar{B}_{s}^{0}}-p_{\Lambda}-p_{\bar{\Lambda}}$ and the form factors $g_{i}$ and $f_{i}(i=$ $1,2, \ldots, 5)$. In the approach of pQCD counting rules [1-4], we are able to count the number of hard gluon propagators within the baryon pair, so that the momentum dependences of $g_{i}$ and $f_{i}$ can be parameterized as [7]
$f_{i}=\frac{D_{f_{i}}}{t^{n}}, \quad g_{i}=\frac{D_{g_{i}}}{t^{n}}$,
with $t \equiv m_{\Lambda \bar{\Lambda}}^{2}=\left(p_{\Lambda}+p_{\bar{\Lambda}}\right)^{2}$. To the leading order, the counting gives $n=3$, in which 2 of them are for the gluons connecting to the valence quarks, while the rest one for the gluon speeding up $\bar{s}$ in $\bar{B}_{s}^{0}$ to be part of $\bar{\Lambda}$. As $t$ approaches the threshold area, the increasing value of $1 / t^{3}$ creates a peak in the $m_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ spectrum of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$, which interprets the threshold enhancement. Under the $\mathrm{SU}(3)$ flavor symmetry, $D_{g_{i}}$ and $D_{f_{i}}$ are related by $D_{g_{1}\left(f_{1}\right)}=D_{\| \|}$, and $D_{g_{k}}=-D_{f_{k}}=D_{\|}^{k}(k=2,3, \ldots, 5)$, in which the reduced constants $D_{\|},\left(D_{\Pi}\right)$ and $D_{\|}^{k}$ can be fitted through the measured baryonic decays [8].

For $\bar{B}_{s}^{0} \rightarrow V p \bar{p}$, because the matrix elements of the $\bar{B}_{s}^{0} \rightarrow$ $(s \bar{s} \rightarrow) p \bar{p}$ transition need the $s \bar{s}$ annihilation to produce $p \bar{p}$, which encounters the OZI suppression, it is not suitable for pQCD counting rules. Consequently, $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)$ is estimated to be smaller than $10^{-9}$ as mentioned early. On the other hand, for the resonant transition of $\bar{B}_{s}^{0} \rightarrow f_{J} \rightarrow$ $p \bar{p}$ as shown in Fig. 1b, $m_{f_{J}} \simeq 2.23 \mathrm{GeV}$ is in the $p \bar{p}$ invariant mass $\left(m_{p \bar{p}}\right)$ spectrum, of which the range of 1.88 $\mathrm{GeV}<m_{p \bar{p}}<2.27 \mathrm{GeV}$ is so confined that the resonance has a complete peak, enhancing the decay branching ratio of $\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}$. The matrix element of the resonant $\bar{B}_{s}^{0} \rightarrow$ $f_{J} \rightarrow p \bar{p}$ transition is given by

$$
\begin{align*}
& \langle p \bar{p}|(\bar{s} b)_{V-A}\left|\bar{B}_{s}^{0}\right\rangle \\
& \quad=\left\langle p \bar{p} \mid f_{J}\right\rangle \frac{i}{\left(t-m_{f_{J}}^{2}\right)+i m_{f_{J}} \Gamma_{f_{J}}}\left\langle f_{J}\right|(\bar{s} b)_{V-A}\left|\bar{B}_{s}^{0}\right\rangle \tag{9}
\end{align*}
$$

where $\Gamma_{f_{J}}\left(m_{f_{J}}\right)$ stands for the decay width (mass) of $f_{J}$. In terms of Eqs. (5), (6), and (9), we can write the amplitude of $\bar{B}_{s}^{0} \rightarrow V\left(f_{J} \rightarrow\right) p \bar{p}$ to be

$$
\mathcal{A}_{R}\left(\bar{B}_{s}^{0} \rightarrow V\left(f_{J} \rightarrow\right) p \bar{p}\right)
$$

$$
\begin{equation*}
=\frac{G_{F}}{\sqrt{2}} V_{c b} V_{q s}^{*} a_{2}^{V} \frac{m_{V} f_{V}}{\left(t-m_{f_{J}}^{2}\right)+i m_{f_{J}} \Gamma_{f_{J}}} \bar{u}\left(a+b \gamma_{5}\right) v \tag{10}
\end{equation*}
$$

with $\left\langle p \bar{p} \mid f_{J}\right\rangle \varepsilon^{\mu *}\left\langle f_{J}\right|(\bar{s} b)_{V-A}\left|\bar{B}_{s}^{0}\right\rangle=\bar{u}\left(a+b \gamma_{5}\right) v$, where the Lorentz indices from four-momentum factors, $\varepsilon^{\mu *}$, and the summations of the spins for the intermediate $f_{J}$ state are coupled to have a scalar quantity, leading $a$ and $b$ to be parameters. Note that $a$ and $b$ are generally momentum dependent; however, as the narrow range of $m_{p \bar{p}}$ is 1.88 2.27 GeV due to the heavy $J / \psi$ mass, $a$ and $b$ can only be changed slightly so that they are nearly constants. Moreover, the dominant contribution to the branching ratio comes from the pole effect, which is even more narrow, fixing the pole at $m_{f_{J}}=2.23 \mathrm{GeV}$. In fact, we see for $\left\langle p \bar{p} \mid f_{J}\right\rangle$, as the strong interaction conserves parity, that it is in the form of either $\bar{u} v$ or $\bar{u} \gamma_{5} v$ for the parity to be even or odd. Hence, while $f_{J}$ has been confirmed to have an even parity as the data indicated [10], $b=0$. In Eq. (10), since $a$ is unknown, it will be fitted with $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)$ and then used to predict $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0} p \bar{p}\right)$ as well as those of $\bar{B}_{s}^{0} \rightarrow J / \psi(\pi \pi, K \bar{K})$ and $\bar{B}_{s}^{0} \rightarrow D^{* 0}(\pi \pi, K \bar{K})$. To integrate over the phase space of the three-body decays, the general equation in the PDG [10] can be referred to, which is given by
$\mathrm{d} \Gamma=\frac{1}{(2 \pi)^{3}} \frac{|\mathcal{A}|^{2}}{32 M_{B}^{3}} \mathrm{~d} m_{12}^{2} \mathrm{~d} m_{23}^{2}$,
where $m_{12}=p_{\mathbf{B}}+p_{\overline{\mathbf{B}}^{\prime}}, m_{23}=p_{\mathbf{B}}+p_{V}$ and $|\mathcal{A}|^{2}$ represents the amplitude squared. By integrating over the variables $m_{12}$ and $m_{23}$, we obtain the total branching ratio. Here we will integrate over $m_{23}$ alone to have the partial branching ratio
as the function of $m_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$, such that the threshold and resonant effects drawn as the peaks in the $m_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ spectra would be in comparison with the future experiments.

## 3 Numerical results and discussions

In our numerical analysis, we adopt $\left(m_{f_{J}}, \Gamma_{f_{J}}\right)=(2231$, 23) MeV and $\left(V_{c b}, V_{c s}, V_{u s}\right)=\left(A \lambda^{2}, 1-\lambda^{2} / 2, \lambda\right)$ with $A=0.811$ and $\lambda=0.225$ in the PDG [10], and take

$$
\begin{align*}
& \left(a_{2}^{D^{*}}, a_{2}^{J / \psi}\right)=(0.33 \pm 0.04,0.17 \pm 0.03) \\
& \left(D_{\|}, D_{\|}\right)=(67.7 \pm 16.3,-280.0 \pm 35.9) \mathrm{GeV}^{5} \\
& \left(D_{\|}^{2}, D_{\|}^{3}, D_{\|}^{4}, D_{\|}^{5}\right)=(-187.3 \pm 26.6,-840.1 \pm 132.1 \\
& \quad-10.1 \pm 10.8,-157.0 \pm 27.1) \mathrm{GeV}^{4} \tag{12}
\end{align*}
$$

from the study of the charmful three-body baryonic $\bar{B}^{0}\left(B^{-}\right)$ decays in Ref. [8]. For the decay constants, we use ( $f_{D^{*}}$, $\left.f_{J / \psi}\right)=(0.23,0.41) \mathrm{GeV}$ [35]. As a result, with $|a|$ fitted to be $1.04 \pm 0.26$ we obtain $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) p \bar{p}\right)=(3.00 \pm$ 1.74) $\times 10^{-6}$ to explain the data in Eq. (1). Consequently, we can calculate the branching ratios of $\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}$ and $\bar{B}_{s}^{0} \rightarrow D^{0 *} p \bar{p}(\Lambda \bar{\Lambda})$, of which the $m_{\mathbf{B}_{\mathbf{B}^{\prime}}}$ spectra are drawn in Fig. 2, while the total branching ratios are listed in Table 1 with the errors coming from the uncertainties in various form factors.

Since $\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}$ and $B^{-} \rightarrow J / \psi \Lambda \bar{p}$ are essentially identical, except for the spectator quarks in $\bar{B}_{s}^{0}$ and $B^{-}$, their branching ratios should be at the same level. Nonetheless, from Table 1 , we see that $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}\right) \simeq$ $0.02 \mathcal{B}\left(B^{-} \rightarrow J / \psi \Lambda \bar{p}\right)$. The reason for this is that $m_{\Lambda \bar{\Lambda}}$ around the threshold area is smaller than $m_{\Lambda \bar{p}}$ by 100 MeV , which causes the more constrained threshold effect. Similarly, for $D^{* 0}$ cases, $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0} \Lambda \bar{\Lambda}\right) \sim 2.25 \times 10^{-6}$ is at least 20 times smaller than $\mathcal{B}\left(B^{-} \rightarrow D^{* 0} \Lambda \bar{p}\right)$ [36]. It is interesting to note that the assumption of the constant parameter of $a$ is demonstrated to be insensitive to the data fitting, while the pole effects via the resonant $f_{J} \rightarrow p \bar{p}$ in $\bar{B}_{s}^{0} \rightarrow J / \psi\left(D^{* 0}\right) p \bar{p}$ are narrow and sharp, as shown in Fig. 2. A further confirmation for the resonant $\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) p \bar{p}$ can depend on the future search for $\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) p \bar{p}$, whose decay branching ratio is predicted to be $(4.70 \pm 2.89) \times 10^{-7}$ (see Table 1). The difference between the threshold effect and the resonant $f_{J}$ peak can be seen from Fig. 2, where the peaks from the threshold effects for $\bar{B}_{s}^{0} \rightarrow J / \psi\left(D^{* 0}\right) \Lambda \bar{\Lambda}$ are drawn to be smooth, whereas the peaks from the $f_{J}$ resonance are sharp with the highest point precisely at $m_{p \bar{p}}=2.23 \mathrm{GeV}$ for $\bar{B}_{s}^{0} \rightarrow J / \psi\left(D^{* 0}\right)\left(f_{J} \rightarrow\right) p \bar{p}$. This can be used for the future experiments to distinguish the threshold effect from the resonance. Except for $\bar{B}^{0} \rightarrow J / \psi p \bar{p}$ with $1.88 \mathrm{GeV}<m_{p \bar{p}}<$ 2.18 GeV away from $m_{J}=2.23 \mathrm{GeV}$, the resonant contri-


Fig. 2 The partial distributions vs. $m_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ in the $\bar{B}_{s}^{0} \rightarrow V \mathbf{B} \overline{\mathbf{B}}^{\prime}$ decays with $V=J / \psi\left(D^{* 0}\right)$, where the left panel is for $V=J / \psi$, while the right one for $V=D^{* 0}$

Table 1 The branching ratios of $\bar{B}_{s}^{0} \rightarrow V \mathbf{B} \overline{\mathbf{B}}^{\prime}$ decays, where the uncertainties arise from $a_{2}^{V}$ and the $\bar{B}_{s}^{0} \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transitions

| Decay mode | Branching ratio |
| :--- | :--- |
| $\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) p \bar{p}$ | $(3.00 \pm 1.74) \times 10^{-6}$ |
| $\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) p \bar{p}$ | $(4.70 \pm 2.89) \times 10^{-7}$ |
| $\bar{B}_{s}^{0} \rightarrow J / \psi \Lambda \bar{\Lambda}$ | $(2.68 \pm 1.23) \times 10^{-7}$ |
| $\bar{B}_{s}^{0} \rightarrow D^{* 0} \Lambda \bar{\Lambda}$ | $(2.25 \pm 0.80) \times 10^{-6}$ |

butions are also possible for the other $\bar{B}^{0}$ and $B^{-}$decays of $B \rightarrow p \bar{p} M$, such as $B \rightarrow p \bar{p} K$ and $B \rightarrow p \bar{p} K^{*}$ [37]. Nonetheless, the ratios of $[10,38,39]$
$\mathcal{B}\left(B^{-} \rightarrow K^{-}\left(f_{J} \rightarrow\right) p \bar{p}\right) / \mathcal{B}\left(B^{-} \rightarrow K^{-} p \bar{p}\right)<0.06-0.08$,
$\mathcal{B}\left(B^{-} \rightarrow K^{*-}\left(f_{J} \rightarrow\right) p \bar{p}\right) / \mathcal{B}\left(B^{-} \rightarrow K^{*-} p \bar{p}\right)<0.18-0.27$,
$\mathcal{B}\left(\bar{B}^{0} \rightarrow \bar{K}^{0}\left(f_{J} \rightarrow\right) p \bar{p}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow \bar{K}^{0} p \bar{p}\right)<0.15-0.19$,
$\mathcal{B}\left(\bar{B}^{0} \rightarrow \bar{K}^{* 0}\left(f_{J} \rightarrow\right) p \bar{p}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow \bar{K}^{* 0} p \bar{p}\right)<0.10-0.15$
are too small to have impacts on the experimental results, due to the fact that the threshold effects in the decays shadow the resonant peaks. Instead, the unexpected large value of $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)$ in Eq. (1) would reveal the existence of $f_{J}(2220)$ due to the suppressed threshold effect in the decay.

In terms of $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow V\left(f_{J} \rightarrow\right) A B\right)=\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow\right.$ $\left.V f_{J}\right) \mathcal{B}\left(f_{J} \rightarrow A B\right)$ with $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow V\left(f_{J} \rightarrow\right) A B\right)$ from Table 1 and the $f_{J} \rightarrow A B$ decays in Eq. (4), where $A B$ can be $p \bar{p}, K \bar{K}$, and $\pi \pi$, we obtain
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi f_{J}\right)=(2.1 \pm 1.7) \times 10^{-3}$,
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) K \bar{K}\right)<1.6 \times 10^{-5}$,
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) \pi \pi\right)=(15.6 \pm 15.2) \times 10^{-6}$
for $V=J / \psi$, and
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0} f_{J}\right)=(3.4 \pm 2.8) \times 10^{-4}$,
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) K \bar{K}\right)<2.5 \times 10^{-6}$,
$\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) \pi \pi\right)=(24.5 \pm 24.4) \times 10^{-7}$
for $V=D^{* 0}$. We then let the $\bar{B}_{s}^{0}$ decays be the new scenario to study the $f_{J}$ state.

## 4 Conclusions

We have studied the roles of $f_{J}(2220)$, considered as the tensor glueball state of $G_{2^{++}}$, in the hadronic $\bar{B}_{s}^{0}$ decays. Explicitly, we have shown that the recent measured large branching ratio by the LHCb for the OZI suppressed decay of $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi p \bar{p}\right)$ can be understood due to the resonant contribution of $f_{J}(2220)$. We have also found that $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow\right.$ $\left.D^{* 0}\left(f_{J} \rightarrow\right) p \bar{p}\right)=(4.70 \pm 2.89) \times 10^{-7}$. Similarly, we have predicted that $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) \pi \pi\right)=(15.6 \pm 15.2) \times$ $10^{-6}$ and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) \pi \pi\right)=(24.5 \pm 24.4) \times$ $10^{-7}$, while $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi\left(f_{J} \rightarrow\right) K \bar{K}\right)<1.6 \times 10^{-5}$ and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{* 0}\left(f_{J} \rightarrow\right) K \bar{K}\right)<2.5 \times 10^{-6}$. In addition, we have obtained the result that the $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow\left(J / \psi, D^{* 0}\right) \Lambda \bar{\Lambda}\right)$ are $(2.68 \pm 1.23) \times 10^{-7}$ and $(2.25 \pm 0.80) \times 10^{-6}$, which are accessible to the experiments at the LHCb.

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## References

1. G.P. Lepage, S.J. Brodsky, Phys. Rev. Lett. 43, 545 (1979). [Erratum-ibid. 43, 1625 (1979)]
2. G.P. Lepage, S.J. Brodsky, Phys. Rev. D 22, 2157 (1980)
3. S.J. Brodsky, G.P. Lepage, S.A.A. Zaidi, Phys. Rev. D 23, 1152 (1981)
4. C.K. Chua, W.S. Hou, S.Y. Tsai, Phys. Rev. D 66, 054004 (2002)
5. C.Q. Geng, Y.K. Hsiao, Phys. Lett. B 619, 305 (2005)
6. C.Q. Geng, Y.K. Hsiao, J.N. Ng, Phys. Rev. Lett. 98, 011801 (2007)
7. C.Q. Geng, Y.K. Hsiao, Phys. Rev. D 74, 094023 (2006)
8. C.H. Chen, H.Y. Cheng, C.Q. Geng, Y.K. Hsiao, Phys. Rev. D 78, 054016 (2008)
9. R. Aaij et al., LHCb Collaboration, JHEP 1309, 006 (2013)
10. K.A. Olive et al., Particle Data Group Collaboration, Chin. Phys. C 38, 090001 (2014)
11. R. Aaij et al., LHCb Collaboration, Eur. Phys. J. C 73, 2462 (2013)
12. P. del Amo Sanchez et al., BaBar Collaboration, Phys. Rev. D 85, 092017 (2012)
13. Y.K. Hsiao, C.Q. Geng, Phys. Lett. B 727, 168 (2013)
14. H.Y. Cheng, C.Q. Geng, Y.K. Hsiao, Phys. Rev. D 89, 034005 (2014)
15. J. Sculli et al., Phys. Rev. Lett. 58, 1715 (1987)
16. G. Bardin et al., Phys. Lett. B 195, 292 (1987)
17. P.D. Barnes et al., Phys. Lett. B 309, 469 (1993)
18. C. Evangelista et al., Phys. Rev. D 56, 3803 (1997)
19. For the review, please see V. Crede and C.A. Meyer, Prog. Part. Nucl. Phys. 63, 74 (2009) arXiv:0812.0600 [hep-ex], and the references therein
20. R.M. Baltrusaitis et al., Mark III Collaboration, Phys. Rev. Lett. 56, 107 (1986)
21. J.Z. Bai et al., BES Collaboration, Phys. Rev. Lett. 76, 3502 (1996)
22. D. Alde et al., Phys. Lett. B 177, 120 (1986)
23. D. Aston et al., Phys. Lett. B 215, 199 (1988)
24. B.V. Bolonkin et al., Nucl. Phys. B 309, 426 (1988)
25. C. Evagelista et al., JETSET Collaboration, Phys. Rev. D 57, 5370 (1998)
26. C. Amsler et al., Crystal Ball Collaboration, Phys. Lett. B 520, 175 (2001)
27. M. Acciarri et al., L3 Collaboration, Phys. Lett. B 501, 173 (2001)
28. K. Benslama et al., CLEO Collaboration, Phys. Rev. D 66, 077101 (2002)
29. For the review, please consult V. Mathieu et al., Int. J. Mod. Phys. E 18, 1 (2009), and the references therein
30. C.J. Morningstar, M.J. Peardon, Phys. Rev. D 60, 034509 (1999)
31. Y. Chen et al., Phys. Rev. D 73, 014516 (2006)
32. Y.B. Yang et al., CLQCD Collaboration, Phys. Rev. Lett. 111, 091601 (2013)
33. P. del Amo Sanchez et al., BABAR Collaboration, Phys. Rev. Lett. 105, 172001 (2010)
34. A.J. Buras, hep-ph/9806471
35. H.M. Choi, Phys. Rev. D 75, 073016 (2007)
36. P. Chen et al., BELLE Collaboration, Phys. Rev. D 84, 071501 (2011)
37. C.K. Chua, W.S. Hou, S.Y. Tsai, Phys. Lett. B 544, 139 (2002)
38. M.Z. Wang et al., BELLE Collaboration, Phys. Lett. B 617, 141 (2005)
39. B. Aubert et al., BABAR Collaboration, Phys. Rev. D 76, 092004 (2007)

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[^1]:    ${ }^{1}$ The result in Ref. [5] has been updated according to the observed value of $\mathcal{B}\left(B^{-} \rightarrow J / \psi \Lambda \bar{p}\right)$.

