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Letter

Neutrinos, axions and conformal symmetry

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Abstract We demonstrate that radiative breaking of conformal symmetry (and simultaneously electroweak symmetry) in the standard model with right-chiral neutrinos and a minimally enlarged scalar sector induces spontaneous breaking of lepton number symmetry, which naturally gives rise to an axion-like particle with some unusual features. The couplings of this 'axion' to standard model particles, in particular photons and gluons, are entirely determined (and computable) via the conformal anomaly, and their smallness turns out to be directly related to the smallness of the masses of the light neutrinos.

1 Introduction

It has been known for some time that classically unbroken conformal symmetry may provide a possible mechanism for explaining the stability of the weak scale with respect to the Planck scale [1]. In such a scheme all observed mass scales must arise from a single scale via the quantum mechanical breaking of conformal invariance induced by the Coleman–Weinberg (CW) effective potential [2]. In [3] (see [4, 5] for an earlier, but different proposal, and also [6, 7] for subsequent work along these lines), we have recently shown that a minimal extension of the standard model (SM) with right-chiral neutrinos and one extra scalar can realize this possibility, provided that [3, 8]:

- There are no intermediate mass scales between the weak scale and the Planck scale M_{P}.
- The RG evolved couplings exhibit neither Landau poles nor instabilities over this whole range of energies.

While the first point concerns an issue that must be decided experimentally, the second assumption is motivated by the expectation that *any* extension of the SM (with or without supersymmetry) that stays within the framework of quantum field theory will eventually fail, and that the main task is therefore to delay the onset of this breakdown to the Planck scale, where a proper theory of quantum gravity is expected to replace quantum field theory. This requirement leads to important restrictions on the SM parameters, which can in principle be tested at LHC.

We study an extension of the model [3] with the usual Higgs doublet $\Phi(x)$, and one extra (weak singlet) scalar field $\phi(x)$, but with the main difference that this extra scalar field is now taken to be *complex*. Writing

$$\phi(x) = \varphi(x) \exp\left(\frac{\mathrm{i}a(x)}{\sqrt{2}\mu}\right), \tag{1}$$

with real fields $\varphi(x)$ and $a(x)$, we will show that $\varphi(x)$ can acquire a non-vanishing vacuum expectation value via radiative corrections. The field $a(x)$ then gives rise to a (pseudo-) Goldstone particle associated with the spontaneous breaking of a new *global* $\mathrm{U}(1)_L$ (modified lepton number) symmetry. A major new result of the present work is that this boson, commonly referred to as the 'majoron' [9], shares many properties with the axion [10–12]. We here explore some of these features, which are mainly due to 'neutrino mediation', especially the effective couplings of this 'axion' to photons and gluons. The latter may have important implications with regard to the potential suitability of the axion as a dark matter candidate and for solving the strong CP problem. Details concerning such phenomenological applications will be given elsewhere.

Apart from its compatibility with the known SM phenomenology, and its implications for the hierarchy problem, the main virtue of the present proposal is that it provides a *single* source of explanation for axion couplings and neutrino masses via the conformal anomaly, thereby tying together in a most economical manner features of the SM previously thought to be unrelated. If it should turn

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out that there are indeed no new scales beyond the weak scale it could thus offer an attractive and economical alternative to MSSM-type models, because low energy supersymmetry may not be needed to stabilize the weak scale with regard to the Planck scale if the two conditions stated above are met [8] (see also [5] for a very similar point of view).

2 Lagrangian

We refer to [13, 14] for the basic properties of the SM, and here only quote the relevant interaction terms in the (classically conformal) Lagrangian, viz.

$$\begin{aligned}\mathcal{L}_{\text{int}} = \big(&\overline{L}^i \Phi Y^E_{ij} E^j + \overline{Q}^i \Phi Y^D_{ij} D^j + \overline{Q}^i \varepsilon \Phi^* Y^U_{ij} U^j \\ &+ \overline{L}^i \varepsilon \Phi^* Y^\nu_{ij} N^j + \phi N^{iT} \mathcal{C}^{-1} Y^M_{ij} N^j + \text{h.c.}\big) \\ &- \frac{\lambda_1}{4}\big(\Phi^\dagger\Phi\big)^2 - \frac{\lambda_2}{2}\big(\Phi^\dagger\Phi\big)\big(\phi^\dagger\phi\big) - \frac{\lambda_3}{4}\big(\phi^\dagger\phi\big)^2. \quad (2)\end{aligned}$$

Here Q^i and L^i are the left-chiral quark and lepton doublets, U^i and D^i the right-chiral up- and down-like quarks, while E^i are the right-chiral electron-like leptons, and $N^i \equiv \nu^i_R$ the right-chiral neutrinos. As in [3] we suppress all indices except the family indices $i, j = 1, 2, 3$. One can use global redefinitions of the fermion fields to transform the Yukawa matrices Y^E_{ij}, Y^U_{ij} and Y^M_{ij} to real diagonal matrices. Furthermore, $Y^D = K_D M_D$, where K_D is a CKM matrix and M_D is real diagonal. Using the remaining freedom we can then set $Y^\nu = K_\nu M_\nu U_N$, where K_ν is a CKM-like matrix (i.e., with three angles and one phase), M_ν is real diagonal and U_N is a unitary matrix with $\det U_N = 1$.

Besides the standard (local) $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ symmetries, the Lagrangian (2) admits two *global* U(1) symmetries. One is the standard baryon-number symmetry $\mathrm{U}(1)_B$,

$$Q^i \to e^{i\beta} Q^i, \qquad U^i \to e^{i\beta} U^i, \qquad D^i \to e^{i\beta} D^i; \quad (3)$$

the other is the modified lepton-number symmetry $U(1)_L$:

$$\begin{aligned} &L^i \to e^{i\alpha} L^i, \qquad E^i \to e^{i\alpha} E^i, \\ &N^i \to e^{i\alpha} N^i, \qquad \phi \to e^{-2i\alpha}\phi \end{aligned} \quad (4)$$

with associated Noether current

$$\mathcal{J}^\mu_L = \overline{L}^i \gamma^\mu L^i + \overline{E}^i \gamma^\mu E^i + \overline{N}^i \gamma^\mu N^i - 2i\phi^\dagger \overset{\leftrightarrow}{\partial^\mu} \phi. \quad (5)$$

Notice that the first three terms add up to a purely vector-like current of the standard form $= \bar{e}^i \gamma^\mu e^i + \bar{\nu}^i \gamma^\mu \nu^i$. An important feature is that the scalar field ϕ also carries lepton charge, which can thus 'leak' from the fermions via the last term in (5); see e.g. [15] and references therein for a discussion of this issue.

3 Minimization of effective potential

While the classical potential (2) does not induce spontaneous symmetry breaking, the effective one-loop potential computed from (2) can develop non-vanishing vacuum expectation values for the scalar fields. This potential is given by the sum of (3) (now with $N = 4$ and $M = 2$) and (6) of [3], to which we also add the contribution from the $\mathrm{SU}(2)_w$ gauge fields which was not taken into account in [3]; explicitly,

$$\begin{aligned}V_{\text{eff}}(H,\varphi) = &\frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2\varphi^2}{2} + \frac{\lambda_3\varphi^4}{4} \\ &+ \frac{9}{16}\alpha_w^2 H^4 \ln\left[\frac{H^2}{v^2}\right] \\ &+ \frac{3}{256\pi^2}\big(\lambda_1 H^2 + \lambda_2\varphi^2\big)^2 \ln\left[\frac{\lambda_1 H^2 + \lambda_2\varphi^2}{v^2}\right] \\ &+ \frac{1}{256\pi^2}\big(\lambda_2 H^2 + \lambda_3\varphi^2\big)^2 \ln\left[\frac{\lambda_2 H^2 + \lambda_3\varphi^2}{v^2}\right] \\ &+ \frac{1}{64\pi^2}F_+^2 \ln\left[\frac{F_+}{v^2}\right] + \frac{1}{64\pi^2}F_-^2 \ln\left[\frac{F_-}{v^2}\right] \\ &- \frac{6}{32\pi^2} g_t^4 H^4 \ln\left[\frac{H^2}{v^2}\right] \\ &- \frac{1}{32\pi^2} Y_M^4 \varphi^4 \ln\left[\frac{\varphi^2}{v^2}\right], \end{aligned} \quad (6)$$

where $\alpha_w \equiv g_2^2/4\pi$ and $H^2 \equiv \Phi^\dagger\Phi$. v is the mass parameter required by dimensional regularization (which breaks conformal invariance). Also, we have defined

$$\begin{aligned} &F_\pm(H,\varphi) \\ &:= \frac{3\lambda_1+\lambda_2}{4}H^2 + \frac{3\lambda_3+\lambda_2}{4}\varphi^2 \\ &\pm \sqrt{\left[\frac{3\lambda_1-\lambda_2}{4}H^2 - \frac{3\lambda_3-\lambda_2}{4}\varphi^2\right]^2 + \lambda_2^2\varphi^2 H^2}. \end{aligned} \quad (7)$$

As a further simplification, only the contributions from the top quark (with coupling g_t) and one massive neutrino (with coupling Y_M) have been included in (6). Next we perform the numerical minimization subject to the requirements stated in the Introduction. As in [3], the numerical search shows that there exists a (small) subset of parameter space compatible with our requirements and, in particular, allowing for the following exemplary set of values:

$$\begin{aligned} &\lambda_1 = 3.77, \qquad \lambda_2 = 3.72, \qquad \lambda_3 = 3.73, \\ &g_t = 1, \qquad Y_M^2 = 0.4 \end{aligned} \quad (8)$$

(the approximate O(6) symmetry of the scalar self-couplings is accidental and is not preserved by quantum corrections;

cf. (13) below). One should note that the natural expansion parameters in the loop expansion are $\lambda_i/(4\pi^2)$, so that the scalar couplings are not very large. For these values, the minimum is located at

$$\langle H\rangle = 2.74\cdot 10^{-5} v, \qquad \langle\varphi\rangle = 1.51\cdot 10^{-4} v. \tag{9}$$

These values are such that the logarithmic corrections in the potential are of the same order of magnitude as the tree-level contributions—as must be the case if the minimum is to be shifted away from zero. In assessing the reliability of this result one must therefore ensure that the higher-order corrections remain small over the relevant range of energies in spite of 'large logarithms'. Generically, the latter might invalidate conclusions based on the minimization of the one-loop potential, but as we show in a separate publication [16], the minimum can nevertheless be stable *if* there are cancellations among the couplings of the theory ensuring its survival up to very large energies – as is the case for our model (by construction!). To set the scale for v, and thereby for all other dimensionful quantities in the model, we impose $\langle H\rangle = 174$ GeV. Hence,

$$\begin{aligned}&\langle H\rangle = 174\ \text{GeV}, \qquad \langle\varphi\rangle = 958\ \text{GeV},\\ &v = 3.65\cdot 10^4\langle \mathrm{H}\rangle.\end{aligned} \tag{10}$$

After symmetry breaking, three degrees of freedom of Φ are converted into longitudinal components of $W^\pm$ and Z^0, so we are left with one real scalar field H and the complex field (1). Defining (at the potential minimum)

$$H' = H\cos\beta + \varphi\sin\beta, \qquad \varphi' = -H\sin\beta + \varphi\cos\beta, \tag{11}$$

the numerical analysis yields the following values (slightly different from [3]):

$$m_{H'} = 207\ \text{GeV}, \qquad m_{\varphi'} = 477\ \text{GeV}, \qquad \sin\beta = 0.179, \tag{12}$$

while the Goldstone field $a(x)$ stays massless. Note that only the components along H of the mass eigenstates couple to the usual SM particles. This leads to a clear and testable prediction of the present model: the decay amplitudes of H' and φ' into SM particles are both proportional to the corresponding decay amplitudes of the usual SM Higgs particle with mass values set equal to $m_{H'}$ and $m_{\varphi'}$, respectively [3].

The effective coupling constants are calculated numerically as the respective fourth-order derivatives of the effective potential at the minimum (now with the factors $1/(4\pi^2)$ appearing in the loop expansion):

$$\begin{aligned}&\frac{\lambda_1^{\text{eff}}}{4\pi^2} = 0.03665, \qquad \frac{\lambda_2^{\text{eff}}}{4\pi^2} = 0.01641,\\ &\frac{\lambda_3^{\text{eff}}}{4\pi^2} = 0.02206.\end{aligned} \tag{13}$$

Checking the consistency of our basic assumptions now amounts to evolving all couplings according to their RG equations, with (13) and the known SM couplings at the weak scale as the initial values. Performing the steps described in [3], and also taking into account the contributions of the weak $SU(2)_w$ coupling α_w in the RG evolutions, we have verified that for the values indicated above there are indeed no Landau poles or instabilities up to the Planck scale.

Although the numbers (12) do not constitute a definitive prediction of our model, it turns out that the 'window' left open by our requirements is fairly small for $m_{H'}$. Preliminary estimates give an approximate range 200 GeV $< m_{H'} <$ 220 GeV, whereas $m_{\varphi'}$ can vary over a larger range of values, $>\mathcal{O}$ (400 GeV), such that the mixing angle β decreases with increasing $m_{\varphi'}$. The comparatively large value for $m_{H'}$ distinguishes the present model from other proposals, and in particular from MSSM-type models predicting $m_{H'} < \mathcal{O}$ (135 GeV) [17].

4 Neutrino propagators

The new effects reported in this Letter all depend crucially on the mixing of the neutrino degrees of freedom for each neutrino species. Inspection of the explicit expressions for the propagators given below in (16) shows that this mixing is *maximal* in the sense that any neutrino degree of freedom can oscillate into any other. Because the results would take a much more cumbersome form in terms of 4-spinors, we temporarily switch to $SL(2,\mathbb{C})$ spinor notation, see e.g. [18] for details and conventions, which proves also more convenient for the computation of loop diagrams. With $\nu_L \equiv \frac{1}{2}(1-\gamma^5)\nu \equiv \bar{\nu}^{\dot\alpha}$ and $\nu_R \equiv \frac{1}{2}(1+\gamma^5)\nu \equiv N_\alpha$, the relevant (free) part of the Lagrangian reads

$$\begin{aligned}\mathcal{L} = {}&\frac{\mathrm{i}}{2}\big(\nu^\alpha \partial\!\!\!/_{\alpha\dot\beta}\bar{\nu}^{\dot\beta} + N^\alpha \partial\!\!\!/_{\alpha\dot\beta}\bar{N}^{\dot\beta}\big) + \text{c.c.}\\ &+ m\big(\nu^\alpha N_\alpha + \bar{\nu}_{\dot\alpha}\bar{N}^{\dot\alpha}\big) + \frac{M}{2}\big(N^\alpha N_\alpha + \bar{N}_{\dot\alpha}\bar{N}^{\dot\alpha}\big),\end{aligned} \tag{14}$$

with the Dirac and Majorana mass parameters $m = Y^\nu\langle H\rangle$ and $M = Y^M\langle\varphi\rangle$, respectively. For simplicity, we here consider only one neutrino generation; in the general case the formulas below will contain additional factors of $Y^\nu Y_M^{-1} Y^\nu$, or traces over family indices (which may alter our estimates below). Rather than diagonalize the fields with respect to these mass terms, we prefer to work with *non-diagonal propagators*, leaving the fields as they are in the interaction vertices. The poles of the propagators are obtained via the standard seesaw formula [19–21]

$$\mu_\pm^2 = m^2 + \frac{1}{2}M^2 \pm \frac{1}{2}M^2\sqrt{1+\frac{4m^2}{M^2}}, \tag{15}$$

whence $\mu_+ \approx M$ and $\mu_- \approx m^2/M \equiv m_\nu$ for $m \ll M$ (so that, assuming $|Y_\nu| < 10^{-5}$ and substituting the values (12) found above, we get $m_\nu < 1$ eV [3]). Defining $\mathcal{D}(p) := [(p^2 - M^2)(p^2 - m_\nu^2)]^{-1}$ we obtain the propagators (in momentum space)

$$\begin{aligned}
\langle \nu_\alpha \nu_\beta\rangle &= \mathrm{i}m^2 M\mathcal{D}(p)\varepsilon_{\alpha\beta},\\
\langle \nu_\alpha \bar\nu_{\dot\beta}\rangle &= \mathrm{i}\big(p^2 - M^2 - m^2\big)\mathcal{D}(p)\not{p}_{\alpha\dot\beta},\\
\langle N_\alpha N_\beta\rangle &= \mathrm{i}Mp^2\mathcal{D}(p)\varepsilon_{\alpha\beta},\\
\langle N_\alpha \bar N_{\dot\beta}\rangle &= \mathrm{i}\big(p^2 - m^2\big)\mathcal{D}(p)\not{p}_{\alpha\dot\beta},\\
\langle \nu_\alpha N_\beta\rangle &= \mathrm{i}m\big(p^2 - m^2\big)\mathcal{D}(p)\varepsilon_{\alpha\beta},\\
\langle \nu_\alpha \bar N_{\dot\beta}\rangle &= -\mathrm{i}mM\mathcal{D}(p)\not{p}_{\alpha\dot\beta},
\end{aligned} \tag{16}$$

together with their complex conjugate components (where, as usual, $\not{p}_{\alpha\dot\beta} \equiv p_\mu \sigma^\mu_{\alpha\dot\beta}$ and $\varepsilon_{\alpha\beta}$ is the $SL(2,\mathbb{C})$ invariant tensor [18]). It will be essential for the UV finiteness of the diagrams to be computed below that some of these propagators fall off like $\sim p^{-3}$, unlike the standard Dirac propagator.

5 Neutrino triangles and 'axion' vertices

With the above propagators we can now proceed to compute various couplings involving the 'axion' a, which are mediated by neutrino mixing via two- or three-loop diagrams. In order to extract the new vertices from (2) (and to have canonically normalized kinetic terms for these scalar fields), we set $\mu = \langle\varphi\rangle$ in (1) and expand

$$\phi(x) = \langle\varphi\rangle + \frac{1}{\sqrt{2}}\big(\varphi(x) + \mathrm{i}a(x)\big) + \cdots. \tag{17}$$

We here only present results for the axion–photon coupling $aF\tilde F$, whose lowest-order contribution is given by the two-loop diagram depicted in Fig. 1. Because parity is (in fact, maximally) broken, there are also 'dilatonic' couplings of type aFF, whose determination will require in addition the consideration of diagrams with photons emanating from the internal W-line. To simplify the calculation we first compute the 'blob' in Fig. 1, which is proportional to the integral (after a Wick rotation)

$$\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{\not{l}_{\alpha\dot\beta}}{(l^2+M^2)[(l+q)^2+M^2]l^2[(l+p_1)^2+m_W^2]} + (p_1 \leftrightarrow p_2), \tag{18}$$

where m_W is the W-boson mass, and where we set $m = m_\nu = 0$ in order to simplify the integrand (a valid approximation, because $m, m_\nu \ll m_W, M$). Here we are interested in the result for small axion momentum $q^\mu = p_1^\mu + p_2^\mu$. Putting in all factors and reverting back to 4-spinor notation, the 'form factor' for the electron–axion vertex for large p_1 and p_2 is well approximated by

$$\mathcal{F}(q, p_1) = \frac{m_\nu Y_M \alpha_w}{8\pi}\frac{1+\gamma^5}{2}\frac{\not{q}}{p_1^2 + m_W^2} \tag{19}$$

(modulo terms of order $\mathcal{O}(M^2/p_1^2)$ and $\mathcal{O}(m_W^2/p_1^2)$), where now $\not{q} \equiv \gamma^\mu q_\mu$. This formula makes obvious one of our main assertions, namely that the effective axionic couplings are proportional to the light-neutrino masses and vanish in the limit $m_\nu \to 0$. The complete expression for the effective axion–electron vertex at small q^μ will be given elsewhere. Inter alia: the above vertex would also determine the rate of energy loss through radiation of 'axions' from charged plasma in the Sun's core.

In order to arrive at an estimate of the axion–photon coupling, we next substitute the result (19) into the expression for the electron triangle shown in Fig. 1. Though superficially similar to the diagram giving rise to the well-known γ^5-anomaly in QED, the integral here is *convergent*, because of the damping of the integrand due to the 'form factor' (19). After some computation we obtain

$$\mathcal{L}_{\text{eff}}^{a\gamma\gamma} = \frac{1}{4f_a}aF^{\mu\nu}\tilde F_{\mu\nu}, \qquad f_a = \frac{2\pi^2 m_W^2}{\alpha_w \alpha_{em} m_\nu Y_M}. \tag{20}$$

Substituting values, we find $f_a = \mathcal{O}\ (10^{15}$ GeV), which is outside the range of existing or planned experiments [22].

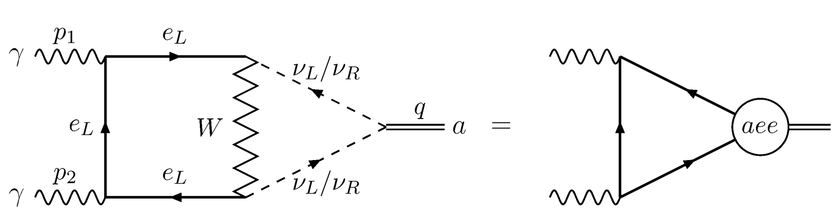

Fig. 1 Axion–photon–photon effective vertex

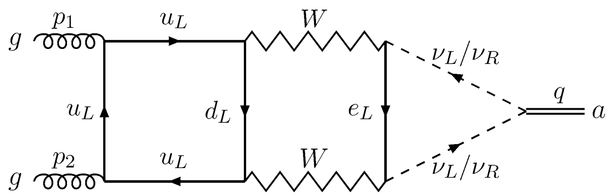

Fig. 2 Axion–gluon–gluon effective vertex

However, apart from the simplifications introduced above which may still affect this estimate, one must keep in mind that the simultaneous presence of aFF couplings may substantially alter the analysis with regard to observable effects.

The gluonic couplings can be analyzed in a similar way. For their determination we must evaluate the three-loop diagram shown in Fig. 2, as well as analogous diagrams that are not shown, with Z-boson exchange and a triangle consisting only of neutrino lines, and with gluons emanating from the quark line connecting two W- or Z-bosons . This calculation is complicated by the fact that the 'quark box' is dominated by small momenta, where α_s is large. A very crude estimate can be derived by replacing α_{em} in (20) by $6\alpha_w\alpha_s/\pi$, which yields

$$\mathcal{L}_{\text{eff}}^{agg} = \frac{\alpha_s}{8\pi g_a} a G^{\mu\nu}\tilde{G}_{\mu\nu}, \quad g_a \cong \frac{\pi^2 m_W^2}{6 Y_M \alpha_w^2 m_\nu}. \tag{21}$$

This gives $g_a = \mathcal{O}$ (10^{16} GeV). Summing over quark flavors as well as taking into account all relevant diagrams could bring this number down, into the range that could make the axion a viable dark matter candidate according to standard reasoning (see e.g. [23], p. 280 ff., but the actual numbers are subject to large uncertainties [24]). However, before starting the actual comparison, conventional lines of reasoning must be re-examined in view of the fact that parity violation now also allows for 'dilatonic' couplings $\propto aGG$, which may vitiate some of the accepted arguments (for instance, concerning periodicity properties of the 'axion' potential and the location of the minimum in this potential). Similar caveats may apply to the use of our 'axion' for solving the strong CP problem: although for this purpose the precise value of g_a does not matter so much, it is not clear whether the present scenario would lead to $\langle a\rangle = 0$, as generally required for the solution of the strong CP problem.

6 Conclusions and outlook

In [3] it was shown that all mass scales of the SM can be generated via radiative corrections and the conformal anomaly *from a single scale* v, which is set here by the choice of $\langle H\rangle$ in (10). In particular, no large scales beyond the SM are needed to explain the smallness of neutrino masses, if one allows for the entries of the Yukawa coupling matrix Y^ν to assume values in the range $\mathcal{O}(1)$–$\mathcal{O}(10^{-5})$ (this is the main difference with more conventional seesaw-type scenarios, for which $m \approx \mathcal{O}$ (100 GeV) and M must be assumed very large, resulting in very large masses for the heavy neutrinos; see e.g. [25]). In this Letter, we have extended these considerations to another sector of the SM, by showing that the replacement of the real field φ by the complex scalar ϕ makes it possible to also accommodate an axion-like degree of freedom in the model. Again, the relevant (large) mass scales emerge rather naturally by radiative corrections and without the need to introduce any new large mass scales by hand.

The main novelty of the present work is the proposal to identify the 'majoron' of [9] with the axion, and to do so in conjunction with the quantum mechanical breaking of (classical) conformal invariance. This entails several unusual features; for instance, the fact that our 'axion' cannot be assigned a definite parity, unlike the standard axion of [11, 12]. The crucial ingredient here is the maximal neutrino mixing in (16), which mediates the axionic couplings (20) and (21). Without such a neutrino mediation the latter couplings would simply be absent if the lepton number symmetry $\mathrm{U}(1)_L$ is non-anomalous [26]. This part of our proposal does not depend on the symmetry-breaking mechanism and might thus also work without the assumption of classical conformal invariance. The cosmological consequences of this scenario (e.g. for leptogenesis in the early universe) remain to be explored.

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