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Superallowed beta-decay between isobaric analog states up to A = 99

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Abstract Isospin symmetry is used to fit the precisely known Q_{EC}-values of lighter $T_Z = 0, -1$ and $T_Z = -1/2$ β-emitters and to extrapolate them up to $Z = 50$. For the T = 1 emitters the half-lives of the pure $0^+ \rightarrow 0^+$ Fermi decays are calculated and compared with experimentally known values, even for the heaviest cases where the experimental uncertainties are still rather large. For the T = 1/2 emitters only the Fermi component of the transitions can be predicted, but with the experimental half-lives the ratio of Gamow–Teller to Fermi strength can be determined.

1 Introduction

The liquid drop mass formula [1] is very successful but comes to its limits because of the structure of nuclear states. To get to higher precision we have to use nuclear structure models, in general. However, if we are interested in mass differences of nuclei with the "same" nuclear structure, the liquid drop model should still work well. Mass differences are needed for β-decay Q-values. And nuclei with the "same" structure are members of an isospin multiplet where just one or several protons are replaced by neutrons in the same configuration. Isospin multiplets are well studied for T = 1, $0^+ \rightarrow 0^+$ β-transitions [2]. Because these are pure Fermi-transitions they are used to test the conserved vector current (CVC) hypothesis and to deduce the first element V_{ud} of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. Experimentally, Q-values and partial half-lives are precisely known up to $A \approx 70$. But these $0^+ \rightarrow 0^+$ β transitions exist up to $A = 98$ (e.g. [3]). Therefore, it is my aim to extrapolate electron capture Q-values (Q_{EC}) and half-lives of T = 1 triplets up to $A = 98$. The situation is similar for T = 1/2 doublets which are also well studied [4]. Here, parent and daughter nuclei, naturally, cannot have spin I = 0 and also a Gamow-Teller transition is allowed. Nevertheless, we can extrapolate the Q_{EC}-values and calculate the partial half-lives for the Fermi-transition up to A = 99.

2 Q-values

Using the liquid drop mass formula [1] with the binding energy of a nucleus ${}^AZ$ e.g. written in the form [12]

$$BE({}^AZ) = a_v A - a_s A^{2/3} - a_c Z(Z-1)/A^{1/3} \\ -a_a(N-Z)^2/A - \delta a_p/A^{1/2}$$

one gets for the mass difference between two neighboring isobaric nuclei or the Q-value for electron capture decay:

$$\Delta M({}^AZ; {}^AZ-1) = Q_{EC} = a_c(2Z_p - 2)/A^{1/3} \\ -4a_a(N_p - Z_p + 1)/A + 2\delta a_p/A^{1/2} + m_H - m_n$$

with the constants for Coulomb-energy a_c, asymmetry energy a_a, pairing energy a_p and $\delta = +1; 0; -1$ for Z,N = odd,odd; $A = odd$; $Z, N =$ even,even, respectively. Z_p and N_p are proton and neutron number of the parent nucleus. If nuclei are homogeniously charged spheres the Coulomb energy constant should be $a_c = 6/5e^2/(4\pi\epsilon_0 r_0)$, i.e. inversely proportional to the radius parameter r_0.

For the Q_{EC} values between members of the $A = 4n + 2$ and $T = 1$ triplets we have to take into account all terms and also that there is a discontinuity for N = Z nuclei, i.e. at $T_Z = 0$, which was postulated already by Wigner in 1937 [13]. The mass of the $T_Z = 0$ (odd,odd) member is modified by $\Delta M(T_Z = 0) = -4a_a/A + a_p/\sqrt{A} - E_W$ where E_W is the Wigner energy. In order to extract the Coulomb term a_c the sum of the Q-values for the $T_Z = -1$ parent and the $T_Z = 0$ parent for a given mass number A was fitted as a linear function of the sum of $(Z_{1p} + Z_{2p} - 2)/A^{1/3} = (A-1)/A^{1/3}$. Data (see Table 1) were used [2,6] for $10 \leq A \leq 66$. The result of that fit is that the $T = 1$, $T_Z = 0$ state is on average

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Table 1 Experimental and calculated Q$_{EC}$-values for superallowed decays of T $=1$, T$_Z=0,-1$, I$^\pi$=0$^+$ parent nuclei, or the corresponding mass difference if the T $=1, 0^+$ is not the ground state of the T$_Z=0$ member of the isospin triplett. Errors are only given if $\Delta Q > 1$ keV. The values for log(f) are taken from the National Nuclear Data Center [5] for the calculated Q-values. With an uncorrected value of $ft = 3030 + Z$ (see Sect. 3) the half-life is calculated for a superallowed decay. Experimental half-lives (uncorrected for branching) are only given, if the branch is > 99%. Errors are given, if $\Delta T_{1/2}/T_{1/2} > 1\%$. The deviation is calculated as $1 - T_{1/2}^{calc}/T_{1/2}^{exp}$

A	Z_p	Q_{exp} [keV]	ΔQ_{exp} [keV]	References	Q_{calc} [keV]	log(f)	$T_{1/2}^{calc}$ [ms]	$T_{1/2}^{exp}$ [ms]	$\Delta T_{1/2}^{exp}$ [ms]	References	Deviation
10	5	1183		[6]	1143	−2.594					
10	6	1908		[2]	1926	0.399					
14	7	2156		[6]	2028	0.576	806,000				
14	8	2832		[2]	2742	1.538	88,000	70619		[2]	−0.246
18	9	2698		[6]	2815	1.605	75,500				
18	10	3403		[2]	3483	2.193	19490				
22	11	3499		[6]	3537	2.225	18,110				
22	12	4124		[2]	4170	2.650	6810				
26	13	4233		[2]	4210	2.650	6812	6346		[2]	−0.073
26	14	4841		[2]	4817	3.000	3044				
30	15	4841		[6]	4846	3.007	2996				
30	16	5465		[2]	5431	3.282	1591				
34	17	5492		[2]	5452	3.291	1559	1527		[2]	−0.021
34	18	6062		[2]	6018	3.516	929				
38	19	6044		[2]	6032	3.514	934	924		[2]	−0.010
38	20	6612		[2]	6583	3.717	585				
42	21	6426		[2]	6591	3.712	592	681		[2]	0.130
42	22	7017		[2]	7128	3.892	391				
46	23	7052		[2]	7131	3.884	399	423		[2]	0.056
46	24	7599	2.6	[2]	7656	4.046	275				
50	25	7634		[2]	7655	4.039	279	283.22		[2]	0.014
50	26	8151	2.9	[2]	8169	4.184	200				
54	27	8244		[2]	8164	4.176	204	193.27		[2]	−0.055
54	28	8725	2.7	[2]	8669	4.309	150				
58	29	8764	0.4	[6]	8660	4.300	153				
58	30	9200	29	[6,7]	9156	4.423	116				
62	31	9181	0.54	[2]	9144	4.413	118	116.12		[2]	−0.018
62	32	9698	37	[7]	9632	4.527	91.0				
66	33	9579	26	[2]	9618	4.517	93.1	95.79		[2]	0.028
66	34	10043	61	[7]	10098	4.623	73.0				
70	35	10504	15	[6,8]	10081	4.613	74.7	78.80		[2]	0.052
70	36	10106	140	[7]	10555	4.713	59.4				
74	37	10417	4	[2]	10535	4.702	60.9	64.78		[2]	0.060
78	39				10981	4.785	50.3	53	8	[9,10]	0.050
82	41				11419	4.863	42.1	50	6	[9,10]	0.158
86	43				11850	4.936	35.6	55	7	[9,10]	0.353
90	45	13190	1330	[11]	12274	5.004	30.5	29	3	[11]	−0.051
94	47	13350	650	[11]	12691	5.069	26.2	27	2	[11]	0.028
98	49	12930	400	[11]	13103	5.131	22.8	30	1	[11]	0.241

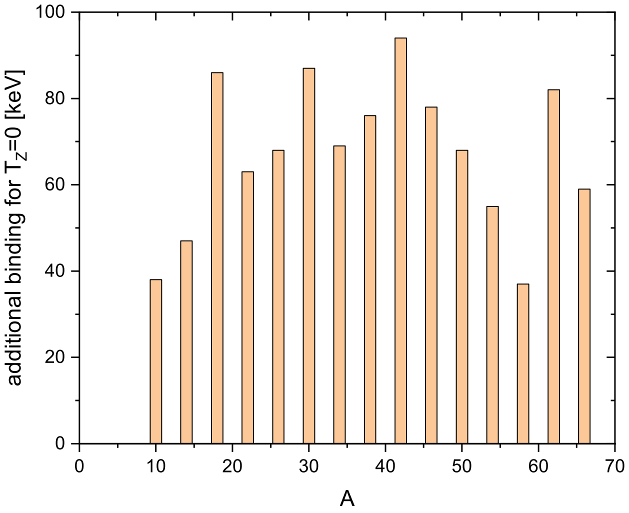

Fig. 1 Additional binding energy of the $T_Z = 0$ member of $T = 1$ triplets compared to the $T_Z = \pm 1$ members as a function of the mass number A

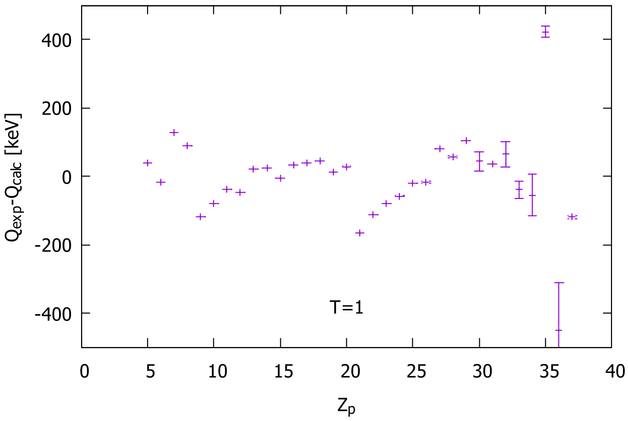

Fig. 2 Residua of the fit to the $T = 1$ Q_{EC}-values as a function of the nuclear charge of the parent nucleus Z_p

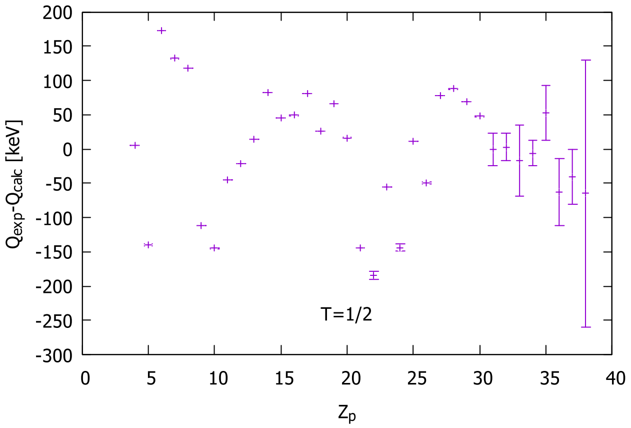

Fig. 3 Residua of the fit to the $T = 1/2$ Q_{EC}-values as a function of the charge number of the parent nucleus Z_p

stronger bound by 67 keV than the average of the $T = 1$, $T_Z = \pm 1$ isospin partners. That means that the Wigner energy more than compensates the pairing term minus the asymmetry term of the $T_Z = \pm 1$ member of the isospin triplet. The individual values for this additional binding are shown in Fig. 1 and I use this constant value independent of A. The result is $Q_{EC} = 2a_c(Z_p - 1)/A^{1/3} + b \pm c$ with $a_c = 699.0$ keV, $b = -1385$ keV and $c = 67$ keV. For the last term the positive (negative) sign is for $T = 1$, $T_Z = -1$ ($T_Z = 0$) parent nuclei. The differences between experimental and fit values are shown in Fig. 2 and their rms deviation amounts to 70 keV. The value of a_c corresponds to a radius parameter for spherical nuclei of $r_0 = 1.236\, fm$. One can recognize discontinuities at $Z_p = 8$ and $Z_p = 20$, the magic numbers, but not at $Z_p = 28$. Apparently, a real theory would have to account for shell effects. It is also obvious that there is a problem at A = 70 or $Z_p = 35, 36$. It appears like the experimental mass of ^{70}Br is too large by some 400 keV. Hardy and Towner [2] already rejected this result [8]. A similar discrepancy has been recognized by Morales et al. [14] in calculating the ft-value for the superallowed decay of ^{70}Br using the Q-value of AME2020 [6,8]. (For ^{70}Br the precise, most recent value [8] deviates by 3.1σ from the older value of Davids [15] and by 7.6σ from that of Karny et al. [16]. Note added in proof: a recent evaluation of Coulomb energy differences (Phys. Rev. C 108:034301) comes to a similar conclusion, namely that ^{70}Br is by 508(22) keV more bound than the adopted value from [16])

For parent nuclei with $Z = N+1$ or $T_Z = -1/2$ the asymmetry and pairing term vanish in Q_{EC}. When the well known Q_{EC} values for $T_Z = -1/2$ parent nuclei (see Table 2) are fitted one recognizes a similar staggering between alternate values which was already described by Jaenecke [17,18]. It can be interpreted as a difference in pairing energy of protons and neutrons. Therefore I fitted the A = 4n + 1 and A = 4n + 3 series separately, as was done already by MacCormick and Audi [19]. The result with a common a_c for $Q_{EC} = 2a_c(Z_p - 1)/A^{1/3} + b \pm c$ using the Q-values for $7 \leq A \leq 69$ is $a_c = 700.45\ keV$, $b = -631.0\ keV$ and $c = 72.9\ keV$. Here, the positive (negative) sign of the last term is for A = 4n + 3 (A = 4n + 1) nuclei. This means that the neutron pairing is stronger than the proton pairing. The rms deviation of the thus calculated values from the input values for $7 \leq A \leq 75$ is 85 keV, and the individual deviations are shown in Fig. 3. Again, discontinuities at the magic numbers 8 and 20 are visible. The calculated Q_{EC}-values and those from experiment [6,11] are given in Table 2.

3 Half-lives

The $0^+ \rightarrow 0^+$ transitions of the $T_Z = 0$ or $T_Z = -1$ parent nuclei are all pure Fermi transitions and their strength is a

Table 2 Calculated and experimental Q$_{EC}$-values for the T = 1/2 mirror decays. The experimental values for $A \le 59$ are from [6], those for $61 \le A \le 75$ are weighted averages from [6,7], and those for $A \ge 91$ are from [11]. The values for the statistical rate function f for $A \le 59$ have been taken from Severijns et al. [4] who used experimental Q-values, and were calculated for $A \ge 61$ from the calculated Q-values. With an uncorrected value of $ft = 2 * (3030 + Z)$ (see Sect. 3) the partial half-life is calculated for the Fermi decay component. The experimental half-lives are those given by [4]. However for $A \ge 63$ they are not corrected for the branches feeding other states than the mirror state. A "p" means that the ground state decays via proton emission. The values for $A \ge 91$ are again from [11]. The last column gives the ratio of Gamow-Teller to Fermi strength (see text)

A	Z_p	Q^{exp}_{EC} [keV]	ΔQ^{exp}_{EC} [keV]	Q^{calc}_{EC} [keV]	log(f)	$T^{calc}_{1/2}$ [s]	$T^{exp}_{1/2}$ [s]	$\Delta T^{exp}_{1/2}$ [s]	ρ^2
7	4	861.89	0.07	857					
9	5	1068.0	0.9	1208			p		
11	6	1981.69	0.06	1809	0.503	1907.7	1223.2	0.3	0.560
13	7	2220.47	0.27	2088	0.887	787.37	599.05	0.23	0.314
15	8	2754.20	0.50	2636	1.550	171.17	122.39	0.06	0.399
17	9	2760.47	0.25	2872	1.547	172.63	64.46	0.026	1.678
19	10	3239.50	0.16	3385	1.994	61.632	17.2767	0.0034	2.567
21	11	3546.92	0.02	3591	2.232	35.618	23.594	0.020	0.510
23	12	4056.18	0.03	4078	2.578	16.069	12.286	0.019	0.308
25	13	4276.81	0.04	4263	2.706	11.968	7.2331	0.0047	0.655
27	14	4812.36	0.10	4730	2.997	6.128	4.1218	0.0018	0.487
29	15	4942.20	0.40	4897	3.056	5.359	4.1774	0.0061	0.283
31	16	5398.00	0.23	5349	3.266	3.302	2.5859	0.0025	0.277
33	17	5582.50	0.40	5502	3.340	2.783	2.5439	0.0038	0.094
35	18	5966.20	0.70	5940	3.494	1.953	1.8081	0.001	0.080
37	19	6147.48	0.23	6081	3.559	1.683	1.2646	0.0012	0.331
39	20	6524.50	0.60	6508	3.695	1.232	0.8611	0.0008	0.431
41	21	6495.55	0.16	6639	3.676	1.286	0.5970	0.0022	1.154
43	22	6873.00	6.00	7057	3.804	0.959	0.5796	0.0095	0.655
45	23	7123.82	0.21	7179	3.882	0.802	0.5720	0.0100	0.401
47	24	7444.00	5.00	7588	3.979	0.642	0.4798	0.0066	0.337
49	25	7712.43	0.23	7702	4.055	0.538	0.417	0.012	0.291
51	26	8054.00	1.40	8104	4.150	0.433	0.3235	0.0054	0.338
53	27	8288.10	0.40	8210	4.210	0.377	0.2588	0.0055	0.456
55	28	8694.00	0.60	8606	4.315	0.296	0.2034	0.0048	0.457
57	29	8774.90	0.40	8706	4.330	0.286	0.2187	0.0021	0.309
59	30	9142.80	0.60	9095	4.418	0.234	0.1941	0.0017	0.203
61	31	9189	24	9190	4.426	0.230	0.1768	0.0033	0.300
63	32	9577	20	9574	4.513	0.188	0.1485	0.0051	0.266
65	33	9647	52	9663	4.529	0.181	0.1303	0.0006	0.392
67	34	10,036	19	10,042	4.611	0.150	0.1330	0.0038	0.128
69	35	10,180	42	10,127	4.624		p		
71	36	10,437	49	1,0,501	4.701	0.122	0.0988	0.0003	0.236
73	37	10,540	40	10581	4.713		p		
75	38	10,885	195	10,951	4.785	0.101	0.0954	0.0038	0.056
77	39			11,027	4.795	0.098	0.065	0.017	0.514
79	40			11,393	4.863	0.084	0.056	0.030	0.502
81	41			11,465	4.872		p		
83	42			11,827	4.937	0.071	0.028	0.019	1.538
85	43			11,896	4.945		p		
87	44			12,254	5.006	0.061	–		

Table 2 continued

A	Z_p	Q_{EC}^{exp} [keV]	ΔQ_{EC}^{exp} [keV]	Q_{EC}^{calc} [keV]	log(f)	$T_{1/2}^{calc}$ [s]	$T_{1/2}^{exp}$ [s]	$\Delta T_{1/2}^{exp}$ [s]	ρ^2
89	45			12,320	5.013		p		
91	46	11,800	2200	12,675	5.072	0.052	0.032	0.003	0.630
93	47			12,737	5.078		p		
95	48	10,200	1700	13,089	5.134	0.045	0.032	0.003	0.415
97	49	10,000	3000	13,149	5.139	0.045	0.036	0.006	0.242
99	50	14,700	3600	13,498	5.193	0.040	0.024	0.004	0.647

constant, when the appropriate radiative, charge dependent and isospin symmetry breaking corrections are applied [2]: $\mathcal{F}t = 3072.24 \pm 1.85s$. Here I want to give extrapolations up to $Z = 50$ and am neglecting the corrections. Instead, for the slowly increasing ft-values $ft = (3030+Z)s$ (compare Fig. 3a of Ref. [2]) is used. I assume this to be correct within 1%. To calculate the half-lives I took the values of $log(f)$ from the National Nuclear Data Center's internet application [5] using the calculated Q_{EC}-values in the case of T = 1 transitions (see Table 1). The experimental uncertainties for $A \geq 74$ are in the order of 10% and I assume that the calculated half-lives give a better estimate. It turns out that Q-values calculated from measured half-lives are more precise than the measured Q-values. This is shown in Fig. 4 for the heaviest $T_Z = 0$ β-emitters with $39 \leq Z_p \leq 49$. The red circles give the values calculated with the Coulomb energies. The blue triangles are calculated from the half-lives with the ft-values mentioned, the errors are those due to the experimental half-lives. And the squares are the directly measured Q-values from the β-endpoints for the three heaviest $T_Z = 0$ emitters [11].

Because the Fermi strength $B(F) = 1$ for $T = 1/2$ transitions and $B(F) = 2$ for $T = 1$ transitions the partial half-life is calculated for the Fermi decay component of $T = 1/2$ transitions with an uncorrected value of $ft = 2 * (3030 + Z)s$, double that of $T = 1$ transitions. The values for the statistical rate function f have been used from experimental Q-values [4] for $A \leq 69$ and for $A \geq 71$ from the calculated Q-values, because then the error bars become ≈ 1% and larger. The experimental half-lives are those given by [4] and for $A \geq 91$ from [11]. The last column gives the Gamow-Teller to Fermi ratio for the transition

$$\rho^2 = T_{1/2}^{Fermi} / T_{1/2}^{exp} - 1 = (g_A/g_V)^2 * B(GT)/B(F)$$

where $T_{1/2}^{Fermi} = T_{1/2}^{calc}$, and $B(F) = 1$ for $T = 1/2$ [3,4]. One has to bear in mind that in nuclear β-transitions the ratio g_A/g_V is usually assumed to be quenched compared to the value from the β-decay of the free neutron $g_A/g_V = -1.2754(13)$ [20]. Typically, in shell model calculations $\mid g_A/g_V \mid \approx 1$ is used, thus accounting for correlations or the truncation of the model space. So, the values of ρ^2 could be compared with theoretical values (e.g. V.I. Isakov [21]).

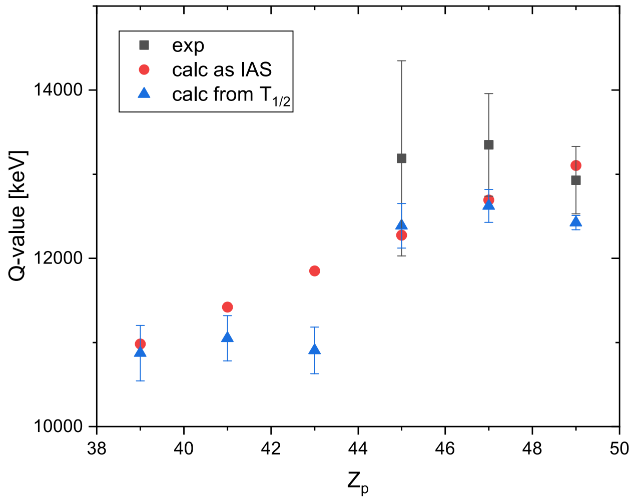

Fig. 4 Q_{EC}-values for the heaviest $T_Z = 0$; $0^+ \rightarrow 0^+$ β-emitters. The red circles give the values calculated with the Coulomb energies. The blue triangles are calculated from the half-lives with the ft-values mentioned, the errors are those due to the experimental half-lives. And the black squares are the directly measured Q-values from the β-endpoints for the three heaviest $T_Z = 0$ emitters

Summarizing, one can conclude that it is possible to extrapolate β-decay Q-values between $T = 1, T_Z = 0, \pm 1$ and $T = 1/2, T_Z = \pm 1/2$ isospin partners from the well measured cases with $Z_P \leq 30$ up to $Z_P \leq 50$. I expect that the uncertainty may increase from about 1% (rms deviation) for the fitted cases to about 2% for the cases with $Z_P \leq 50$, i.e. less than 300 keV. Because Fermi β-decay is so well understood, the half-lives of the heaviest $T = 1, 0^+ \rightarrow 0^+ \beta$-emitters and the partial half-life for the Fermi component of the T = 1/2 emitters can be well predicted. Since the Q_{EC}-value enters the statistical rate function f with about the $5th$ to $7th$ power, I expect an uncertainty of about 15% for the predicted half-lives.

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