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Coupled-channel description for mirror mass-11 nuclei compared to shell-model structures

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Abstract The spectra of mass-11 nuclei are unusual, and so pose a challenge for nuclear-structure theory. Relating to nucleon emission, the set of isobars range from being well-bound (^{11}B,^{11}C) through weakly bound (^{11}Li, ^{11}Be), to being proton unstable (^{11}N,^{11}O). To add complexity, the weakly bound ^{11}Li takes the form of a two-nucleon halo nucleus. A self-consistent approach to understand this set of nuclei is especially important as the mirror pair ^{11}Be-^{11}N exhibit a parity-inverted ground state compared to their neighboring nuclei. Herein, the Multi-Channel Algebraic Scattering method (MCAS) has been used to describe the low excitation spectra of those isobars in terms of nucleon-nucleus clusters. A collective model description of the low-excitation states of the mass-10 mass-10 core nuclei has been used to form the coupled-channel interactions required in the method. For comparison, and to understand the underlying configurations, a shell model approach has been used to obtain those spectra with no-core $(0+2+4)\hbar\omega$ and $(0+2)\hbar\omega$ shell-model spaces for the mass 10 and mass 11 nuclei respectively. The results of the calculations suggest the need of a strong coupling in the collective coupled-channel vibrational model. In particular, the strong coupling of the collective 2^+_1 state of ^{10}Be to the valence neutron plays a decisive role in forming the positive parity ground state in ^{11}Be; an effect confirmed by the shell-model results.

1 Introduction

The set of mass-11 isobars, from ^{11}Li to ^{11}O and encompassing three mirror pairs, span the region from the proton to the neutron drip lines, with ^{11}N and ^{11}O being beyond the proton drip line. A systematic study of this system is of interest given the change in isospin from one end of the range to the other, but also given that the mirror pair ^{11}Be and ^{11}N have ground states of positive parity, opposite to those of the other nuclei in the system. That has been the subject of extensive studies (see, for example, Refs. [1–3]), as ^{11}Be is also considered to be a single-neutron halo. While the determination of the nature of the positive-parity ground states is of interest, it is important to understand those ground states also with regards to the neighbouring mass-11 nuclei. In that context, a self-consistent many-body interpretation across the mass-11 isobars is necessary. In so doing, it becomes necessary to understand the nature of the ground state of ^{11}Be beyond the extreme single-particle picture as is suggested by the halo, which would require that its ground state wave function has significant neutron occupancy in the $1s_{1/2}$ orbit.

These mass-11 nuclei have quite diverse nucleon emission thresholds as is shown in Table 1. They range from 22.39 to − 1.49 MeV. Recently [4], an observation of the unbound ^{11}O was made and of its decay by two proton emission. Describing the spectra of such a set of odd-mass nuclei self-consistently presents a challenge. Herein we compare spectra found using a coupled-channel description for the mirror mass-11 nuclei with results from complete-space shell-model calculations. We have used the coupled-channel approach [5] that, previously, was taken to similarly analyse the spectra of the full set of mass-7 isobars [6].

Those analyses assumed two types of coupling: ones involving two clusters of composite nuclei (mass-3 and mass-4) for only a single channel, and the other, coupling of nucleons to low-lying states in ^{6}He and ^{6}Be. For the former, the assumption of only a single channel contributing to the spec-

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trum is adequate given that there are no excited states in the core nuclei, save for the resonances above 20 MeV in ^{4}He [7], which do not affect the low-lying spectra of the compound.

Our interest is to describe the low excitation spectra of the set of mirror pairs, ^{11}C–^{11}B and ^{11}Be–^{11}N, within a coupled-channel approach. Too little is known about the third mirror pair, ^{11}Li–^{11}O, and their underlying mass-10 cores for the method to be used with any confidence. We seek to establish the extent to which a nearly-uniform interaction may account for the spectra of four of these isobars. For these cases, we treat the mirror pairs as the coupling of a nucleon with the appropriate mass-10 core, which is described by a collective model (either vibrational or rotational).

The method we use solves the coupled-channel problems of two-cluster systems, usually nucleon-nucleus, algebraically. The coupling interactions in this approach have been formed using a number of low excitation states of the core nuclei. In those coupled-channel calculations, the basic nucleon-nucleus interaction used has the form

$$v_0(r) = [V_0 + V_{ll}\,\{\mathbf{l}\cdot\mathbf{l}\}]\,w(r) + 2\lambda_\pi^2\,V_{ls}\,\frac{1}{r}\frac{\partial w(r)}{\partial r}\,\{\mathbf{l}\cdot\mathbf{s}\}\ , \quad (1)$$

in which the Woods-Saxon (WS) function,

$$w(r) = \left[1 + \exp\left(\frac{r - R_0}{a_0}\right)\right]^{-1} ,$$

has been used. The vector operators **l** and **s** denote orbital and nucleon intrinsic spin, and V_0, V_{ll}, V_{ls} are potential-strength parameters. R_0 is nuclear radius and a_0 is the diffusivity. All details of the approach, first used in [5], and a collection of the results of applications made to date are given in Ref. [9].

The Pauli principle is taken into account in this method by adding orthogonalizing pseudo-potentials (OPP) to the nucleon-nucleus potentials chosen for the cluster coupled-channels interactions in the defining Hamiltonian for the cluster [10]. The OPP are formed as nonlocal separable products of bound single nucleon wave functions in the chosen nucleon-nucleus potentials. Thus the interaction matrix of potentials used has the non-local form,

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\,\delta(r - r') + \lambda A_c(r) A_c(r')\,\delta_{c,c'}\ , \quad (2)$$

in which $V_{cc'}$ are the collective-model derived nuclear potentials, $A_c(r)$ is the radial part of the single particle wave function in channel c, determined by solving the radial Schrödinger equation, and λ is a scaling energy. A sufficiently large value for these energy parameters (about 10^6 MeV) implies that completely-occupied nucleon sub-shells in the target become inaccessible to occupancy by a projectile nucleon. Smaller non-zero values lead to a Pauli-hindrance effect involving a partially occupied sub-shell, as has been extensively discussed [9,11,12]. Pauli-allowed states imply the scaling parameter being set to zero.

For completeness, we have also evaluated the spectra of some of the mass-10 and mass-11 isobars of relevance using a complete space, no-core, shell model. For mass-10 we have used a $(0 + 2 + 4)\hbar\omega$ shell model while for mass-11 a no-core $(0 + 2)\hbar\omega$ shell model was used. Use of the smaller basis model for the odd-mass nuclei was due to the much larger matrices involved in using the $(0+2+4)\hbar\omega$ model for mass-11 nuclei rendering those calculations impractical. In all cases, the OXBASH shell-model program [13] was used. The results are free of any spurious center of mass effects since the bases are complete.

Results are presented in two sections. In Sect. 2 we consider the structure of three sets of mass-11 mirror pairs as found using MCAS to solve coupled-channel cluster systems. Results of using this approach are given in three subsections. In Sect. 3, spectra of the mass 10 and 11 isobars that we have found using no-core shell models are presented and discussed. Conclusions are given in Sect. 4.

2 Mass-11 isobars as cluster systems

The MCAS method has been used to solve couple-channel equations to give low-energy spectra of a set of mass-11 isobars. These systems are described by the effective coupling of a nucleon to a ^{10}Be, a ^{10}B, or a ^{10}C core. The interactions for the channel couplings were defined using collective models for the core nuclei. The spectra for the mirror pair ^{10}Be and ^{10}C exhibit structures somewhat consistent with a vibrational model (being a 0^+ ground state, followed by almost-equal spacings to a 2^+ excited state and then a near-degenerate triplet). However, given the lack of knowledge of the spectrum of ^{10}C, we restrict the couplings to involve the first four states in each only. The low-lying spectrum of ^{10}B does not show features consistent with a vibrational model. We use the lowest six states of ^{10}B to define the nucleon-nucleus channel couplings in a rotational model.

Results from those calculations are presented and discussed in the following three subsections for the mirror systems ^{11}C (described as the coupling of a neutron to ^{10}C) and ^{11}B ($p+{}^{10}$Be); ^{11}N ($p+{}^{10}$C) and ^{11}Be ($n+{}^{10}$Be); and, ^{11}C ($p+{}^{10}$B) and ^{11}B ($n+{}^{10}$B), sequentially.

2.1 The mirror systems, ^{11}C and ^{11}B

Initially, we consider ^{11}C and ^{11}B as modelled by couplings of the clusters $n+{}^{10}$C and $p+{}^{10}$Be, respectively. The spectra of these mirror systems have many bound states given the relevant emission thresholds as shown in Table 1. In our case, using the coupled-channel approach, good results have been found assuming a vibration collective model and for the rel-

Table 1 General properties of mass-11 isobars [8]. Uncertain/unknown values shown in brackets. Energies are in MeV

	^{11}Li	^{11}Be	^{11}B	^{11}C	^{11}N (*)	^{11}O
J^π (gs)	$\frac{3}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^+$	$(\frac{3}{2}^-)$
Decay	β^-	β^-	Stable	β^+	$1p$	$2p$
$1p$ thr.	15.73	20.16	11.23	8.69	−1.49	(–)
$1n$ thr.	0.396	0.502	11.45	13.12	22.39	(–)

*In [6] a proton separation threshold of − 1.54 MeV was suggested for ^{11}N

Table 2 The parameter values of Eqs. (1), (2) and (3) that have been used in our coupled-channel evaluations of the ^{11}C (as n+^{10}C) and ^{11}B (as p+^{10}Be) spectra. Strengths are in MeV, geometry radii and diffusivities are in fm

Nuclear (WS)	$R_0 = 2.8$	$a_0 = 0.65$	
	$V_0 = -53.3$	$V_{ls} = 6.2$	$V_{ll} = 0.0$
			$\beta_2 = -0.83$
Coulomb (3pF)	$R_c = 2.39$	$a_c = 0.8$	$w_c = -0.06$

Table 3 OPP strengths for relevant nucleon interactions with states in ^{10}C (^{10}Be). All energies [8] are in MeV and those for the mass-10 states are the experimental ones

State	E_x, ^{10}C (^{10}Be)	Widths ^{10}C	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$
g.s.	0.000 (0.000)	–	10^6	5.2	9.0
2_1^+	3.354 (3.368)	–	10^6	10.0	9.0
2_2^+	5.220 (5.958)	0.225	10^6	0.0	0.0
0_2^+	5.380 (6.180)	0.3	10^6	0.0	0.0

evant nucleon interactions, with the low-lying states of ^{10}C and ^{10}Be. We have used the four known positive states to $\sim$ 6 MeV excitation in each core nucleus. For the vibrational model, the assumed target states are: the ground states (0^+), which are taken as the vacuum, the first excited states (2^+), assumed to be one quadrupole phonon excitation on the vacuum, while the other excited states (2_2^+, 0_2^+) are taken as two quadrupole excitations.

The parameter values defining the interactions are listed in Table 2. Therein, all energies are in MeV and the radii and diffusivities are in fm.

It should be noted that in Table 2 the geometric Coulomb parameters are constrained by analyses of the charge densities of the cores [14] as given by a three-parameter Fermi distribution function 3pF [9], namely

$$\rho_{\rm ch}(r) = \frac{1 + w_c \left(r^2/R_c^2\right)}{1 + \exp\left[(r - R_c)/a_c\right]} . \quad (3)$$

In calculations, the parameters, radius, R_c, diffuseness a_c, and scaling parameter w_c, are chosen to give the rms Coulomb charge radius listed in Ref. [14].

The OPP, $\lambda_{nlj}^{(I)}$, required for those interactions with states in the mass-10 core nuclei are listed in Table 3.

In Fig. 1, the results found for the low excitation spectra of ^{11}C and ^{11}B are compared with the known spectra [8]. Good agreement is evident over almost 10 MeV in excitation. As good an agreement has been found with the same Hamiltonian plus a Coulomb interaction, when used in calculating the ^{11}B spectrum.

Our calculated results reproduce the observed nucleon separation energies, find the proper sequence of low excitation negative parity states (energy gaps and spins) and give the

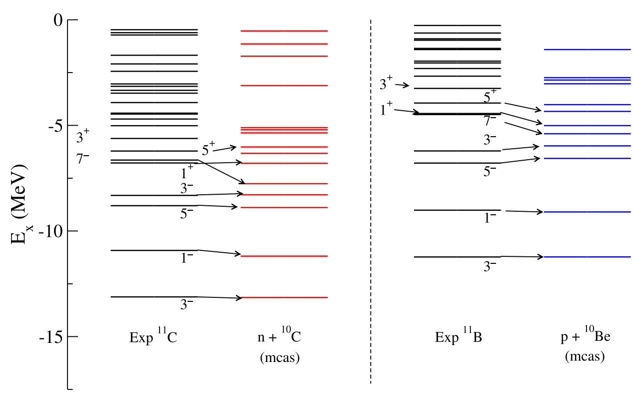

Fig. 1 Experimental spectra of ^{11}C and of ^{11}B [8] compared with the results of MCAS calculations for the n+^{10}C and p+^{10}Be clusters. The zero energy in each is the relevant nucleon separation energy. The spin-parities of the states are indicated as $2J^\pi$

known excitation energies of the first positive parity excited states.

2.2 The mirror systems, ^{11}Be and ^{11}N

We have described ^{11}N and ^{11}Be by the coupling of the clusters p+^{10}C and n+^{10}Be, respectively. They pose very different problems for evaluation to those of the ^{11}C and ^{11}B mirrors. Largely those different problems relate to ^{11}Be being weakly bound while ^{11}N is unbound as is indicated by the relevant nucleon thresholds listed in Table 1. Further, both ground states (a resonance in the case of ^{11}N) have assigned spin-parities of $\frac{1}{2}^+$.

Our coupled-channel calculations for the spectra of ^{11}N and ^{11}Be used the parameter values for the interactions that

Table 4 The parameter values used in the coupled-channel evaluations of the ^{11}N (as p+^{10}C) and ^{11}Be (as n+^{10}Be) spectra. Strengths are in MeV, radii and diffusivities are in fm

Nuclear (WS)	$R_0 = 2.8$	$a_0 = 0.625$	
	$V_0 = -42.2$	$V_{ls} = 6.0$	$V_{ll} = 0.7$
			$\beta_2 = -0.83$
Coulomb (3pF)	$R_c = 2.4$	$a_c = 0.8$	$w_c = 0.0$

Table 5 OPP strengths for relevant nucleon interactions with states in ^{10}C (^{10}Be) to get the results shown in Fig. 2. All energies are in MeV and core nuclei excitation energies are the experimental values

State	E_x, ^{10}C (^{10}Be)	Widths ^{10}C	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$
g.s.	0.000 (0.000)	–	10^6	10^6	5.3 (5.0)
2_1^+	3.354 (3.368)	–	10^6	83.0	13.7
2_2^+	5.220 (5.958)	0.225	10^6	4.4	3.0 (2.8)
0_2^+	5.380 (6.180)	0.3	10^6	4.3	0.0

are listed in Table 4, with a deformation parameter $\beta_2 = -0.83$.

While most of these parameter values are similar to those given in Table 2 for the ^{11}C and ^{11}B cases, the values of the central strengths are $\sim$ 30% smaller. This change in the central potential strength reflects the changed number of like nucleon versus unlike nucleon interactions between the nucleons in each mass-10 core with the added nucleon forming the clusters.

The OPP strengths needed in these cases also differ from the values for the clusters due to the changed numbers of like nucleons (to the external one) in the core nuclei. In particular, in the current configurations, the two ground states have the $0p_{\frac{3}{2}}$ orbits fully occupied. The OPP strengths required to obtain the low excitation spectra of ^{11}N and ^{11}Be are listed in Table 5; with that of the $0p_{\frac{1}{2}}$ orbit in the ground state of ^{10}Be being slightly larger to give the observed excitation energies of the $\frac{1}{2}^-|_1$ states.

For optimal results in these two cases, slightly differing central potentials of strength of -42.2 and -41.8 MeV and diffusivities of 0.625 and 0.7 fm have been used to get the spectra of ^{11}Be and ^{11}N respectively. The need of these small adjustments for mirror conditions, in this case, is not surprising given the small differences in spectral properties of the two mass-10 core nuclei as well as the simplistic collective model used to describe the coupling interactions between the nucleon and the four states of those core nuclei chosen for the coupled-channel calculations.

As shown in Fig. 2, our coupled-channel calculations gave the nucleon separation energies and the approximate energies and spin-parities of many of the known spectral states in the low excitation spectra. But there are calculated levels that are not observed and known levels that are not matched in this (and higher) excitation energy range. Possibly there is a state at 3.06 MeV in the spectrum of ^{11}N [8] that is not found in the calculation. But there has not been any spin-parity assigned to it in the tabulations. Properties of the spectra are listed in Table 6.

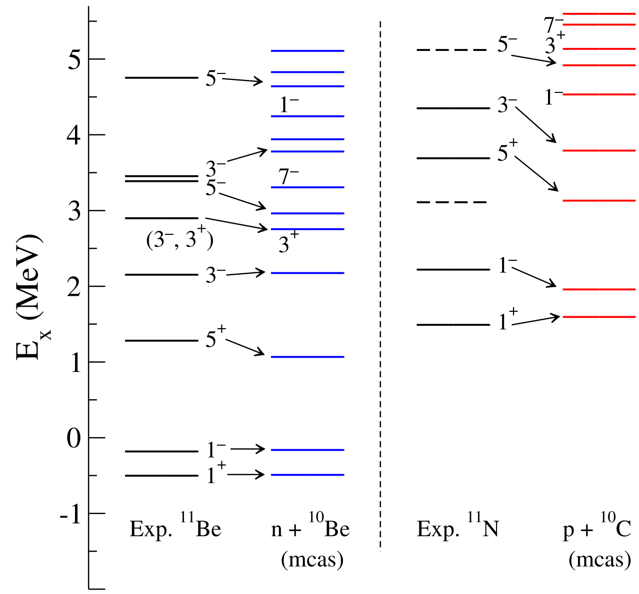

Fig. 2 Experimental spectra of ^{11}Be and of ^{11}N [8] compared with the results of MCAS calculations for the n+^{10}Be and p+^{10}C clusters. The zero energy in each is the relevant nucleon separation energy. The spin-parities of the states are indicated as $2J^{\pi}$

Therein the experimental energies and widths of known states in the low excitation spectra of ^{11}Be and ^{11}N are compared.

Those spectral levels are displayed also in Fig. 2, from which it is evident that, in the low energy excitation range, the calculated excitation energies agree well overall with observation. Table 6 also shows that most calculated level widths compare with observations usually to within a factor of 2.

Of note is that there are calculated $\frac{7}{2}^-$ states at $\sim$ 4 MeV excitation in both ^{11}Be and ^{11}N that have no empirical counterpart in the range of energies shown. While there are no known experimental states at all in ^{11}N above 5.08 MeV excitation, there is a possible $\frac{7}{2}^-$ state in the spectrum of ^{11}Be at 6.705 MeV excitation. $\frac{7}{2}^-$ states exist in ^{11}C and ^{11}B with excitation energies of 6.48 and 6.74 MeV respectively with the calculated MCAS values being $\sim$ 1 MeV less as shown in Fig. 1.

These calculations involve very strong coupling, as evidenced by the value of $\beta_2 = -0.83$. Varying that value has a drastic effect, with a small decrease resulting in a significant raising of the centroid energy of the $\frac{1}{2}^+$ resonance for it to no longer be the ground state of ^{11}N. On the other hand a small increase makes the $\frac{1}{2}^+$ ground state centroid decrease

Table 6 The known and MCAS results for ^{11}Be and ^{11}N as given in the Fig. 2. The experimental and MCAS energies and widths are compared. Level energies are in MeV, widths (in brackets) are in keV

State	^{11}Be		State	^{11}N	
J^π	Experiment	MCAS	J^π	Experiment	MCAS
$\frac{1}{2}^+$	− 0.501	− 0.501	$\frac{1}{2}^+$	1.49 (830)	1.49 (2760)
$\frac{1}{2}^-$	− 0.18	− 0.182	$\frac{1}{2}^-$	2.22 (600)	2.21 (10)
				3.06(< 100)	
$\frac{5}{2}^+$	1.28 (100)	1.05 (118)	$\frac{5}{2}^+$	3.69 (540)	3.19 (573)
$\frac{3}{2}^-$	2.15 (206)	2.15 (96)	$\frac{3}{2}^-$	4.36 (340)	4.08 (172)
$\frac{1}{2}^-$	–	2.74 (11)	$\frac{1}{2}^-$	–	4.44 (3)
$(\frac{3}{2}^{(\pm)})$	2.95 (122)	2.95 (275)	$\frac{3}{2}^+$	–	4.87 (3200)
$\frac{5}{2}^-$	3.39 (< 8)	3.29 (< 1)	$\frac{5}{2}^-$	5.12 (< 220)	5.09 (165)
$\frac{7}{2}^-$	–	3.76 (12)	$\frac{7}{2}^-$	–	5.57 (340)
$\frac{3}{2}^-$	3.46 (10)	3.92 (131)	$\frac{5}{2}^-$	5.91 (?)	6.41 (150)
$\frac{1}{2}^-$	–	4.22 (107)	$(\frac{3}{2}^-)$	6.57 (100)	6.51 (258)
$\frac{5}{2}^-$	4.76 (45)	4.62 (88)			

in energy. At the same time the width of that ground state resonance also decreases and a value of 800 keV can be found, in agreement with that listed [8]. But then, on weakening the central potential strength to recover the wanted centroid energy of 1.49 MeV, the width increases back to over 3 MeV.

Allowing the chosen states of the core nucleus, ^{10}C, to be orthonormal admixtures including two quadrupole phonon components, also can give a $\frac{1}{2}^+$ ground state resonance for ^{11}N with a width of $\sim$ 800 keV. But then the centroid energy lies between 0.8 and 1.0 MeV. Again, weakening the central potential strengths to place that centroid at 1.49 MeV leads to an increase in those widths to between 3 and 4 MeV.

In Fig. 3, we show how the strong coupling, particularly to the 2_1^+ state of the core, ^{10}Be, leads to the ground state in ^{11}Be having a spin-parity $\frac{1}{2}^+$. Therein the low excitation known spectrum of ^{11}Be is compared to the full coupled-channel result, as displayed previously in Fig. 2, and here labelled as "MCAS 4" with the numeral being the number of states of ^{10}Be used in forming the coupled-channel interaction matrix of potentials, to wit the ground 0^+, and the 2_1^+, 2_2^+ and 0_2^+ in sequence. The adjacent columns show the spectra formed with just the first three of those ("MCAS 3"), with only the ground and 2_1^+ ("MCAS 2"), and as a single channel, ground state, calculation ("MCAS 1"). Crucially all other details involved in these calculations were unchanged.

Clearly the major spectral change with the evaluated ground state being a $\frac{1}{2}^+$ level arises from the coupling involving the 2_1^+ state but the other terms play important roles in adjusting evaluated energies downward with improvement of the energy gaps in the low lying ^{11}Be spectrum. Notably, the

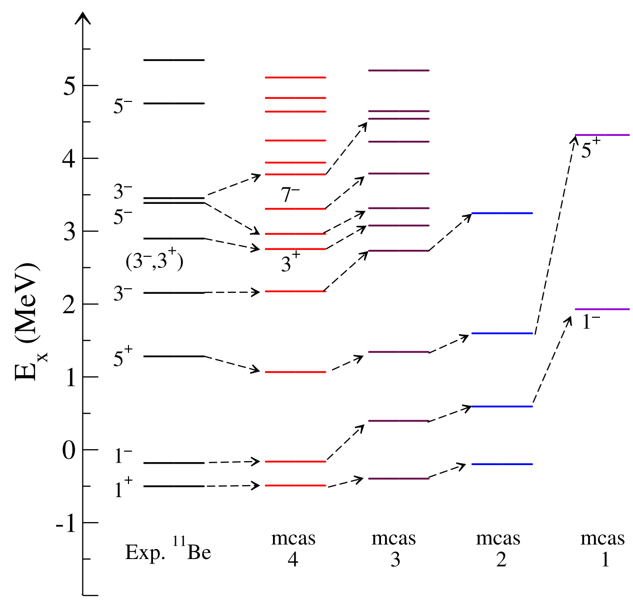

Fig. 3 Experimental spectra of ^{11}Be [8] compared with the results of diverse calculations for the n+^{10}Be cluster. The spin-parities of the states are indicated as $2J^\pi$. The numerals under "MCAS" indicate reduction in the number of target states considered in each calculation as is indicated in the text

Table 7 The parameter values used in the coupled-channel evaluations of the ^{11}C (as p+^{10}B) and ^{11}B (as n+^{10}B) spectra. Strengths are in MeV, radii and diffusivities are in fm

Nuclear (WS)	$R_0 = 2.8$	$a_0 = 0.65$	$V_0 = -41.7$
			$V_{ls} = 6.2$
			$\beta_2 = -0.8$
Coulomb (3pF)	$R_c = 2.355$	$a_c = 0.522$	$w_c = -0.15$

Table 8 OPP strengths for nucleon interactions with states in ^{10}B required to get the results shown in Fig. 4. All energies are in MeV

State	E_x, ^{10}B	$0s_{1/2}$	$0p_{3/2}$	$0p_{1/2}$
g.s. 3^+	0.000	10^6	20.0	10.0
1_1^+	0.718	10^6	4.0	10.0
0_1^+	1.740	13.5	4.0	9.0
1_2^+	2.154	10^6	0.0	0.0
2_1^+	3.587	10^6	0.0	0.0
3_2^+	4.774	10^6	0.0	14.0

shell model calculations presented in Sect. 3.2, Table 9, also indicate this large contribution of the 2_1^+ state.

2.3 The mirror systems, ^{11}C and ^{11}B, with ^{10}B as core

Treating these nuclei as the clusters of a proton and of a neutron on the core nucleus, ^{10}B, is a major problem. First the low energy spectrum of ^{10}B does not readily map to any simple collective model of structure with its ground state spin-parity of 3^+ being a challenge to be found even in larger space shell-model studies (see below).

The six lowest lying states in ^{10}B have the spin-parity sequence $3_1^+, 1_1^+, 0^+, 1_2^+, 2^+$ and 3_2^+; a sequence which is not easily identifiable with collective structure. Thus, a simple rotation scheme with solely a quadrupole coupling between all states was used to see what might result from our coupled-channel approach. The Woods-Saxon potential and 3pF Coulomb charge distribution parameter values used are shown in Table 7.

The OPP strengths needed to achieve the final results are listed in Table 8.

These values reflect underlying nucleon shell occupancies disparate to those in the core nuclei of the other clusters considered. Most notably we need to have some inner core ($0s$-shell) breaking with the monopole. This breaking is essential to get the known $\frac{1}{2}^+$ in the calculated result. Comparisons with the known spectra [8] (of ^{11}C and ^{11}B) are made in Fig. 4.

The low excitation spectra are reasonably in agreement, with only the calculated $\frac{5}{2}^-$ state in the calculated spectrum of ^{11}C being over an MeV too low and too close to the first excited $\frac{1}{2}^-$ state.

3 Shell-model study of mass-10 and mass-11 isobars

As a comparison to the spectra for the mass-11 pairs obtained from the coupled-channel evaluations, alternate spectra were obtained using the shell model. For the mass-10 nuclei, a complete $(0+2+4)\hbar\omega$ shell-model calculation was performed using the Zheng G-matrix interaction [15]. For the mass-11 systems, the Millener-Kurath interaction [16] was used in a complete $(0+2)\hbar\omega$ model space. The positive parity states of ^{11}Be and ^{11}N were obtained in a $(1+3)\hbar\omega$ model space. The OXBASH shell-model program [13] was used to obtain both the wave functions and spectra.

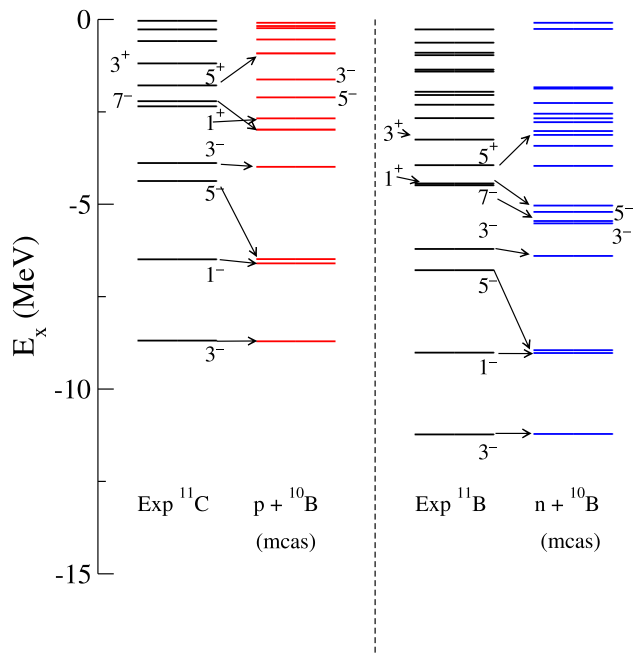

Fig. 4 Experimental spectra of ^{11}C and of ^{11}B compared with the results of MCAS calculations for the $p+^{10}$B and $n+^{10}$B clusters. The zero energy in each is the relevant nucleon separation energy. The spin-parities of the states are indicated as $2J^\pi$

3.1 Mass 10 isobars

In Figs. 5 and 6 the low-energy spectra for ^{10}B and the mirror pair ^{10}Be-^{10}C are shown, respectively. Therein, the results of the shell-model calculations are compared to the known spectra [17].

The nucleus ^{10}B is a special case, being one of the few light odd-odd mass nuclei. Its ground state has a spin-parity of 3^+ which presents a problem for shell-model descriptions. It is not a simple matter of the coupling of the odd $0p_{3/2}$ proton to the odd $0p_{3/2}$ neutron. The result from the $(0+2+4)\hbar\omega$ shell model gives a ground state of 1^+, but with a binding energy only 319 keV below that of the predicted 3_1^+ state. The wave functions of those states are, as given by the shell-model calculation:

$$\left|1_1^+\right\rangle = 69.21\%\ |0\hbar\omega\rangle + 17.18\%\ |2\hbar\omega\rangle + 13.61\%\ |4\hbar\omega\rangle$$
$$\left|3_1^+\right\rangle = 67.79\%\ |0\hbar\omega\rangle + 17.75\%\ |2\hbar\omega\rangle + 14.28\%\ |4\hbar\omega\rangle\ , \quad (4)$$

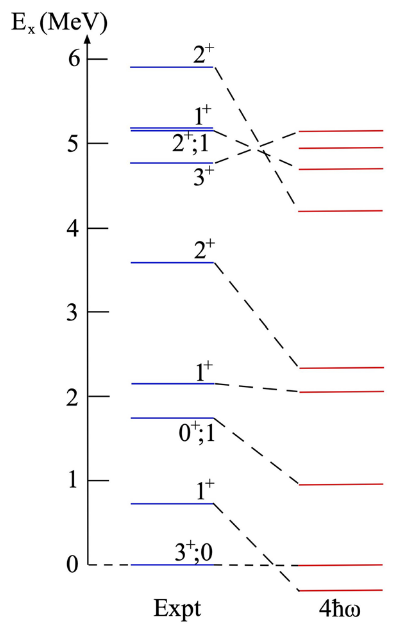

Fig. 5 Low-energy spectrum of ^{10}B. The experimental spectrum [17] is compared to the results of the shell-model calculation as described in the text

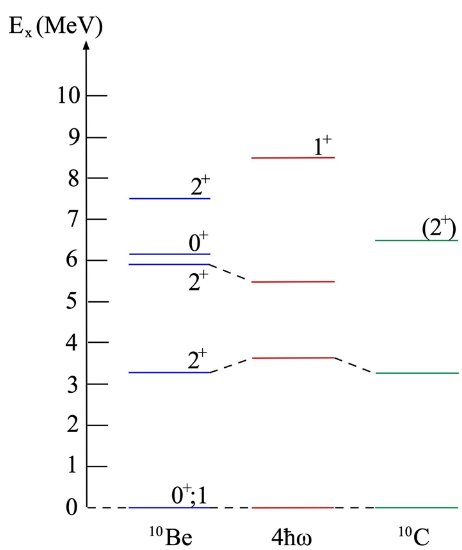

Fig. 6 Low-energy spectra of the mirror pair ^{10}Be and ^{10}C. The data [17] are compared to the results of the shell-model calculation as described in the text

while the dominant $0\hbar\omega$ configurations are

$$
\begin{aligned}
|1_1^+\rangle = {} & 28.63\% \left|(0p_{3/2})^5\,(0p_{1/2})\right\rangle \\
& + 28.61\% \left|(0p_{3/2})^4\,(0p_{1/2})^2\right\rangle \\
& + 7.87\% \left|(0p_{3/2})^3\,(0p_{1/2})^3\right\rangle + 3.21\% \left|(0p_{3/2})^6\right\rangle \\
|3_1^+\rangle = {} & 26.00\% \left|(0p_{3/2})^6\right\rangle + 21.26\% \left|(0p_{3/2})^5\,(0p_{1/2})\right\rangle \\
& + 18.42\% \left|(0p_{3/2})^4\,(0p_{1/2})^2\right\rangle . \qquad (5)
\end{aligned}
$$

A closed $0s_{1/2}$ shell forms part of each component listed in Eq. (5). Other configurations not explicitly given in Eq. (5) are $\sim$ 1% or less, leading to both wave functions being $\sim$ 70% $0\hbar\omega$ with the rest of the wave functions formed from higher order $\hbar\omega$ components.

There is significant occupation of the $0p_{1/2}$ orbit in both wave functions, while a dominant $(0p_{3/2})^6$ component is only in the wave function for the 3_1^+ state. While a 1^+ ground state is favored in such a model, as is suggested by the dominant configurations involving the $0p_{1/2}$ orbit in each wave function, the small excitation energy of the 3^+ state encourages the idea that a small change in the shell-model interaction matrix elements could invert the levels for the 3^+ to be the ground state. This problem has also been considered using other shell models, such as by those classified as *ab initio* [18,19]. Both of these studies consider very large basis spaces but do not account for all possible states contained therein. Thus the spaces are incomplete and so require adjusting methods, such as that of the projection method of Gloeckner and Lawson [20], to account for spurious center of mass motion. Nonetheless, both studies obtained good spectra and electromagnetic properties of the systems studied, for the mass-10 isobars [18] and for the isotopes of Boron [19]. The first of these [18] used the CD-Bonn and Argonne realistic NN interactions, but in the case of ^{10}B, even allowing changes in the oscillator energy defining the single particle states, still gave a 1^+ ground state. They considered that as an indication of the need for true three-body forces to describe the low-lying structure in complex nuclei. That was considered in the more recent study [19] and found to be the case but reproduced using one of the set of starting NN interactions, the INOY NN interaction.

In our $(0+2+4)\hbar\omega$ calculation, the dominant component of the 0_1^+ state in ^{10}B is 70% $\left|(0s_{1/2})^4\,(0p_{3/2})^6\right\rangle$, suggesting a closed $0s$ core. However, the occupation number is 1.9 for both the protons and neutrons in the $0s$ shell; an effect of the other 30% of the wave function which contains either an open $0s_{1/2}$ shell and/or occupancy in the $0d1s$ shell. Such result provides a microscopic justification of a small OPP strength parameter for for the 0_1^+ level of the ^{10}B system, as reported in Table 8.

The low-energy spectra for the mirror nuclei ^{10}Be and ^{10}C are shown in Fig. 6, with the result of the calculation using the $(0+2+4)\hbar\omega$ shell model. The spectra are reasonably well reproduced, reflecting the mirror symmetry, although there is not much experimental information about the spectrum of ^{10}C. The first excited 0^+ state in ^{10}Be is predicted to be at 10.779 MeV, well above the measured energy. The wave

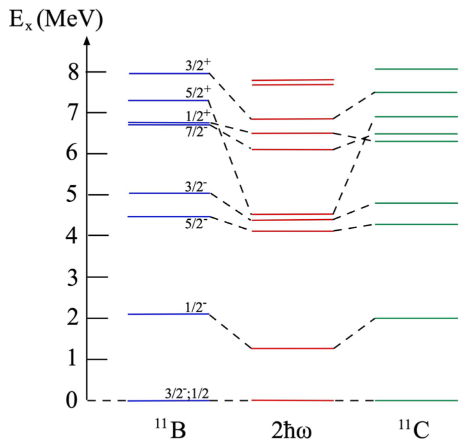

Fig. 7 Low-energy spectra of the mirror system $^{11}B/^{11}C$. The results of the $(0+2)\hbar\omega$ shell-model calculation are compared to the data [8] for both nuclei

function of this particular 0^+ state is

$$\left|0_2^+\right\rangle = 71.42\% \left|0\hbar\omega\right\rangle + 15.15\% \left|2\hbar\omega\right\rangle + 13.43\% \left|4\hbar\omega\right\rangle . \quad (6)$$

By comparison, the energy of that state is predicted to be between 9.51 and 9.78 MeV in the model of Caurier et al. [18], and is also largely a $0\hbar\omega$ state. However, this 0^+ state, at 6.179 MeV, is considered to be predominantly $2\hbar\omega$ in nature [21].

3.2 Mass 11 isobars

The spectra of the mass-11 isobars of interest, the mirror pairs $^{11}B/^{11}C$ and $^{11}Be/^{11}N$, have been obtained from MCAS evaluations as shown in Figs. 1, 2, and 4. The mirror pair $^{11}Li/^{11}O$ presents a different problem for the shell model, given the very loose binding of ^{11}Li (the two neutron separation energy is 369.1 keV while the single neutron separation energy is 395.5 keV [8]), and ^{11}O is unbound. The spectra of these nuclei of interest have also been obtained using a $(0+2)\hbar\omega$ shell model. In the instances where odd-parity states are required, the state have been obtained in a $(1+3)\hbar\omega$ model. The MK3W interaction of Warburton and Millener [16], a modification of the earlier shell-model interaction of Millener and Kurath, was used.

The low-energy spectra for the mirror pair $^{11}B/^{11}C$ is shown in Fig. 7.

Therein, the results of the $(0+2)\hbar\omega$ shell model calculation agree reasonably well with the measured spectra, with the exception of the $\frac{5}{2}^+$ state at 7.29 MeV in ^{11}B and 6.91 MeV in ^{11}C. The prevalence of negative parity states in this region of excitation of both these nuclei would indicate the need for larger shell-model spaces and the inclusion of higher $\hbar\omega$ components.

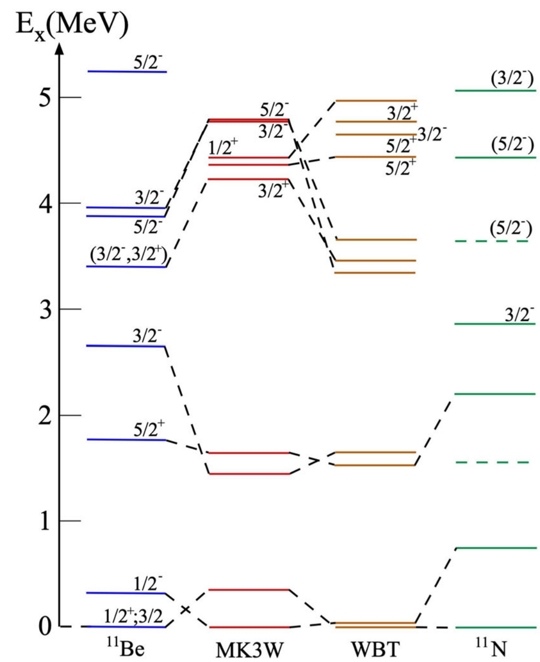

Fig. 8 Low-energy spectra for the mirror pair $^{11}Be/^{11}N$. The data [8] are compared to the results of the shell-model calculations made in a $(0+2)\hbar\omega$ model space using the MK3W [16] and WBT [22] shell-model interactions

Figure 8 displays the low-energy spectra for the mirror pair $^{11}Be/^{11}N$. Not much is known of the spectrum of ^{11}N, but the spectrum of ^{11}Be is well-established as a result of measurements of the β-decay of ^{11}Li [8].

The presence of the $\frac{1}{2}^+$ ground state in both ^{11}Be and ^{11}N presents a problem for the shell model. The result of the $(0+2)\hbar\omega$ shell-model calculation using the MK3W interaction predicts a $\frac{1}{2}^-$ ground state, with the $\frac{1}{2}^+$ state 320 keV above it. For comparison, we have also performed a calculation in the same model space using the WBT interaction of Warburton and Brown [22]. It gives the $\frac{1}{2}^+$ ground state with the $\frac{1}{2}^-$ state only 20 keV in excitation. The ground state configuration in the WBT model is

$$\begin{aligned}\left|\frac{1}{2}^+_{gs}\right\rangle &= 38.54\% \left|(0s_{1/2})^4 (0p_{3/2})^6 (1s_{1/2})\right\rangle \\ &+ 26.44\% \left|(0s_{1/2})^4 (0p_{3/2})^4 (0p_{1/2})^2 (1s_{1/2})\right\rangle \\ &+ 5.78\% \left|(0s_{1/2})^4 (0p_{3/2})^5 (0p_{1/2}) (1s_{1/2})\right\rangle \\ &+ 3.09\% \left|(0s_{1/2})^4 (0p_{3/2})^2 (0p_{1/2})^4 (1s_{1/2})\right\rangle \\ &+ 26.15\% \text{ configurations not involving the } 1s \text{ orbit,} \end{aligned} \quad (7)$$

with the breakdown in $\hbar\omega$ being

$$\left|\frac{1}{2}^+_{gs}\right\rangle = 94.29\% \left|1\hbar\omega\right\rangle + 5.71\% \left|3\hbar\omega\right\rangle . \quad (8)$$

Table 9 Shell-model spectroscopic amplitudes, S_j, for the coupling of a neutron to states in ^{10}Be forming the $\frac{1}{2}^+$ ground state in ^{11}Be

State	Orbit	Interaction	S_j
g.s (0^+)	$1s_{1/2}$	WBT	− 1.2293
		MK3W	1.1097
2_1^+	$0d_{5/2}$	WBT	0.5538
		MK3W	0.6317
2_1^+	$0d_{3/2}$	MK3W	− 0.1122

The configurations involving a nucleon in the $1s$ orbit account for 73.85% of the total wave function within which the occupation numbers for the $1s_{1/2}$ orbit are 0.787 for neutrons and 0.006 for protons. These components may be viewed as forming a $1s$ neutron halo in ^{11}Be, but there is a substantial component (26.15%) that has no $1s_{1/2}$ involvement. Strong configuration mixing is suggested by Eq. (7); a result enhanced with the shell model calculations made using the MK3W interaction from which configurations involving the $1s_{1/2}$ orbit account for only 49.48% of the total wave function.

The relevant couplings of the valence neutron to states in ^{10}Be forming the ground state in ^{11}Be may also be obtained from the shell model calculation. The spectroscopic amplitudes for forming the ground state of ^{11}Be as obtained from the shell model calculations using the WBT and MK3W interactions are given in Table 9.

The results of the shell model calculations show strong coupling of the neutron not only to the ground state of ^{10}Be but also to the 2_1^+ state. In the case of the WBT shell model calculation the coupling to the 2^+ state is almost purely from the neutron occupying the $0d_{5/2}$ orbit. The MK3W result, however, indicates an additional coupling via the neutron occupying the $0d_{3/2}$ orbit. The spectroscopic amplitude in that case is of opposite sign, indicating an overall weaker coupling to the 2^+ state in the model. This indication of a strong coupling to the ground and 2^+ states supports the necessity of such coupling in the MCAS calculation (cf. Table 4).

As with the ground state of ^{10}B, a small change in the matrix elements in both the MK3W and WBT interactions may give the correct inversion with the correct energy separation. Note that the states in ^{11}Be were not used in the determination of the WBT interaction [22].

The spectrum obtained using the WBT interaction also confirms the results obtained in Ref. [1]. Therein, the authors note that it is possible for a shell model to reproduce the parity inversion in ^{11}Be, but also ascribe the inversion to the coupling of the valence neutron to the 2^+ state in the ^{10}Be core. This supports the inclusion of that state as part of the target spectrum used in our coupled-channel calculations of ^{11}Be and ^{11}N. Though less is known about the resonance states in ^{11}N, while being indicative of some mirror symmetry with ^{11}Be, the results of the shell-model calculations show tentative agreement with some of the known spin-parities of those resonances in the spectrum.

The parity inversion (of the ground and first excited states) observed in both ^{11}Be and ^{11}N has also been the subject of other shell model studies [1,2]; the latter of which noted that the effects of quadrupole core excitation and pairing are important to find the parity inversion in ^{11}Be. The former however performed essentially a traditional microscopic calculation but over many shells, and with their optimised interaction for the chosen model space, found the desired inversion.

4 Conclusions

The low-energy spectra of four mass-11 nuclei (^{11}Be, ^{11}B, ^{11}C, and ^{11}N) have been evaluated, initially, by using the multi-channel algebraic scattering approach treating the nuclei as the coupling of a nucleon with states of a mass-10 nuclear core. Coupled-channel Hamiltonians were formed assuming a collective model of the nuclear cores and of their interactions with the extra-core nucleon. A vibration model was used to specify the target states of ^{10}Be and ^{10}C as their low excitation spectra have the essential sequence of states for that description. Mirror symmetries were assumed and the Coulomb potentials for interactions with protons were taken from three-parameter Fermi descriptions of the nuclear charge distributions. In contrast, with ^{10}B as the core, a rotation model was used to specify the interaction potential matrices input to MCAS calculations since the low-excitation spectra of ^{10}B does not show any semblance of a vibration model structure. However the spectra of ^{11}B and ^{11}C, described by the rotational model with the odd–odd ^{10}B core, set the second excited state, the $\frac{5}{2}^-$ level, too close to the first excited state, $\frac{1}{2}^-$, making them to appear almost degenerate. This characteristic is not observed in the measured spectra of both mirror nuclei.

The vibration couplings to the collective motion of the even-even core states were important especially in the description of the states in ^{11}Be and ^{11}N. Those nuclei have positive-parity ground states, opposite to those of the ground states of the neighbouring mass-11 nuclei, with ^{11}Be having a single-neutron halo ground state. From phenomenological descriptions of that nucleus, significant coupling to the 2^+ state in ^{10}Be is required in order to obtain the $\frac{1}{2}^+$ ground state. Such coupling is naturally included in the coupled-channel approach used.

The low-energy spectra of both the mass-10 and mass-11 nuclei considered were also described using complete basis space shell models. Specifically no-core $(0+2)\hbar\omega$ and $(0+2+4)\hbar\omega$ space shell-model studies were made. The complete $(0+2+4)\hbar\omega$ space was used to define the spectra of the mass-10 systems with the Zheng G-matrix interaction. For the mass-11 systems, the fitted interactions (MK3W and WBT) were used in the complete $(0+2)\hbar\omega$ model space. The low-energy spectrum for ^{11}Be obtained from the shell model calculation using the WBT interaction gave the correct spin-parity of the ground state, but not the energy of the first excited state. However, it is important to note that in that model significant occupation of a single neutron in the $1s_{1/2}$ orbit is found, consistent with the single-neutron halo description of the ground state of ^{11}Be. Also, the spectroscopic amplitudes for the coupling of a neutron to states in ^{10}Be as obtained by the shell model support the significant coupling of the neutron to both the ground and 2^+ states in ^{10}Be as required by MCAS.

The results of our calculations of the mirror systems, ^{11}C and ^{11}B treated as the clusters $n+^{10}$C and $p+^{10}$Be respectively, gave excellent agreement with the known spectra of these nuclei to over 10 MeV excitation and with the known nucleon separation energy. For ^{11}N and ^{11}Be treated as the clusters $p+^{10}$C and $n+^{10}$Be, however, the calculated results are not perfectly matched to the known spectra. But the sequencing and, notably, nucleon emission thresholds, are given correctly. Unlike ^{11}C and ^{11}B, ^{11}Be is weakly bound, while ^{11}N is unbound. These results, involving clusters coupling to form states in the weakly bound ^{11}Be and unbound ^{11}N, may indicate inadequacy in the simple collective model used to specify the coupled-channels Hamiltonian. The nuclei, ^{11}C and ^{11}B, considered as the clusters $p+^{10}$B and $n+^{10}$B respectively, were found to give reasonable spectra again to over 10 MeV excitation, notwithstanding a breaking of the $0s_{1/2}$ shell in the specification of the OPP of the excited 0_1^+ state of the core which was found necessary to have the correct location of the $\frac{1}{2}^+$ state. That requirement is, in part, supported by the results of the shell-model calculation for ^{10}B.

Aside from there being insufficient data to provide constraints on the details of the structure model for ^{10}C, with even the spin-parities of two of those four states selected in the coupled-channel calculations being not listed in the data sheets, the assumed vibration model of structure for the core nuclei may be too simplistic. Improved structure models for ^{10}Be and ^{10}C need to be used before a higher quality agreement between known spectra with these cluster model results may be achieved.

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