Eur. Phys. J. A (2016) **52**: 117
DOI 10.1140/epja/i2016-16117-y

THE EUROPEAN PHYSICAL JOURNAL A

Letter

Estimation of the shear viscosity from 3FD simulations of Au + Au collisions at $\sqrt{s_{NN}} = 3.3$–39 GeV

Yu.B. Ivanov[1,2,a] and A.A. Soldatov[2,b]

[1] National Research Centre "Kurchatov Institute", 123182 Moscow, Russia
[2] National Research Nuclear University "MEPhI" (Moscow Engineering Physics Institute), 115409 Moscow, Russia

Received: 15 January 2016 / Revised: 4 April 2016
Published online: 2 May 2016

Communicated by G. Torrieri

Abstract. An effective shear viscosity in central Au+Au collisions is estimated in the range of incident energies 3.3 GeV $\leq \sqrt{s_{NN}} \leq$ 39 GeV. The simulations are performed within a three-fluid model employing three different equations of state with and without the deconfinement transition. In order to estimate this effective viscosity, we consider the entropy produced in the 3FD simulations as if it is generated within the conventional one-fluid viscous hydrodynamics. It is found that the effective viscosity within the different considered scenarios is very similar at the expansion stage of the collision: as a function of temperature (T) the viscosity-to-entropy ratio behaves as $\eta/s \sim 1/T^4$; as a function of the net-baryon density (n_B), $\eta/s \sim 1/s$, *i.e.* it is mainly determined by the density dependence of the entropy density. The above dependences take place along the dynamical trajectories of Au+Au collisions. At the final stages of the expansion the η/s values are ranged from ~ 0.05 at the highest considered energies to ~ 0.5 at the lowest ones.

Dissipation in strongly interacting matter is an important property of the produced matter and is crucial for understanding the dynamics of heavy-ion collisions. Observables that are the most sensitive to the dissipation at the expansion stage of the reaction are the elliptic flow and other anisotropic flow coefficients. This dissipation deduced from the analysis of experimental data at the Large Hadron Collider (LHC) at CERN and at top energies of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) amounts to $\eta/s \approx 0.1$–0.2 in terms of the viscosity-to-entropy ratio [1]. The analysis of the STAR data in the RHIC Beam Energy Scan (BES) range $\sqrt{s_{NN}} = 7.7$–200 GeV [2], recently performed within a hybrid model [3], indicated that the η/s ratio remains approximately in the same range even at lower BES-RHIC energies. This is definitely in contrast to common expectations that at the BES-RHIC energies the viscosity of the matter should rapidly rise because the system spends most of its time in the hadronic phase [4].

In our recent paper [5] we found that the model of the three-fluid dynamics (3FD) [6] equally well describes the STAR data [2] on the momentum-integrated elliptic flow of charged particles at energies from $\sqrt{s_{NN}} = 7.7$ to 39 GeV within very different scenarios characterized by very different equations of state (EoSs) —from a purely hadronic EoS [7] to the EoSs involving deconfinement transition [8], *i.e.* a first-order phase transition and a smooth crossover one. We assumed that the main reason of this good description is that the dissipation in the 3FD dynamics with different EoSs is very similar. However, we did not present a proof of this assumption because the 3FD model does not include viscosity in its formulation. The dissipation in the 3FD model is present through a friction interaction between participated fluids rather than a viscosity. It is highly difficult to quantitatively express the 3FD dissipation in terms of the effective viscosity, because this dissipation depends on the dynamics of the collisions rather then only on the parameters of the friction. In the present paper, in order to estimate this dissipation in terms of an effective viscosity, we consider the entropy produced in the 3FD simulations as if it is generated within the conventional one-fluid viscous hydrodynamics.

A three-fluid approximation [6] is a minimal way to simulate a finite stopping power of colliding nuclei at high incident energies. Within this approximation a generally nonequilibrium distribution of baryon-rich matter is modeled by counter-streaming baryon-rich fluids initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the midrapidity region, are associated with a

[a] e-mail: Y.Ivanov@gsi.de
[b] e-mail: saa@ru.net

separate net-baryon-free fluid which is called a "fireball" fluid (f-fluid). A certain formation time τ_f is allowed for the f-fluid, during which the matter of the fluid propagates without interactions. The formation time is associated with a finite time of string formation. The physical input of the present 3FD calculations is described in detail in ref. [9].

The proper (*i.e.* in a local rest frame) entropy density of a separate fluid (α) can be calculated by means of the thermodynamic relation

$$s_\alpha = \frac{1}{T_\alpha}(\varepsilon_\alpha + P_\alpha - n_\alpha \mu_\alpha), \tag{1}$$

where, T_α, ε_α, P_α, n_α and μ_α are the temperature, the energy density, the pressure, the baryon density and the baryon chemical potential of the α-fluid. All these quantities are known from the solution of the 3FD equations. The total entropy (S) is then calculated by integration of the sum of these s-densities over the volume of the system

$$S = \int \mathrm{d}V \sum_\alpha u^0_\alpha s_\alpha, \tag{2}$$

where u^0_α is the 0-component of the α-fluid 4-velocity, which is introduced to transform the proper s-density into a common frame of the calculation.

The main idea of estimating an effective viscosity in a nuclear collision consists in associating the entropy production within the 3FD simulation with the effect of the viscous dissipation within the standard viscous hydrodynamics.

In fact, the 3FD dissipation is directly related neither to the shear viscosity nor to other transport coefficients, *i.e.* the bulk viscosity (ζ) and thermal conductivity (κ). The dissipation due to these transport coefficients takes place only when gradients of the collective velocity, temperature and chemical potential exist [10,11]. The 3FD dissipation can, in principal, occur even without any gradients, *e.g.* in two homogeneous counter-streaming media. Though the real evolution of the nuclear collision gives rise to such gradients. Thus, we can express the 3FD dissipation in familiar terms by associating it with the shear viscosity. The shear viscosity is chosen among other transport coefficients only because the dissipation in heavy-ion collisions is traditionally discussed in terms of this quantity.

Under the assumption that only the shear viscosity is nonzero among the transport coefficient, the standard viscous fluid dynamics results in the following equation for the entropy production [10,11]:

$$\partial_\mu s^\mu = \frac{1}{T}\pi_{\mu\nu}\partial^\mu u^\nu, \tag{3}$$

where s^μ, u^μ and T are the entropy four-current, fluid four-velocity and temperature, respectively. The stress tensor, $\pi^{\mu\nu}$, reads

$$\pi^{\mu\nu} = \eta\left(\partial^\mu u^\nu + \partial^\nu u^\mu - u^\mu u_\lambda \partial^\lambda u^\nu - u^\nu u_\lambda \partial^\lambda u^\mu\right) - \frac{2}{3}\eta\left(g^{\mu\nu} - u^\mu u^\nu\right)\partial_\lambda u^\lambda \tag{4}$$

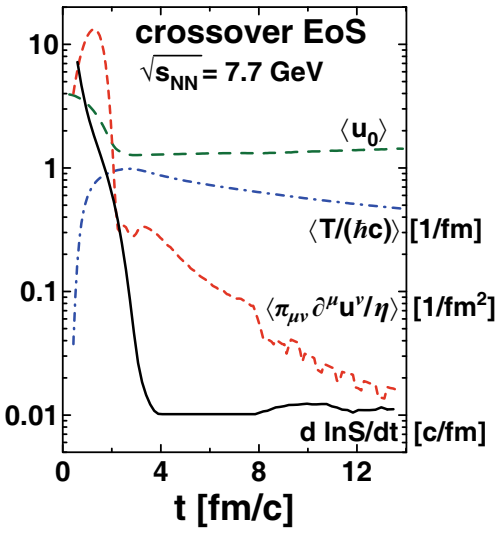

Fig. 1. Time evolution of different factors in eq. (7) at $\sqrt{s_{NN}} = 7.7$ GeV within the crossover scenario.

with η being the shear viscosity. We have to put all other coefficients, *i.e.* ζ and κ, to be zero because we can determine only a single quantity from a single equation.

If the thermal conductivity is zero, the heat flow also vanishes. Then we have no other choice but to associate the hydrodynamic velocity u^μ with the baryon flow [10] in the 3FD model

$$n_B u^\mu = n_p u^\mu_p + n_t u^\mu_t, \tag{5}$$

where n_p, n_t, u^μ_p and u^μ_t are the net-baryon densities and 4-velocities of the p- and t-fluids within the 3FD model, respectively, and n_B and u^μ those quantities of the unified baryon-rich fluid. The mean temperature, T, that is also required by eq. (3), is defined proceeding from common sense, *i.e.* it is defined as a local energy-density-weighted temperature

$$T = \sum_\alpha T_\alpha \varepsilon_\alpha \Big/ \sum_\alpha \varepsilon_\alpha. \tag{6}$$

Integrating eq. (3) over volume, V, we arrive at

$$\frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{V}{S}\left\langle \frac{1}{T}\pi_{\mu\nu}\partial^\mu u^\nu \right\rangle \approx \frac{\langle\eta\rangle}{\langle s\rangle}\frac{1}{\langle T\rangle\langle u^0\rangle}\left\langle \frac{1}{\eta}\pi_{\mu\nu}\partial^\mu u^\nu \right\rangle, \tag{7}$$

where $\langle\ldots\rangle$ denotes averaging over the volume. Here we also took into account that $s^\mu = s u^\mu$, where s is the proper entropy density, and hence $S = V\langle s u^0\rangle \approx V\langle s\rangle\langle u^0\rangle$. From this equation, together with definitions (5) and (6), we easily obtain the estimation of the η/s ratio. In order to facilitate the numerical evaluation of terms with time derivatives, we also used the approximation $\langle \pi_{\mu\nu}\partial^\mu u^\nu\rangle \approx \langle\pi_{\mu\nu}\rangle\langle\partial^\mu u^\nu\rangle$.

Figure 1 illustrates the relative importance of different factors in eq. (7). At the initial, highly nonequilibrium stage, when the concept of the viscosity is hardly applicable, all the factors reveal fast changes in time. At the late expansion stage ($t > 4\,\mathrm{fm}/c$) only the gradient term $\langle\pi_{\mu\nu}\partial^\mu u^\nu\rangle$ manifests fast changes, the entropy-production

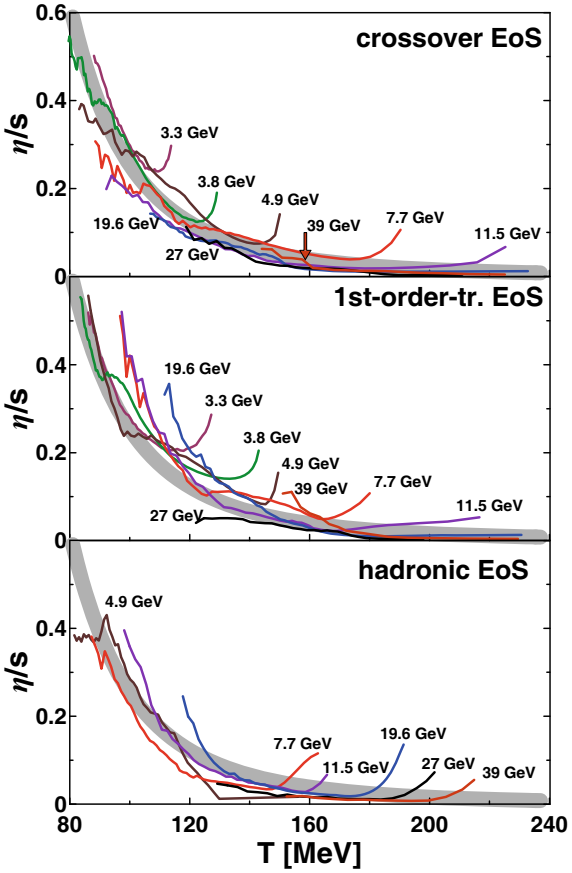

Fig. 2. The η/s ratio as a function of temperature along the trajectories of central Au+Au collisions at various collision energies $\sqrt{s_{NN}}$ within different scenarios. The gray band in all the panels is the function $(T_0/T)^4$, where $T_0 = 71\,\mathrm{MeV}$.

factor defines the basic scale of the η/s ratio, while the two remaining factors, $\langle T\rangle$ and $\langle u^0\rangle$, are responsible for only relatively insignificant corrections.

The 3FD simulations of central Au+Au collisions at energies $3.3\,\mathrm{GeV} \leq \sqrt{s_{NN}} \leq 39\,\mathrm{GeV}$ were performed without freeze-out. The freeze-out in the 3FD model removes the frozen out matter from the hydrodynamical evolution [12,13]. Therefore, in order to keep all the matter in the consideration the freeze-out was turned off.

At the initial stage of the reaction, all three fluids coexist in the same space-time region, thus describing a certain *nonequilibrium* state of the matter. This short initial stage is followed by a longer stage at which the p- and t-fluids are either spatially separated or unified, while the f-fluid still overlaps with the baryon-rich (p- and t-) fluids to a lesser (at high energies) or grater (at lower energies) extent. Therefore, the friction between the f-fluid and the baryon-rich fluids still causes the dissipation and hence the entropy growth.

In fig. 2 the results on the η/s ratio are presented as a function of the mean temperature $\langle T\rangle$ (cf. eqs. (5)–(7)) along the dynamical trajectories of central Au+Au collisions at various collision energies $\sqrt{s_{NN}}$ within different scenarios based on a purely hadronic EoS [7] and those involving the deconfinement transition [8], *i.e.* a first-order phase transition and a smooth crossover one. We plot the η/s ratio as a function of the temperature rather than time because the temperature is a natural argument of the η/s quantity. The mean temperature is also a function of time. The viscosity is meaningful when nonequilibrium is weak. Therefore, it should be analyzed at the expansion stage of the collision following the fast highly nonequilibrium stage. In terms of the η/s ratio of fig. 2, the expansion stage takes place at lower temperatures up to the minimum of the η/s ratio. The η/s curves are continued to higher temperatures after the minimum only for the sake of convenience of labeling them.

The results that manifest fluctuations exceeding the scale of the plot are omitted. These fluctuations are a consequence of the numerical calculation of the derivatives that causes a loss of accuracy. In view of this low accuracy and a very approximate nature of eq. (7) itself, the present results on the η/s ratio should be considered as an order-of-magnitude estimation. For the sake of the graphic representation, we apply a running-average procedure to the results of the direct calculation in such a way that the η/s ratio is averaged over each sequential five time steps. Though these running-average results are not completely smooth, they exhibit much weaker fluctuations.

At high temperatures $T \gtrsim 160\,\mathrm{GeV}$ in collisions with $\sqrt{s_{NN}} > 10\,\mathrm{GeV}$, the η/s ratio turns out to be noticeably smaller that the conjectured lowest bound for this quantity $1/(4\pi)$ [14]. Even in view of the above-discussed roughness of the present estimate, that small values η/s should be attributed to the 3FD model itself. This is certainly a theoretical shortcoming of the model. At the final stages of the expansion[1] the η/s values are ranged from ~ 0.05 at the highest considered energies to ~ 0.5 at the lowest ones.

We further focus on the qualitative properties of the deduced η/s ratio. As seen from fig. 2, the temperature dependence of the η/s ratio at the expansion stages of collisions at various collision energies is very similar within different scenarios. This dependence is approximately described by $1/T^4$ low, *i.e.* this ratio decreases with the temperature rise, as is commonly expected. It is important to emphasize that this is the T-dependence along the dynamical trajectories of collisions, along which the mean net-baryon density, n_B, also changes. The density dependence of the η/s ratio is also very similar within different scenarios, though it does not follow any universal low in terms of n_B. In the case of another representation, *i.e.* in terms of the kinematic viscosity η/n_B, it is more spectacular, as is seen from fig. 3. The n_B-dependence of the kinematic viscosity along dynamical trajectories of collisions approximately follows the law of $1/n_B$. Of course, different numeric factors are required for different collision energies: from $\sim 2n_0$ at low $\sqrt{s_{NN}}$ to $\sim 0.5n_0$ at the highest considered $\sqrt{s_{NN}}$, where $n_0 = 0.15\,\mathrm{fm}^{-3}$ is the normal nuclear density. Thus, we get $\eta/s \sim 1/s$ in terms of the η/s ratio. This is in agreement with the result of

[1] We avoid the term of freeze-out because the freeze-out within the 3FD model is an extended in time process which continues over the whole expansion stage [12,13].

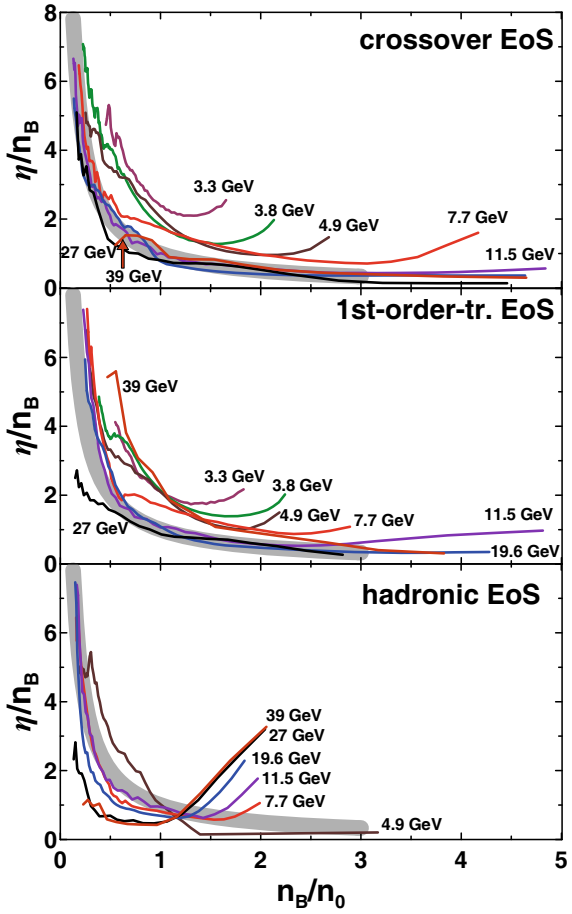

Fig. 3. The kinematic viscosity, η/n_B, as a function of the net-baryon density, n_B, along the trajectories of central Au+Au collisions at various collision energies $\sqrt{s_{NN}}$ within different scenarios. The gray band in all the panels is the function n_0/n_B, where $n_0 = 0.15\,\mathrm{fm}^{-3}$ is the normal nuclear density.

refs. [15,16], where it was found that the reduction of the η/s ratio with the n_B rise happens mostly because of s increase.

Apparently, the similarity of the deduced effective viscosity within different scenarios is the main reason why all considered scenarios equally well reproduce the measured integrated elliptic flow of charged particles [5]. In this respect, the estimated η/s can be considered as that deduced from experimental data [2] by means of the 3FD analysis. Of course, this η/s is model dependent and is not unique.

In conclusion, we estimated the effective viscosity in central Au+Au collisions at collision energies from $\sqrt{s_{NN}} = 3.3\,\mathrm{GeV}$ to $39\,\mathrm{GeV}$ within different scenarios in order to quantify the dissipation in the 3FD model. To estimate this dissipation in terms of the effective shear viscosity (more precisely, the η/s ratio), we considered the entropy produced in the 3FD model as if it was generated within the conventional one-fluid viscous hydrodynamics.

It is found that the effective viscosity within the different considered scenarios (with and without deconfinement transition) is very similar at the expansion stage of the collision: as a function of temperature (T), $\eta/s \sim 1/T^4$ and quantitatively it is very similar within different scenarios; as a function of the net-baryon density (n_B), $\eta/s \sim 1/s$, *i.e.* it is mainly determined by the density dependence of the entropy density. The above dependences take place along the dynamical trajectories of Au+Au collisions. In the hadronic scenario, the reported small values of the η/s ratio at high collision energies, $\sqrt{s_{NN}} > 10\,\mathrm{GeV}$, were achieved due to an artificial enhancement [6,9] of the friction forces estimated on the basis of experimental proton-proton cross sections [17]. This enhancement was required to reproduce the observed baryon stopping at high energies.

At the final stages of the expansion the η/s values are ranged from ~ 0.05 at the highest considered energies to ~ 0.5 at the lowest ones. This result does not contradict the finding of ref. [3], where the average η/s over the expansion stage values were reported, because in our case the η/s ratio turns out to be strongly temperature dependent.

Fruitful discussions with D.N. Voskresensky are gratefully acknowledged. The calculations were performed at the computer cluster of GSI (Darmstadt). This work was partially supported by grant NS-932.2014.2.

References

1. U. Heinz, R. Snellings, Annu. Rev. Nucl. Part. Sci. **63**, 123 (2013).
2. STAR Collaboration (L. Adamczyk *et al.*), Phys. Rev. C **86**, 054908 (2012).
3. I.A. Karpenko, P. Huovinen, H. Petersen, M. Bleicher, Phys. Rev. C **91**, 064901 (2015).
4. G. Kestin, U.W. Heinz, Eur. Phys. J. C **61**, 545 (2009).
5. Yu.B. Ivanov, A.A. Soldatov, Phys. Rev. C **91**, 024914 (2015).
6. Yu.B. Ivanov, V.N. Russkikh, V.D. Toneev, Phys. Rev. C **73**, 044904 (2006).
7. V.M. Galitsky, I.N. Mishustin, Sov. J. Nucl. Phys. **29**, 181 (1979).
8. A.S. Khvorostukhin, V.V. Skokov, K. Redlich, V.D. Toneev, Eur. Phys. J. C **48**, 531 (2006).
9. Yu.B. Ivanov, Phys. Rev. C **87**, 064904 (2013).
10. L.D. Landau, E.M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1979).
11. D.H. Rischke, Lect. Notes Phys. **516**, 21 (1999).
12. V.N. Russkikh, Yu.B. Ivanov, Phys. Rev. C **76**, 054907 (2007).
13. Yu.B. Ivanov, V.N. Russkikh, Phys. At. Nucl. **72**, 1238 (2009).
14. P. Kovtun, D.T. Son, A.O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
15. A.S. Khvorostukhin, V.D. Toneev, D.N. Voskresensky, Nucl. Phys. A **845**, 106 (2010).
16. G.S. Denicol, C. Gale, S. Jeon, J. Noronha, Phys. Rev. C **88**, 064901 (2013).
17. L.M. Satarov, Yad. Fiz. **52**, 412 (1990) (Sov. J. Nucl. Phys. **52**, 264 (1990)).