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OBITUARY

Vladimir Mikhailovich Millionshchikov

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On March 19, 2009, Vladimir Mikhailovich Millionshchikov, a distinguished specialist in the field of ordinary differential equations, Doctor of Sciences in physics and mathematics, Professor of the Department of Differential Equations of the Faculty of Mechanics and Mathematics of Moscow State University, died at the age of 70 after a serious, long illness.

Millionshchikov was born in Moscow on October 22, 1939. His father, Mikhail Dmitrievich Millionshchikov, a distinguished Soviet scientist, was a specialist in the field of mechanics and applied physics, Academician, Vice-President of the Academy of Sciences of USSR. His mother, Lyudmila Mikhailovna Millionshchikova (Mukhina), is the author of several books of poetry. As early as in secondary school, Vladimir Millionshchikov became interested in mathematics and studied it on his own by reading popular books; three times he won second prizes at Moscow Mathematical Olympiads.

After graduating from secondary school with excellence in 1956, Millionshchikov entered the Faculty of Mechanics and Mathematics of Moscow State University at the same year, graduated from it with excellence in 1961, and began postgraduate studies under V.V. Nemytskii. During his student years, Millionshchikov began to participate in the scientific seminar held by Nemytskii, who suggested him two topics for investigation, differential equations in locally convex spaces and topological dynamics. In Millionshchikov's papers published on the first topic during his student years, he generalized a number of theorems of the theory of ordinary differential equations to the case of linear topological or locally convex spaces. In the second topic, he was the author of one of first publications on so-called nonautonomous topological dynamics. In less than two years, developing the approach used in that paper, Millionshchikov devised a method of metric (probabilistic) investigation of linear nonautonomous differential systems and constructed their metric theory.

Millionshchikov defended his Ph.D. thesis "On the Spectral Theory of Nonautonomous Linear Systems of Differential Equations" in 1966 and the doctoral thesis "On the Theory of Linear Systems of Ordinary Differential Equations" in 1968.

Since the mid-1960s, Millionshchikov completely switched to the theory of Lyapunov exponents. The problem on the attainability of central exponents and the related problem on necessary and sufficient stability conditions for Lyapunov exponents under small perturbations of the system coefficient matrix were the main unsolved problems of the theory of exponents at the time. In one of his first papers on the theory of exponents, he constructed an example of a system with unstable Lyapunov exponents whose instability has a nature other than that in the Perron classical example or Vinograd examples of unstable exponents and substantially clarified the mechanism of their instability. In 1967, this paper started the most fruitful period of Millionshchikov's activity: in three years (1967–1969), he published 19 scientific papers, which enriched the theory of exponents with a number of distinguished results and determined the research directions in this field for several decades.

Let us only list some most important results obtained by him in these years. Obviously, the proof of the attainability of central exponents is the first of them. To this end, Millionshchikov created an essentially new constructive way for the construction of linear and nonlinear systems with given properties, which was later referred to as the Millionshchikov rotation method. (This method goes back to the above-mentioned publication in 1967.) The simplicity of this method, in combination with its universality and flexibility, provides its wide scope and makes it an irreplaceable tool in the field of the theory of exponents. By using it, Millionshchikov himself and other mathematicians solved numerous problems of the theory of Lyapunov exponents, which had remained unsolved and even seemed to be unsolvable for a long time. For example, Millionshchikov obtained a necessary and sufficient stability condition for Lyapunov exponents under small perturbations of the coefficient matrix; this problem goes back to O. Perron (1930). By continuing this result, he showed that systems with central separation form an open (which was known earlier) and everywhere dense set in the space of linear differential systems with the topology of uniform convergence of the coefficients on the half-line and form the interior of the set of systems with stable Lyapunov exponents in this space. He also derived an important description of systems with central separation via the character of the response of their solutions to small perturbations of the coefficient matrices.

One more series of Millionshchikov's papers deals with the above-mentioned construction of a metric theory of linear differential systems on the basis of the metric theory of dynamical systems (ergodic theory). The main result of these papers is as follows: for almost every (in the sense of any normalized invariant measure) point of a smooth dynamical system, the system of variational equations along the trajectory passing through this point is Lyapunov proper, and its upper Lyapunov exponent coincides with the upper central exponent and hence is upper semistable. Developing this result, Millionshchikov introduced the class of absolutely regular systems, which take an intermediate position between almost reducible and proper systems, and showed that almost every (in the sense of an arbitrary invariant measure on the dynamical system of shifts) linear system is absolutely regular. Later, he detected an interesting phenomenon of stochastic stability of Lyapunov exponents of absolutely regular systems: they experience small changes under a small perturbation of the coefficients by nonzero independent white noises. Without describing Millionshchikov's other deep results on the metric theory of linear differential systems, we note one of his results, which is used in the theory of almost periodic systems: suppose that the dynamical system of shifts generated by the matrix of a linear system is strictly ergodic; then the probabilistic spectrum of the linear system is stable if and only if this system is almost reducible.

Another series of Millionshchikov's papers is related to the investigation of systems with almost periodic and quasi-periodic coefficients. For example, he gave the positive solution of the Erugin problem (1946) on the existence of Lyapunov improper systems with almost periodic coefficients. Later, he also proved the existence of improper systems with quasiperiodic coefficients. One of the main results obtained by Millionshchikov in the theory of almost periodic systems is stated in the following theorem, which is a consequence of his above-mentioned result in the metric theory of linear systems: an almost periodic system is almost reducible if and only if its exponents are stable. Without considering other fine results on almost periodic systems, we note the following result on almost reducible systems. He constructed a system with unstable singular exponents, which leads to the negative answer to the long-standing (early 1950s) Bylov problem on the symmetry of the relation of almost reducibility.

In the mid-1970s, Millionshchikov obtained a deep and important result on the derivation of an entropy formula for a dynamical system via Lyapunov exponents of the variational system: the en-

tropy of a C^2-smooth dynamical system on a compact smooth manifold which has a normalized invariant measure induced by some Riemannian metric is equal to the phase mean of the sum of positive Lyapunov exponents of the system of variational equations. In addition, the entropy treated as a function on the above-mentioned set of dynamical systems equipped with the C^1-topology is upper semicontinuous.

Millionshchikov's whole further creative activity was related to the investigation of Baire typical properties of Lyapunov exponents and other asymptotic characteristics of differential systems. The systematic investigation of Baire typical properties was started in his two large series of papers ("Baire Classes of Functions and Lyapunov Exponents." I–XII and "On Typical Properties of Conditional Exponential Stability." I–XX). In these papers, he used the characteristic exponents of families of morphisms of vector bundles to suggest a common approach to the investigation of Lyapunov exponents, which covers all cases of their appearance and application in the theory of differential equations, diffeomorphisms of smooth manifolds, and equations in finite differences.

The main results of these series of papers are as follows. The Lyapunov exponents (in the above-mentioned wide sense) are functions of the second Baire class on their domain, and the set of semicontinuity points of all exponents is Baire typical. He also proved theorems (more precisely, series of theorems corresponding to different situations) on the typicalness of exponential boundedness, of the coincidence of Lyapunov exponents with central exponents, and of the conditional exponential stability in the first approximation. A detailed development of a new system of notions that permits one to investigate objects of various nature in the theory of exponents from a common viewpoint and to analyze constructions related to the use of that theory is an essential integral part of the papers of these series.

By using the methods suggested and developed in these papers, later Millionshchikov extracted a large number of various asymptotic properties and characteristics of linear systems of diffeomorphisms reflecting various aspects of stability, whose mere list would be quite long, and proved their typicalness (in the Baire sense or in the sense of measure) in various situations.

We should note one more side of Millionshchikov's talent, his unique ability for the study of languages, which is extraordinary even for polyglots.

Millionshchikov's whole life was closely related to Moscow State University, where he began to teach when he was a postgraduate student. Since 1964, he was working at the Department of Differential Equations (since 1970, as a Professor). For almost forty years, Vladimir Mikhailovich Millionshchikov read the obligatory course of differential equations at the Faculty of Mechanics and Mathematics and a number of special courses on various topics in the theory of Lyapunov exponents. He always had a lot of students and treated them with benevolence and care. More than 25 philosophy doctor theses were defended under his supervision, and several doctoral theses were prepared. For more than 30 years, he, together with V.A. Kondrat'ev and N.Kh. Rozov, headed the scientific seminar on the qualitative theory of differential equations at Moscow State University. Since 1978, the abstracts of reports on this seminar have been regularly published in the journal *Differentsial'nye Uravneniya* (*Differential Equations*).

Since 1973, Millionshchikov combined his university position with work in the Division of Mathematical Physics at the Steklov Mathematical Institute of the Russian Academy of Sciences. For a long time, he was member of the commission of experts and advisory committee in mathematics and mechanics at the Higher Certifying Commission of the USSR. For about 20 years, Vladimir Mikhailovich Millionshchikov was member of the Editorial Board of the Journal *Differentsial'nye Uravneniya* (*Differential Equations*) and for ten years, Vice Editor-in-Chief of the Journal *Matematicheskie Zametki* (*Mathematical Notes*), and in the recent time, he was member of the Editorial Board of the journal *Izvestiya RAN. Seriya Matematicheskaya* (*Proceedings of Russian Academy of Sciences. Mathematical Series*).

Vladimir Mikhailovich Millionshchikov's wide knowledge, erudition, intelligence, diligence, modesty, benevolence to all people, and self-exactingness won him the respect of everyone who knew him. The fond memory of Vladimir Mikhailovich Millionshchikov, a distinguished mathematician, an outstanding teacher, and a good person will be always kept in the hearts of all who were lucky to work and communicate with him.

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I.T. Kiguradze, V.A. Kondrat'ev, V.V. Kozlov, L.D. Kudryavtsev,
A.A. Martynyuk, E.F. Mishchenko, V.A. Pliss, N.Kh. Rozov,
A.M. Samoilenko, I.N. Sergeev, T.K. Shemyakina, and V.S. Vladimirov

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357. On Power-Law Auxiliary Exponents, *Differ. Uravn.*, 2000, vol. 36, no. 11, p. 1571.

358. On Exponentially Invariant Systems, *Differ. Uravn.*, 2000, vol. 36, no. 11, p. 1572.

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360. On Lyapunov–Perron Systems, *Differ. Uravn.*, 2000, vol. 36, no. 11, p. 1573.

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366. Upper Lyapunov Exponent of a Linear System Treated as a Function of Complex Parameters, *Differ. Uravn.*, 2001, vol. 37, no. 6, p. 848.

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378. On the Izobov Upper Exponential Exponent, *Differ. Uravn.*, 2001, vol. 37, no. 11, p. 1579.

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382. On Some Relations Between Extraordinary Lyapunov Exponents, *Differ. Uravn.*, 2005, vol. 41, no. 11, p. 1578.

[1] The beginning of the list was published in the journal *Differentsial'nye Uravneniya* (*Differential Equations*), 1999, vol. 35, no. 10, pp. 1299–1312.