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ERRATA

Erratum to: Observer-Aided Output Feedback Synthesis as an Optimization Problem

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The formula for the gradient $\nabla_L f(K, L, \alpha)$ in Lemma 3 is given inaccurately; it should read

$$\frac{1}{2}\nabla_L f(K, L, \alpha) = \rho_L L + M_2^{\mathrm{T}} Y P N_2^{\mathrm{T}} - \frac{1}{\alpha}\begin{pmatrix} 0 & I \end{pmatrix} Y \begin{pmatrix} D \\ D - LD_1 \end{pmatrix} D_1^{\mathrm{T}}. \tag{1}$$

Below find the full derivation of this result.

For differentiation of the function $f(K, L, \alpha)$ with respect to L under a constraint in the form of the Lyapunov equation

$$\Big(A_{K,L} + \frac{\alpha}{2} I\Big) P + P\Big(A_{K,L} + \frac{\alpha}{2} I\Big)^{\mathrm{T}} + \frac{1}{\alpha}\begin{pmatrix} D \\ D - LD_1 \end{pmatrix}\begin{pmatrix} D \\ D - LD_1 \end{pmatrix}^{\mathrm{T}} = 0, \tag{2}$$

we give an increment of ΔL to the quantity L and denote the corresponding increment in P by ΔP,

$$\begin{aligned}&\Big(\mathcal{A} + M_1 K N_1 + M_2(L + \Delta L) N_2 + \frac{\alpha}{2} I\Big)(P + \Delta P)\\ &\qquad + (P + \Delta P)\Big(\mathcal{A} + M_1 K N_1 + M_2(L + \Delta L) N_2 + \frac{\alpha}{2} I\Big)^{\mathrm{T}}\\ &\qquad\qquad + \frac{1}{\alpha}\begin{pmatrix} D \\ D - (L + \Delta L) D_1 \end{pmatrix}\begin{pmatrix} D \\ D - (L + \Delta L) D_1 \end{pmatrix}^{\mathrm{T}} = 0.\end{aligned}$$

Leaving the notation ΔP for the principal part of the increment, we obtain

$$\begin{aligned}&\Big(A_{K,L} + M_2 \Delta L N_2 + \frac{\alpha}{2} I\Big) P + P\Big(A_{K,L} + M_2 \Delta L N_2 + \frac{\alpha}{2} I\Big)^{\mathrm{T}}\\ &\qquad + \Big(A_{K,L} + \frac{\alpha}{2} I\Big)\Delta P + \Delta P\Big(A_{K,L} + \frac{\alpha}{2} I\Big)^{\mathrm{T}}\\ &\qquad\qquad + \frac{1}{\alpha}\Bigg[\begin{pmatrix} D \\ D - LD_1 \end{pmatrix}\begin{pmatrix} D \\ D - LD_1 \end{pmatrix}^{\mathrm{T}} - \begin{pmatrix} 0 \\ \Delta L D_1 \end{pmatrix}\begin{pmatrix} D \\ D - LD_1 \end{pmatrix}^{\mathrm{T}}\\ &\qquad\qquad\qquad - \begin{pmatrix} D \\ D - LD_1 \end{pmatrix}\begin{pmatrix} 0 \\ \Delta L D_1 \end{pmatrix}^{\mathrm{T}}\Bigg] = 0.\end{aligned}$$

After subtracting Eq. (2) from this equation, we have

$$\begin{aligned}\Big(A_{K,L}+\frac{\alpha}{2}I\Big)\,\Delta P+\Delta P\Big(A_{K,L}+\frac{\alpha}{2}I\Big)^{\mathrm T}+M_2\Delta LN_2P+P(M_2\Delta LN_2)^{\mathrm T}\\ -\frac{1}{\alpha}\left[\begin{pmatrix}0\\ \Delta LD_1\end{pmatrix}\begin{pmatrix}D\\ D-LD_1\end{pmatrix}^{\mathrm T}+\begin{pmatrix}D\\ D-LD_1\end{pmatrix}\begin{pmatrix}0\\ \Delta LD_1\end{pmatrix}^{\mathrm T}\right]=0.\end{aligned}\qquad(3)$$

Let us calculate the increment of the functional $f(K,L,\alpha)$ with respect to L by linearizing the relevant quantities,

$$\Delta_L f(K,L,\alpha)=\operatorname{tr}\mathcal{C}_2\Delta P\mathcal{C}_2^{\mathrm T}+\rho_L\operatorname{tr}L^{\mathrm T}\Delta L+\rho_L\operatorname{tr}(\Delta L)^{\mathrm T}L=\operatorname{tr}\Delta P\mathcal{C}_2^{\mathrm T}\mathcal{C}_2+2\rho_L\operatorname{tr}L^{\mathrm T}\Delta L.$$

Consider the Lyapunov equation

$$\Big(A_{K,L}+\frac{\alpha}{2}I\Big)^{\mathrm T}Y+Y\Big(A_{K,L}+\frac{\alpha}{2}I\Big)+\mathcal{C}_2^{\mathrm T}\mathcal{C}_2=0.\qquad(4)$$

From the dual equations (3) and (4), we have

$$\begin{aligned}\Delta_L f(K,L,\alpha)&=2\operatorname{tr}Y\left[M_2\Delta LN_2P-\frac{1}{\alpha}\begin{pmatrix}0\\ \Delta LD_1\end{pmatrix}\begin{pmatrix}D\\ D-LD_1\end{pmatrix}^{\mathrm T}\right]+2\rho_L\operatorname{tr}L^{\mathrm T}\Delta L\\ &=2\operatorname{tr}\left[N_2PYM_2\Delta L-\frac{1}{\alpha}D_1\begin{pmatrix}D\\ D-LD_1\end{pmatrix}^{\mathrm T}Y\begin{pmatrix}0\\ I\end{pmatrix}\Delta L\right]+2\rho_L\operatorname{tr}L^{\mathrm T}\Delta L\\ &=2\Big\langle\rho_L L+M_2^{\mathrm T}YPN_2^{\mathrm T}-\frac{1}{\alpha}\begin{pmatrix}0 & I\end{pmatrix}Y\begin{pmatrix}D\\ D-LD_1\end{pmatrix}D_1^{\mathrm T},\Delta L\Big\rangle,\end{aligned}$$

whence formula (1) follows.

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