## ERRATA —

## Erratum to: "Regularized trace of the perturbed Laplace-Beltrami operator on two-dimensional manifolds with closed geodesics" [*Mathematical Notes* 93 (3), 397-411 (2013)]

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In [1], a technical error of the author in the description of the asymptotics of the series following formula (3.1) led to the omission of a term, which resulted in its absence in the final formula.

The phrase following formula (3.21): "Consider the second summand on the right-hand side. It follows from [19] that for the inner series, as  $k \to \infty$ ,

$$\sum_{i=0}^{2k} (\lambda_{ki} - k(k+1))^2 \sim (2\pi)^{-2} \left( k \int_{S^*ML} (q^{av})^2 \, dv + \cdots \right),$$

must be replaced by "Consider the second summand on the right-hand side. It follows from [19] and [21] that, for the inner series, as  $k \to \infty$ 

$$\sum_{i=0}^{2k} (\lambda_{ki} - k(k+1))^2 \sim (2\pi)^{-2} k \left( \int_{S^*ML} (q^{av})^2 \, dv + \int_{S^*ML} (\sigma^{av})^2 \, dv \right) + O(1),$$

The reference to the paper [21] is given by the reference [2] at the end of the text of this Erratum, and the function  $\sigma^{av}$  is defined below in the theorem.

In the present text, the revised statement of the main theorem is presented, taking into account the missing term.

**Theorem 1.** Let ML be a manifold given by a functional family of smooth almost Liouville metrics on the sphere and defined by formulas (2.2). If q is an infinitely differentiable complex-valued function on ML, then the eigenvalues of the operator  $-\Delta + q$  satisfy the equality

$$\begin{split} \sum_{k=0}^{\infty} \sum_{i=0}^{2k} & \left( \lambda_{ki} - k(k+1) - \frac{1}{4\pi} \iint_{ML} q(v_2, v_3) \sqrt{\det g} \, dv_2 \, dv_3 \right) \\ &= \frac{1}{16\pi^2} \int_{S^*ML} (q^{av})^2 \, dv + \frac{1}{16\pi^2} \int_{S^*ML} (\sigma^{av})^2 \, dv \\ &\quad - \frac{1}{60\pi} \iint_{ML} (\Delta \gamma(ML) + \gamma^2(ML) - \gamma(ML)) \sqrt{\det g} \, dv_2 \, dv_3 \\ &\quad - \frac{1}{24\pi} \iint_{ML} (-\Delta q(v_2, v_3) + 3q^2(v_2, v_3) - 2q(v_2, v_3)(\gamma(ML) - 1)) \sqrt{\det g} \, dv_2 \, dv_3, \end{split}$$

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where

$$\gamma(ML) = \frac{2(R(v_3) - Q(v_2)) + (v_2 - v_3)(R'(v_3) + Q'(v_2))}{(v_2 - v_3)^3}$$

is the Gaussian curvature of ML and

$$\sqrt{\det g} = \frac{v_2 - v_3}{4\sqrt{-Q(v_2)R(v_3)}}$$

is the root of the determinant of the matrix of the metric tensor,  $S^*ML$  is the total space of the unit sphere bundle in the cotangent space, dv is the canonical form of the volume on  $S^*ML$ , and

$$q^{av} = \frac{1}{2\pi} \int_0^{2\pi} (\exp t\Xi)^*(q) \, dt;$$

here  $\Xi$  is the Hamiltonian vector field on the cotangent bundle  $T^*ML \setminus \{0\}$  defined by the Riemannian structure on ML,

$$\sigma^{av} = \frac{1}{2\pi} \int_0^{2\pi} (\exp t\Xi)^*(\sigma) \, dt,$$

where

$$\sigma = \frac{1}{4} \bigg( \gamma(ML) - 1 + \bigg[ \frac{1}{3} (\gamma(ML))_{\mathsf{v}} u^3 \int_0^r (\gamma(ML))_{\mathsf{v}} J^3 \, ds - (\gamma(ML))_{\mathsf{v}} u^2 J \int_0^r (\gamma(ML))_{\mathsf{v}} u J^2 \, ds \bigg] \bigg),$$

v is the unit vector of the normal to the geodesic  $\tau$ ,  $J(r, \omega)$  is the volume density in geodesic polar coordinates, i.e.,  $dvol(\tau) = J(r, \omega) dr d\omega$ , and u and v are the fundamental solutions of the Jacobi equation along the geodesic  $\tau$  satisfying the conditions

$$\begin{pmatrix} u(0) & v(0) \\ \dot{u}(0) & \dot{v}(0) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

## REFERENCES

- 1. T. V. Zykova, Math. Notes 93 (3), 397-411 (2013).
- 2. S. Zelditch, J. Funct. Anal. 143 (2), 415 (1997).

837