# Erratum to: "Regularized trace of the perturbed Laplace-Beltrami operator on two-dimensional manifolds with closed geodesics" <br> [Mathematical Notes 93 (3), 397-411 (2013)] 

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In [1], a technical error of the author in the description of the asymptotics of the series following formula (3.1) led to the omission of a term, which resulted in its absence in the final formula.

The phrase following formula (3.21): "Consider the second summand on the right-hand side. It follows from [19] that for the inner series, as $k \rightarrow \infty$,

$$
\sum_{i=0}^{2 k}\left(\lambda_{k i}-k(k+1)\right)^{2} \sim(2 \pi)^{-2}\left(k \int_{S^{*} M L}\left(q^{a v}\right)^{2} d v+\cdots\right), "
$$

must be replaced by "Consider the second summand on the right-hand side. It follows from [19] and [21] that, for the inner series, as $k \rightarrow \infty$

$$
\sum_{i=0}^{2 k}\left(\lambda_{k i}-k(k+1)\right)^{2} \sim(2 \pi)^{-2} k\left(\int_{S^{*} M L}\left(q^{a v}\right)^{2} d v+\int_{S^{*} M L}\left(\sigma^{a v}\right)^{2} d v\right)+O(1), "
$$

The reference to the paper [21] is given by the reference [2] at the end of the text of this Erratum, and the function $\sigma^{a v}$ is defined below in the theorem.

In the present text, the revised statement of the main theorem is presented, taking into account the missing term.

Theorem 1. Let ML be a manifold given by a functional family of smooth almost Liouville metrics on the sphere and defined by formulas (2.2). If $q$ is an infinitely differentiable complex-valued function on $M L$, then the eigenvalues of the operator $-\Delta+q$ satisfy the equality

$$
\begin{aligned}
\sum_{k=0}^{\infty} \sum_{i=0}^{2 k} & \left(\lambda_{k i}-k(k+1)-\frac{1}{4 \pi} \iint_{M L} q\left(v_{2}, v_{3}\right) \sqrt{\operatorname{det} g} d v_{2} d v_{3}\right) \\
= & \frac{1}{16 \pi^{2}} \int_{S^{*} M L}\left(q^{a v}\right)^{2} d v+\frac{1}{16 \pi^{2}} \int_{S^{*} M L}\left(\sigma^{a v}\right)^{2} d v \\
& \quad-\frac{1}{60 \pi} \iint_{M L}\left(\Delta \gamma(M L)+\gamma^{2}(M L)-\gamma(M L)\right) \sqrt{\operatorname{det} g} d v_{2} d v_{3} \\
& \quad-\frac{1}{24 \pi} \iint_{M L}\left(-\Delta q\left(v_{2}, v_{3}\right)+3 q^{2}\left(v_{2}, v_{3}\right)-2 q\left(v_{2}, v_{3}\right)(\gamma(M L)-1)\right) \sqrt{\operatorname{det} g} d v_{2} d v_{3}
\end{aligned}
$$

[^0]where
$$
\gamma(M L)=\frac{2\left(R\left(v_{3}\right)-Q\left(v_{2}\right)\right)+\left(v_{2}-v_{3}\right)\left(R^{\prime}\left(v_{3}\right)+Q^{\prime}\left(v_{2}\right)\right)}{\left(v_{2}-v_{3}\right)^{3}}
$$
is the Gaussian curvature of $M L$ and
$$
\sqrt{\operatorname{det} g}=\frac{v_{2}-v_{3}}{4 \sqrt{-Q\left(v_{2}\right) R\left(v_{3}\right)}}
$$
is the root of the determinant of the matrix of the metric tensor, $S^{*} M L$ is the total space of the unit sphere bundle in the cotangent space, $d v$ is the canonical form of the volume on $S^{*} M L$, and
$$
q^{a v}=\frac{1}{2 \pi} \int_{0}^{2 \pi}(\exp t \Xi)^{*}(q) d t
$$
here $\Xi$ is the Hamiltonian vector field on the cotangent bundle $T^{*} M L \backslash\{0\}$ defined by the Riemannian structure on $M L$,
$$
\sigma^{a v}=\frac{1}{2 \pi} \int_{0}^{2 \pi}(\exp t \Xi)^{*}(\sigma) d t,
$$
where
$$
\sigma=\frac{1}{4}\left(\gamma(M L)-1+\left[\frac{1}{3}(\gamma(M L))_{\mathrm{v}} u^{3} \int_{0}^{r}(\gamma(M L))_{\mathrm{v}} J^{3} d s-(\gamma(M L))_{\mathrm{v}} u^{2} J \int_{0}^{r}(\gamma(M L))_{\mathrm{v}} u J^{2} d s\right]\right),
$$
v is the unit vector of the normal to the geodesic $\tau, J(r, \omega)$ is the volume density in geodesic polar coordinates, i.e., $\operatorname{dvol}(\tau)=J(r, \omega) d r d \omega$, and $u$ and $v$ are the fundamental solutions of the Jacobi equation along the geodesic $\tau$ satisfying the conditions
\[

\left($$
\begin{array}{ll}
u(0) & v(0) \\
\dot{u}(0) & \dot{v}(0)
\end{array}
$$\right)=\left($$
\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}
$$\right) .
\]

## REFERENCES

1. T. V. Zykova, Math. Notes 93 (3), 397-411 (2013).
2. S. Zelditch, J. Funct. Anal. 143 (2), 415 (1997).

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