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Erratum on "Boundary Conditions for Scalar Conservation Laws, from a Kinetic Point of View"

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This note corrects a Gronwall argument in the proof of the uniform L^∞ bound on $u_\epsilon(t,x) := \int f_\epsilon(t,x,v)\,dv$ given in ref. 2. The main result of the paper, i.e., Theorem 1, still holds and now even in an appropriate L^∞ setting not requiring the earlier *BV* assumption of ref. 2. In the statement of Theorem 1, "Under some technical assumptions (of Proposition 4), the function u_ϵ converges in $L^\infty(0,T; L^1_{\mathrm{loc}}(\Omega))$ to a function $u \in L^\infty(0,T; BV(\Omega))$..." should be replaced by "Under the assumption $|\{v\in\mathbb{R}; a(v)\cdot\sigma = u\}| = 0$, $\sigma\in S^{N-1}$, $u\in\mathbb{R}$ and in the setting of Theorem 2, the function u_ϵ converges in $L^\infty(0,T; L^1_{\mathrm{loc}}(\Omega))$ to a function $u\in L^\infty((0,T)\times\Omega)$... ." Theorem 2 should be completed by the following.

Finally, under the assumption that f_0 and $\tilde f$ are L^∞ functions with $\|f_0\|_{L^\infty}\leqslant 1$, $\|\tilde f\|_{L^\infty}\leqslant 1$, $f_0(\cdot,v)\,\mathrm{sgn}(v)\geqslant 0$, $\tilde f(\cdot,v)\,\mathrm{sgn}(v)\geqslant 0$, and compact supports in v, u_ϵ is bounded in $L^\infty((0,T)\times\Omega)$ uniformly w.r.t. ϵ.

The following is a proof for $\Omega =]0,+\infty[$. In the proof of Theorem 2, a Banach fixed point argument in $L^\infty(0,T; L^1(\Omega\times\mathbb{R}))$ was performed for the map $\mathscr{T}$ that maps f into F solution to $\partial_t F + a_1(v)\,\partial_x F = \frac{1}{\epsilon}(\chi_{u_f} - F)$ with the same initial and boundary data as in (10). Under the previous assumptions on f_0 and $\tilde f$, there is a positive number M such that $\chi_{-M}(v)\leqslant f_0(x,v)\leqslant \chi_M(v)$ and $\chi_{-M}(v)\leqslant \tilde f(t,v)\leqslant\chi_M(v)$. Prove that if $\|u_f\|_\infty\leqslant M$ then $\|u_{\mathscr{T}(f)}\|_\infty\leqslant M$.

For $x - ta_1(v) > 0$,

$$F(t,x,v) = f_0(x - ta_1(v), v)\, e^{-\frac{t}{\epsilon}} + \int_0^t \frac{1}{\epsilon} e^{\frac{s-t}{\epsilon}} \chi_{u_f(s,\, x+(s-t)\,a_1(v))}(v)\, ds.$$

For $x - ta_1(v) < 0$,

$$F(t,x,v) = \tilde f\left(t - \frac{x}{a_1(v)}, v\right) e^{-\frac{x}{\epsilon a_1(v)}} + \int_{t-\frac{x}{a_1(v)}}^t \frac{1}{\epsilon} e^{\frac{s-t}{\epsilon}} \chi_{u_f(s,\, x+(s-t)\,a_1(v))}(v)\, ds.$$

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And so,

$$\begin{aligned}
\int F(t,x,v)\,dv &\leqslant e^{-\frac{t}{\epsilon}}\int_{a_1(v)<\frac{x}{t}} f_0(x-ta_1(v),v)\,dv \\
&\quad+\int_0^t \frac{1}{\epsilon}\,e^{\frac{s-t}{\epsilon}}\int_{a_1(v)<\frac{x}{t}} \chi_{u_f(s,\,x+(s-t)\,a_1(v))}(v)\,dv\,ds \\
&\quad+\int_{a_1(v)>\frac{x}{t}} \tilde{f}\left(t-\frac{x}{a_1(v)},v\right)e^{-\frac{x}{\epsilon a_1(v)}}\,dv \\
&\quad+\int_{a_1(v)>\frac{x}{t}}\int_{t-\frac{x}{a_1(v)}}^t \frac{1}{\epsilon}\,e^{\frac{s-t}{\epsilon}}\chi_{u_f(s,\,x+(s-t)\,a_1(v))}(v)\,ds\,dv \\
&\leqslant e^{-\frac{t}{\epsilon}}\int_{a_1(v)<\frac{x}{t}} \chi_M(v)\,dv+(1-e^{-\frac{t}{\epsilon}})\int_{a_1(v)<\frac{x}{t}} \chi_M(v)\,dv \\
&\quad+\int_{a_1(v)>\frac{x}{t}} \chi_M(v)\,e^{-\frac{x}{\epsilon a_1(v)}}\,dv \\
&\quad+\int_{a_1(v)>\frac{x}{t}} \chi_M(v)(1-e^{-\frac{x}{\epsilon a_1(v)}})\,dv=M.
\end{aligned}$$

Similarly, $\int F(t,x,v)\,dv \geqslant -M$.

Proposition 3(iii), Propositions 4 and 5 are skipped. The second and third lines of p. 795 no more follow from Proposition 5 but from the strong convergence in L^1 of f_ϵ to χ_u derived from now classical arguments. Indeed, (f_ϵ) being uniformly bounded in $L^1 \cap L^\infty$, converges (up to a subsequence) to some f in L^2 weak. By an averaging lemma (ref. 1, p. 23), u_ϵ converges strongly in L^1 to $u(t,x) := \int f(t,x,v)\,dv$. By the integral representation of f_ϵ, f_ϵ converges strongly in L^1 to χ_u (ref. 3, p. 81). The reference to Helly's theorem p. 795 should be skipped. The rest of the paper is unchanged.

REFERENCES

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